

R S Aggarwal Solutions for Class 11 Maths Chapter 27 Limits

Exercise 27A

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Question 1: Evaluate

$$\lim_{x \rightarrow 2} (5 - x)$$

Solution:

$$\text{We know, } \lim_{x \rightarrow a} f(x) = f(a)$$

$$\text{As } x \rightarrow 2$$

$$\lim_{x \rightarrow 2} (5 - x) = 5 - 2 = 3$$

The value of $\lim_{x \rightarrow 2} (5 - x)$ is 3.

Question 2: Evaluate

$$\lim_{x \rightarrow 1} (6x^2 - 4x + 3)$$

Solution:

$$\text{We know, } \lim_{x \rightarrow a} f(x) = f(a)$$

$$\text{As } x \rightarrow 1$$

$$\lim_{x \rightarrow 1} (6x^2 - 4x + 3) = 6(1^2) - 4(1) + 3 = 5$$

Question 3: Evaluate

$$\lim_{x \rightarrow 3} \left(\frac{x^2 + 9}{x + 3} \right)$$

Solution:

$$\text{We know, } \lim_{x \rightarrow a} f(x) = f(a)$$

$$\text{As } x \rightarrow 3$$

$$\lim_{x \rightarrow 3} \frac{x^2 + 9}{x + 3} = \frac{3^2 + 9}{3 + 3} = \frac{18}{6} = 3$$

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Question 4: Evaluate

$$\lim_{x \rightarrow 3} \left(\frac{x^2 - 4x}{x - 2} \right)$$

Solution:

We know, $\lim_{x \rightarrow a} f(x) = f(a)$

As $x \rightarrow 3$

$$\lim_{x \rightarrow 3} \frac{x^2 - 4x}{x - 2} = \frac{3^2 - 4(3)}{3 - 2}$$

$$= \frac{3^2 - 4(3)}{3 - 2}$$

$$= -3$$

Question 5: Evaluate

$$\lim_{x \rightarrow 5} \left(\frac{x^2 - 25}{x - 5} \right)$$

Solution:

We know, $\lim_{x \rightarrow a} f(x) = f(a)$

As $x \rightarrow 5$

$$\lim_{x \rightarrow 5} \frac{x^2 - 25}{x - 5} = \lim_{x \rightarrow 5} \frac{(x + 5)(x - 5)}{x - 5}$$

$$= \lim_{x \rightarrow 5} (x + 5)$$

$$= 5 + 5$$

$$= 10$$

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Question 6: Evaluate

$$\lim_{x \rightarrow 1} \left(\frac{x^3 - 1}{x - 1} \right)$$

Solution:

We know, $\lim_{x \rightarrow a} f(x) = f(a)$

As $x \rightarrow 1$

$$\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1} = \lim_{x \rightarrow 1} \frac{(x - 1)(x^2 + x + 1)}{x - 1}$$

$$= \lim_{x \rightarrow 1} (x^2 + x + 1)$$

$$= 1 + 1 + 1$$

$$= 3$$

Question 7: Evaluate

$$\lim_{x \rightarrow -2} \left(\frac{x^3 + 8}{x + 2} \right)$$

Solution:

We know, $\lim_{x \rightarrow a} f(x) = f(a)$

As $x \rightarrow -2$

$$\lim_{x \rightarrow -2} \frac{x^3 + 8}{x + 2} = \lim_{x \rightarrow -2} \frac{(x + 2)(x^2 - 2x + 4)}{x + 2}$$

$$= \lim_{x \rightarrow -2} (x^2 - 2x + 4)$$

$$= 4 + 4 + 4$$

$$= 12$$

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Question 8: Evaluate

$$\lim_{x \rightarrow 3} \left(\frac{x^4 - 81}{x - 3} \right)$$

Solution:

We know, $\lim_{x \rightarrow a} f(x) = f(a)$

As $x \rightarrow 3$

$$\lim_{x \rightarrow 3} \frac{x^4 - 81}{x - 3} = \lim_{x \rightarrow 3} \frac{(x^2 + 9)(x^2 - 9)}{x - 3}$$

$$= \lim_{x \rightarrow 3} \frac{(x^2 + 9)(x + 3)(x - 3)}{x - 3}$$

$$= \lim_{x \rightarrow 3} (x^2 + 9)(x + 3)$$

$$= (9 + 9)(3 + 3)$$

$$= 108$$

Question 9: Evaluate

$$\lim_{x \rightarrow 3} \left(\frac{x^2 - 4x + 3}{x^2 - 2x - 3} \right)$$

Solution:

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We know, $\lim_{x \rightarrow a} f(x) = f(a)$

As $x \rightarrow 3$

$$\lim_{x \rightarrow 3} \frac{x^2 - 4x + 3}{x^2 - 2x - 3} = \lim_{x \rightarrow 3} \frac{(x-3)(x-1)}{(x-3)(x+1)}$$

$$= \lim_{x \rightarrow 3} \frac{(x-1)}{(x+1)}$$

$$= \frac{2}{4} \text{ or } \frac{1}{2}$$

Question 10: Evaluate

$$\lim_{x \rightarrow \frac{1}{2}} \left(\frac{4x^2 - 1}{2x - 1} \right)$$

Solution:

We know, $\lim_{x \rightarrow a} f(x) = f(a)$

As $x \rightarrow \frac{1}{2}$

$$\lim_{x \rightarrow \frac{1}{2}} \frac{4x^2 - 1}{2x - 1} = \lim_{x \rightarrow \frac{1}{2}} \frac{(2x+1)(2x-1)}{2x-1}$$

$$= \lim_{x \rightarrow \frac{1}{2}} (2x+1)$$

$$= 2$$

Question 11: Evaluate

$$\lim_{x \rightarrow 4} \left(\frac{x^3 - 64}{x^2 - 16} \right)$$

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Solution:

$$\lim_{x \rightarrow 4} \frac{x^3 - 64}{x^2 - 16} = \lim_{x \rightarrow 4} \frac{(x - 4)(x^2 + 4x + 16)}{(x + 4)(x - 4)}$$

$$= \lim_{x \rightarrow 4} \frac{(x^2 + 4x + 16)}{(x + 4)}$$

$$= \frac{48}{8}$$

$$= 6$$

Question 12: Evaluate

$$\lim_{x \rightarrow 2} \left(\frac{x^5 - 32}{x^3 - 8} \right)$$

Solution:

$$\lim_{x \rightarrow 2} \frac{x^5 - 32}{x^3 - 8} = \lim_{x \rightarrow 2} \frac{x^5 - 2^5}{x^3 - 2^3}$$

We know,

$$\lim_{x \rightarrow a} \frac{x^m - a^m}{x - a} = m a^{m-1}$$

so,

$$\lim_{x \rightarrow 2} \frac{x^5 - 32}{x^3 - 8} = \lim_{x \rightarrow 2} \frac{\frac{x^5 - 2^5}{x - 2}}{\frac{x^3 - 2^3}{x - 2}}$$

$$= \lim_{x \rightarrow 2} \frac{5(2)^4}{3(2)^2}$$

$$= \frac{20}{3}$$

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Question 13: Evaluate

$$\lim_{x \rightarrow a} \left(\frac{x^{5/2} - a^{5/2}}{x - a} \right)$$

Solution:

We know,

$$\lim_{x \rightarrow a} \frac{x^m - a^m}{x - a} = m a^{m-1}$$

$$\lim_{x \rightarrow a} \frac{x^{5/2} - a^{5/2}}{x - a} = \frac{5}{2} a^{5/2-1} = \frac{5}{2} a^{3/2}$$

Question 14: Evaluate

$$\lim_{x \rightarrow a} \left\{ \frac{(x+2)^{5/3} - (a+2)^{5/3}}{x - a} \right\}$$

Solution:

$$\lim_{x \rightarrow a} \left\{ \frac{(x+2)^{5/3} - (a+2)^{5/3}}{x - a} \right\} = \lim_{x \rightarrow a} \left\{ \frac{(x+2)^{5/3} - (a+2)^{5/3}}{(x+2) - (a+2)} \right\}$$

We know,

$$\lim_{x \rightarrow a} \frac{x^m - a^m}{x - a} = m a^{m-1}$$

$$\lim_{x \rightarrow a} \left\{ \frac{(x+2)^{5/3} - (a+2)^{5/3}}{x - a} \right\} = \frac{5}{3} (a+2)^{5/3-1} = \frac{5}{3} (a+2)^{2/3}$$

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Question 15: Evaluate

$$\lim_{x \rightarrow 1} \left(\frac{x^n - 1}{x - 1} \right)$$

Solution:

$$\lim_{x \rightarrow a} \frac{x^n - 1}{x - 1} = \lim_{x \rightarrow a} \frac{x^n - 1^n}{x - 1}$$

We know,

$$\lim_{x \rightarrow a} \frac{x^m - a^m}{x - a} = m a^{m-1}$$

$$\lim_{x \rightarrow a} \frac{x^n - 1}{x - 1} = n$$

Question 16: Evaluate

$$\lim_{x \rightarrow a} \left(\frac{\sqrt{x} - \sqrt{a}}{x - a} \right)$$

Solution:

$$\lim_{x \rightarrow a} \left(\frac{\sqrt{x} - \sqrt{a}}{x - a} \right) = \lim_{x \rightarrow a} \frac{x^{\frac{1}{2}} - a^{\frac{1}{2}}}{x - a}$$

We know,

$$\lim_{x \rightarrow a} \frac{x^m - a^m}{x - a} = m a^{m-1}$$

$$\lim_{x \rightarrow a} \frac{x^{\frac{1}{2}} - a^{\frac{1}{2}}}{x - a} = \frac{1}{2} a^{\frac{1}{2}-1} = \frac{1}{2\sqrt{a}}$$

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Question 17: Evaluate

$$\lim_{h \rightarrow 0} \left(\frac{\sqrt{x+h} - \sqrt{x}}{h} \right)$$

Solution:

$$\begin{aligned} \lim_{h \rightarrow 0} \left(\frac{\sqrt{x+h} - \sqrt{x}}{h} \right) &= \lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \times \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} \\ &= \lim_{h \rightarrow 0} \frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})} \\ &= \lim_{h \rightarrow 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} \\ &= \frac{1}{2\sqrt{x}} \end{aligned}$$

Question 18: Evaluate

$$\lim_{h \rightarrow 0} \frac{1}{h} \left\{ \frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}} \right\}$$

Solution:

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{1}{h} \left\{ \frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}} \right\} &= \lim_{h \rightarrow 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h} \\ &= \lim_{h \rightarrow 0} \frac{1}{h\sqrt{x+h}} - \frac{1}{h\sqrt{x}} \\ &= \lim_{h \rightarrow 0} \frac{\sqrt{x}}{h\sqrt{x+h}\sqrt{x}} - \frac{\sqrt{x+h}}{h\sqrt{x+h}\sqrt{x}} \\ &= \lim_{h \rightarrow 0} \frac{\sqrt{x} - \sqrt{x+h}}{h\sqrt{x}(x+h)} \times \frac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x} + \sqrt{x+h}} \\ &= \lim_{h \rightarrow 0} \frac{x - (x+h)}{(h\sqrt{x}(x+h))(\sqrt{x} + \sqrt{x+h})} \\ &= \frac{-1}{2x\sqrt{x}} \end{aligned}$$

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Question 19: Evaluate

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x}$$

Solution:

$$\begin{aligned}\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x} &= \lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x} \times \frac{\sqrt{1+x} + 1}{\sqrt{1+x} + 1} \\&= \lim_{x \rightarrow 0} \frac{1+x-1}{x(\sqrt{1+x}+1)} \\&= \frac{1}{2}\end{aligned}$$

Question 20: Evaluate

$$\lim_{x \rightarrow 0} \frac{\sqrt{2-x} - \sqrt{2+x}}{x}$$

Solution:

$$\begin{aligned}\lim_{x \rightarrow 0} \frac{\sqrt{2-x} - \sqrt{2+x}}{x} &= \lim_{x \rightarrow 0} \frac{\sqrt{2-x} - \sqrt{2+x}}{x} \times \frac{\sqrt{2-x} + \sqrt{2+x}}{\sqrt{2-x} + \sqrt{2+x}} \\&= \lim_{x \rightarrow 0} \frac{2-x-2-x}{x(\sqrt{2-x} + \sqrt{2+x})} \\&= \frac{-1}{\sqrt{2}}\end{aligned}$$

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Exercise 27B

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Question 1: Evaluate

$$\lim_{x \rightarrow 0} \frac{\sin 4x}{6x}$$

Solution:

$$\lim_{x \rightarrow 0} \frac{\sin 4x}{6x} = \lim_{x \rightarrow 0} \left(\frac{\sin 4x}{4x} \right) \times \frac{4}{6}$$

$$\text{We know, } \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{\sin 4x}{6x} = \frac{4}{6} = \frac{2}{3}$$

Question 2: Evaluate

$$\lim_{x \rightarrow 0} \frac{\sin 5x}{\sin 8x}$$

Solution:

$$\lim_{x \rightarrow 0} \frac{\sin 5x}{\sin 8x} = \lim_{x \rightarrow 0} \left(\frac{\sin 5x}{5x} \right) \times \lim_{x \rightarrow 0} \frac{8x}{\sin 8x} \times \lim_{x \rightarrow 0} \frac{5x}{8x}$$

$$\text{We know, } \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{\sin 5x}{\sin 8x} = \frac{5}{8}$$

Question 3: Evaluate

$$\lim_{x \rightarrow 0} \frac{\tan 3x}{\tan 5x}$$

Solution:

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$$\lim_{x \rightarrow 0} \frac{\tan 3x}{\tan 5x} = \lim_{x \rightarrow 0} \left(\frac{\tan 3x}{3x} \right) \times \lim_{x \rightarrow 0} \frac{5x}{\tan 5x} \times \lim_{x \rightarrow 0} \frac{3x}{5x}$$

We know, $\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$

$$\lim_{x \rightarrow 0} \frac{\tan 3x}{\tan 5x} = \frac{3}{5}$$

Question 4: Evaluate

$$\lim_{x \rightarrow 0} \frac{\tan \alpha x}{\tan \beta x}$$

Solution:

$$\lim_{x \rightarrow 0} \frac{\tan \alpha x}{\tan \beta x} = \lim_{x \rightarrow 0} \left(\frac{\tan \alpha x}{\alpha x} \right) \times \lim_{x \rightarrow 0} \frac{\beta x}{\tan \beta x} \times \lim_{x \rightarrow 0} \frac{\alpha x}{\beta x}$$

We know, $\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$

$$\lim_{x \rightarrow 0} \frac{\tan \alpha x}{\tan \beta x} = \frac{\alpha}{\beta}$$

Question 5: Evaluate

$$\lim_{x \rightarrow 0} \frac{\sin 4x}{\tan 7x}$$

Solution:

$$\lim_{x \rightarrow 0} \frac{\sin 4x}{\tan 7x} = \lim_{x \rightarrow 0} \left(\frac{\sin 4x}{4x} \right) \times \lim_{x \rightarrow 0} \frac{7x}{\sin 7x} \times \lim_{x \rightarrow 0} \frac{4x}{7x}$$

We know, $\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$ and $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

$$\lim_{x \rightarrow 0} \frac{\sin 4x}{\tan 7x} = \frac{4}{7}$$

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Question 6: Evaluate

$$\lim_{x \rightarrow 0} \frac{\tan 3x}{\sin 4x}$$

Solution:

$$\lim_{x \rightarrow 0} \frac{\tan 3x}{\sin 4x} = \lim_{x \rightarrow 0} \left(\frac{\tan 3x}{3x} \right) \times \lim_{x \rightarrow 0} \frac{4x}{\sin 4x} \times \lim_{x \rightarrow 0} \frac{3x}{4x}$$

$$\text{We know, } \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1 \text{ and } \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{\tan 3x}{\sin 4x} = \frac{3}{4}$$

Question 7: Evaluate

$$\lim_{x \rightarrow 0} \frac{\sin mx}{\tan nx}$$

Solution:

$$\lim_{x \rightarrow 0} \frac{\sin mx}{\tan nx} = \lim_{x \rightarrow 0} \left(\frac{\sin mx}{mx} \right) \times \lim_{x \rightarrow 0} \frac{nx}{\tan nx} \times \lim_{x \rightarrow 0} \frac{mx}{nx}$$

$$\text{We know, } \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1 \text{ and } \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{\sin mx}{\tan nx} = \frac{m}{n}$$

Question 8: Evaluate

$$\lim_{x \rightarrow 0} \frac{\sin x - 2 \sin 3x + \sin 5x}{x}$$

Solution:

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$$\begin{aligned}\lim_{x \rightarrow 0} \frac{\sin x - 2 \sin 3x + \sin 5x}{x} &= \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} - \frac{2 \sin 3x}{x} + \frac{\sin 5x}{x} \right) \\ &= \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} - \frac{6 \sin 3x}{3x} + \frac{5 \sin 5x}{5x} \right)\end{aligned}$$

We know, $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

$$\lim_{x \rightarrow 0} \frac{\sin x - 2 \sin 3x + \sin 5x}{x} = 1 - 6 + 5 = 0$$

Question 9: Evaluate

$$\lim_{x \rightarrow \pi/6} \frac{(2 \sin^2 x + \sin x - 1)}{(2 \sin^2 x - 3 \sin x + 1)}$$

Solution:

Let $\sin x = y$

Numerator: $2y^2 + y - 1 = (2y - 1)(y + 1)$

Denominator: $2y^2 - 3y + 1 = (2y - 1)(y - 1)$

Eliminate common term from numerator and denominator which is $(2y - 1)$.

$$\lim_{x \rightarrow \pi/6} \frac{(2 \sin^2 x + \sin x - 1)}{(2 \sin^2 x - 3 \sin x + 1)} = \lim_{x \rightarrow \pi/6} \frac{(\sin x + 1)}{(\sin x - 1)}$$

We know, $\lim_{x \rightarrow a} f(x) = f(a)$

$$\lim_{x \rightarrow \pi/6} \frac{(2 \sin^2 x + \sin x - 1)}{(2 \sin^2 x - 3 \sin x + 1)} = \frac{(\sin \frac{\pi}{6} + 1)}{(\sin \frac{\pi}{6} - 1)}$$

We know, $\sin \frac{\pi}{6} = \frac{1}{2}$

$$\lim_{x \rightarrow \pi/6} \frac{(2 \sin^2 x + \sin x - 1)}{(2 \sin^2 x - 3 \sin x + 1)} = -3$$

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Question 10: Evaluate

$$\lim_{x \rightarrow 0} \frac{(\sin 2x + 3x)}{(2x + \sin 3x)}$$

Solution:

$$\lim_{x \rightarrow 0} \frac{(\sin 2x + 3x)}{(2x + \sin 3x)} = \lim_{x \rightarrow 0} \frac{\frac{2x \sin(2x)}{(2x)} + 3x}{2x + \frac{(3x \sin 3x)}{3x}}$$

$$\text{We know, } \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{(\sin 2x + 3x)}{(2x + \sin 3x)} = \frac{2x + 3x}{2x + 3x} = 1$$

Question 11: Evaluate

$$\lim_{x \rightarrow 0} \frac{(\tan 2x - x)}{(3x - \tan x)}$$

Solution:

$$\lim_{x \rightarrow 0} \frac{(\tan 2x - x)}{(3x - \tan x)} = \lim_{x \rightarrow 0} \frac{\frac{2x \tan 2x}{(2x)} - x}{3x - \frac{x \tan x}{x}}$$

$$\text{We know, } \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{(\tan 2x - x)}{(3x - \tan x)} = \frac{2x - x}{3x - x} = \frac{1}{2}$$

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Question 12: Evaluate

$$\lim_{x \rightarrow 0} \frac{(x^2 - \tan 2x)}{\tan x}$$

Solution:

$$\lim_{x \rightarrow 0} \frac{(x^2 - \tan 2x)}{\tan x} = \lim_{x \rightarrow 0} \frac{x^2 - \frac{2x \tan 2x}{(2x)}}{\frac{x \tan x}{x}}$$

$$\text{We know, } \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{(x^2 - \tan 2x)}{\tan x} = \lim_{x \rightarrow 0} \frac{x^2 - 2x}{x} = -2$$

Question 13: Evaluate

$$\lim_{x \rightarrow 0} \frac{x \cos x + \sin x}{x^2 + \tan x}$$

Solution:

$$\lim_{x \rightarrow 0} \frac{x \cos x + \sin x}{x^2 + \tan x} = \lim_{x \rightarrow 0} \frac{x(\cos x + \frac{\sin x}{x})}{x(x + \frac{\tan x}{x})}$$

$$\text{We know, } \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1 \text{ and } \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{x \cos x + \sin x}{x^2 + \tan x} = \lim_{x \rightarrow 0} \frac{\cos x + 1}{x + 1} = 2$$

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Question 14: Evaluate

$$\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{\sin^3 x}$$

Solution:

$$\frac{\tan x - \sin x}{\sin^3 x} = \lim_{x \rightarrow 0} \frac{\frac{\sin x}{\cos x} - \sin x}{\sin^3 x}$$

$$= \lim_{x \rightarrow 0} \frac{\sin x - \sin x \cos x}{\cos x \sin^3 x}$$

$$= \lim_{x \rightarrow 0} \frac{1 - \cos x}{\sin^2 x \cos x}$$

We know, $\sin x = 1 - \cos x = (1 - \cos x)(1 + \cos x)$

$$= \lim_{x \rightarrow 0} \frac{1 - \cos x}{\cos x (1 - \cos x)(1 + \cos x)}$$

$$= \lim_{x \rightarrow 0} \frac{1}{(\cos x)(1 + \cos x)}$$

$$= \frac{1}{2}$$

Question 15: Evaluate

$$\lim_{x \rightarrow 0} x \operatorname{cosec} x$$

Solution:

$$\lim_{x \rightarrow 0} x \operatorname{cosec} x = \lim_{x \rightarrow 0} \frac{x}{\sin x}$$

$$= \frac{1}{\lim_{x \rightarrow 0} \frac{\sin x}{x}}$$

We know, $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

$$\lim_{x \rightarrow 0} x \operatorname{cosec} x = 1$$

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Question 16: Evaluate

$$\lim_{x \rightarrow 0} (x \cot 2x)$$

Solution:

$$\lim_{x \rightarrow 0} (x \cot 2x) = \lim_{x \rightarrow 0} x \frac{\cos 2x}{\sin 2x}$$

$$= \lim_{x \rightarrow 0} x \frac{\cos 2x}{\sin 2x} \cdot \frac{2x}{2x}$$

$$= \lim_{x \rightarrow 0} x \frac{\cos 2x}{2x} \cdot \frac{2x}{\sin 2x}$$

$$\text{We know, } \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$= \lim_{x \rightarrow 0} \frac{\cos 2x}{2}$$

$$\lim_{x \rightarrow 0} (x \cot 2x) = \frac{1}{2}$$

Question 17: Evaluate

$$\lim_{x \rightarrow 0} \frac{\sin x \cos x}{3x}$$

Solution:

$$\lim_{x \rightarrow 0} \frac{\sin x \cos x}{3x} = \lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \frac{\cos x}{3}$$

$$\text{We know, } \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$= \lim_{x \rightarrow 0} \frac{\cos x}{3}$$

$$= \frac{1}{3}$$

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Question 18: Evaluate

$$\lim_{x \rightarrow 0} \frac{\sin(x/4)}{x}$$

Solution:

$$\lim_{x \rightarrow 0} \frac{\sin(x/4)}{x} = \lim_{x \rightarrow 0} \frac{\sin(x/4)}{4 \cdot x/4}$$

$$\text{We know, } \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$= \frac{1}{4}$$

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Exercise 27C

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Question 1: If $f(x) = |x - 3|$, find $\lim_{x \rightarrow 3} f(x)$ **Solution:**

$$\lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} |x - 3| = \lim_{x \rightarrow 3^+} (x - 3) = 3 - 3 = 0$$

$$\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} |x - 3| = \lim_{x \rightarrow 3^-} -(x - 3) = 3 - 3 = 0$$

$$\text{So, } \lim_{x \rightarrow 3} f(x) = 0$$

Question 2:

$$\text{Let } f(x) = \begin{cases} \frac{x}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Show that $\lim_{x \rightarrow 0} f(x)$ does not exist.**Solution:**

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \frac{x}{|x|} = \lim_{x \rightarrow 0^-} \frac{x}{(-x)} = -1$$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \frac{x}{|x|} = \lim_{x \rightarrow 0^+} \frac{x}{(+x)} = 1$$

$$\Rightarrow \lim_{x \rightarrow 0^-} f(x) \neq \lim_{x \rightarrow 0^+} f(x)$$

So, $\lim_{x \rightarrow 0} f(x)$ does not exist.**Question 3:**

$$\text{Let } f(x) = \begin{cases} \frac{|x - 3|}{(x - 3)}, & x \neq 3 \\ 0, & x = 3 \end{cases}$$

Show that $\lim_{x \rightarrow 3} f(x)$ does not exist

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Solution:

$$\lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} \frac{|x-3|}{x-3} = \lim_{x \rightarrow 3^+} \frac{(x-3)}{x-3} = 1$$

$$\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} \frac{|x-3|}{x-3} = \lim_{x \rightarrow 3^-} \frac{-(x-3)}{x-3} = -1$$

$$\Rightarrow \lim_{x \rightarrow 3^+} f(x) \neq \lim_{x \rightarrow 3^-} f(x)$$

So, $\lim_{x \rightarrow 3} f(x)$ does not exist.

Question 4:

$$\text{Let } f(x) = \begin{cases} 1+x^2, & 0 \leq x \leq 1 \\ 2-x, & x > 1 \end{cases}$$

Show that $\lim_{x \rightarrow 1} f(x)$ does not exist

Solution:

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} 2-x = 2-1 = 1$$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} 1+x^2 = 1+1 = 2$$

$$\lim_{x \rightarrow 1^-} f(x) \neq \lim_{x \rightarrow 1^+} f(x)$$

So, $\lim_{x \rightarrow 1} f(x)$ does not exist

Question 5:

$$\text{Let } f(x) = \begin{cases} \frac{x-|x|}{x}, & x \neq 0 \\ 2, & x = 0 \end{cases}$$

Show that $\lim_{x \rightarrow 0} f(x)$ does not exist

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Solution:

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \frac{x - |x|}{x} = \lim_{x \rightarrow 0^+} \frac{x - (x)}{x} = 0$$

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \frac{x - |x|}{x} = \lim_{x \rightarrow 0^-} \frac{x - (-x)}{x} = 2$$

$$\lim_{x \rightarrow 0^-} f(x) \neq \lim_{x \rightarrow 0^+} f(x)$$

$\lim_{x \rightarrow 0} f(x)$ does not exist

Question 6:

$$\text{Let } f(x) = \begin{cases} 5x - 4, & 0 < x \leq 1 \\ 4x^3 - 3x, & 1 < x < 2 \end{cases}$$

Find $\lim_{x \rightarrow 1} f(x)$

Solution:

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} 4x^3 - 3x = 4 - 3 = 1$$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} 5x - 4 = 5 - 4 = 1$$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x)$$

Therefore, $\lim_{x \rightarrow 1} f(x) = 1$

Question 7:

$$\text{Let } f(x) = \begin{cases} 4x - 5, & x \leq 2 \\ x - a, & x > 2 \end{cases}$$

If $\lim_{x \rightarrow 2} f(x)$ exists then find the value of a .

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Solution:

$$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} x - a = 2 - a$$

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} 4x - 5 = 8 - 5 = 3$$

As, $\lim_{x \rightarrow 2} f(x)$ exists (given)

$$\text{Then, } 2 - a = 3$$

$$\text{or } a = -1$$

Question 8:

$$\text{Let } f(x) = \begin{cases} \frac{3x}{|x| + 2x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Show that $\lim_{x \rightarrow 0} f(x)$ does not exist

Solution:

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \frac{3x}{|x| + 2x} = \lim_{x \rightarrow 0^+} \frac{3x}{(x) + 2x} = 1$$

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \frac{3x}{|x| + 2x} = \lim_{x \rightarrow 0^-} \frac{3x}{(-x) + 2x} = 3$$

$$\lim_{x \rightarrow 0^-} f(x) \neq \lim_{x \rightarrow 0^+} f(x)$$

So, $\lim_{x \rightarrow 0} f(x)$ does not exist.