R S Aggarwal Solutions for Class 11 Maths Chapter 28 Differentiation

Exercise 28A

Question 1: Differentiate the following functions:
(i) x^{-3} (ii) $x^{1/3}$

Solution:

We know, $d/dx\ (x^m) = m(x^{m-1})$

(i) $d/dx\ (x^{-3}) = -3x^{-4}$

(ii) $d/dx\ (x^{1/3}) = 1/3\ x^{-2/3}$

Question 2: Differentiate the following functions:
(i) $1/x$ (ii) $1/\sqrt{x}$ (iii) $1/x^{1/3}$

Solution:
We know, $d/dx\ (x^m) = m(x^{m-1})$

(i) $d/dx\ (1/x) = x^{-1} = -x^{-2} = -1/x^2$

(ii) $d/dx\ (1/\sqrt{x}) = x^{-1/2} = -1/2 * x^{-3/2}$

(iii) $d/dx\ (1/x^{1/3}) = x^{-1/3} = -1/3 * x^{-4/3}$

Question 3: Differentiate the following functions:
(i) $3x^{-5}$ (ii) $1/5x$ (iii) $6\ (x^2)^{3/2}$

Solution:
We know, $d/dx\ (x^m) = m(x^{m-1})$

(i) $d/dx\ (3x^{-5}) = 3(-5)x^{-6} = -15\ x^{-6}$

(ii) $d/dx\ (1/5x) = 1/5 * (d/dx\ (1/x)) = 1/5\ (-1/x^2) = -1/5x^2$

(iii) $d/dx\ (6\ (x^2)^{1/3}) = 6\ d/dx\ (x^{2/3}) = 6(2/3 * x^{-1/3}) = 4x^{-1/3}$

Question 4: Differentiate the following functions:
(i) $6x^5 + 4x^3 - 3x^2 + 2x - 7$
(ii) $5x^{-3/2} + 4/\sqrt{x} + \sqrt{x} - 7/x$
(iii) $ax^3 + bx^2 + cx + d$, where a, b, c, d are constants

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Solution:
We know, $d/dx\,(x^m) = m(x^{m-1})$

(i) $d/dx(6x^5 + 4x^3 – 3x^2 + 2x – 7) = 30x^4 + 12x^2 – 6x + 2$

(ii) $d/dx(5x^{-3/2} + 4/\sqrt{x} + \sqrt{x} – 7/x)$ ……(1)

$d/dx(5x^{-3/2}) = 5 * -3/2 * x^{-5/2} = -15/2 * x^{-5/2}$

$d/dx(4/\sqrt{x}) = 4\, d/dx(1/\sqrt{x}) = 4 * -1/2 * x^{-3/2} = -2\, x^{-3/2}$

$d/dx(\sqrt{x}) = 1/2 * x^{-1/2}$

$d/dx(7/x) = 7\, d/dx(1/x) = 7 * -1/x^2$

(1)=>

$$d/dx(5x^{-3/2} + 4/\sqrt{x} + \sqrt{x} – 7/x) = -\frac{15}{2}x^{-\frac{5}{2}} - 2x^{-\frac{3}{2}} + \frac{1}{2}x^{\frac{-1}{2}} + 7x^{-2}$$

(iii) $ax^3 + bx^2 + cx + d$, where a, b, c, d are constants

$d/dx(ax^3 + bx^2 + cx + d) = d/dx(ax^3) + d/dx(bx^2) + d/dx(cx) + d/dx(d)$

$= 3ax^2 + 2bx + c + 0$

We know derivative of a constant is zero.

$d/dx(ax^3 + bx^2 + cx + d) = 3ax^2 + 2bx + c$

Question 5: Differentiate the following functions:

(i) $4x^3 + 3.2^x + 6.\sqrt[8]{x^{-4}} + 5\cot x$

(ii) $\frac{x}{3} - \frac{3}{x} + \sqrt{x} - \frac{1}{\sqrt{x}} + x^2 - 2^x + 6x^{-2/3} - \frac{2}{3}x^6$

Solution:
We know,
$d/dx\,(x^m) = m(x^{m-1})$
$d/dx \cot x = -\operatorname{cosec}^2 x$

d/dx $a^x = \log_n (a)a^x$

(i)

$$\frac{d}{dx}(4x^3 + 3.2^x + 6x^{-\frac{1}{2}} + 5\cot x)$$

$= 12x^2 + 3\log_n(2)2^x + 6 * -1/2 * (x^{-3/2}) + 5(-\operatorname{cosec}^2 x)$

$= 12x^2 + (3\log_n 2)\,2^x - 3x^{-3/2} - 5\operatorname{cosec}^2 x$

(ii) $$\frac{d}{dx}\left(\frac{x}{3} - 3x^{-1} + x^{\frac{1}{2}} - x^{-\frac{1}{2}} + x^2 - 2^x + 6x^{-\frac{2}{3}} - \frac{2}{3}x^6\right)$$

$$= \frac{1}{3} - (-1) \times 3x^{-1-1} + \frac{1}{2}x^{\frac{1}{2}-1} - \left(-\frac{1}{2}\right)x^{-\frac{1}{2}-1} + 2x^{2-1} - \log(2).2^x + 6\left(-\frac{2}{3}\right)x^{-\frac{2}{3}-1} - \frac{2}{3} \times 6x^{6-1}$$

$$= \frac{1}{3} + 3x^{-2} + \frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}x^{-\frac{3}{2}} + 2x^1 - \log(2).2^x - 4x^{-\frac{5}{3}} - 4x^5$$

Question 6: Differentiate the following functions:

(i) $$4\cot x - \frac{1}{2}\cos x + \frac{2}{\cos x} - \frac{3}{\sin x} + \frac{6\cot x}{\operatorname{cosec} x} + 9$$

(ii) $-5\tan x + 4\tan x\cos x - 3\cot x\sec x + 2\sec x - 13$

Solution:

We know,

d/dx sin x = cos x

d/dx cos x = - sin x

d/dx tan x = sec2x

d/dx cosec x = -cosec x cot x

d/dx sec x = secx tanx

d/dx cot x = - cosec2 x

d/dx k = 0

(k is any constant)

(i)

$$\frac{d}{dx}(4\cot x - \frac{1}{2}\cos x + 2\sec x - 3\operatorname{cosec} x + 6\cos x + 9)$$

$$= 4(-\operatorname{cosec}^2 x) - \frac{1}{2}(-\sin x) + 2\sec x \tan x - 3(-\operatorname{cosec} x \cot x) + 6(-\sin x) + 0$$

$$= -4\operatorname{cosec}^2 x + \frac{1}{2}\sin x + 2\sec x \tan x + 3\operatorname{cosec} x \cot x - 6\sin x$$

(ii) -5 tan x + 4 tan x cos x – 3 cot x sec x + 2sec x – 13

= -5 tan x + 4 sinx/cosx * cosx – 3 cosx/sinx * 1/cosx + 2sec x – 13

= -5 tan x + 4 sinx – 3 cosec x + 2sec x – 13

Now,
d/dx (-5 tan x + 4 sinx – 3 cosecx + 2sec x – 13)

= -5 $\sec^2$x + 4cosx -3(- cosecx cotx) + 2 secx tanx – 0

= -5 $\sec^2$x + 4cosx + 3 cosecx cotx + 2 secx tanx

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Find the derivation of each of the following from the first principle:

First Principle:

we know that,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$
……………..(1)

Question 1: Derivate (ax + b)

Solution:

Let f(x) = ax + b ….(i)

Find f'(x) using first principle.

Now,

f(x + h) = a(x + h) + b = ax + ah + b …..(ii)

Subtract (i) form (ii)

f(x + h) – f(x) = ax + ah + b – ax – b = ah

From (1), we get

f'(x) = lim(h->0) {ah/h} = a

Question 2: Derivate

$$\left(ax^2 + \frac{b}{x}\right)$$

Solution:

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Let $f(x) = ax^2 + \frac{b}{x}$ ....(i)

Find f'(x) using first principle.

$f(x+h) = a(x+h)^2 + \frac{b}{(x+h)}$ .....(ii)

Subtract (i) form (ii)

$$f(x+h) - f(x) = \left[a(x+h)^2 + \frac{b}{(x+h)}\right] - \left[ax^2 + \frac{b}{x}\right]$$

$$= a[x^2 + h^2 + 2xh - x^2] + b\left[\frac{x-(x+h)}{x(x+h)}\right]$$

$$= a[h^2 + 2xh] + b\left[\frac{-h}{x(x+h)}\right]$$

From (1), we get

$$f'(x) = \lim_{h\to 0}\left[\frac{ah(h+2x)}{h} + \frac{b(-h)}{hx(x+h)}\right]$$

$$= \lim_{h\to 0}\left[a(h+2x) - \frac{b}{x(x+h)}\right]$$

$$= 2ax - \frac{b}{x^2}$$

Question 3: Derivate $3x^2 + 2x - 5$

Solution:

Let $f(x) = 3x^2 + 2x - 5$....(i)

Find f'(x) using first principle.

Now,

$f(x + h) = 3(x+h)^2 + 2(x+h) - 5$

$= 3x^2 + 3h^2 + 6xh + 2x + 2h - 5$ .....(ii)

Subtract (i) form (ii)

$f(x + h) - f(x) = 3x^2 + 3h^2 + 6xh + 2x + 2h - 5 - 3x^2 - 2x + 5$

$= 3h^2 + 6xh + 2h$

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From (1), we get

f'(x) = lim(h->0) {($3h^2$ + 6xh + 2h)/h} = 6x + 2

Question 4: Derivate $x^3 - 2x^2 + x + 3$
Solution:
Let f(x) = $x^3 - 2x^2 + x + 3$ ....(i)

Find f'(x) using first principle.

Now,
f(x + h) = $(x+h)^3 - 2(x+h)^2 + (x + h) + 3$ .....(ii)

Subtract (i) form (ii)

f(x + h) – f(x) = $(x+h)^3 - 2(x+h)^2 + (x + h) + 3 - x^3 + 2x^2 - x - 3$

= $[(x+h)^3 - x^3] - 2[(x+h)^2 - x^2] + [x + h - x]$

[Using the identities:
$(a + b)^3 = a^3 + b^3 + 3ab^2 + 3a^2b$
$(a + b)^2 = a^2 + b^2 + 2ab$]

= $h^3 + 3xh^2 + 3x^2h - 2[h^2 + 2xh] + h$

From (1), we get

$$f'(x) = \lim_{h\to 0} \frac{h[h^2 + 3xh + 3x^2] - 2h[h + 2x] + h}{h}$$

$$= \lim_{h\to 0} h^2 + 3xh + 3x^2 - 2h - 4x + 1$$

$$= 3x^2 - 4x + 1$$

Question 5: Derivate x^8
Solution:
Let f(x) = x^8

Find f'(x) using first principle.

Now,
f(x + h) = $(x+h)^8$

From (1), we get

$$f'(x) = \lim_{h\to 0} \frac{(x+h)^8 - x^8}{h}$$

$$= \lim_{h\to 0} \frac{(x+h)^8 - x^8}{(x+h) - x}$$

We know, $\lim_{x\to a} \frac{x^n - a^n}{x-a} = na^{n-1}$

$$f'(x) = 8x^7$$

Question 6: Derivate $1/x^3$
Solution:
Let f(x) = $1/x^3$

Find f'(x) using first principle.

Now,
f(x + h) = $1/(x+h)^3$

From (1), we get

$$f'(x) = \lim_{h\to 0} \frac{\frac{1}{(x+h)^3} - \frac{1}{x^3}}{h}$$

$$= \lim_{h\to 0} \frac{(x+h)^{-3} - x^{-3}}{(x+h) - x}$$

We know, $\lim_{x\to a} \frac{x^n - a^n}{x-a} = na^{n-1}$

$$f'(x) = -3x^{-4}$$

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Question 7: Derivate $1/x^5$
Solution:
Let $f(x) = 1/x^5$

Find f'(x) using first principle.

Now,
$f(x + h) = 1/(x+h)^5$

From (1), we get

$$f'(x) = \lim_{h\to 0} \frac{\frac{1}{(x+h)^5} - \frac{1}{x^5}}{h}$$

$$= \lim_{h\to 0} \frac{(x+h)^{-5} - x^{-5}}{(x+h) - x}$$

We know, $\lim_{x\to a} \frac{x^n - a^n}{x-a} = na^{n-1}$

$$f'(x) = -\frac{5}{x^6}$$

Question 8: Derivate √(ax+b)
Solution:
Let f(x) = √(ax+b)

Find f'(x) using first principle.

Now,
f(x + h) = √(a(x+h) +b)

From (1), we get

$$f'(x) = \lim_{h\to 0}\frac{\sqrt{ax+ah+b}-\sqrt{ax+b}}{h}$$

$$= \lim_{h\to 0}\frac{\sqrt{ax+ah+b}-\sqrt{ax+b}}{h}\times\frac{\sqrt{ax+ah+b}+\sqrt{ax+b}}{\sqrt{ax+ah+b}+\sqrt{ax+b}}$$

$$= \lim_{h\to 0}\frac{ax+ah+b-ax-b}{h(\sqrt{ax+ah+b}+\sqrt{ax+b})}$$

$$= \lim_{h\to 0}\frac{a}{\sqrt{ax+ah+b}+\sqrt{ax+b}}$$

$$= \frac{a}{\sqrt{ax+a(0)+b}+\sqrt{ax+b}}$$

$$= \frac{a}{\sqrt{ax+b}+\sqrt{ax+b}}$$

$$= \frac{a}{2\sqrt{ax+b}}$$

Question 9: Derivate √(5x-4)
Solution:
Let f(x) = √(5x-4)

Find f'(x) using first principle.

Now,
f(x + h) = √(5(x+h) - 4)

From (1), we get

$$f'(x) = \lim_{h\to 0}\frac{\sqrt{5x+5h-4}-\sqrt{5x-4}}{h}$$

$$= \lim_{h\to 0}\frac{\sqrt{5x+5h-4}-\sqrt{5x-4}}{h}\times\frac{\sqrt{5x+5h-4}+\sqrt{5x-4}}{\sqrt{5x+5h-4}+\sqrt{5x-4}}$$

$$= \lim_{h\to 0}\frac{5x+5h-4-5x+4}{h(\sqrt{5x+5h-4}+\sqrt{5x-4})}$$

$$= \lim_{h\to 0}\frac{5}{\sqrt{5x+5h-4}+\sqrt{5x-4}}$$

$$= \frac{5}{2\sqrt{5x-4}}$$

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Question 10: Derivate 1/√(x+2)

Solution:

Let f(x) = 1/√(x+2)

Find f'(x) using first principle.

Now,
f(x + h) = 1/√((x+h) +2)

From (1), we get

$$f'(x) = \lim_{h\to 0}\frac{\frac{1}{\sqrt{x+h+2}} - \frac{1}{\sqrt{x+2}}}{h}$$

$$= \lim_{h\to 0}\frac{\frac{\sqrt{x+2} - \sqrt{x+h+2}}{(\sqrt{x+h+2})(\sqrt{x+2})}}{h}$$

$$= \lim_{h\to 0}\frac{\sqrt{x+2} - \sqrt{x+h+2}}{h(\sqrt{x+h+2})(\sqrt{x+2})} \times \frac{\sqrt{x+2} + \sqrt{x+h+2}}{\sqrt{x+2} + \sqrt{x+h+2}}$$

$$= \lim_{h\to 0}\frac{(\sqrt{x+2})^2 - (\sqrt{x+h+2})^2}{h(\sqrt{x+h+2})(\sqrt{x+2})(\sqrt{x+2} + \sqrt{x+h+2})}$$

$$= \lim_{h\to 0}\frac{-1}{(\sqrt{x+h+2})(\sqrt{x+2})(\sqrt{x+2} + \sqrt{x+h+2})}$$

$$= \frac{-1}{(\sqrt{x+2})^2(2\sqrt{x+2})}$$

$$= \frac{-1}{2(\sqrt{x+2})^3}$$

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Question 11: Derivate 1/√(2x+3)
Solution:

Let f(x) = 1/√(2x+3)

Find f'(x) using first principle.

Now,
f(x + h) = 1/√(2(x+h) +3)

From (1), we get

$$f'(x) = \lim_{h\to 0} \frac{\frac{1}{\sqrt{2x+2h+3}} - \frac{1}{\sqrt{2x+3}}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{\sqrt{2x+3} - \sqrt{2x+2h+3}}{(\sqrt{2x+2h+3})(\sqrt{2x+3})}}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{2x+3} - \sqrt{2x+2h+3}}{h(\sqrt{2x+2h+3})(\sqrt{2x+3})} \times \frac{\sqrt{2x+3} + \sqrt{2x+2h+3}}{\sqrt{2x+3} + \sqrt{2x+2h+3}}$$

$$= \lim_{h\to 0} \frac{(\sqrt{2x+3})^2 - (\sqrt{2x+2h+3})^2}{h(\sqrt{2x+2h+3})(\sqrt{2x+3})(\sqrt{2x+3} + \sqrt{2x+2h+3})}$$

$$= \lim_{h\to 0} \frac{-2}{(\sqrt{2x+2h+3})(\sqrt{2x+3})(\sqrt{2x+3} + \sqrt{2x+2h+3})}$$

$$= \frac{-2}{(\sqrt{2x+3})^2(2\sqrt{2x+3})}$$

$$= \frac{-1}{(\sqrt{2x+3})^3}$$

Question 12: Derivate 1/√(6x-5)
Solution:
Let f(x) = 1/√(6x-5)

Find f'(x) using first principle.

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Now,

f(x + h) = 1/√(6(x+h) - 5)

From (1), we get

$$f'(x) = \lim_{h\to 0}\frac{\frac{1}{\sqrt{6x+6h-5}} - \frac{1}{\sqrt{6x-5}}}{h}$$

$$= \lim_{h\to 0}\frac{\frac{\sqrt{6x-5}-\sqrt{6x+6h-5}}{(\sqrt{6x+6h-5})(\sqrt{6x-5})}}{h}$$

$$= \lim_{h\to 0}\frac{\sqrt{6x-5}-\sqrt{6x+6h-5}}{h(\sqrt{6x+6h-5})(\sqrt{6x-5})} \times \frac{\sqrt{6x-5}+\sqrt{6x+6h-5}}{\sqrt{6x-5}+\sqrt{6x+6h-5}}$$

$$= \lim_{h\to 0}\frac{(\sqrt{6x-5})^2-(\sqrt{6x+6h-5})^2}{h(\sqrt{6x+6h-5})(\sqrt{6x-5})(\sqrt{6x-5}+\sqrt{6x+6h-5})}$$

$$= \lim_{h\to 0}\frac{-6}{(\sqrt{6x+6h-5})(\sqrt{6x-5})(\sqrt{6x-5}+\sqrt{6x+6h-5})}$$

$$= \frac{-6}{(\sqrt{6x-5})^2(2\sqrt{6x-5})}$$

$$= \frac{-3}{(\sqrt{6x-5})^3}$$

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Exercise 28C

Question 1: Differentiate: x^2 sin x
Solution:
By Product Rule: (uv)′ = u′v + uv′

Here u = x^2 and v = sin x

(x^2 sin x)′ = 2x (sinx) + x^2 (cosx)

= 2xsinx + x^2cosx

Question 2: Differentiate e^x cos x

Solution:
By Product Rule: (uv)′ = u′v + uv′

Here u = e^x and v = cos x

(e^x cosx)′ = e^x (cosx) + e^x (-sinx)

= e^xcosx - e^xsinx

= e^x (cosx - sinx)

Question 3: Differentiate e^x cot x

Solution:
By Product Rule: (uv)′ = u′v + uv′

Here u = e^x and v = cot x

(e^x cotx)′ = e^x (cotx) + e^x (-cosec^{2}x)

= e^x (cotx) - e^x cosec^{2}x

= e^x (cotx - cosec^{2}x)

Question 4: Differentiate x^n cot x
Solution:

By Product Rule: (uv)′ = u′v + uv′

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Here u = x^n and v = cot x

$(x^n \cot x)' = nx^{n-1} (\cot x) + x^n (-\operatorname{cosec}^2 x)$

$= nx^{n-1} (\cot x) - x^n (\operatorname{cosec}^2 x)$

$= x^{n-1} (n \cot x – x \operatorname{cosec}^2 x)$

Question 5: Differentiate x^3 sec x

Solution:
By Product Rule: (uv)′ = u′v + uv′

Here u = x^3 and v = sec x

$(x^3 \sec x)' = 3x^2 (\sec x) + x^3 (\sec x \tan x)$

$= x^2 \sec x(3 + x \tan x)$

Question 6: Differentiate $(x^2 + 3x + 1)$ sin x

Solution:
By Product Rule: (uv)′ = u′v + uv′

Here u = $(x^2 + 3x + 1)$ and v = sin x

$[(x^2 + 3x + 1) \sin x]' = (2x + 3) \times \sin x + (x^2 + 3x + 1) \times \cos x$

$= (2x + 3) \sin x + (x^2 + 3x + 1) \cos x$

Question 7: Differentiate x^4 tan x

Solution:

By Product Rule: (uv)′ = u′v + uv′

Here u = x^4 and v = tan x

$(x^4 \tan x)' = 4x^3 \times \tan x + x^4 \times \sec^2 x$

$= 4x^3 \tan x + x^4 \sec 2x$

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Exercise 28D

Solve all the questions using quotient rule.

Quotient Rule:

$$\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$$

.........(1)

Question 1: Differentiate $2^x/x$

Solution:

Here $u = 2^x$ and $v = x$

$u' = d/dx\ (2^x) = 2^x \log 2$

$v' = d/dx\ (x) = 1$

(1)=>

$$\left(\frac{2^x}{x}\right)' = \frac{2^x \log 2 \times x - 2^x \times 1}{(x)^2}$$

$$= \frac{2^x(x\log 2 - 1)}{x^2}$$

Question 2: Differentiate log x/x

Solution:

Here u = log x and v = x

u' = d/dx (log x) = 1/x

v' = d/dx (x) = 1

(1)=>

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$$\left(\frac{\log x}{x}\right)' = \frac{\frac{1}{x} \times x - \log x \times 1}{(x)^2}$$

$$= \frac{1 - \log x}{x^2}$$

Question 3: Differentiate $e^x/(1+x)$

Solution:
Here $u = e^x$ and $v = 1+x$

$u' = d/dx\ (e^x) = e^x$

$v' = d/dx\ (1+x) = 1$

(1)=>

$$\left(\frac{e^x}{(1+x)}\right)' = \frac{e^x \times (1+x) - e^x \times 1}{(1+x)^2}$$

$$= \frac{xe^x}{(1+x)^2}$$

Question 4: Differentiate $e^x/(1+x^2)$

Solution:
Here $u = e^x$ and $v = 1+x^2$

Using Quotient rule, we have

$$\left(\frac{e^x}{(1+x^2)}\right)' = \frac{e^x \times (1+x^2) - e^x \times 2x}{(1+x^2)^2}$$

$$= \frac{e^x(x^2-2x+1)}{(1+x^2)^2}$$

$$= \frac{e^x(x-1)^2}{(1+x^2)^2}$$

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Question 5: Differentiate

$$\left(\frac{2x^2-4}{3x^2+7}\right)$$

Solution:

Here $u = 2x^2 - 4$ and $v = 3x^2 + 7$

Using Quotient rule, we have

$$\left[\frac{(2x^2-4)}{(3x^2+7)}\right]' = \frac{4x \times (3x^2+7) - (2x^2-4) \times 6x}{(3x^2+7)^2}$$

$$= \frac{12x^3 + 28x - 12x^3 + 24x}{(3x^2+7)^2}$$

$$= \frac{52x}{(3x^2+7)^2}$$

Question 6: Differentiate

$$\left(\frac{x^2+3x-1}{x+2}\right)$$

Solution:

Here $u = x^2 + 3x - 1$ and $v = x + 2$

Using Quotient rule, we have

$$\left(\frac{x^2 + 3x - 1}{x + 2}\right)' = \frac{(2x + 3) \times (x + 2) - (x^2 + 3x - 1) \times 1}{(x + 2)^2}$$

$$= \frac{2x^2+7x+6-x^2-3x+1}{(x + 2)^2}$$

$$= \frac{x^2+4x+7}{(x + 2)^2}$$

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Question 7: Differentiate

$$\frac{(x^2-1)}{(x^2+7x+1)}$$

Solution:

Here u = x^2 - 1 and v = x^2 + 7x + 1

Using Quotient rule, we have

$$\left[\frac{(x^2-1)}{(x^2+7x+1)}\right]' = \frac{2x \times (x^2+7x+1) - (x^2-1) \times (2x+7)}{(x^2+7x+1)^2}$$

$$= \frac{2x^3 + 14x^2 + 2x - 2x^3 - 7x^2 + 2x + 7}{(x^2+7x+1)^2}$$

$$= \frac{7x^2 + 4x + 7}{(x^2+7x+1)^2}$$

Question 8: Differentiate

$$\left(\frac{5x^2+6x+7}{2x^2+3x+4}\right)$$

Solution:

Here u = $5x^2$ +6x + 7 and v = $2x^2$ + 3x + 4

Using Quotient rule, we have

$$\left(\frac{5x^2+6x+7}{2x^2+3x+4}\right)' = \frac{(10x+6)\times(2x^2+3x+4) - (5x^2+6x+7)\times(4x+3)}{(2x^2+3x+4)^2}$$

$$= \frac{20x^3+30x^2+40x+12x^2+18x+24-20x^3-15x^2-24x^2-18x-28x-21}{(2x^2+3x+4)^2}$$

$$= \frac{3x^2+12x+3}{(2x^2+3x+4)^2}$$

$$= \frac{3(x^2+4x+1)}{(2x^2+3x+4)^2}$$

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Question 9: Differentiate

$$\frac{x}{(a^2+x^2)}$$

Solution:

Here u = x and v = $a^2 + x^2$

Using Quotient rule, we have

$$\left[\frac{x}{(a^2+x^2)}\right]' = \frac{1 \times (a^2+x^2) - (x) \times (2x)}{(a^2+x^2)^2}$$

$$= \frac{a^2+x^2-2x^2}{(a^2+x^2)^2}$$

$$= \frac{a^2-x^2}{(a^2+x^2)^2}$$

Question 10: Differentiate x^4/sinx

Solution:

Here u = x^4 and v = sinx

Using Quotient rule, we have

$$\left[\frac{x^4}{\sin x}\right]' = \frac{4x^3 \times (\sin x) - (x^4) \times (\cos x)}{(\sin x)^2}$$

$$= \frac{x^3[4(\sin x) - x(\cos x)]}{(\sin x)^2}$$

Question 11: Differentiate

$$\frac{\sqrt{a}+\sqrt{x}}{\sqrt{a}-\sqrt{x}}$$

Solution:

Here u = √a + √x and v = √a - √x

Using Quotient rule, we have

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$$\left[\frac{\sqrt{a}+\sqrt{x}}{\sqrt{a}-\sqrt{x}}\right]' = \frac{\frac{1}{2\sqrt{x}} \times (\sqrt{a}-\sqrt{x}) - (\sqrt{a}+\sqrt{x}) \times -\frac{1}{2\sqrt{x}}}{(\sqrt{a}-\sqrt{x})^2}$$

$$= \frac{\frac{\sqrt{a}}{2\sqrt{x}} - \frac{1}{2} + \frac{\sqrt{a}}{2\sqrt{x}} + \frac{1}{2}}{(\sqrt{a}-\sqrt{x})^2}$$

$$= \frac{\sqrt{a}}{\sqrt{x}(\sqrt{a}-\sqrt{x})^2}$$

Question 12: Differentiate cosx/log x
Solution:

Here u = cos x and v = log x

Using Quotient rule, we have

$$\left[\frac{\cos x}{\log x}\right]' = \frac{-\sin x \times (\log x) - (\cos x) \times \left(\frac{1}{x}\right)}{(\log x)^2}$$

$$= \frac{-x\sin x(\log x) - (\cos x)}{x(\log x)^2}$$

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Exercise 28E

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Differentiate the following with respect to x:
Differentiate below functions using “Chain Rule”:
If y = f(t) and t = g(x) then dy/dx = dy/dt x dt/dx

Question 1: Differentiate sin 4x

Solution:
Let y = sin 4x

If 4x = t then sin4x = sin t or y = sin t

dy/dt = cos t

dt/dx = 4

Now,
dy/dx = dy/dt x dt/dx = 4 cos t

Substituting the value of t back, we get

dy/dx = 4 cos 4x

Question 2: Differentiate cos 5x

Solution:
Let y = cos 5x

If t = 5x then cos 5x = cos t or y = cos t

dy/dt = -sin t

dt/dx = 5

Now,
dy/dx = dy/dt x dt/dx = -5 sin t

Substituting the value of t back, we get

dy/dx = -5 sin 5x

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Question 3: Differentiate tan 3x

Solution:
Let y = tan 3x

If t = 3x then tan 3x = tan t or y = tan t

dy/dt = $\sec^2 t$

dt/dx = 3

Now,
dy/dx = dy/dt x dt/dx = 3 $\sec^2 t$

Substituting the value of t back, we get

dy/dx = 3$\sec^2 3x$

Question 4: Differentiate cos x^3

Solution:
Let y = cos x^3

If t = x^3 then cos x^3= cos t or y = cos t

dy/dt = -sin t

dt/dx = $3x^2$

Now,
dy/dx = dy/dt x dt/dx = 3 x^2 (-sin t)

Substituting the value of t back, we get

dy/dx = 3 x^2 (-sin x^3)

= -3 x^2 sin x^3

Question 5: Differentiate $\cot^2 x$

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Solution:

Let $y = \cot^2 x$

$dy/dx = - 2\cot x\,[d/dx(\cot x)]$

$= - 2 \cot x \operatorname{cosec}^2 x$

Question 6: Differentiate $\tan^3 x$

Solution:

Let $y = \tan^3 x$

$dy/dx = 3 \tan^2 x\,[d/dx(\tan x)]$

$= 3 \tan^2 x \sec^2 x$

Question 7: Differentiate $\tan \sqrt{x}$

Solution:

Let $y = \tan \sqrt{x}$

$dy/dx = \sec^2\sqrt{x}\; d/dx(\sqrt{x})$

$= \sec^2\sqrt{x}\,\{1/(2\sqrt{x})\}$

$= \sec^2\sqrt{x} / 2\sqrt{x}$

Question 8: Differentiate e^{x^4}

Solution:

$$\frac{d}{dx} e^{x^4} = e^{x^4} \times \frac{d}{dx}(x^4) = 4x^3\ e^{x^4}$$

Question 9: Differentiate $e^{\cot x}$

Solution:

$$\frac{d}{dx} e^{\cot x} = e^{\cot x} \times \frac{d}{dx}(\cot x)$$

$$= e^{\cot x} \times -\operatorname{cosec}^2 x$$

$$= -e^{\cot x} \operatorname{cosec}^2 x$$

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Question 10: Differentiate √(sin x)

Solution:

d/dx √(sin x) = d/dx $(\sin x)^{1/2}$

= 1/2 * $(\sin x)^{-1/2}$ * d/dx (sin x)

= 1/2 * $(\sin x)^{-1/2}$ * cos x

= cosx / 2√(sin x)

Question 11: Differentiate $(5 + 7x)^6$

Solution:
d/dx$(5 + 7x)^6$ = $6(5 + 7x)^5$ * d/dx(5 + 7x)

= $6(5 + 7x)^5$ * 7

= $42(5 + 7x)^5$

Question 12: Differentiate $(3 - 4x)^5$

Solution:

d/dx$(3 - 4x)^5$ = $5(3 - 4x)^4$ * d/dx (3 - 4x)

= $5(3 - 4x)^4$ * (- 4)

= $- 20(3 - 4x)^4$

Question 13: Differentiate $(3x^2 - x + 1)^4$

Solution:

d/dx $(3x^2 - x + 1)^4$ = $4(3x^2 - x + 1)^3$ * d/dx $(3x^2 - x + 1)$

= $4(3x^2 - x + 1)^3$ * (6x - 1)

= $4(3x^2 - x + 1)^3$ (6x - 1)

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Question 14: Differentiate $(ax^2 + bx + c)^n$

Solution:

d/dx $(ax^2 + bx + c)^n = n(ax^2 + bx + c)^{n-1}$ * d/dx $(ax^2 + bx + c)$

$= n(ax^2 + bx + c)^{n-1} (2ax + b)$

Question 15: Differentiate

$$\frac{1}{(x^2 - x + 3)^3}$$

Solution:

d/dx $(x^2 - x + 3)^{-3} = -3(x^2 - x + 3)^{-4}$ * d/dx $(x^2 - x + 3)$

$= -3(x^2 - x + 3)^{-4}(2x^2 - 1)$

Question 16: Differentiate $\sin^2 (2x + 3)$

Solution:

d/dx $\sin^2 (2x + 3) = 2 \sin (2x + 3)$ * d/dx $\{\sin(2x + 3)\}$

$= 2 \sin (2x + 3)$ * $\cos(2x + 3)$ * d/dx $(2x + 3)$

$= 2 \sin (2x + 3)$ * $\cos(2x + 3)$ * 2

$= 4 \sin(2x + 3) \cos(2x + 3)$

$= 2 \sin (4x + 6)$

[Using identity: $2\sin x \cos x = \sin 2x$]

Question 17: Differentiate $\cos^2(x^3)$

Solution:

d/dx $\cos^2(x^3) = 2 \cos(x^3)$ * d/dx $\{ \cos (x^3)\}$

$= 2 \cos(x^3)$ * $(- \sin (x^3))$ * d/dx (x^3)

= 2 cos(x^3) * (- sin (x^3)) * $3x^2$

= - $6x^2$ cos(x^3) sin x^3

Question 18: Differentiate √(sin (x^3))
Solution:

$$\frac{d}{dx}\sqrt{\sin x^3} = \frac{1}{2\sqrt{\sin x^3}} \times \frac{d}{dx}(\sin x^3) \times \frac{d}{dx}(x^3)$$

$$= \frac{1}{2\sqrt{\sin x^3}} \times (\cos x^3) \times 3x^2$$

$$= \frac{3x^2(\cos x^3)}{2\sqrt{\sin x^3}}$$

Question 19: Differentiate √(x sin x)
Solution:

$$\frac{d}{dx}\sqrt{x \sin x} = \frac{1}{2\sqrt{x\sin x}} \times \frac{d}{dx}(x\sin x)$$

$$= \frac{1}{2\sqrt{x\sin x}} \times (\sin x + x\cos x)$$

$$= \frac{(\sin x + x\cos x)}{2\sqrt{x\sin x}}$$

Question 20: Differentiate

$$\sqrt{\cot\sqrt{x}}$$

Solution:

$$\frac{d}{dx}\sqrt{\cot\sqrt{x}} = \frac{1}{2\sqrt{\cot\sqrt{x}}} \times \frac{d}{dx}\cot\sqrt{x}$$

$$= \frac{1}{2\sqrt{\cot\sqrt{x}}} \times (-\operatorname{cosec}^2\sqrt{x}) \times \frac{1}{2\sqrt{x}}$$

$$= \frac{-\operatorname{cosec}^2\sqrt{x}}{4\sqrt{x}\ \sqrt{\cot(\sqrt{x})}}$$

R S Aggarwal Solutions for Class 11 Maths Chapter 29 Mathematical Reasoning

(vii) 'Every relation is a function' is a statement.
There are relations which are not functions.
Therefore, the sentence is false.

(viii) 'The sum of any two sides of a triangle is always greater than the third side'
It is a statement and mathematically proven result.
Hence the statement is true.

(ix) 'May God bless you!' is an exclamation sentence. Hence it is not a statement.

Question 2: Which of the following sentences are statements? In case of a statement, mention whether it is true or false.

(i) Paris is in France.
(ii) Each prime number has exactly two factors.
(iii) The equation $x^2 + 5|x| + 6 = 0$ has no real roots.
(iv) $(2 + \sqrt{3})$ is a complex number.
(v) Is 6 a positive integer?
(vi) The product of -3 and -2 is -6.
(vii) The angles opposite the equal sides of an isosceles triangle are equal.
(viii) Oh! it is too hot.
(ix) Monika is a beautiful girl.
(x) Every quadratic equation has at least one real root.

Solution:
(i) Paris is in France, is a statement.
Paris is located in France, so the sentence is true.
So, the statement is true.

(ii) Each prime number has exactly two factors, is a statement.
This is a mathematically proven fact.
So, the statement is true.

(iii) The equation $x^2 + 5|x| + 6 = 0$ has no real roots.

Find the roots of $x^2 + 5|x| + 6 = 0$:

Case 1: $x \geq 0$
$x^2 + 5x + 6 = (x+2)(x+3) = 0 \Rightarrow x = -2, -3$ but we already assumed $x \geq 0$, which is a contradiction.