R S Aggarwal Solutions for Class 11 Maths Chapter 21 Circles

Exercise 21A

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Question 1: Find the equation of a circle with centre (2, 4) and radius 5.
Solution:
The general form of the equation of a circle is:

$(x - h)^2 + (y - k)^2 = r^2$ .....(1)

Where, r is the radius of the circle and (h, k) is the centre of the circle.

Here, r = 5, h = 2 and k = 4

Equation (1)=>

$(x - 2)^2 + (y - 4)^2 = 5^2$

or $(x - 2)^2 + (y - 4)^2 = 25$

or $x^2 + y^2 – 4x – 8y - 5 = 0$

Which is the required equation.

Question 2: Find the equation of a circle with centre (-3, -2) and radius 6.
Solution:
The general form of the equation of a circle is:

$(x - h)^2 + (y - k)^2 = r^2$ .....(1)

Where, r is the radius of the circle and (h, k) is the centre of the circle.

Here, r = 6, h = -3 and k = -2

Equation (1)=>

$(x + 3)^2 + (y + 2)^2 = 6^2$

or $(x + 3)^2 + (y + 2)^2 = 36$

or $x^2 + y^2 + 6x + 4y – 23 = 0$

Which is the required equation.

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Question 3: Find the equation of a circle with centre (a, a) and radius √2.
Solution:
The general form of the equation of a circle is:

$(x - h)^2 + (y - k)^2 = r^2$ .....(1)

Where, r is the radius of the circle and (h, k) is the centre of the circle.

Here, $r = \sqrt{2}$, h = a and k = a

Equation (1)=>

$(x - a)^2 + (y - a)^2 = (\sqrt{2})^2$

or $(x - a)^2 + (y - a)^2 = 2$

or $x^2 + y^2 - 2ax - 2ay + (2a^2 - 2) = 0$

Which is the required equation.

Question 4: Find the equation of a circle with centre (a cos ∝, a sin ∝) and radius a

Solution:
The general form of the equation of a circle is:

$(x - h)^2 + (y - k)^2 = r^2$ .....(1)

Where, r is the radius of the circle and (h, k) is the centre of the circle.

Here, r = a, h = a cos ∝ and k = a sin ∝

Equation (1)=>

$(x - a \cos \propto)^2 + (y - a \sin \propto)^2 = (a)^2$

or $(x - a \cos \propto)^2 + (y - a \sin \propto)^2 = a^2$

or $x^2 + y^2 - 2a \cos \propto x - 2a \sin \propto y + a^2 \cos^2 \propto + a^2 \sin^2 \propto = a^2$

or $x^2 + y^2 - 2a \cos \propto x - 2a \sin \propto y = 0$
$[\text{Because } \cos^2 \propto + a^2 \sin^2 \propto = 1]$
Which is the required equation.

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Question 5: Find the equation of a circle with centre (-a, -b) and radius $\sqrt{a^2 - b^2}$.

Solution:
The general form of the equation of a circle is:

$(x - h)^2 + (y - k)^2 = r^2$ .....(1)

Where, r is the radius of the circle and (h, k) is the centre of the circle.

Here, $r = \sqrt{a^2 - b^2}$, h = -a and k = -b

Equation (1)=>

$(x + a)^2 + (y + b)^2 = (\sqrt{a^2 - b^2})^2$

or $(x + a)^2 + (y + b)^2 = a^2 - b^2$

or $x^2 + y^2 + 2ax + 2ay + a^2 + b^2 = a^2 - b^2$

or $x^2 + y^2 + 2ax + 2ay + 2b^2 = 0$

Which is the required equation.

Question 6: Find the equation of a circle with centre at the origin and radius 4.

Solution:
The general form of the equation of a circle is:

$(x - h)^2 + (y - k)^2 = r^2$ .....(1)

Where, r is the radius of the circle and (h, k) is the centre of the circle.

Here, r = 4, h = 0 and k = 0

Equation (1)=>

$(x - 0)^2 + (y - 0)^2 = (4)^2$

or $x^2 + y^2 = 16$

or $x^2 + y^2 - 16 = 0$
Which is the required equation.

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Question 7: Find the centre and radius of each of the following circles:

(i) $(x - 3)^2 + (y - 1)^2 = 9$
(ii)

$$\left(x - \frac{1}{2}\right)^2 + \left(y + \frac{1}{3}\right)^2 = \frac{1}{16}$$

(iii) $(x + 5)^2 + (y - 3)^2 = 20$

(iv) $x^2 + (y - 1)^2 = 2$

Solution:

(i) The general form of the equation of a circle is:

$(x - h)^2 + (y - k)^2 = r^2$

Where, (h, k) is the centre of the circle.

r = radius of the circle.

Given equation is $(x - 3)^2 + (y - 1)^2 = 9$

Comparing the given equation of circle with general form we get:

$h = 3$, $k = 1$, $r^2 = 9$

Centre = (3, 1) and radius = 3 units.

(ii) The general form of the equation of a circle is:

$(x - h)^2 + (y - k)^2 = r^2$

Where, (h, k) is the centre of the circle.

r = radius of the circle.

Given equation is

$$\left(x - \frac{1}{2}\right)^2 + \left(y + \frac{1}{3}\right)^2 = \frac{1}{16}$$

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Comparing the given equation of circle with general form we get:

$h = 1/2$, $k = - 1/3$, $r^2 = 1/16$

So, Centre = (1/2, - 1/3) and radius = 1/4 units.

(iii) The general form of the equation of a circle is:

$(x - h)^2 + (y - k)^2 = r^2$

Where, (h, k) is the centre of the circle.

r = radius of the circle.

Given equation is

$(x + 5)^2 + (y - 3)^2 = 20$

Comparing the given equation of circle with general form we get:

$h = - 5$, $k = 3$, $r^2 = 20$

Centre = (-5, 3) and radius = $\sqrt{20}$ or $2\sqrt{5}$ units.

(iv) The general form of the equation of a circle is:

$(x - h)^2 + (y - k)^2 = r^2$

Where, (h, k) is the centre of the circle.

r = radius of the circle.

Given equation is
$x^2 + (y - 1)^2 = 2$

Comparing the given equation of circle with general form we get:

$h = 0$, $k = 1$, $r^2 = 2$

So, Centre = (0, 1) and radius = $\sqrt{2}$units.

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Question 8: Find the equation of the circle whose centre is (2, - 5) and which passes through the point (3, 2).

Solution:

The general form of the equation of a circle is:

$(x - h)^2 + (y - k)^2 = r^2$ .....(1)

Where, (h, k) is the centre of the circle.

r = radius of the circle.

We are given with, centre = (2, - 5)

Or (h, k) = (2, - 5)

Find the radius of circle:

Since the circle passes through (3, 2), so it must satisfy the equation.
Put x = 3 and y = 2 in (1)

$(3 - 2)^2 + (2 + 5)^2 = r^2$

$1 + 49 = r^2$

Or $r^2 = 50$
Now,
Equation of circle is:

$(x - 2)^2 + (y + 5)^2 = 50$

Which is required equation.

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Exercise 21B

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Question 1: Show that the equation $x^2 + y^2 - 4x + 6y - 5 = 0$ represents a circle. Find its centre and radius.

Solution:

Given equation is $x^2 + y^2 - 4x + 6y - 5 = 0$

The general equation of a circle is as follows:

$x^2 + y^2 + 2gx + 2fy + c = 0$

Where g, f and c are constants

With

Centre: (-g, -f)

$$\text{Radius} = \sqrt{g^2 + f^2 - c}$$

On comparing given equation with general form of circle, we have

2g = - 4 => g = -2

2f = 6 => f = 3 and

c = - 5

Centre: (-g, -f) = (2, -3)

$$\text{Radius} = \sqrt{g^2 + f^2 - c}$$

$$= \sqrt{(-2)^2 + 3^2 - (-5)}$$

$$= \sqrt{4 + 9 + 5} = \sqrt{18} = 3\sqrt{2}$$

Question 2: Show that the equation $x^2 + y^2 + x - y = 0$ represents a circle. Find its centre and radius.

Solution:

Given equation is $x^2 + y^2 + x - y = 0$

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The general equation of a circle is as follows:

$x^2 + y^2 + 2gx + 2fy + c = 0$
Where g, f and c are constants

With
Centre: (-g, -f)

Radius = $\sqrt{g^2 + f^2 - c}$

On comparing given equation with general form of circle, we have
2g = 1 => g = 1/2

2f = - 1 => f = -1/2 and

c = 0

Now,
Centre: (-g, -f) = (-1/2, 1/2)

Radius = $\sqrt{g^2 + f^2 - c}$

$$= \sqrt{\frac{1}{2}^2 + (-\frac{1}{2}^2) - 0}$$

$$= \sqrt{\frac{1}{4} + \frac{1}{4}} = \sqrt{\frac{1}{2}}$$

Question 3: Show that the equation $3x^2 + 3y^2 + 6x - 4y - 1 = 0$ represents a circle. Find its centre and radius.

Solution:
Given equation is $3x^2 + 3y^2 + 6x - 4y - 1 = 0$
Or $x^2 + y^2 + 2x - 4/3\ y - 1/3 = 0$

The general equation of a circle is as follows:

$x^2 + y^2 + 2gx + 2fy + c = 0$
Where g, f and c are constants.

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With
Centre: (-g, -f)

Radius = $\sqrt{g^2 + f^2 - c}$

On comparing given equation with general form of circle, we have
2g = 2 => g = 1

2f = -4/3 => f = -2/3 and

c = -1/3

Now,

Centre (-g, -f) = (-1, 2/3)

Radius = $\sqrt{g^2 + f^2 - c}$

$$= \sqrt{1^2 + (-\frac{2}{3})^2 - (-\frac{1}{3})}$$

$$= \sqrt{1 + \frac{4}{9} + \frac{1}{3}} = \sqrt{\frac{16}{9}} = \frac{4}{3}$$

Question 4: Show that the equation $x^2 + y^2 + 2x + 10y + 26 = 0$ represents a point circle. Also, find its centre.

Solution:
Given equation is $x^2 + y^2 + 2x + 10y + 26 = 0$

The general equation of a circle:

$x^2 + y^2 + 2gx + 2fy + c = 0$
where c, g, f are constants.

On comparing given equation with general equation of circle, we have

2g = 2 => g = 1

2f = 10 => f = 5 and

c = 26

Now,

Centre (- g, - f) = (- 1, - 5).

Radius = $\sqrt{g^2 + f^2 - c}$

$= \sqrt{1^2 + 5^2 - 26}$

$= \sqrt{26 - 26} = 0$

Since radius is zero, Thus it is a point circle with radius zero.

Question 5: Show that the equation $x^2 + y^2 - 3x + 3y + 10 = 0$ does not represent a circle.

Solution:
Given equation is $x^2 + y^2 - 3x + 3y + 10 = 0$

We know that, any equation with negative radius(complex number) does not represent a circle.

Find radius of given equation:

Radius = $\sqrt{g^2 + f^2 - c}$

$= \sqrt{(-\frac{3}{2})^2 + (-\frac{3}{2}^2) - 10}$

$= \sqrt{\frac{9}{2} - 10} = \sqrt{-\frac{11}{2}}$

Radius is a complex number. Therefore, given equation does not represent a circle.

Question 6: Find the equation of the circle passing through the points

(i) (0, 0), (5, 0) and (3, 3)
(ii) (1, 2), (3, - 4) and (5, - 6)
(iii) (20, 3), (19, 8) and (2, - 9)

Also, find the centre and radius in each case.

Solution:
Before we start solving listed problems, students are advised to keep below information in mind.
The general equation of a circle is as follows:

$x^2 + y^2 + 2gx + 2fy + c = 0$
Where g, f and c are constants.
With
Centre: (-g, -f)

Radius = $\sqrt{g^2 + f^2 - c}$

(i) (0, 0), (5, 0) and (3, 3)

The Circle equation is:

$$\begin{vmatrix} x^2+y^2 & x & y & 1 \\ 0^2+0^2 & 0 & 0 & 1 \\ 5^2+0^2 & 5 & 0 & 1 \\ 3^2+3^2 & 3 & 3 & 1 \end{vmatrix} = 0$$

Let us apply Laplace Expansion to solve this problem:

$$(x^2 + y^2)\begin{vmatrix} 0 & 0 & 1 \\ 5 & 0 & 1 \\ 3 & 3 & 1 \end{vmatrix} - x\begin{vmatrix} 0 & 0 & 1 \\ 25 & 0 & 1 \\ 18 & 3 & 1 \end{vmatrix} + y\begin{vmatrix} 0 & 0 & 1 \\ 25 & 5 & 1 \\ 18 & 3 & 1 \end{vmatrix} - \begin{vmatrix} 0 & 0 & 0 \\ 25 & 5 & 0 \\ 18 & 3 & 3 \end{vmatrix} = 0$$

$$15(x^2 + y^2) - 75x - 15y = 0$$

$$x^2 + y^2 - 5x - y = 0$$

On comparing above equation with the general form of circle, we get
2g = -5 => g = -2.5
2f = -1 => f = -0.5
c = 0

Now,
centre = (2.5, 0.5)

$$\text{Radius} = \sqrt{g^2 + f^2 - c}$$

$$= \sqrt{(-2.5^2) + (-0.5)^2 - 0}$$

$$= 2.549$$

(ii) (1, 2), (3, - 4) and (5, - 6)

The Circle equation is:

$$\begin{vmatrix} x^2+y^2 & x & y & 1 \\ 1^2+2^2 & 1 & 2 & 1 \\ 3^2+(-4)^2 & 3 & -4 & 1 \\ 5^2+(-6)^2 & 5 & -6 & 1 \end{vmatrix} = 0$$

Let us apply Laplace Expansion to solve this problem:

$$(x^2 + y^2)\begin{vmatrix} 1 & 2 & 1 \\ 3 & -4 & 1 \\ 5 & -6 & 1 \end{vmatrix} - x\begin{vmatrix} 5 & 2 & 1 \\ 25 & -4 & 1 \\ 61 & -6 & 1 \end{vmatrix} + y\begin{vmatrix} 5 & 1 & 1 \\ 25 & 3 & 1 \\ 61 & 5 & 1 \end{vmatrix} - \begin{vmatrix} 5 & 1 & 2 \\ 25 & 3 & -4 \\ 61 & 5 & -6 \end{vmatrix} = 0$$

$$8(x^2 + y^2) - 176x - 32y - 200 = 0$$

$$x^2 + y^2 - 22x - 4y - 25 = 0$$

On comparing above equation with the general form of circle, we get

2g = -22 => g = -11
2f = -4 => f = -2
c = -25

Now,
Centre = (11, 2)

$$\text{Radius} = \sqrt{g^2 + f^2 - c}$$

$$= \sqrt{(-11)^2 + (-2)^2 - 25}$$

$$= 10$$

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(iii) (20, 3), (19, 8) and (2, - 9)

The Circle equation is:

$$\begin{vmatrix} x^2+y^2 & x & y & 1 \\ 20^2+3^2 & 20 & 3 & 1 \\ 19^2+8^2 & 19 & 8 & 1 \\ 2^2+(-9)^2 & 2 & -9 & 1 \end{vmatrix} = 0$$

Let us apply Laplace Expansion to solve this problem:

$$(x^2+y^2)\begin{vmatrix} 20 & 3 & 1 \\ 19 & 8 & 1 \\ 2 & -9 & 1 \end{vmatrix} - x\begin{vmatrix} 409 & 3 & 1 \\ 425 & 8 & 1 \\ 85 & -9 & 1 \end{vmatrix} + y\begin{vmatrix} 409 & 20 & 1 \\ 425 & 19 & 1 \\ 85 & 2 & 1 \end{vmatrix} - \begin{vmatrix} 409 & 20 & 3 \\ 425 & 19 & 8 \\ 85 & 2 & -9 \end{vmatrix} = 0$$

$$102(x^2+y^2) - 1428x - 612y - 11322 = 0$$

$$x^2+y^2-14x-6y-111 = 0$$

On comparing above equation with the general form of circle, we get
2g = -14 => g = -7
2f = -6 => f = -3
c = -111

Now,
Centre = (7, 3)

Radius = $\sqrt{g^2+f^2-c}$

$= \sqrt{(-7)^2+(-3)^2-(-111)}$

$= 13$

Question 7: Find the equation of the circle which is circumscribed about the triangle whose vertices are A(- 2, 3), b(5, 2) and C(6, - 1). Find the centre and radius of this circle.

Solution:

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Since circle is circumscribed about the triangle whose vertices are A(- 2, 3), B(5, 2) and C(6, - 1), which implies points A, B and C are lie on circumference of circle and satisfy its equation.

The general equation of a circle: $(x - h)^2 + (y - k)^2 = r^2$ ...(i)

where (h, k) is the centre and r is the radius.

Putting A(-2, 3), B(5, 2) and C(6, -1) in above equation, we get

$h^2 + k^2 + 4h - 6k + 13 = r^2$ ....(ii)

$h^2 + k^2 - 10h - 4k + 29 = r^2$ ....(iii)

$h^2 + k^2 - 12h + 2k + 37 = r^2$ ....(iv)

Subtract (ii) from (iii)

$- 14h + 2k + 16 = 0$

or $- 7h + k + 8 = 0$ ....(v)

Subtract (ii) from (iv)

$- 16h + 8k + 24 = 0$

or $-2h + k + 3 = 0$ ......(vi)

Solving (v) and (vi), we have

$h = 1$

(vi) => $-2 \times 1 + k + 3 = 0$

=> $k= -1$

Therefore,

Centre = (1, - 1)

And,
Equation (ii) => r = 5
[using values of h and k]

Thus, required equation of the circle is

$(x - 1)^2 + (y + 1)^2 = 5^2$

$(x - 1)^2 + (y + 1)^2 = 25$

Question 8: Find the equation of the circle concentric with the circle x2 + y2 + 4x + 6y + 11 = 0 and passing through the point P(5, 4).

Solution:

Since circles are concentric, which means circles have common centre and different radii.

Equation of given circle, $x^2 + y^2 + 4x + 6y + 11 = 0$

The concentric circle will have the equation

$x^2 + y^2 + 4x + 6y + d = 0$ ....(1)

As it passes through P(5, 4),

Put x = 5 and y = 4

$5^2 + 4^2 + 20 + 24 + d= 0$

25 + 16 + 20 + 24 + d = 0

d = - 85

Equation (1) => $x^2 + y^2 + 4x + 6y - 85 = 0$

Which is required equation.

Question 9: Show that the points A(1, 0), B(2, - 7), C(8, 1) and D(9, - 6) all lie on the same circle. Find the equation of this circle, its centre and radius.

Solution:

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The general equation of a circle: $(x - h)^2 + (y - k)^2 = r^2$ ...(i)

where (h, k) is the centre and r is the radius.

Consider points (1, 0), (2, - 7) and (8, 1) lie on the circle.

Putting (1, 0), (2, - 7) and (8, 1) in (i)

Putting (1, 0) => $h^2 + k^2 + 1 - 2h = r^2$ ......(ii)

Putting (2, - 7) => $h^2 + k^2 + 53 - 4h + 14k = r^2$ ....(iii)

Putting (8, 1) => $(8 - h)^2 + (1 - k)^2 = r^2$
$h^2 + k^2 + 65 – 16h – 2k = r^2$ ....(iv)

Subtract (ii) from (iii), we get

h - 7k - 26 = 0 ......(v)

Subtract (ii) from (iv), we get

7h + k - 32 = 0 ......(vi)

Solving (v) and (vi)
h = 5 and k = - 3

Equation (iv) => r = 25
[using h = 5 and k = - 3]

Therefore,
Centre (5, - 3)
Radius = 25

Check for (9, - 6):
To check if (9, - 6) lies on the circle,
$(9 - 5)^2 + (- 6 + 3)^2 = 5^2$

25 = 25
Which is true.
Hence, all the points are lie on circle.