R S Aggarwal Solutions for Class 11 Maths Chapter 24 Hyperbola

Exercise 24

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Question 1: Find the (i) lengths of axes, (ii) coordinates of the vertices, (iii) coordinates of the foci, (iv) eccentricity, and (v) length of the latus rectum of each of the following hyperbola. (Question 1 to question 10)

Horizontal Hyperbola:
For general form of Hyperbola:
$x^2/a^2 - y^2/b^2 = 1$ .....(1)

Length of Transverse axis	Length of Conjugate axis	vertices	foci	eccentricity	latus rectum
2a	2b	(±a, 0)	(±c, 0)	e = c/a	$2b^2/a$

Vertical Hyperbola:
For general form of Hyperbola:
$y^2/a^2 - x^2/b^2 = 1$ .....(2)

Length of Transverse axis	Length of Conjugate axis	vertices	foci	eccentricity	latus rectum
2a	2b	(0, ±a)	(0, ±c)	e = c/a	$2b^2/a$

Question 1: $x^2/9 - y^2/16 = 1$
Solution:
Given equation is of the form $x^2/a^2 - y^2/b^2 = 1$

On comparing given equation with (1), we get

a = 3 and b = 4

Then,
$c^2 = a^2 + b^2$

$c^2 = 9 + 16 = 25$

or c = 5

Now,

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(i) Lengths of the axes:

Length of Transverse axis = 2a = 6 units

Length of Conjugate axis = 2b = 8 units

(ii) Coordinates of the vertices: (±a, 0) = (±3, 0)

(iii) Coordinates of the foci: (±c, 0) = (±5, 0)

(iv) Eccentricity: e = c/a = 5/3

(v) Length of the latus rectum: $2b^2/a = 2(16)/5 = 32/5$ units

Question 2: $x^2/25 - y^2/4 = 1$
Solution:

Given equation is of the form $x^2/a^2 - y^2/b^2 = 1$

On comparing given equation with (1), we get

a = 5 and b = 2

Then,

$c^2 = a^2 + b^2$

$c^2 = 25 + 4 = 29$

or $c = \sqrt{29}$

Now,
(i) Lengths of the axes:

Length of Transverse axis = 2a = 10 units

Length of Conjugate axis = 2b = 4 units

(ii) Coordinates of the vertices: (±a, 0) = (±5, 0)

(iii) Coordinates of the foci: (±c, 0) = (±√29, 0)

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(iv) Eccentricity: $e = c/a = \sqrt{29}/5$

(v) Length of the latus rectum: $2b^2/a = 8/5$ units

Question 3: $x^2 - y^2 = 1$

Solution:
Given equation is of the form $x^2/a^2 - y^2/b^2 = 1$

On comparing given equation with (1), we get

$a = 1$ and $b = 1$

Then,

$c^2 = a^2 + b^2$

$c^2 = 1 + 1 = 2$

or $c = \sqrt{2}$

Now,
(i) Lengths of the axes:

Length of Transverse axis = $2a = 2$ units

Length of Conjugate axis = $2b = 2$ units

(ii) Coordinates of the vertices: $(\pm a, 0) = (\pm 1, 0)$

(iii) Coordinates of the foci: $(\pm c, 0) = (\pm\sqrt{2}, 0)$

(iv) Eccentricity: $e = c/a = \sqrt{2}$

(v) Length of the latus rectum: $2b^2/a = 2$ units

Question 4: $3x^2 - 2y^2 = 6$

Solution:
Divide each side by 6
$x^2/2 - y^2/3 = 1$
Given equation is of the form $x^2/a^2 - y^2/b^2 = 1$

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On comparing given equation with (1), we get

$a = \sqrt{2}$ and $b = \sqrt{3}$

Then,

$c^2 = a^2 + b^2$

$c^2 = 2 + 3 = 5$

or $c = \sqrt{5}$

Now,
(i) Lengths of the axes:

Length of Transverse axis = $2a = 2\sqrt{2}$ units

Length of Conjugate axis = $2b = 2\sqrt{3}$ units

(ii) Coordinates of the vertices: $(\pm a, 0) = (\pm\sqrt{2}, 0)$

(iii) Coordinates of the foci: $(\pm c, 0) = (\pm\sqrt{5}, 0)$

(iv) Eccentricity: $e = c/a = \sqrt{(5/2)}$

(v) Length of the latus rectum: $2b^2/a = 3\sqrt{2}$ units

Question 5: $25x^2 - 9y^2 = 225$

Solution:
Divide both sides by 225

$x^2/9 - y^2/25 = 1$

Given equation is of the form $x^2/a^2 - y^2/b^2 = 1$

On comparing given equation with (1), we get

$a = 3$ and $b = 5$

Then,

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$c^2 = a^2 + b^2$

$c^2 = 9 + 25 = 34$

or $c = \sqrt{34}$

Now,
(i) Lengths of the axes:

Length of Transverse axis = 2a = 6 units

Length of Conjugate axis = 2b = 10 units

(ii) Coordinates of the vertices: $(\pm a, 0) = (\pm 3, 0)$

(iii) Coordinates of the foci: $(\pm c, 0) = (\pm\sqrt{34}, 0)$

(iv) Eccentricity: $e = c/a = \sqrt{34}/3$

(v) Length of the latus rectum: $2b^2/a = 50/3$ units

Question 6: $24x^2 - 25y^2 = 600$

Solution:

Divide both sides by 600

$x^2/25 - y^2/24 = 1$

Given equation is of the form $x^2/a^2 - y^2/b^2 = 1$

On comparing given equation with (1), we get

$a = 5$ and $b = 2\sqrt{6}$

Then,

$c^2 = a^2 + b^2$

$c^2 = 25 + 24 = 49$

or $c = 7$

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Now,
(i) Lengths of the axes:

Length of Transverse axis = 2a = 10 units

Length of Conjugate axis = 2b = 4√6 units

(ii) Coordinates of the vertices: (±a, 0) = (±5, 0)

(iii) Coordinates of the foci: (±c, 0) = (±7, 0)

(iv) Eccentricity: e = c/a = 7/5

(v) Length of the latus rectum: $2b^2/a$ = 48/5 units

Question 7: $y^2/16 - x^2/49 = 1$

Solution:

Given equation is of the form $y^2/a^2 - x^2/b^2 = 1$

On comparing given equation with (2), we get

a = 4 and b = 7

Then,

$c^2 = a^2 + b^2$

$c^2 = 16 + 49 = 65$

or c = √65

Now,
(i) Lengths of the axes:

Length of Transverse axis = 2a = 8 units

Length of Conjugate axis = 2b = 14 units

(ii) Coordinates of the vertices: (0, ±a) = (0, ±4)

(iii) Coordinates of the foci: (0, ±c) = (0, ±√65)

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(iv) Eccentricity: $e = c/a = \sqrt{65}/4$

(v) Length of the latus rectum: $2b^2/a = 49/2$ units

Question 8: $y^2/9 - x^2/27 = 1$
Solution:
Given equation is of the form $y^2/a^2 - x^2/b^2 = 1$

On comparing given equation with (2), we get

$a = 3$ and $b = \sqrt{27} = 3\sqrt{3}$

Then,

$c^2 = a^2 + b^2$

$c^2 = 9 + 27 = 36$

or $c = 6$

Now,
(i) Lengths of the axes:

Length of Transverse axis = $2a = 6$ units

Length of Conjugate axis = $2b = 6\sqrt{3}$ units

(ii) Coordinates of the vertices: $(0, \pm a) = (0, \pm 3)$

(iii) Coordinates of the foci: $(0, \pm c) = (0, \pm 6)$

(iv) Eccentricity: $e = c/a = 2$

(v) Length of the latus rectum: $2b^2/a = 18$ units

Question 9: $3y^2 - x^2 = 108$

Solution:
Divide each side by 108
$y^2/36 - x^2/108 = 1$

Which is of the form $y^2/a^2 - x^2/b^2 = 1$

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On comparing given equation with (2), we get

$a = 6$ and $b = 6\sqrt{3}$

Then,

$c^2 = a^2 + b^2$

$c^2 = 36 + 108 = 144$

or $c = 12$

Now,
(i) Lengths of the axes:

Length of Transverse axis = $2a = 12$ units

Length of Conjugate axis = $2b = 12\sqrt{3}$ units

(ii) Coordinates of the vertices: $(0, \pm a) = (0, \pm 6)$

(iii) Coordinates of the foci: $(0, \pm c) = (0, \pm 12)$

(iv) Eccentricity: $e = c/a = 2$

(v) Length of the latus rectum: $2b^2/a = 36$ units

Question 10: $5y^2 - 9x^2 = 36$

Solution:
Divide each side by 36, we get
$y^2/(36/5) - x^2/4 = 1$

Which is of the form $y^2/a^2 - x^2/b^2 = 1$

On comparing given equation with (2), we get

$a = 6/\sqrt{5}$ and $b = 2$

Then,

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$c^2 = a^2 + b^2$

$c^2 = 36/5 + 4 = 56/5$

or $c = 2\sqrt{(14/5)}$

Now,
(i) Lengths of the axes:

Length of Transverse axis = $2a = 12/\sqrt{5}$ units

Length of Conjugate axis = $2b = 4$ units

(ii) Coordinates of the vertices: $(0, \pm a) = (0, \pm 6/\sqrt{5})$

(iii) Coordinates of the foci: $(0, \pm c) = (0, \pm 2\sqrt{(14/5)})$

(iv) Eccentricity: $e = c/a = \sqrt{14}/3$

(v) Length of the latus rectum: $2b^2/a = 4\sqrt{5}/3$ units

Question 11: Find the equation of hyperbola whose vertices at (±6, 0) and foci at (±8, 0).

Solution:
Vertices are of the form (±a, 0), hence it is a horizontal hyperbola.

So, the equation is of the form: $x^2/a^2 - y^2/b^2 = 1$ .....(1)

Given: vertices of hyperbola (±6, 0)

Here $a = 6$

Again,

Foci is of the form (±c, 0)

Given: foci of hyperbola at (±8, 0)

$\Rightarrow c = 8$

Find b:
We know, $c^2 = a^2 + b^2$

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$64 = 36 + b^2$

or $b^2 = 64 – 36 = 28$

Equation (1)=>

$x^2/36 - y^2/28 = 1$

Which is required equation.

Question 12: Find the equation of the hyperbola with vertices at (0, ±5) and foci at (0, ±8).

Solution:

Vertices are of the form (0, ±a), hence it is a vertical hyperbola.

So, the equation is of the form: $y^2/a^2 - x^2/b^2 = 1$ .....(1)

Given: vertices of hyperbola (0, ±5)

Here a = 5

Again,

Foci is of the form (0, ±c)

Given: foci of hyperbola at (0, ±8)

=> c = 8

Find b:
We know, $c^2 = a^2 + b^2$

$64 = 25 + b^2$

or $b^2 = 39$

Equation (1)=>

$y^2/25 - x^2/39 = 1$

Which is required equation.

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Question 13: Find the equation of hyperbola whose foci are (±√29, 0) and the transverse axis is of the length 10.

Solution: Major axis of the given hyperbola lie on x-axis.

Equation of the hyperbola will be of the form: $x^2/a^2 - y^2/b^2 = 1$
Whose foci at (±c, 0) and transverse axis is 2a

Given: transverse axis = 10
=> 2a = 10 => a = 5

Given: foci (±√29, 0)
(±c, 0) = (±√29, 0)

=> c = √29

We know, $c^2 = a^2 + b^2$

$29 = 25 + b^2$

or $b^2 = 4$

Equation (1)=>

$x^2/25 - y^2/4 = 1$

Which is required equation.

Question 14: Find the equation of hyperbola whose foci are (±5, 0) and the conjugate axis is of the length 8. Also, find its eccentricity.

Solution: Foci of the equation is in the form (±c, 0)

Equation of the hyperbola will be of the form: $x^2/a^2 - y^2/b^2 = 1$
Whose foci at (±c, 0) and conjugate axis is 2b

On comparing, we get
2b = 8 => b = 4
And c = 5

Again, We know, $c^2 = a^2 + b^2$

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$25 = 16 + a^2$

or $a^2 = 9$

Equation (1)=>

$x^2/9 - y^2/16 = 1$

Which is required equation.

Again,

Eccentricity (e) = c/a = 5/3