Straight Lines

EXERCISE 20A

Q. 1. Find the distance between the points:

(i) A(2, -3) and B(-6, 3)
(ii) C(-1, -1) and D(8, 11)
(iii) P(-8, -3) and Q(-2, -5)
(iv) R(a + b, a – b) and S(a – b, a + b)

Solution : (i) Formula Used:

Distance between any two points $A(x_1, y_1)$ and $B(x_2, y_2)$=

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Distance between A(2, -3) and B(-6, 3)

$$= \sqrt{(-6 - 2)^2 + (3 - (-3))^2}$$

$$= \sqrt{64 + 36} = \sqrt{100}$$

= 10 units

Therefore, the distance between points A and B is 10 units.

(ii) Distance between C(-1, -1) and D(8, 11) =

$$\sqrt{(8 - (-1))^2 + (11 - (-1))^2}$$

$$= \sqrt{81 + 144} = \sqrt{225}$$

= 15 units

Therefore, the distance between points C and D is 10 units.

(iii) Distance between P(-8, -3) and Q(-2, -5)=

$\sqrt{(-2 - (-8))^2 + (-5 - (-3))^2}$

$= \sqrt{36 + 4} = \sqrt{40}$

$= 2\sqrt{10}$ units

Therefore, the distance between the points P and Q is $2\sqrt{10}$ units.

(iv) Distance between R(a + b, a - b) and S(a - b, a + b) $\sqrt{((a - b) - (a + b))^2 + ((a + b) - (a - b))^2}$

$= \sqrt{4b^2 + 4b^2}$

$= 2b\sqrt{2}$ units

Therefore, the distance between the points R and S is $2b\sqrt{2}$ units.

Q. 2. Find the distance of the point P(6, -6) from the origin.

Solution : Distance of point P(6, -6) from origin (0, 0) =

$\sqrt{(0 + 6)^2 + (0 - 6)^2}$

$= \sqrt{36 + 36}$

$= 6\sqrt{2}$ units

Therefore, the distance of the point P from the origin is $6\sqrt{2}$ units.

Q. 3. If a point P(x, y) is equidistant from the points A(6, -1) and B(2, 3), find the relation between x and y.

Solution : Given: Point P(x, y) is equidistant from points A(6, -1) and B(2, 3)

i.e., distance of P from A = distance of P from B

$\Rightarrow \sqrt{(x - 6)^2 + (y + 1)^2} = \sqrt{(x - 2)^2 + (y - 3)^2}$

Squaring both sides,

$\Rightarrow (x - 6)^2 + (y - 1)^2 = (x - 2)^2 + (y - 3)^2$

$\Rightarrow x^2 - 12x + 36 + y^2 - 2y + 1 = x^2 - 4x + 4 + y^2 - 6y + 9$

$\Rightarrow -12x + 36 + 2y + 1 = -4x + 4 - 6y + 9$

$\Rightarrow -8x + 8y = -24$

$\Rightarrow x - y = 3$

Therefore, $x - y = 3$ is the required relation.

Q. 4. Find a point on the x-axis which is equidistant from the points A(7, 6) and B(-3, 4).

Solution : Let the point on x-axis be P(x, 0).

Given: Point P(x, 0) is equidistant from points A(7, 6) and B(-3, 4)

i.e., distance of P from A = distance of P from B

$\Rightarrow \sqrt{(x - 7)^2 + 36} = \sqrt{(x + 3)^2 + 16}$

Squaring both sides,

$\Rightarrow (x - 7)^2 + 36 = (x + 3)^2 + 16$

$\Rightarrow x^2 - 14x + 49 + 36 = x^2 + 6x + 9 + 16$

$\Rightarrow -20x = -60$

$\Rightarrow x = 3$

Therefore, the point on the x-axis is (3, 0).

Q. 5., Find the distance between the points A(x_1, y_1) and B(x_2, y_2), when

(i) AB is parallel to the x-axis
(ii) AB is parallel to the y-axis.

Solution : **(i) Given:** AB is parallel to the x-axis.

When AB is parallel to the x-axis, the y co-ordinate of A and B will be the same.

i.e., $y_1 = y_2$

Distance

$= \sqrt{(x_2 - x_1)^2 + (y_1 - y_1)^2}$

$\Rightarrow |x_2 - x_1|$

Therefore the distance between A and B when AB is parallel to x-axis is $|x_2 - x_1|$

(ii) Given: AB is parallel to the y-axis.

When AB is parallel to the y-axis, the x co-ordinate of A and B will be the same.

i.e., $x_2 = x_1$

Distance

$= \sqrt{(x_1 - x_1)^2 + (y_2 - y_1)^2}$

$\Rightarrow |y_2 - y_1|$

Therefore the distance between A and B when AB is parallel to y-axis is $|y_2 - y_1|$

Q. 6. A is a point on the x-axis with abscissa -8 and B is a point on the y-axis with ordinate 15. Find the distance AB.

Solution : Given: The two points are A(-8, 0) and B(0,

15) Distance between A and B

$= \sqrt{(0 + 8)^2 + (15 - 0)^2}$

$\Rightarrow \sqrt{64 + 225}$

$\Rightarrow \sqrt{289}$

$\Rightarrow$ 17 units

Therefore, the distance between A and B is 17 units.

Q. 7. Find a point on the y-axis which is equidistant from A(-4, 3) and B(5, 2).

Solution : Let the point on the y-axis be P(0, y)

Given: P is equidistant from A(-4, 3) and B(5, 2).

i.e., PA = PB

$$\Rightarrow \sqrt{(-4-0)^2+(3-y)^2} = \sqrt{(5-0)^2+(2-y)^2}$$

Squaring both sides, we get

$\Rightarrow (-4-0)^2 + (3-y)^2 = (5-0)^2 + (2-y)^2$

$\Rightarrow 16 + 9 - 6y + y^2 = 25 + 4 - 4y + y^2$

$\Rightarrow 25 - 6y = 29 - 4y$

$\Rightarrow 2y = -4$

$\Rightarrow y = -2$

Therefore, the required point on the y-axis is (0, -2).

Q. 8. Using the distance formula, show that the points A(3, -2), B(5, 2) and C(8,8) are collinear.

Solution : Given: The 3 points are A(3, -2), B(5, 2) and C(8, 8).

$$AB = \sqrt{(5-3)^2+(2+2)^2}$$

$$= \sqrt{4+16}$$

$= 2\sqrt{5}$ units .....(1)

$$BC = \sqrt{(8-5)^2+(8-2)^2}$$

$$= \sqrt{9+36}$$

$= 3\sqrt{5}$ units .....(2)

$$AC = \sqrt{(8-3)^2+(8+2)^2}$$

$$= \sqrt{25+100}$$

= 5√5 units …..(3)

From equations 1, 2 and 3, we have

⇒ AC = AB + BC

This is possible only if the points are collinear.

Therefore, the points A, B and C are collinear.

Hence, proved.

Q. 9. Show that the points A(7, 10), B(-2, 5) and C(3, -4) are the vertices of an isosceles right-angled triangle.

Solution : Given: The 3 points are A(7, 10), B(-2, 5) and C(3, -4)

$AB = \sqrt{(-2-7)^2 + (5-10)^2}$

$= \sqrt{81 + 25}$

= √106 units …..(1)

$BC = \sqrt{(3+2)^2 + (-4-5)^2}$

$= \sqrt{25 + 81}$

= √106 units …..(2)

$AC = \sqrt{(3-7)^2 + (-4-10)^2}$

$= \sqrt{16 + 196}$

= √212 units

From equations 1 and 2, we have

⇒ AB = BC

Therefore, Δ ABC is an isosceles triangle …..(3)

Also, AB^2 = 106 units …..(4)

$BC^2 = 106$ units …..(5)

$AC^2 = 212$ units …..(6)

From equations 4, 5 and 6, we have

$AB^2 + BC^2 = AC^2$

So, it satisfies the Pythagoras theorem.

Δ ABC is right angled triangle …..(7)

From 3 and 7, we have

Δ ABC is an isosceles right angled triangle.

Hence, proved.

Q. 10. Show that the points A(1, 1), B(-1, -1) and C(-√3, √3) are the vertices of an equilateral triangle each of whose sides is 22 units.

Solution : Given: The 3 points are A(1, 1), B(-1, -1) and C(-√3, √3).

$AB = \sqrt{(-1 - 1)^2 + (-1 - 1)^2}$

$= \sqrt{4 + 4}$

= 2√2 units …..(1)

$BC = \sqrt{(-\sqrt{3} + 1)^2 + (\sqrt{3} + 1)^2}$

$= \sqrt{3 - 2\sqrt{3} + 1 + 3 + 2\sqrt{3} + 1}$

= 2√2 units …..(2)

$AC = \sqrt{(-\sqrt{3} - 1)^2 + (\sqrt{3} - 1)^2}$

$= \sqrt{3 + 2\sqrt{3} + 1 + 3 - 2\sqrt{3} + 1}$

= $2\sqrt{2}$ units …..(3)

From equations 1, 2 and 3, we have

AB = BC = AC = $2\sqrt{2}$ units.

Therefore, Δ ABC is an equilateral triangle each of whose sides is $2\sqrt{2}$ units.

Hence, proved.

Q. 11. Show that the points A(2, -2), B(8, 4), C(5, 7) and D(-1, 1) are the angular points of a rectangle.

Solution : Given: The 4 points are A(2, -2), B(8, 4), C(5, 7) and D(-1, 1).

Note: For a quadrilateral to be a rectangle, the opposite sides of the quadrilateral must be equal and the diagonals must be equal as well.

AB = $\sqrt{36 + 36}$

= $6\sqrt{2}$ units .....(1)

BC = $\sqrt{9 + 9}$

= $3\sqrt{2}$ units .....(2)

CD = $\sqrt{36 + 36}$

= 6√2 units .....(3)

AD = $\sqrt{9 + 9}$

= 3√2 units .....(4)

From equations 1, 2, 3 and 4, we have

AB = CD and BC = AD .....(5)

Also, AC = $\sqrt{9 + 81}$

= 3√10 units

BD = $\sqrt{81 + 9}$

= 3√10 units

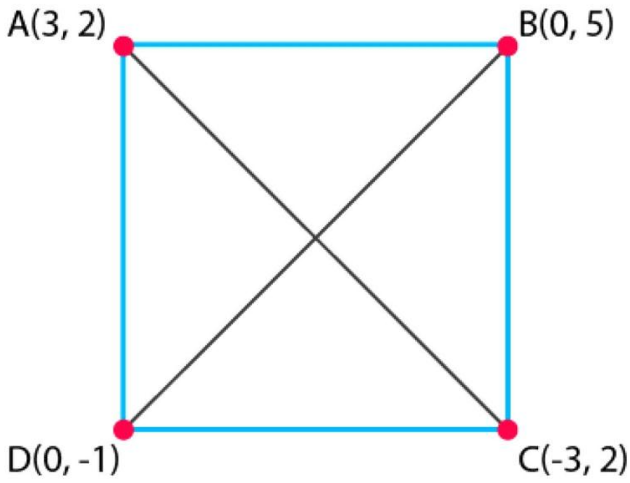
Thus, AC = BD .....(6)

From equations 5 and 6, we can conclude that the opposite sides of quadrilateral ABCD are equal and the diagonals of ABCD are equal as well.

Therefore, point A, B, C and D are the angular points of a rectangle.

Q. 12. Show that A(3, 2), B(0, 5), C(-3, 2) and D(0, -1) are the vertices of a square.

Solution :



Given: The points are A(3, 2), B(0, 5), C(-3, 2) and D(0, -1).

Note: For a quadrilateral to be a square, all the sides of the quadrilateral must be equal in length and the diagonals must be equal in length as well.

$$AB = \sqrt{(0-3)^2 + (5-2)^2} = \sqrt{9+9}$$

$= 3\sqrt{2}$ units

$$BC = \sqrt{(-3-0)^2 + (2-5)^2} = \sqrt{9+9}$$

$= 3\sqrt{2}$ units

$$CD = \sqrt{(0+3)^2 + (-1-2)^2} = \sqrt{9+9}$$

$= 3\sqrt{2}$ units

$$DA = \sqrt{(3-0)^2 + (2+1)^2} = \sqrt{9+9}$$

$= 3\sqrt{2}$ units

Therefore, AB = BC = CD = DA …..(1)

$$AC = \sqrt{(-3 - 3)^2 + (2 - 2)^2}$$

= 6 units

$$BD = \sqrt{(0 - 0)^2 + (-1 - 5)^2}$$

= 6 units

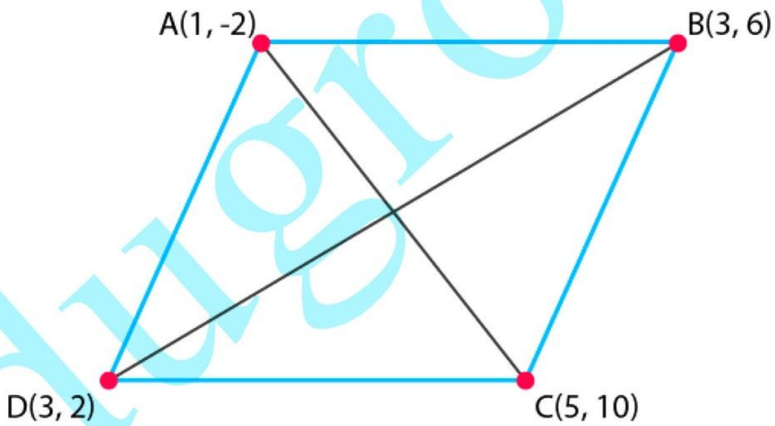
Therefore, AC = BD …..(2)

From 1 and 2, we have all the sides of ABCD are equal and the diagonals are equal in length as well.

Therefore, ABCD is a square.

Hence, the points A, B, C and D are the vertices of a square.

Q. 13. Show that A(1, -2), B(3, 6), C(5, 10) and D(3, 2) are the vertices of a parallelogram.



Solution :

Given: Vertices of the quadrilateral are A(1, -2), B(3, 6), C(5, 10) and D(3, 2).

Note: For a quadrilateral to be a parallelogram opposite sides of the quadrilateral must be equal in length, and the diagonals must not be equal.

$AB = \sqrt{(3 - 1)^2 + (6 + 2)^2} = \sqrt{4 + 64}$

= 2√17 units

$BC = \sqrt{(5 - 3)^2 + (10 - 6)^2} = \sqrt{4 + 16}$

= 2√5 units

$CD = \sqrt{(3 - 5)^2 + (2 - 10)^2} = \sqrt{4 + 64}$

= 2√17 units

$DA = \sqrt{(1 - 3)^2 + (-2 - 2)^2} = \sqrt{4 + 16}$

= 2√5 units

Therefore, AB = CD and BC = DA …..(1)

$AC = \sqrt{(5 - 1)^2 + (10 + 2)^2} = \sqrt{16 + 144}$

= 4√10 units

$BD = \sqrt{(3 - 3)^2 + (2 - 6)^2}$

= 4 units

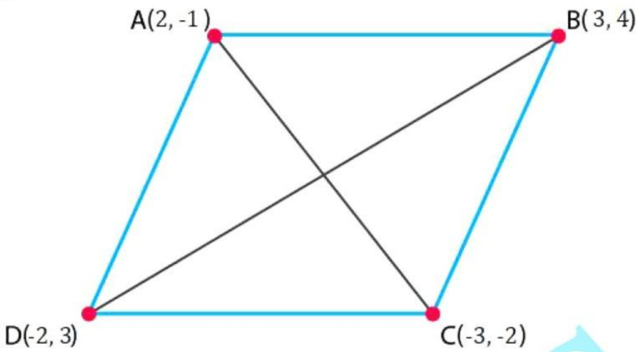
Therefore, AC ≠ BD …..(2)

From 1 and 2, we have

Opposite sides of ABCD are equal, and diagonals are not equal. Hence, points A, B, C and D are the vertices of a parallelogram.

Q. 14. Show that the points A(2, -1), B(3, 4), C(-2, 3) and D(-3, -2) are the vertices of a rhombus.

Solution :



Given: Vertices of the quadrilateral are A(2, -1), B(3, 4), C(-2, 3) and D(-3, -2).
Note: For a quadrilateral to be a rhombus, all the sides must be equal in length and the diagonals must not be equal.

$$AB = \sqrt{(3-2)^2 + (4+1)^2} = \sqrt{1+25}$$

$= \sqrt{26}$ units

$$BC = \sqrt{(-2-3)^2 + (3-4)^2} = \sqrt{25+1}$$

$= \sqrt{26}$ units

$$CD = \sqrt{(-3+2)^2 + (-2-3)^2} = \sqrt{1+25}$$

$= \sqrt{26}$ units

$$DA = \sqrt{(2+3)^2 + (-1+2)^2} = \sqrt{25+1}$$

= √26 units

Therefore, AB = BC = CD = DA .....(1)

AC = $\sqrt{(-2 - 2)^2 + (3 + 1)^2} = \sqrt{16 + 16}$

= 4√2 units

BD = $\sqrt{(-3 - 3)^2 + (-2 - 4)^2} = \sqrt{36 + 36}$

= 6√2 units

Also, AC ≠ BD .....(2)

From 1 and 2, we have all the sides are equal and diagonals are not equal.

Hence, the points A, B, C and D are the vertices of a rhombus.

Q. 15. If the points A (-2, -1), B(1, 0), C(x, 3) and D(1, y) are the vertices of a parallelogram, find the values of x and y.

Solution : Given: Vertices of the parallelogram are A(-2, -1), B(1, 0), C(x, 3) and D(1, y). To find: values of x and y.

Since, ABCD is a parallelogram, we have AB = CD and BC = DA.

AB = $\sqrt{(1 + 2)^2 + (0 + 1)^2} = \sqrt{9 + 1}$

= √10 units

BC = $\sqrt{(x - 1)^2 + 9}$

$CD = \sqrt{(1-x)^2 + (y-3)^2}$

$DA = \sqrt{9 + (1+y)^2}$

Since AB = CD,

$\Rightarrow \sqrt{10} = \sqrt{(1-x)^2 + (y-3)^2}$

Squaring both sides, we get

$\Rightarrow 10 = (1 - x)^2 + (y - 3)^2$

$\Rightarrow 10 = 1 - 2x + x^2 + y^2 - 6y + 9$

$\Rightarrow x^2 + y^2 - 2x - 6y = 0$ …..(1)

Since BC = DA,

$\Rightarrow \sqrt{(x-1)^2 + 9} = \sqrt{9 + (1+y)^2}$

Squaring both sides,

$\Rightarrow (x - 1)^2 + 9 = 9 + (1 + y)^2$

$\Rightarrow x^2 - 2x + 1 = 1 + 2y + y^2$

$\Rightarrow x^2 - y^2 - 2x - 2y = 0$ …..(2)

Equation 1 – Equation 2 gives us,

$\Rightarrow 2y^2 - 4y = 0$

$\Rightarrow y^2 - 2y = 0$

$\Rightarrow y(y - 2) = 0$

$\Rightarrow y = 0$ or $y = 2$

But $y \neq 0$ because then point D(1, 0) is same as B(1, 0)

Therefore, y = 2

When y = 2, from equation 1,

⇒ $x^2 + 4 – 2x – 12 = 0$

⇒ $x^2 – 2x – 8 = 0$

⇒ (x – 4) × (x + 2) = 0

⇒ x = 4 or x = -2

So, the possible set of values for x and y are:

x = 4, y = 2

x = -2, y = 2

But when x = -2, then C(-2, 3). Then ABCD does not form a parallelogram.

Therefore, the only solution is x = 4 and y = 2.

Q. 16. Find the area of ΔABC whose vertices are A(-3, -5), B(5, 2) and C(-9, -3).

Solution : Given: The vertices of the triangle are A(-3, -5), B(5, 2) and C(-9, -3).

Formula: Area of Δ ABC $= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

Here,

$x_1 = -3$, $y_1 = -5$

$x_2 = 5$, $y_2 = 2$

$x_3 = -9$, $y_3 = -3$

Putting the values,

Area of Δ ABC $= \frac{1}{2}[-3(2 + 3) + 5(-3 + 5) - 9(-5 - 2)]$

$= \frac{1}{2}[-15 + 10 + 63]$

= 29 square units.

Therefore, the area of Δ ABC is 29 square units.

Q. 17. Show that the points A(-5, 1), B(5, 5) and C(10, 7) are collinear.

Solution : Given: The points are A(-5, 1), B(5, 5) and C(10, 7).

Note: Three points are collinear if the sum of lengths of any sides is equal to the length of the third side.

$$AB = \sqrt{(5+5)^2 + (5-1)^2} = \sqrt{100+16}$$

= 2√29 units .....(1)

$$BC = \sqrt{(10-5)^2 + (7-5)^2} = \sqrt{25+4}$$

= √29 units .....(2)

$$AC = \sqrt{(10+5)^2 + (7-1)^2} = \sqrt{225+36}$$

= 3√29 units …..(3)

From equations 1, 2 and 3, we have

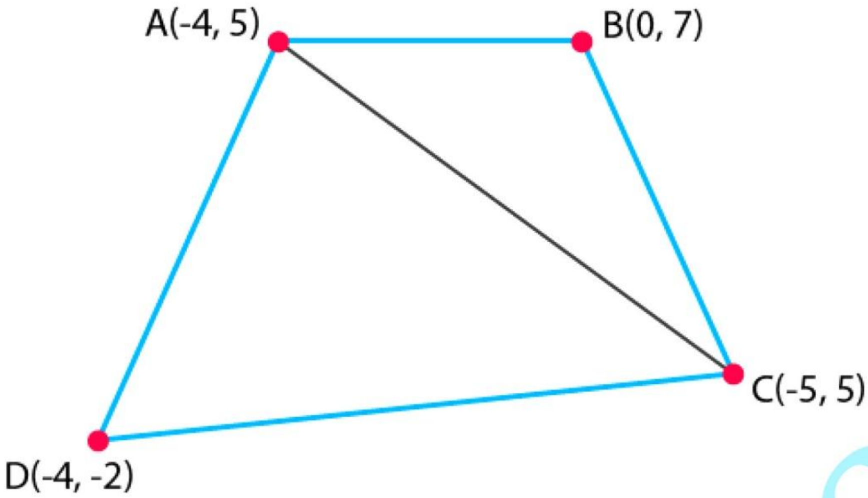
AB + BC = AC

Therefore, the three points are collinear.

Q. 18. Find the value of k for which the points A(-2, 3), B(1, 2) and C(k, 0) are collinear.

Solution : Given: The points are A(-5, 1), B(1, 2) and C(k, 0)

To find: value of k



$AB = \sqrt{(1 + 5)^2 + (2 - 1)^2} = \sqrt{36 + 1}$

$= \sqrt{37}$ units

$BC = \sqrt{(k - 1)^2 + 4}$

$AC = \sqrt{(k + 5)^2 + 1}$

Since the points are collinear, AB + BC = AC

$\Rightarrow \sqrt{37} + \sqrt{(k-1)^2 + 4} = \sqrt{(k+5)^2 + 1}$

Squaring both sides and rearranging,

$\Rightarrow 37 + (k-1)^2 + 4 - (k+5)^2 - 1 = -2\sqrt{37}\sqrt{(k-1)^2 + 4}$

On simplifying,

$\Rightarrow 40 - 2k + 1 - 10k - 25 = -2\sqrt{37}\sqrt{(k-1)^2 + 4}$

$\Rightarrow 16 - 12k = -2\sqrt{37}\sqrt{(k-1)^2 + 4}$

$\Rightarrow 8 - 6k = -\sqrt{37}\sqrt{(k-1)^2 + 4}$

Squaring both sides,

$\Rightarrow 64 - 96k + 36k^2 = 37 \times (k^2 - 2k + 5)$

$\Rightarrow 64 - 96k + 36k^2 = 37k^2 - 74k + 185$

Rearranging,

$\Rightarrow 37k^2 - 74k + 185 = 36k^2 - 96k + 64$
$\Rightarrow k^2 + 22k + 121 = 0$

$\Rightarrow (k + 11)^2 = 0$

$\Rightarrow k = -11$

Therefore, the value of k for which the points A, B and C are collinear is –11.

Q. 19. Find the area of the quadrilateral whose vertices are A(-4, 5), B(0, 7), C(5, -5) and D(-4, -2).

Solution : Given: The vertices of the quadrilateral are A(-4, 5), B(0, 7), C(5, -5) and D(-4, -2).

Formula: Area of a triangle $= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

Area of quadrilateral ABCD = Area of Δ ABC + Area of Δ ADC

$= \frac{1}{2}[-4(7 + 5) + 0 + 5(5 - 7)]$

$= \frac{1}{2}[-48 - 10]$

$= -29$

Taking modulus (∵ area is always positive),

Area of Δ ABC = 29 sq. units …..(1)

Area of Δ ADC $= \frac{1}{2}[-4(-2 + 5) + -4(-5 - 5) + 5(5 + 2)]$

$= \frac{1}{2}[-12 + 40 + 35]$

= 31.5 sq. units …..(2)

From 1 and 2,

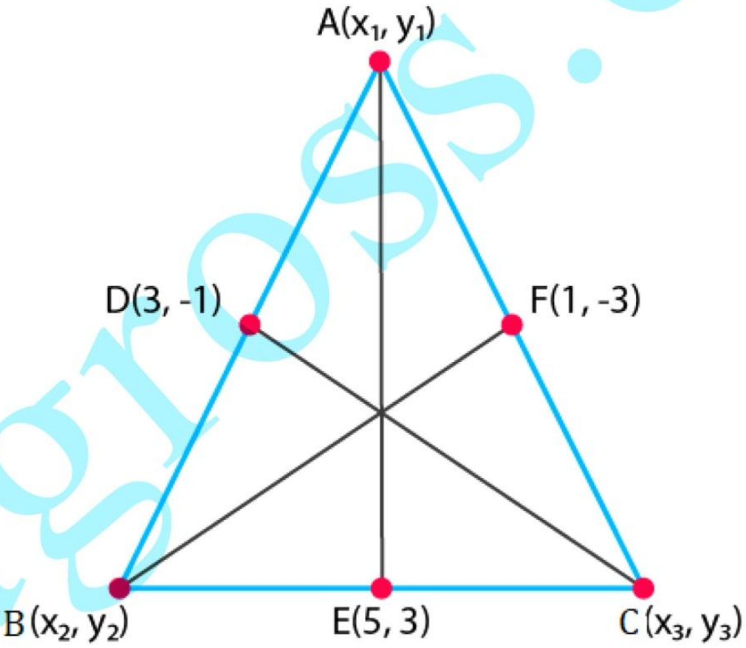
Area of quadrilateral ABCD = 29 + 31.5

= 60.5 square units.

Therefore, the area of quadrilateral ABCD is 60.5 square units.

Q. 20. Find the area of ΔABC, the midpoints of whose sides AB, BC and CA are D(3, -1), E(5, 3) and F(1, -3) respectively.

Solution :



The figure is as shown above.

$x_1 + x_2 = 2 \times 3 = 6$ …..(1)

$x_1 + x_3 = 2 \times 1 = 2$ …..(2)

$x_2 + x_3 = 2 \times 5 = 10$ …..(3)

Equation 1 – Equation 2 gives us

$x_2 - x_3 = 4$ …..(4)

Equation 3 + Equation 4,

$2x_2 = 14 \Rightarrow x_2 = 7$

∴ $x_1 = -1$ and $x_3 = 3$

Similarly,

$y_1 + y_2 = 2 \times -1 = -2$ …..(5)

$y_1 + y_3 = 2 \times -3 = -6$ …..(6)

$y_2 + y_3 = 2 \times 3 = 6$ …..(7)

Equation 5 – Equation 6 gives us

$y_2 - y_3 = 4$ …..(8)

Equation 7 + Equation 8,

$2y_2 = 10 \Rightarrow y_2 = 5$

∴ $y_1 = -7$ and $y_3 = 1$

$$\text{Area of } \Delta\, ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}[-1(5 - 1) + 7(1 + 7) + 3(-7 - 5)]$$

$$= \frac{1}{2}[-4 + 56 - 36]$$

Q. 21. Find the coordinates of the point which divides the join of A(-5, 11) and B(4, -7) in the ratio 2 : 7.

Solution : Let P(x, y) be the point that divides the join of A(-5, 11) and B(4, -7) in the ratio 2 : 7

Formula: If $m_1 : m_2$ is the ratio in which the join of two points is divided by another point (x, y), then

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$$

$$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

Here, x_1 = -5, x_2 = 4, y_1 = 11 and y_2 = -7

Substituting,

$$x = \frac{2 \times 4 + 7 \times -5}{2 + 7}$$

$$x = \frac{8 - 35}{9}$$

$$x = \frac{-27}{9}$$

⇒ x = -3

$$y = \frac{2 \times -7 + 7 \times 11}{2 + 7}$$

$$y = \frac{-14 + 77}{9}$$

$$y = \frac{63}{9}$$

⇒ y = 8

Therefore, the coordinates of the point which divided the join of A(-5, 11) and B(4, -7) in the ratio 2 : 7 is (-3, 8).

Q. 22. Find the ratio in which the x-axis cuts the join of the points A(4, 5) and B(-10, -2). Also, find the point of intersection.

Solution : Let the point which cuts the join of A(4, 5), and B(-10, -2) in the ratio k : 1 be P(x, 0)

Formula: If k : 1 is the ratio in which the join of two points is divided by another point (x, y), then

$$x = \frac{kx_2 + x_1}{k + 1}$$

$$y = \frac{ky_2 + y_1}{k + 1}$$

Taking for the y co-ordinate,

$$0=\frac{k \times -2+5}{k+1}$$

⇒ 2k = 5

$$\Rightarrow k=\frac{5}{2}$$

Therefore,

$$x=\frac{\frac{5}{2}\times -10+4}{\frac{5}{2}+1}$$

$$x=\frac{-50+8}{5+2}$$

$$x=\frac{-42}{7}$$

x = -6

Therefore, the ratio in which x-axis cuts the join of the points A(4, 5) and B(-10, -2) is 5 : 2and the point of intersection is (-6, 0).

Q. 23. In what ratio is the line segment joining the points A(-4, 2) and B(8, 3) divided by the y-axis? Also, find the point of intersection.

Solution : Let the point which cuts the join of A(-4, 2) and B(8, 3) in the ratio k : 1 be P(0, y)

Formula: If k : 1 is the ratio in which the join of two points are divided by another point (x, y), then

$$x=\frac{kx_2+x_1}{k+1}$$

$$y=\frac{ky_2+y_1}{k+1}$$

Taking for the x co-ordinate,

$$0 = \frac{k \times 8 + (-4)}{k + 1}$$

⇒ 8k = 4

$$\Rightarrow k = \frac{1}{2}$$

Therefore,

$$y = \frac{\frac{1}{2} \times 3 + 2}{\frac{1}{2} + 1}$$

$$y = \frac{3 + 4}{1 + 2}$$

$$y = \frac{7}{3}$$

Therefore, the ratio in which the line segment joining the points A(-4, 2) and B(8, 3) divided by the y-axis is 1 : 2 and the point of intersection is $\left(0, \frac{7}{3}\right)$

EXERCISE 20B

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Q. 1. Find the slope of a line whose inclination is

(i) 30°
(ii) 120°
(iii) 135°
(iv) 90°

Solution : We know that the slope of a given line is given by

Slope = tanθ Where θ = angle of inclination

(i) Given that θ = 30°

$$\text{Slope} = \tan(30°) = \frac{1}{\sqrt{3}}$$

(ii) Given that θ = 120°

Slope = tan(120°) = tan(90°+30°) = -cot(30°) = $-\sqrt{3}$

(iii) Given that θ = 135°

Slope = tan(135°) = tan(90°+45°) = -cot(45°)-1

(iv) Given that θ = 90°

Slope = tan(90°) = ∞

Q. 2. Find the inclination of a line whose slope is

(i) $\sqrt{3}$

(ii) $\frac{1}{\sqrt{3}}$

(iii) 1

(iv) -1

(v) $-\sqrt{3}$

Solution : We know that the slope of a given line is given by

Slope = tanθ Where θ angle of inclination

$$\text{(i)}\tan\theta = \sqrt{3}$$
$$\Rightarrow \theta = \tan^{-1}(\sqrt{3})$$
$$\Rightarrow \theta = 60°$$

$$\text{(ii)}\tan\theta = \frac{1}{\sqrt{3}}$$
$$\Rightarrow \theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$
$$\Rightarrow \theta = 30°$$

$$\text{(iii)}\tan\theta = 1$$
$$\Rightarrow \theta = \tan^{-1}(1)$$
$$\Rightarrow \theta = 45°$$

(iv) $\tan\theta = -1$
$\Rightarrow \theta = \tan^{-1}(-1)$
$\Rightarrow \theta = -45° = 315°$

(v) $\tan\theta = -\sqrt{3}$
$\Rightarrow \theta = \tan^{-1}(-\sqrt{3})$
$\Rightarrow \theta = -60° = 300°$

Q. 3. Find the slope of a line which passes through the points

(i) (0, 0) and (4, -2)
(ii) (0, -3) and (2, 1)
(iii) (2, 5) and (-4, -4)
(iv) (-2, 3) and (4, -6)

Solution :

If a line passing through $(x_1, y_1) \& (x_2, y_2)$ then slope of the line is given by $\text{slope} = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$

(i) Given points are (0,0) and (4,-2)

$$\text{slope} = \left(\frac{-2-0}{4-0}\right)$$
$$= \frac{-1}{2}$$

(ii) Given points are (0, -3) and (2, 1)

$$\text{slope} = \left(\frac{1-(-3)}{2-0}\right)$$
$$= 2$$

(iii) Given points are (2, 5) and (-4, -4)

$$\text{slope} = \left(\frac{-4-5}{-4-2}\right)$$
$$= \frac{3}{2}$$
$$= 1.5$$

(iv) Given points are (-2, 3) and (4, -6)

$$\text{slope} = \left(\frac{-6-3}{4+2}\right)$$
$$= \frac{-3}{2}$$
$$= -1.5$$

Q. 4. If the slope of the line joining the points A(x, 2) and B(6, -8) is $\frac{-5}{4}$, find the value of x.

Solution :

If a line passing through $(x_1,y_1)\&(x_2,y_2)$ then slope of the line is given by $\text{slope} = \left(\frac{y_2-y_1}{x_2-x_1}\right)$.

Given points are A(x,2) and B(6,-8), and the slope is

$$\frac{-5}{4}$$

$$\Rightarrow \left(\frac{-8-2}{6-x}\right) = \frac{-5}{4}$$

$$\Rightarrow \left(\frac{-10}{6-x}\right) = \frac{-5}{4} \Rightarrow -40 = -30+5x$$
$\Rightarrow 5x = -10$
$\Rightarrow x = -2$

Q. 5. Show that the line through the points (5, 6) and (2, 3) is parallel to the line through the points (9, -2) and (6, -5)

Solution : We know that for two lines to be parallel, their slope must be the

same. Given points are A(5,6),B(2,3) and C(9,-2),D(6,-5)

$$\text{slope} = \left(\frac{y_2-y_1}{x_2-x_1}\right)$$

$\Rightarrow \left(\frac{3-6}{2-5}\right) = \left(\frac{-5+2}{6-9}\right)$

$\Rightarrow \left(\frac{-3}{-3}\right) = \left(\frac{-3}{-3}\right) \Rightarrow 1 = 1$

Hence proved.

Q. 6. Find the value of x so that the line through (3, x) and (2, 7) is parallel to the line through (-1, 4) and (0, 6).

Solution : We know that for two lines to be parallel, their slope must be the same.
The given points are A(3,x),B(2,7) and C(-1,4),D(0,6)

$\text{slope} = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$

$\Rightarrow \left(\frac{6-4}{0+1}\right) = \left(\frac{7-x}{2-3}\right)$

$\Rightarrow \left(\frac{2}{1}\right) = \left(\frac{7-x}{-1}\right) \Rightarrow -2 = 7 - x$

$\Rightarrow x = 9$

Q. 7. Show that the line through the points (-2, 6) and (4, 8) is perpendicular to the line through the points (3, -3) and (5, -9).
Solution : For two lines to be perpendicular, their product of slope must be equal to -1.

Given points are A(-2,6),B(4,8) and C(3,-3),D(5,-9)

$\text{slope} = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$

Slope of line AB×slope of line CD = -1

$\Rightarrow \left(\frac{8-6}{4+2}\right) \times \left(\frac{-9+3}{5-3}\right) = -1$

$\Rightarrow \left(\frac{2}{6}\right) \times \left(\frac{-6}{2}\right) = -1 \Rightarrow -1 = -1$

$\Rightarrow$ LHS = RHS

Q. 8. If A(2, -5), B(-2, 5), C(x, 3) and D(1, 1) be four points such that AB and CD are perpendicular to each other, find the value of x.

Solution : For two lines to be perpendicular, their product of slope must be equal to -1.

Given points are A(2, -5),B(-2, 5) and C(x, 3),D(1, 1)

$$\text{slope} = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$

⇒Slope of line AB is equal to

$$\left(\frac{5+5}{-2-2}\right)$$
$$= \left(\frac{10}{-4}\right)$$
$$= \left(\frac{-5}{2}\right)$$
$$= -2.5$$

And the slope of line CD is equal to

$$\left(\frac{1-3}{1-x}\right)$$

$$= \left(\frac{-2}{1-x}\right)$$

Their product must be equal to -1

the slope of line AB × Slope of line CD = -1

$$\Rightarrow -2.5 \times \left(\frac{-2}{1-x}\right) = -1 \Rightarrow 5 = x-1$$

⇒x = 6

Q. 9. Without using Pythagora's theorem, show that the points A(1, 2), B(4, 5) and C(6, 3) are the vertices of a right-angled triangle.

Solution : The **ΔABC** is made up of three lines, AB,BC and CA

For a right angle triangle, two lines must be at 90° so they are perpendicular to each other.

Checking for lines AB and BC

For two lines to be perpendicular, their product of slope must be equal to -1.

Given points A(1, 2), B(4, 5) and C(6, 3)

$$\text{slope} = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$

$$\text{Slope of AB} = \left(\frac{5-2}{4-1}\right) = \frac{3}{3} = 1$$

$$\text{Slope of BC} = \left(\frac{3-5}{6-4}\right) = \frac{-2}{2} = -1$$

$$\text{Slope of CA} = \left(\frac{3-2}{6-1}\right) = \frac{1}{5} = 0.2$$

Checking slopes of line AB and BC

1×-1 = -1

So AB is Perpendicular to BC .

So it is a right angle triangle.

Q. 10. Using slopes show that the points A(6, -1), B(5, 0) and C(2, 3) are collinear.

Solution : For three points to be collinear, the slope of all pairs must be equal, that is the slope of AB = slope of BC = slope of CA

Given points are A(6, -1), B(5, 0) and C(2, 3)

$$\text{slope} = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$

Slope of AB = $\left(\frac{0+1}{5-6}\right) = \frac{1}{-1} = -1$

Slope of BC = $\left(\frac{3-0}{2-5}\right) = \frac{3}{-3} = -1$

Slope of CA = $\left(\frac{3+1}{2-6}\right) = \frac{4}{-4} = -1$

Therefore slopes of AB, BC and CA are equal, so Points A,B,C are collinear.

Q. 11. Using slopes, find the value of x for which the points A(5, 1), B(1, -1) and C(x, 4) are collinear.

Solution : For three points to be collinear, the slope of all pairs must be equal, that is the slope of AB = slope of BC = slope of CA

Given points are A(5, 1), B(1, -1) and C(x, 4)

$$\text{slope} = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$

Slope of AB = $\left(\frac{-1-1}{1-5}\right) = \frac{-2}{-4} = \frac{1}{2} = 0.5$

The slope of BC = $\left(\frac{4+1}{x-1}\right) = \left(\frac{5}{x-1}\right)$

Slope of CA = $\left(\frac{4-1}{x-5}\right) = \left(\frac{3}{x-5}\right)$

The slope of all lines must be the same

$$\Rightarrow 0.5 = \left(\frac{5}{x-1}\right)$$

$$\Rightarrow 0.5x - 0.5 = 5$$

$$\Rightarrow 0.5x = 5.5$$

$\Rightarrow x = 11$

Note:- We can use any two points to get the value of “x”.

Q. 12. Using slopes show that the points A(-4, -1), B(-2, -4), C(4, 0) and D(2, 3) taken in order, are the vertices of a rectangle.

Solution : A rectangle has all sides perpendicular to each other, so the product of slope of every adjacent line is equal to -1.

Given point in order are A(-4, -1), B(-2, -4), C(4, 0) and D(2, 3)

$$\text{slope} = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$

$$\text{Slope of AB} = \left(\frac{-4+1}{-2+4}\right) = \frac{-3}{2}$$

$$\text{Slope of BC} = \left(\frac{0+4}{4+2}\right) = \frac{4}{6} = \frac{2}{3}$$

$$\text{The slope of CD} = \left(\frac{3-0}{2-4}\right) = \frac{3}{-2}$$

$$\text{Slope of DA} = \left(\frac{3+1}{2+4}\right) = \frac{4}{6} = \frac{2}{3}$$

⇒slope of AB × slope of BC

$$\Rightarrow \frac{-3}{2} \times \frac{2}{3} = -1$$

Hence AB is perpendicular to BC

Slope of BC × slope of CD

$$\frac{2}{3} \times \frac{3}{-2} = -1$$

Hence BC is perpendicular to CD

Slope of CD × slope of DA

$$\Rightarrow \frac{3}{-2} \times \frac{2}{3} = -1$$

Hence CD is perpendicular to DA

Slope of DA × slope of AB

$$\Rightarrow \frac{2}{3} \times \frac{-3}{2} = -1$$

Hence DA is perpendicular to AB.

All angles are 90°.

So this is a rectangle ABCD.

Q. 13. Using slopes. Prove that the points A(-2, -1), B(1,0), C(4, 3) and D(1, 2) are the vertices of a parallelogram.

Solution : The property of parallelogram states that opposite sides are equal.

We have 4 sides as AB,BC,CD,DA

Given points are A(-2,-1),B(1,0),C(4,3) and D(1,2)

AB and CD are opposite sides, and BC and DA are the other two opposite sides.

So slopes of AB = CD and slopes BC = DA

$$\text{slope} = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$

$$\text{Slope of AB} = \left(\frac{0+1}{1+2}\right) = \frac{1}{3}$$

The slope of BC = $\left(\frac{3-0}{4-1}\right) = \frac{3}{3} = 1$

The slope of CD = $\left(\frac{2-3}{1-4}\right) = \frac{-1}{-3} = \frac{1}{3}$

Slope of DA = $\left(\frac{2+1}{1+2}\right) = \frac{3}{3} = 1$

Therefore the Slope of AB = Slope of CD and

The slope of BC = Slope of DA

Also, the product of slope of two adjacent sides is not equal to -1, therefore it is not a rectangle.

Hence ABCD is a parallelogram.

Q. 14. If the three points A(h, k), B(x_1, y_1) and C(x_2, y_2) lie on a line then show that (h – x_1)(y_2 – y_1) = (k – y_1)(x_2 – x).

Solution : For the lines to be in a line, the slope of the adjacent lines should be the same.

Given points are A(h,k),B(x_1,y_1) and C(x_2,y_2)

So slope of AB = BC = CA

$$\text{slope} = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$

Slope of AB = $\left(\frac{y_1-k}{x_1-h}\right)$

Slope of BC = $\left(\frac{y_2-y_1}{x_2-x_1}\right)$

Slope of CA = $\left(\frac{y_2-k}{x_2-h}\right)$

$$\Rightarrow \left(\frac{y_1-k}{x_1-h}\right) = \left(\frac{y_2-y_1}{x_2-x_1}\right) = \left(\frac{y_2-k}{x_2-h}\right)$$

Now Cross multiplying the first two equality,

$$(y_1 - k)(x_2 - x_1) = (x_1 - h)(y_2 - y_1)$$

$$\Rightarrow (h - x_1)(y_2 - y_1) = (k - y_1)(x_2 - x_1)$$

Hence proved.

Q. 15 If the points A(a, 0), B(0, b) and P(x, y) are collinear, using slopes, prove that

$$\frac{x}{a} + \frac{y}{b} = 1$$

Solution : Given points are A(a,0),B(0,b) and P(x,y)

For three points to be collinear, the slope of all pairs must be equal, that is the slope of AB = slope of BP = slope of PA.

$$\text{slope} = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$

$$\text{Slope of AB} = \left(\frac{b-0}{0-a}\right) = \frac{b}{-a}$$

$$\text{Slope of BP} = \left(\frac{y-b}{x-0}\right) = \frac{y-b}{x}$$

$$\text{Slope of PA} = \left(\frac{y-0}{x-a}\right) = \frac{y}{x-a}$$

Now Slope of AB = BP = PA

$$\frac{b}{-a} = \frac{y-b}{x} = \frac{y}{x-a}$$

Using the first two equality

$$\Rightarrow \frac{b}{-a} = \frac{y-b}{x}$$

$$\Rightarrow bx = -a(y - b)$$

$$\Rightarrow bx = -ay + ab$$

Dividing the equation by “ab”, We get

$$\frac{x}{a} = -\frac{y}{b} + 1$$

$$\Rightarrow \frac{x}{a} + \frac{y}{b} = 1$$

Hence proved.

Q. 16. **A line passes through the points A(4, -6) and B(-2, -5). Show that the line AB makes an obtuse angle with the x-axis.**

Solution : For the line to make an obtuse angle with X-axis, the angle of the line should be greater than 90

For the angle to be greater than 90°, tanθ must be negative

Where tanθ is the slope of the line.

Given points are A(4, -6) and B(-2, -5)

$$\text{slope} = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$

The slope of line AB is $\left(\frac{-5+6}{-2-4}\right) = \frac{1}{-6} = \frac{-1}{6}$

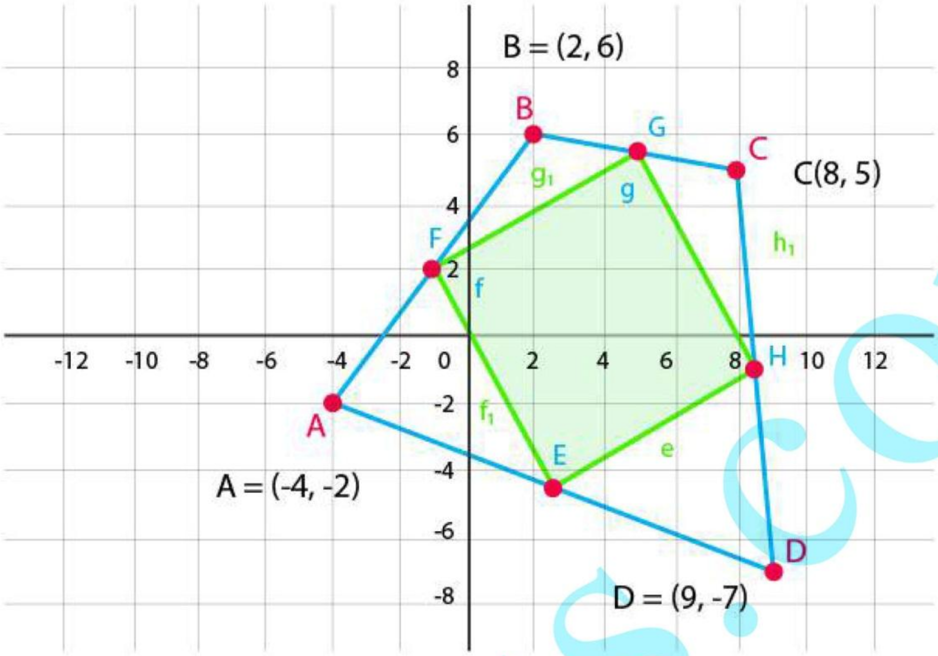
Which is less than 0, hence negative.

$\Rightarrow \tan\theta = \frac{-1}{6} \langle 0$,tanθ is negative in 2nd quadrant whose angle is >90°.

So line AB makes obtuse angle(>90) with the X-axis.

Q. 17. The vertices of a quadrilateral are A(-4, -2), B(2, 6), C(8, 5) and D(9, -7). Using slopes, show that the midpoints of the sides of the quad. ABCD from a parallelogram.

Solution :



The vertices of the given quadrilateral are A(-4,-2) B(2, 6), C(8, 5) and D(9, -7)

The mid point of a line $A(x_1,y_1)$ and $B(x_2,y_2)$ is found out by $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

Now midpoint of AB $=\left(\frac{-4+2}{2},\frac{-2+6}{2}\right) = (-1,2)$

The midpoint of BC $=\left(\frac{2+8}{2},\frac{6+5}{2}\right) = (5,5.5)$

The midpoint of CD $=\left(\frac{8+9}{2},\frac{5-7}{2}\right) = (8.5,-1)$

Midpoint of DA $=\left(\frac{-4+9}{2},\frac{-2-7}{2}\right) = (2.5,-4.5)$

So now we have four points

P(-1,2),Q(5,5.5),R(8.5,-1),S(2.5,-4.5)

$$\text{slope} = \left(\frac{y_2-y_1}{x_2-x_1}\right)$$

$$\text{Slope of PQ} = \left(\frac{5.5-2}{5+1}\right) = \frac{3.5}{6} = \frac{7}{12}$$

$$\text{Slope of QR} = \left(\frac{-1-5.5}{8.5-5}\right) = \frac{-6.5}{3.5} = \frac{-1.3}{0.7} = \frac{-13}{7}$$

$$\text{Slope of RS} = \left(\frac{-4.5+1}{2.5-8.5}\right) = \frac{-3.5}{-6} = \frac{7}{12}$$

$$\text{Slope of SP} = \left(\frac{-4.5-2}{2.5+1}\right) = \frac{-6.5}{3.5} = \frac{-13}{7}$$

Now we can observe that slope of PQ = RS and slope of QR = SP

Which shows that line PQ is parallel to RS and line QR is parallel to SP

Also, the product of two adjacent lines is not equal to -1

Therefore PQRS is a parallelogram.

Q. 18. Find the slope of the line which makes an angle of 30^0 with the positive direction of the y-axis, measured anticlockwise.

Solution : According to the given figure, the angle made by the line from X-axis is 90 +30 = 120°

$$\text{slope} = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$

We also know that slope of a line is equal to tanθ, Where

θ = 120°

tan(120°) = tan(90°+30°) = -cot(30°) = -√3

Therefor the slope of the given line is -√3.

Q.19.

Find the angle between the lines whose slopes are $\sqrt{3}$ and $\frac{1}{\sqrt{3}}$.

Solution : To find out the angle between two lines, the angle is equal to the difference in θ.

The slope of a line = $\tan\theta = \left(\frac{y_2-y_1}{x_2-x_1}\right)$

So slope of the first line = $\sqrt{3} = \tan\theta_1 \Rightarrow \tan\theta_1 = \sqrt{3}$

$\Rightarrow \theta_1 = \tan^{-1}(\sqrt{3})$

$\Rightarrow \theta_1 = 60°$

The slope of the second line = $\frac{1}{\sqrt{3}} = \tan\theta_2 \Rightarrow \theta_2 = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\Rightarrow \theta_2 = 30°$

Now the difference between the two lines is θ_1-θ_2

= 60°-30°

= 30°

Q. 20. Find the angle between the lines whose slopes are

$\left(2-\sqrt{3}\right)$ **and** $\left(2+\sqrt{3}\right)$

Solution : We know that if slope of two lines are m1 and m2 respectively, then the angle between them is given by

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$

Here $m_2 = 2+\sqrt{3}$ and $m_1 = 2-\sqrt{3}$

$$\tan\theta = \frac{(2+\sqrt{3})-(2-\sqrt{3})}{1+(2+\sqrt{3})(2-\sqrt{3})}$$

$$= \frac{2\sqrt{3}}{1 + \left(2^2 - (\sqrt{3})^2\right)}$$

$$= \frac{2\sqrt{3}}{1 + 1} = \sqrt{3}$$

$$\tan\theta = \sqrt{3}$$

$$\Rightarrow \theta = \tan^{-1}(\sqrt{3})$$

$$\Rightarrow \theta = 60°$$

Where θ is the angle between two lines.

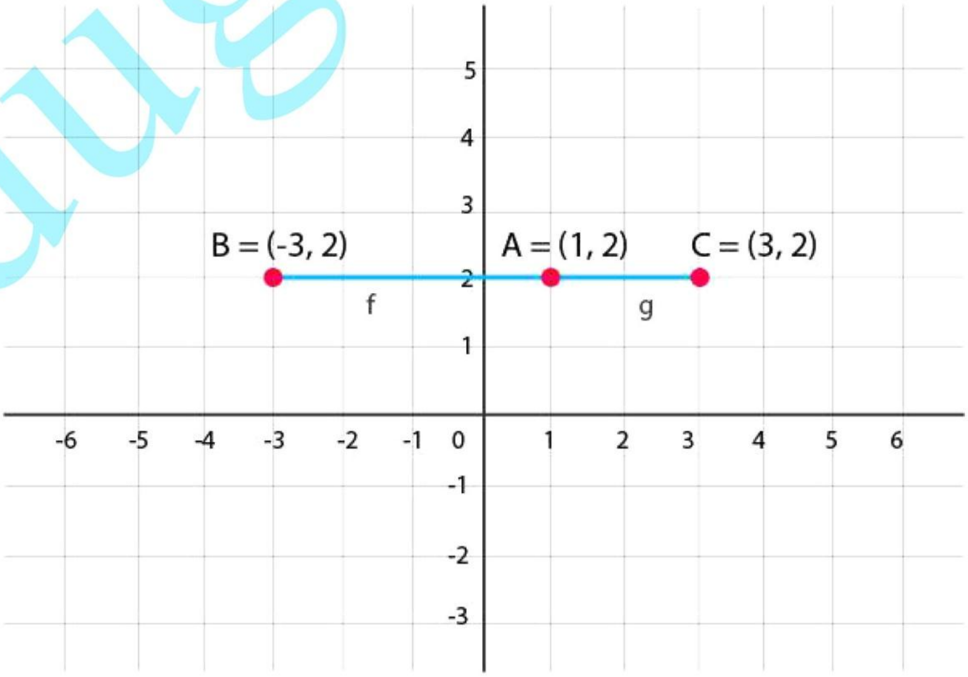
Q. 21. If A(1, 2), B(-3, 2) and C(3, 2) be the vertices of a ΔABC, show that

(i) $\tan A = 2$

(ii) $\tan B = \frac{2}{3}$

(iii) $\tan C = \frac{4}{7}$

Solution : Points A,B,C lie on a same line, therefore the slope of each line is same and hence it does not form a triangle.



Q. 22. If θ is the angle between the lines joining the points (0, 0) and B(2, 3), and the points C(2, -2) and D(3, 5), show that

$$\tan\theta = \frac{11}{23}.$$

Solution : The given points are A(0,0),B(2,3) and C(2,-2),D(3,5).

$$\text{slope} = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$

The slope of line AB is $\left(\frac{3-0}{2-0}\right) = \frac{3}{2} = m_1$

And the slope of line CD is $\left(\frac{5+2}{3-2}\right) = 7 = m_2$

We know that angle between two lines with their slopes as m_1 and m_2 is given by

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$

$$= \frac{7 - \frac{3}{2}}{1 + 7 \times \frac{3}{2}}$$

$$= \frac{\frac{14-3}{2}}{\frac{2+21}{2}}$$

$$= \frac{11}{23}$$

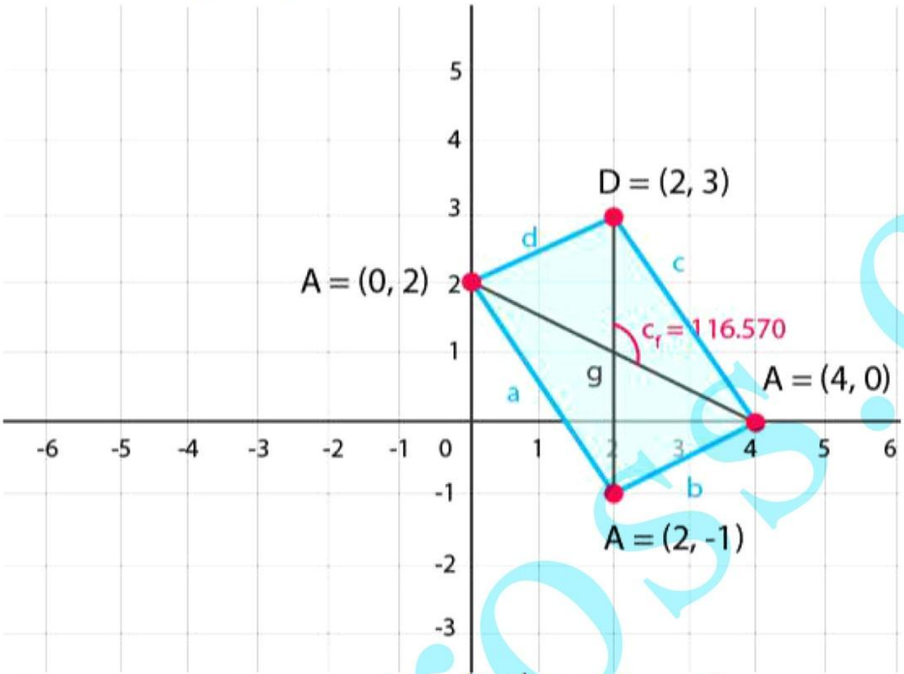
$$\Rightarrow \tan\theta = \frac{11}{23}$$

Hence proved.

Q. 23. If θ is the angle between the diagonals of a parallelogram ABCD whose vertices are A(0, 2), B(2,-1), C(4, 0) and D(2,3). Show that tan θ = 2

Solution : Given points of the parallelogram are A(0, 2), B(2,-1), C(4, 0) and D(2, 3)

$$\text{slope} = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$



The slope of diagonal AC = $\left(\frac{0-2}{4-0}\right) = \frac{-2}{4} = \frac{-1}{2} = m_1$

The slope of diagonal BD = $\left(\frac{3+1}{2-2}\right) = \frac{4}{0} = \infty = m_2$

So diagonal BD is perpendicular to X-axis. Hence it is parallel to Y-axis. Product of slope of two diagonals is equal to -1.

$$m_1 \times m_2 = -1$$

$$\Rightarrow \left(\frac{-1}{2}\right) \times \tan\theta = -1$$

$$\Rightarrow \tan\theta = 2$$

Hence proved.

Q. 24. Show that the points A(0, 6), B(2, 1) and C(7, 3) are three corners of a square ABCD. Find (i) the slope of the diagonal BD and (ii) the coordinates of the fourth vertex D.

Solution : In a square, all sides are perpendicular to the adjacent side, so the product of slope of two adjacent sides is -1.

Let the position of point D(a,b).

Given points of the square are A(0, 6),B(2, 1),C(7, 3) and D(a,b).

$$\text{slope} = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$

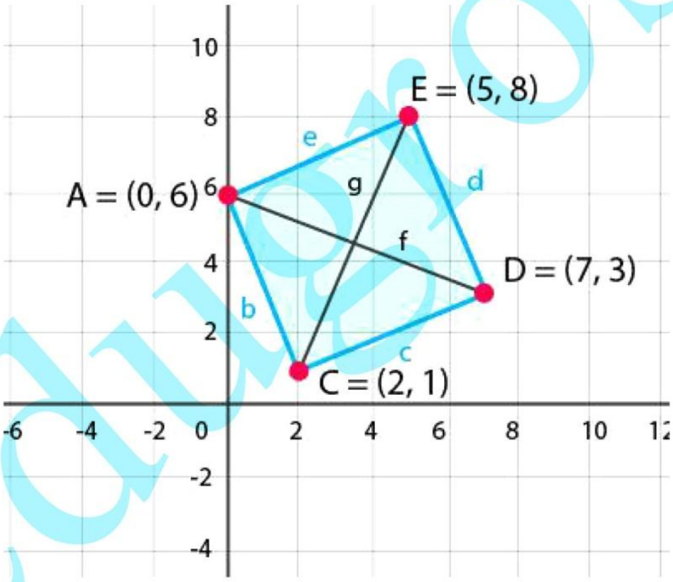
The slope of line AB = $\left(\frac{1-6}{2-0}\right) = \frac{-5}{2} = m_1$

The slope of line BC = $\left(\frac{3-1}{7-2}\right) = \frac{2}{5} = m_2$

The slope of line CD = $\left(\frac{b-3}{a-7}\right) = m_3$

The slope of line DA = $\left(\frac{b-6}{a-0}\right) = \frac{b-6}{a} = m_4$

The slope of diagonal AC = $\left(\frac{3-6}{7-0}\right) = \frac{-3}{7}$



The slope of diagonal BD = m_5

(i) We know that in a square, two diagonals are perpendicular to each other, therefore
The slope of diagonal AC×slope of diagonal BD = -1

$$m_5 \times \frac{-3}{7} = -1$$

$$\Rightarrow m_5 = \frac{7}{3}$$

So the slope of diagonal BD is 7/3.

(ii) We know that midpoint of diagonal AC = midpoint of diagonal BD

$O\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ and comparing x and y coordinates respectively.

$$\left(\frac{7+0}{2}, \frac{3+6}{2}\right) = \left(\frac{a+2}{2}, \frac{b+1}{2}\right)$$

$$\Rightarrow \left(\frac{7}{2}, \frac{9}{2}\right) = \left(\frac{a+2}{2}, \frac{b+1}{2}\right)$$

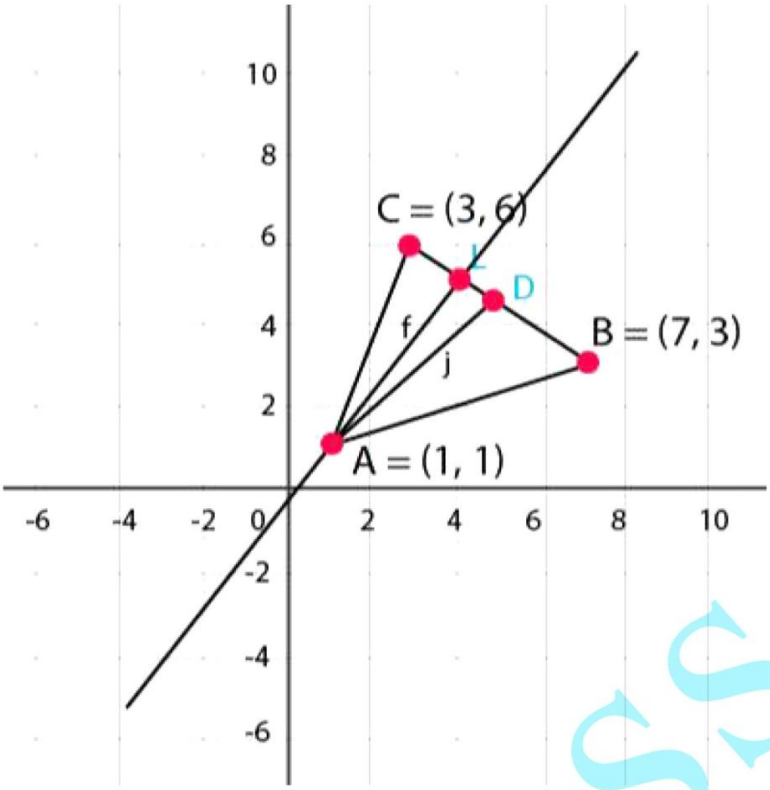
$$\Rightarrow \frac{7}{2} = \frac{a+2}{2} \& \frac{9}{2} = \frac{b+1}{2}$$

$$\Rightarrow a = 5 \& b = 8$$

So coordinate of the point D(5,8).

Q. 25. A(1, 1), B(7, 3) and C(3, 6) are the vertices of a ΔABC. If D is the midpoint of BC and AL ⊥ BC, find the slopes of (i) AD and (ii) AL.

Solution :



Given points are

A(1, 1), B(7, 3) and C(3, 6)

$$\text{slope} = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$

Slope of line BC = $\left(\frac{3-6}{7-3}\right) = \frac{-3}{4}$

(i) As D is the midpoint of BC, coordinate of D are $D\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$$= \left(\frac{7+3}{2}, \frac{3+6}{2}\right) = \left(5, \frac{9}{2}\right)$$

Now the slope of AD = $\left(\frac{\frac{9}{2}-1}{5-1}\right) = \left(\frac{\frac{7}{2}}{4}\right) = \frac{3.5}{4}$

(ii) As AL is perpendicular to BC

The slope of AL×slope of BC = -1

Let slope of AL be m_1

$$\frac{-3}{4} \times m_1 = -1$$

$$\Rightarrow m_1 = \frac{4}{3}$$

EXERCISE 20C

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Q. 1. Find the equation of a line parallel to the x - axis at a distance of

(i) 4 units above it

(ii) 5 units below it

Solution : (i) Equation of line parallel to x - axis is given by y = constant, as the y - coordinate of every point on the line parallel to x - axis is 4,i.e. constant. Now the point lies above x - axis means in positive direction of y - axis,

So, the equation of line is given as y = 4.

(ii) Equation of line parallel to x - axis is given by y = constant, as the y - coordinate of every point on the line parallel to x - axis is - 5 i.e. constant. Now the point lies below x - axis means in negative direction of y - axis,

So, the equation of line is given as y = - 5.

Q. 2. Find the equation of a line parallel to the y - axis at a distance of

(i) 6 units to its right

(ii) 3 units to its left

Solution : (i) Equation of line parallel to y - axis is given by x = constant, as the x - coordinate of every point on the line parallel to y - axis is 6 i.e. constant. Now the point lies to the right of y - axis means in the positive direction of x - axis,

So, required equation of line is x = 6.

(ii) Equation of line parallel to y - axis is given by x = constant, as the x - coordinate of every point on the line parallel to y - axis is - 3. Now point lies to the left of y - axis means in the negative direction of x - axis, So, required equation of line is given as x = - 3.

Q. 3. Find the equation of a line parallel to the x - axis and having intercept - 3 on the y - axis.

Solution: Equation of line parallel to x - axis is given by y = constant, as x - coordinate of every point on the line parallel to y - axis is - 3 i.e. constant.

So, the required equation of line is y = - 3.

Q. 4. Find the equation of a horizontal line passing through the point (4, - 2).

Solution : Equation of line parallel to x - axis (horizontal) is y = constant, as y - coordinate of every point on the line parallel to x - axis is - 2 i.e. constant. Therefore equation of the line parallel to x - axis and passing through (4, - 2) is y = - 2.

Q. 5. Find the equation of a vertical line passing through the point (- 5, 6).

Solution : Equation of line parallel to y - axis (vertical) is given by x = constant, as x - coordinate is constant for every point lying on line i.e. 6.

So, the required equation of line is given as x = 6.

Q. 6. Find the equation of a line which is equidistant from the lines x = - 2 and x = 6.

Solution : For the equation of line equidistant from both lines, we will find point through which line passes and is equidistant from both line.

As any point lying on x = - 2 line is (- 2, 0) and on x = 6 is (6, 0), so mid - point is

$$(x, y) = \left(\frac{-2+6}{2}, \frac{0+0}{2}\right)$$

(x, y) = (2,0)

So, equation of line is x = 2.

Q. 7. Find the equation of a line which is equidistant from the lines y = 8 and y = -2.

Solution : For the equation of line equidistant from both lines, we will find point through which line passes and is equidistant from both line.
As any point lying on y = 8 line is (0, 8) and on y = - 2 is (0, - 2), so mid - point is

$$(x, y) = \left(\frac{0+0}{2}, \frac{8-2}{2}\right)$$

(x, y) = (0, 3)

So, equation of line is y = 3.

Q. 8 A. Find the equation of a line

whose slope is 4 and which passes through the point (5, - 7)

Solution : As slope is given m = 4 and passing through (5, - 7).using slope - intercept form of equation of line, we will find value of intercept first

y = mx + c.............(1)

- 7 = 4(5) + c

- 7 = 20 + c

c = - 7 - 20

c = - 27

Putting the value of c in equation (1), we have

y = 4x + (- 27)

y = 4x - 27

4x - y - 27 = 0

So, the required equation of line is 4x - y - 27 = 0.

Q. 8 B. Find the equation of a line

whose slope is - 3 and which passes through the point (- 2, 3);

Solution : As slope is given m = - 3 and line is passing through point (- 2, 3).Using slope - intercept form of equation of line, we will find intercept first

y = mx + c......................(1)

3 = - 3(- 2) + c

3 = 6 + c

c = 3 - 6

c = - 3

Putting the value of c in equation (1), we have

y = - 3x + (- 3)

y = - 3x - 3

3x + y + 3 = 0

So, the required equation of line is 3x + y + 3 = 0.

Q. 8 C. Find the equation of a line

which makes an angle of $\frac{2\pi}{3}$ with the positive direction of the x – axis and passes through the point (0, 2)

Solution : We have given angle so we have to find slope first given by m = tanθ.

$$m = \tan\theta \Rightarrow \tan\left(\frac{2\pi}{3}\right) = \tan\left(\pi - \frac{\pi}{3}\right)$$

$$m \Rightarrow -\tan\left(\frac{\pi}{3}\right) = -\left(\sqrt{3}\right)$$ (tanx is negative in II quadrant)

$$m = -\sqrt{3}$$

Now the line is passing through the point (0, 2).Using the slope - intercept form of the equation of the line, we will find intercept

y = mx + c.......................(1)

$$2 = -\left(\sqrt{3}\right)(0) + c \Rightarrow c = 2$$

Putting the value of c in equation(1),we have

$$y = -\left(\sqrt{3}\right)x + 2$$

$$-\left(\sqrt{3}\right)x - y + 2 = 0$$

So, required equation of line is $-(\sqrt{3})x - y + 2 = 0.$

Q. 9. Find the equation of a line whose inclination with the x - axis is 30⁰ and which passes through the point (0, 5).

Solution : As angle is given so we have to find slope first given by m =

tanθ m = tan30°

$$m = \frac{1}{\sqrt{3}}$$

Now the line is passing through the point (0, 5).using slope - intercept form of the equation of the line, we will find the intercept

y = mx + c......................(1)

$$5 = \frac{1}{\sqrt{3}}(0) + c \Rightarrow c = 5$$

Putting the value of c in equation (1),we have

$$y = \frac{1}{\sqrt{3}}x + 5$$

$$x - (\sqrt{3})y + 5\sqrt{3} = 0$$

So, required equation of line is $x - (\sqrt{3})y + 5\sqrt{3} = 0.$

Q. 10. Find the equation of a line whose inclination with the x - axis is 150⁰ and which passes through the point (3, - 5).

Solution : As angle is given so we have to find slope first give by m =

tanθ m = tan150°

$m = \tan(180° - 30°) \Rightarrow -\tan 30° = -\frac{1}{\sqrt{3}}$ (tan (180° - θ) is in II quadrant, tanx is negative)

Now the line is passing through the point (3, - 5).Using the slope - intercept form of the equation of the line, we will find the intercept y = mx + c.......................(1)

$$-5 = -\frac{1}{\sqrt{3}}(3) + c \Rightarrow c = -5 + \sqrt{3}$$

Putting the value of c in equation (1),we have

$$y = -\frac{1}{\sqrt{3}}x + \left(-5 + \sqrt{3}\right)$$

$$x + \left(\sqrt{3}\right)y + 5\sqrt{3} - 3 = 0$$

So, required equation of line is $x + \left(\sqrt{3}\right)y + 5\sqrt{3} - 3 = 0.$

Q. 11. Find the equation of a line passing through the origin and making an angle of 120^0 with the positive direction of the x - axis.

Solution : As angle is given so we have to find slope first give by m =

tanθ m = tan120°

$$m = \tan(180° - 60°) \Rightarrow -\tan 60° = -(\sqrt{3})$$

(tan (180° - θ) is in II quadrant, tanx is negative)

Now equation of line passing through origin is given as y = mx

$$y = -(\sqrt{3})x$$

$$(\sqrt{3})x + y = 0$$

So, required equation of line is $(\sqrt{3})x + y = 0$

Q. 12. Find the equation of a line which cuts off intercept 5 on the x - axis and makes an angle of 600 with the positive direction of the x - axis.

Solution : As intercept is given i.e. c = 5 and angle given so first we will find slope of line.

m = tanθ

$$m = \tan 60° \Rightarrow \sqrt{3}$$

Now using slope intercept form of the equation of a line

y = mx + c

$$y = (\sqrt{3})x + 5$$

$$(\sqrt{3})x - y + 5 = 0$$

So, the required equation of line is $(\sqrt{3})x - y + 5 = 0$

Q. 13. Find the equation of the line passing through the point P(4, - 5) and parallel to the line joining the points A(3, 7) and B(- 2, 4).

Solution :. As two points passing through a line parallel to the line are given, we will calculate slope using two points(slope of parallel lines is equal).

$$m = \frac{y_{2-y_1}}{x_2 - x_1} \Rightarrow \frac{4-7}{-2-3} = \frac{-3}{-5}$$

$$m = \frac{3}{5}$$

Now using the slope - intercept form, we will find intercept for a line passing through (4, - 5)

y = mx + c.......................... (1)

$$-5 = \frac{3}{5}(4) + c \Rightarrow -5 - \frac{12}{5} = c$$

$$c = \frac{-25-12}{5} \Rightarrow c = -\frac{37}{5}$$

Putting value in equation (1)

$$y = \frac{3}{5}(x) + \left(\frac{-37}{5}\right) \Rightarrow 3x - 5y - 37 = 0$$

So, the required equation of line is $3x - 5y - 37 = 0$

Q. 14. Find the equation of the line passing through the point P(- 3, 5) and perpendicular to the line passing through the points A(2, 5) and B(- 3, 6)

Solution : As two points passing through line perpendicular to the line are given, we will calculate slope using two points. Let slopes of the two lines be m1 and m2.

$$m_1 = \frac{y_{2-y1}}{x_2 - x_1} \Rightarrow \frac{6-5}{-3-2} = -\frac{1}{5}$$

$$m_1 = -\frac{1}{5}$$

Now the slope of the equation can be found using

$m_1 m_2 = -1$ where m_1, m_2 are slopes of two perpendicular lines

$$\frac{-1}{5}.m_2 = -1 \Rightarrow m_2 = 5$$

Using slope - intercept form we will find intercept for line passing through (- 3, 5)

y = mx + c......................... (1)

5 = 5(- 3) + c

c = 5 + 15

c = 20

Putting value in equation (1)

y = 5x + 20

5x - y + 20 = 0

So, the required equation of line 5x - y + 20 = 0.

Q. 15 A. Find the slope and the equation of the line passing through the points:

(i) (3, - 2) and (- 5, - 7)

Solution : Slope of equation can be calculated using

$$m = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \frac{-7-(-2)}{-5-3} = \frac{-5}{-8}$$

$$m = \frac{5}{8}$$

Now using two point form of the equation of a line

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \text{ where } \frac{y_2 - y_1}{x_2 - x_1} = \text{slope of line}$$

$$y - (-2) = \frac{5}{8}(x-3) \Rightarrow 8(y+2) = 5(x-3)$$

8y + 16 = 5x - 15

5x - 8y - 16 - 15 = 0

5x - 8y - 31 = 0

So, required equation of line is 5x - 8y - 31 = 0.

Q. 15 B. Find the slope and the equation of the line passing through the points:

(- 1, 1) and (2, - 4)

Solution : The slope of the equation can be calculated using

$$m = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \frac{-4-1}{2-(-1)} = \frac{-5}{3}$$

$$m = -\frac{5}{3}$$

Now using two point form of the equation of a line

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \text{ where } \frac{y_2 - y_1}{x_2 - x_1} = \text{slope of line}$$

$$y - 1 = \frac{-5}{3}(x - (-1)) \Rightarrow 3(y-1) = -5(x+1)$$

3y - 3 + 5x + 5 = 0

5x + 3y + 2 = 0

So, required equation of line is 5x - 8y - 31 = 0.

Q. 15 C

Find the slope and the equation of the line passing through the points:

(5, 3) and (- 5, - 3)

Solution : The slope of the equation can be calculated using

$$m = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \frac{-3-3}{-5-5} = \frac{-6}{-10}$$

$$m = \frac{3}{5}$$

Now using two point form of the equation of a line

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \text{ where } \frac{y_2 - y_1}{x_2 - x_1} = \text{slope of line}$$

$$y - 3 = \frac{3}{5}(x - 5) \Rightarrow 5(y - 3) = 3(x - 5)$$

3x - 15 - 5y + 15 = 0

3x - 5y = 0

So, required equation of line is 3x - 5y = 0.

Q. 15 D. Find the slope and the equation of the line passing through the points:

(a, b) and (- a, b)

Solution : The slope of the equation can be calculated using

$$m = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \frac{b - b}{-a - a} = 0$$

m = 0 (Horizontal line)

Now using two point form of the equation of a line

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

y - b = 0(x - a)

y = b

So, required equation of line is y = b.

Q. 16. Find the angle which the line joining the points $(1, \sqrt{3})$ and $(\sqrt{2}, \sqrt{6})$ makes with the x - axis.

Solution : To find angle, we will find slope using two points

$$m = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \frac{(\sqrt{6}) - (\sqrt{3})}{(\sqrt{2}) - 1} = \frac{(\sqrt{3})((\sqrt{2}) - 1)}{((\sqrt{2}) - 1)}$$

$m = \sqrt{3}$

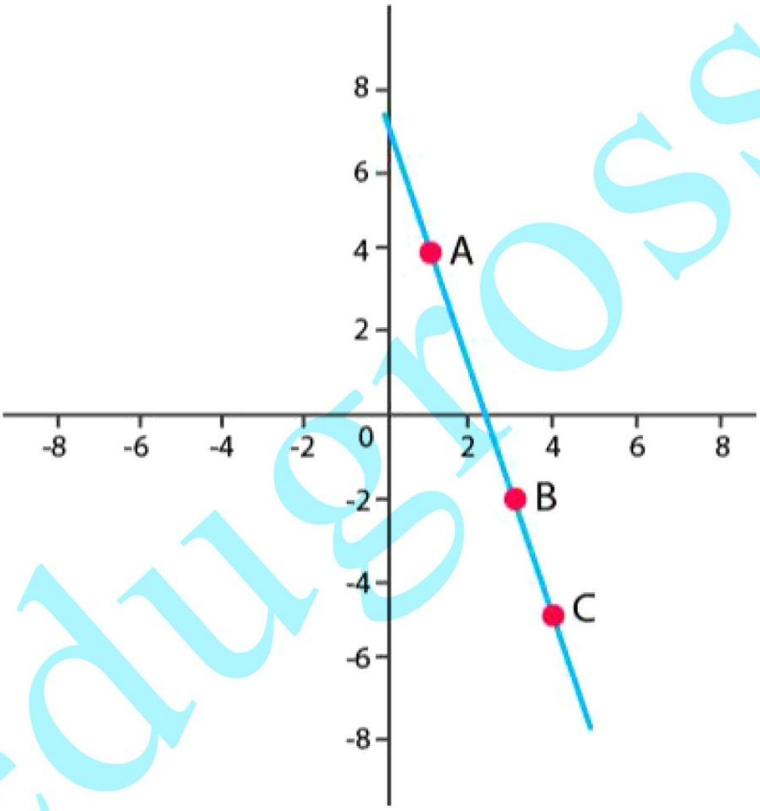
Now as we have m = tanθ

$$\tan\theta = \left(\sqrt{3}\right) \Rightarrow \theta = 60^\circ$$

So, angle line makes with the positive x - axis is 60°.

Q. 17. Prove that the points A(1, 4), B(3, - 2) and C(4, - 5) are collinear. Also, find the equation of the line on which these points lie.

Solution : If two lines having the same slope pass through a common point, then two lines will coincide. Hence, if A, B and C are three points in the XY - plane, then they will lie on a line, i.e., three points are collinear if and only if slope of AB = slope of BC.



Slope of AB = slope of BC

$$\frac{-2-4}{3-1} = \frac{-5-(-2)}{4-3} \Rightarrow \frac{-6}{2} = \frac{-3}{1}$$

- 3 = - 3

Hence verified, i.e. points are collinear. Now using two point form of the equation

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \text{ where } \frac{y_2 - y_1}{x_2 - x_1} = \text{slope of line}$$

y - 4 = - 3(x - 1)

y - 4 + 3x - 3 = 0

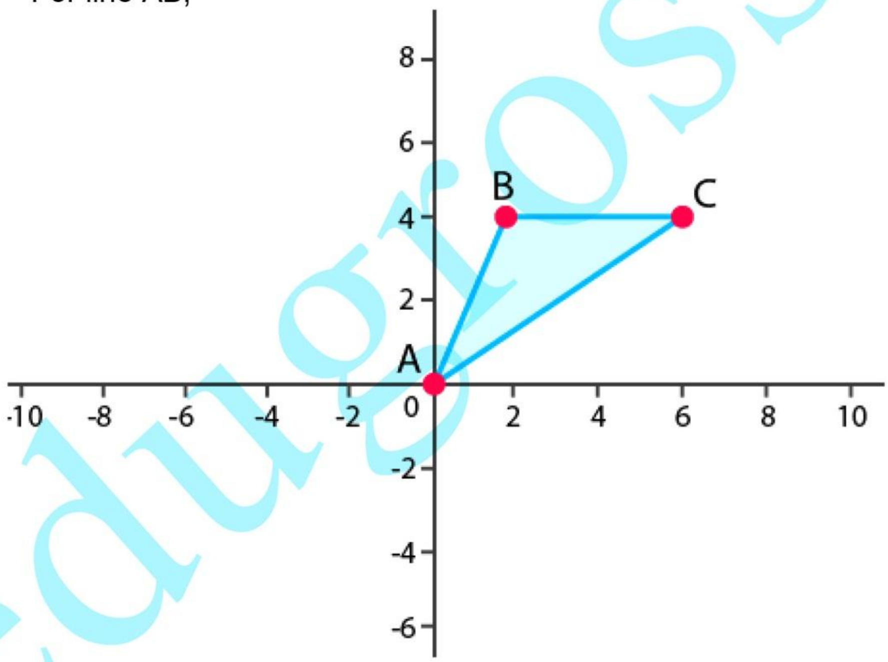
3x + y - 7 = 0

So, required equation of line is 3x + y - 7.

Q. 18. If A(0, 0), b(2, 4) and C(6, 4) are the vertices of a ΔABC, find the equations of its sides.

Solution : Using two point form equation of lines AB, BC and AC can be find. Now A is origin so the lines passing through A (origin) are simply y = mx so we have to find slope of AB and AC.

For line AB,



m = 2

So, the equation of line AB is y = 2x.

For line AC,

$$m = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \frac{4-0}{6-0} = \frac{4}{6}$$

$$m = \frac{2}{3}$$

Now using y = mx

$$y = \frac{2}{3}x \Rightarrow 2x - 3y = 0$$

So, the equation of line AC is 2x - 3y = 0.

Now for line BC, the y coordinate of both is same means horizontal line (parallel to the x - axis) then the equation of line BC is given as

y = 4

So, the required equations of lines for AB: y = 2x

AC: 2x - 3y = 0

BC: y = 4

Q. 19. If A (- 1, 6), B(- 3, - 9) and C(5, - 8) are the vertices of a ΔABC, find the equations of its medians.

Solution : **Construction: -** Draw median from vertices A, B and C on lines BC,AC and AC respectively .Let the mid - points of lines BC,AC and AB be L,M and N respectively.

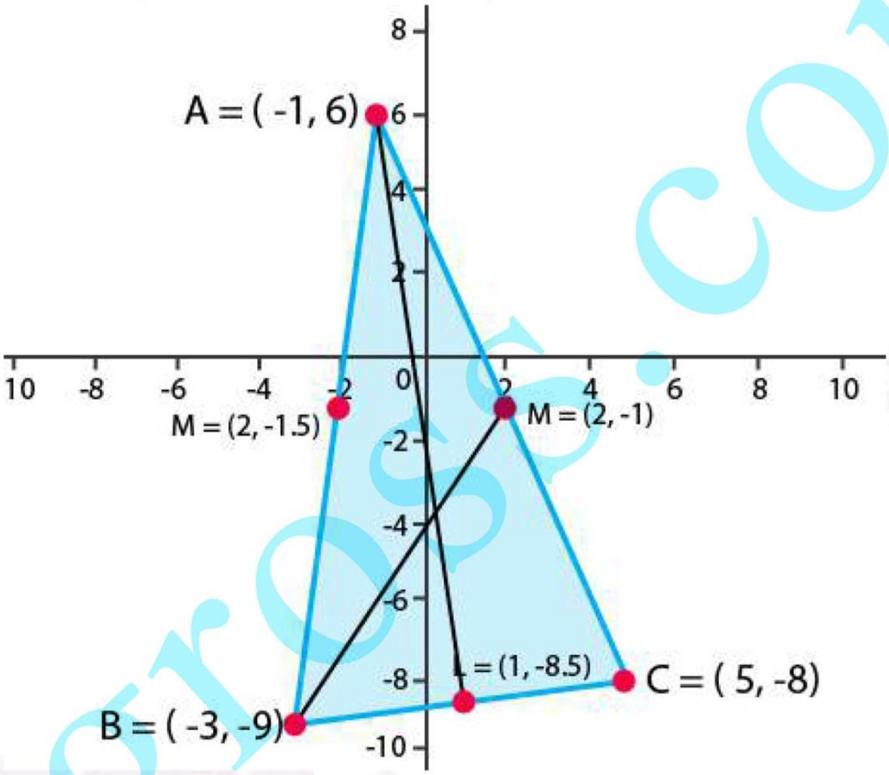
Now find the coordinate of L, M and N using mid - point theorem.

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\text{coordinates of L} = \left(\frac{-3+5}{2}, \frac{-9+(-8)}{2}\right) \Rightarrow \left(1, \frac{-17}{2}\right)$$

$$\text{coordinates of M} = \left(\frac{-1+5}{2}, \frac{6+(-8)}{2}\right) \Rightarrow (2,-1)$$

$$\text{coordinates of N} = \left(\frac{-1+(-3)}{2}, \frac{6+(-9)}{2}\right) \Rightarrow \left(-2, \frac{-3}{2}\right)$$



For median AL,

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 6 = \frac{\frac{-17}{2} - 6}{1-(-1)}(x-(-1))$$

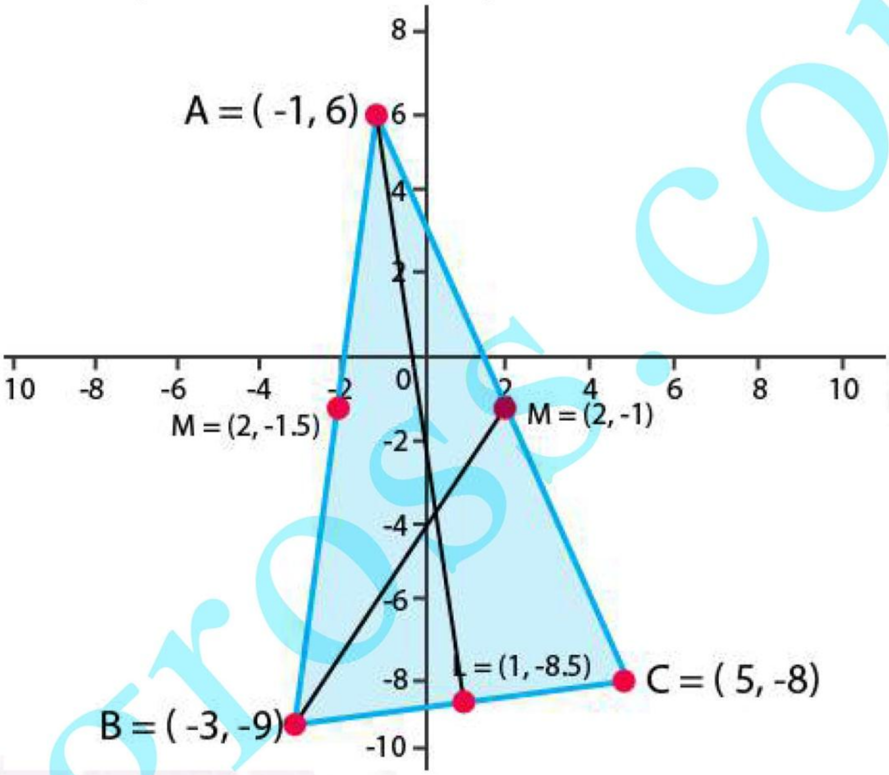
$$y - 6 = \frac{\frac{-17-12}{2}}{2}(x+1) \Rightarrow y - 6 = \frac{-29}{4}(x+1)$$

4(y - 6) = - 29(x + 1)

4y - 24 + 29x + 29 = 0

$$\text{coordinates of M} = \left(\frac{-1+5}{2}, \frac{6+(-8)}{2}\right) \Rightarrow (2,-1)$$

$$\text{coordinates of N} = \left(\frac{-1+(-3)}{2}, \frac{6+(-9)}{2}\right) \Rightarrow \left(-2, \frac{-3}{2}\right)$$



For median AL,

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 6 = \frac{\frac{-17}{2} - 6}{1-(-1)}(x-(-1))$$

$$y - 6 = \frac{\frac{-17-12}{2}}{2}(x+1) \Rightarrow y - 6 = \frac{-29}{4}(x+1)$$

4(y - 6) = - 29(x + 1)

4y - 24 + 29x + 29 = 0

So, the required line of equations for medians are for AL: 29x + y + 5 = 0

For BM: 8x - 5y - 21 = 0

For CN: 13x - 4y - 97 = 0

Q. 20. Find the equation of the perpendicular bisector of the line segment whose end points are A(10, 4) and B(- 4, 9).

Solution : **Perpendicular bisector:** A perpendicular bisector is a line segment which is perpendicular to the given line segment and passes through its mid - point (or we can say bisects the line segment).

Now to find the equation of perpendicular bisector first, we will find mid - point of the given line using mid - point formula (call it midpoint as M),

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\text{coordinates of M} = \left(\frac{10 + (-4)}{2}, \frac{4+9}{2}\right) \Rightarrow \left(3, \frac{13}{2}\right)$$

Now we will calculate the slope of the given line and since lines are perpendicular, so the slope of two is related as m1.m2 = - 1.

$$\text{Slope of AB}: m_1 = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \frac{9-4}{-4-10} = -\frac{5}{14}$$

Now the slope of perpendicular bisector is

$$m_1.m_2 = -1 \Rightarrow -\frac{5}{14}.m_2 = -1$$

$$m_2 = \frac{14}{5}$$

Now equation of perpendicular bisector using two point form,

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - \frac{13}{2} = \frac{14}{5}(x - 3) \Rightarrow 5(2y - 13) = 28(x - 3)$$

10y - 65 = 28x - 84

28x - 10y - 84 + 65 = 0

28x - 10y - 19 = 0

So, required equation of perpendicular bisector 28x - 10y - 19 = 0.

Q. 21. Find the equations of the altitudes of a ΔABC, whose vertices are A(2, - 2), B(1, 1) and C(- 1, 0).

Solution : **Altitude:** A line drawn from the vertex that meets the opposite side at right angles. It determines the height of the triangle.

In triangle ABC, let the altitudes from vertices A, B and C are AL, BM and CN on sides BC,AC and AB respectively.

Now we will find slope of sides and using the relation between the slopes of perpendicular lines i.e. m1.m2 = - 1 we will find the slopes of altitudes.

$$\text{Slope of BC}: m_1 = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \frac{0 - 1}{-1 - 1} = \frac{-1}{-2}$$

$$m_1 = \frac{1}{2}$$

$$\text{Slope of AC}: m_2 = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \frac{0 - (-2)}{-1 - 2} = -\frac{2}{3}$$

$$\text{Slope of AB}: m_3 = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \frac{1 - (-2)}{1 - 2} = -3$$

$$\text{Slope of AL}: m_1.m_1' = -1 \Rightarrow \frac{1}{2}.m_1' = -1$$

$$m_1' = -2$$

$$\text{Slope of BM}: m_2.m_2' = -1 \Rightarrow \frac{-2}{3}.m_2' = -1$$

$$m_2' = \frac{3}{2}$$

$$\text{Slope of CN}: m_3.m_3' = -1 \Rightarrow -3.m_3' = -1$$

$$m_3' = \frac{1}{3}$$

Now equation of altitudes using two point form

For altitude AL,

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

y - (- 2) = - 2(x - 2)

y + 2 + 2x - 4 = 0

2x + y - 2 = 0

For altitude BM,

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

y - 1 = - 1(x - 1)

y - 1 + x - 1 = 0

x + y - 2 = 0

For altitude CN,

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 0 = \frac{1}{3}(x - (-1)$$

3y = x + 1

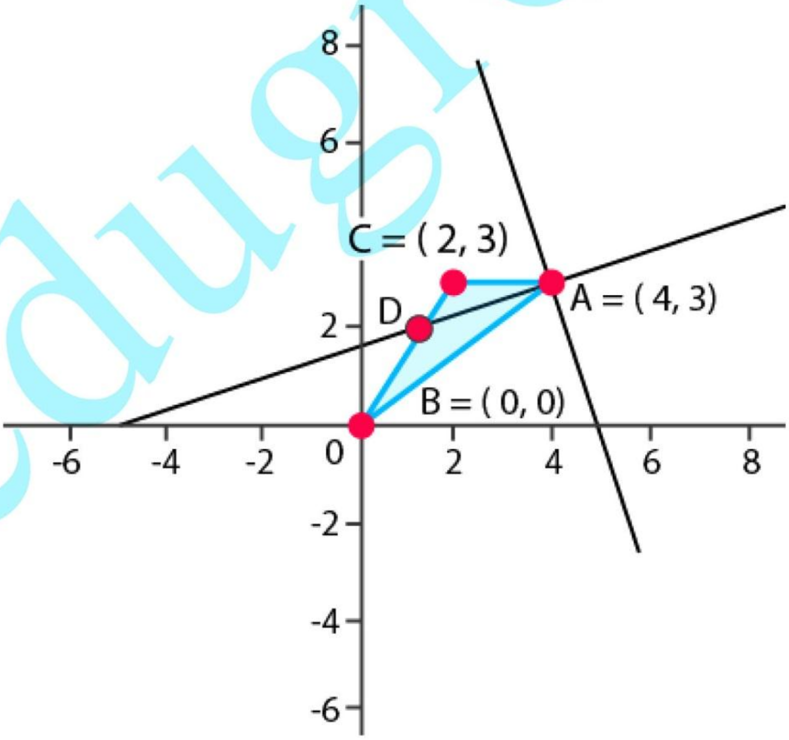
x - 3y + 1 = 0

So, the required equations of altitudes are for AL: 2x + y - 2 = 0

For BM: x + y - 2 = 0
For CN: x - 3y + 1 = 0

Q. 22. If A(4, 3), B(0, 0) and C(2, 3) are the vertices of a ΔABC, find the equation of the bisector of ∠A.

Solution : Construction: Draw a line from vertex A intersecting side BC of the triangle at D (as there is one bisector for exterior angle also but it is the default that we have to find interior angle bisector).



As the angle between the sides AB and angle bisector AD and side AC and angle bisector AD is equal.

$$\angle A = 2\theta \Rightarrow \angle BAD = \angle CAD = \theta$$

Then using the angle between two lines, if the slope of AD be m and slope of AB

$$\text{Slope of AB} = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \frac{3-0}{4-0} = \frac{3}{4}$$

Putting the values in the equation

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1.m_2} \quad \text{.................(1)}$$

$$\Rightarrow \frac{\frac{3}{4} - m}{1 + m.\frac{3}{4}} = \frac{\frac{3-4m}{4}}{\frac{4+3m}{4}}$$

$$\tan\theta = \frac{3-4m}{4+3m} \quad \text{....................(2)}$$

Again for side AC slope

$$\text{Slope of AC} = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \frac{3-3}{2-4} = 0$$

Putting in equation (1)

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1.m_2} \Rightarrow \frac{m-0}{1+0.m} = m \quad \text{....................(3)}$$

From equation (2) and (3),we have

$$m = \frac{3-4m}{4+3m} \Rightarrow 4m + 3m^2 + 4m - 3 = 0$$

$3m^2 + 8m - 3 = 0$

From equation we have two values of $m - 3, \frac{1}{3}$

tanθ = - 3 as tanx is negative in II and IV quadrant means it is obtuse angle either way(exterior here)we require interior angle so will consider the positive value of m.

$$m = \tan\theta = \frac{1}{3}$$

As we obtained the slope of angle bisector which passes through A vertex so using slope intercept form first calculate the value of the intercept

y = mx + c......................(4)

$$3 = \frac{1}{3}(4) + c \Rightarrow c = 3 - \frac{4}{3} \Rightarrow c = \frac{9-4}{3} \Rightarrow \frac{5}{3}$$

Putting the value of c in equation (4),we have

$$y = \frac{1}{3}x + \frac{5}{3} \Rightarrow x - 3y + 5 = 0$$

So, the required equation of angle bisector is x - 3y + 5 = 0.

Q. 23. the midpoints of the sides BC, CA and AB of a ΔABC are D(2, 1), B(- 5, 7) and P(- 5, - 5) respectively. Find the equations of the sides of ΔABC.

Solution : Let us consider the coordinates of vertices of triangle A, B, C be (a, b), (c, d) and (e, f). Now using mid - point formula

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\text{For side BC (midpoint D)}: (2,1) = \frac{c + e}{2}, \frac{d + f}{2}$$

$$\text{For side AC(midpoint E)}: (-5,7) = \frac{a+e}{2}, \frac{b+f}{2}$$

$$\text{For side AB(midpoint F)}: (-5,-5) = \frac{a+c}{2}, \frac{b+d}{2}$$

Now from above equations, we have

c + e = 4, d + f = 2 (i)

a + e = - 10, b + f = 14 (ii)

a + c = - 10, b + d = - 10 (iii)

From subtract (i) from (ii),we get

a - c = - 14, b - d = 12 (iv)

Adding (iii) and (iv)

$$2a = -24 \Rightarrow a = -12, 2b = 2 \Rightarrow b = 1$$

Putting values of a, b in equation (iii)

$$c = -10-(-12) \Rightarrow c = 2,\ d = -10-1 \Rightarrow d = -11$$

Again putting values in (i)

$$e = 4-2 \Rightarrow e = 2,\ f = 2-(-11) \Rightarrow f = 13$$

So coordinates of A (- 12,1), B(2, - 11) and C(2,13).

Using two point form of the equation

Equation of side AB:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 1 = \frac{-11-1}{2-(-12)}(x-(-12)) \Rightarrow y - 1 = \frac{-12}{14}(x+12)$$

14(y - 1) = - 12(x + 12)

14y - 14 + 12x + 144 = 0

12x + 14y + 130 = 0

6x + 7y + 65 = 0

Equation of side BC:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - (-11) = \frac{13-(-11)}{2-2}(x-2) \Rightarrow y + 11 = \frac{24}{0}(x-2)$$

y = - 11(slope is not defined i.e. line is vertical)

Equation of side CA:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 13 = \frac{1-13}{-12-2}(x-2) \Rightarrow y - 13 = \frac{-12}{-14}(x-2)$$

14(y - 13) = 12(x - 2)

12x - 24 - 14y + 182 = 0

12x - 14y + 158 = 0

6x - 7y + 79 = 0

So, the required equations of sides for AB: 6x + 7y + 65 = 0

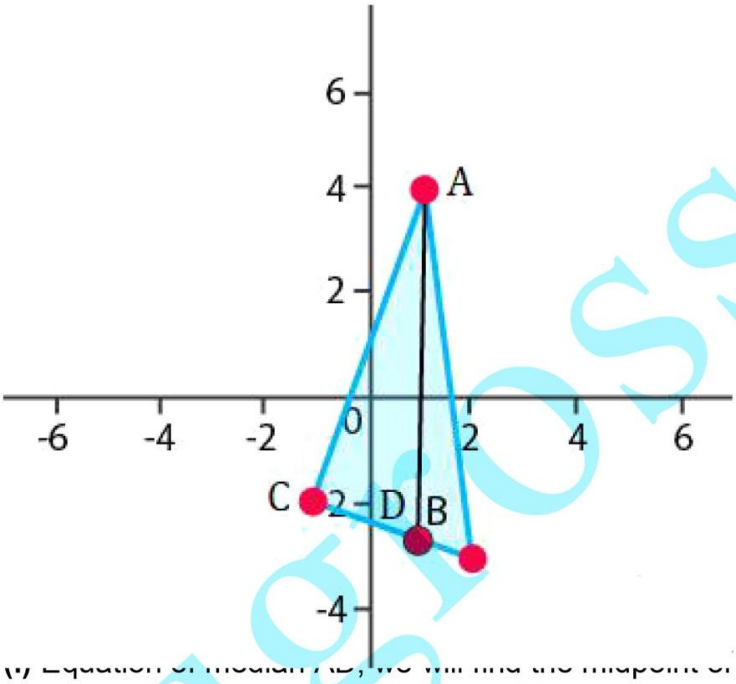
For BC: y = - 11

For CA: 6x - 7y + 79 = 0

Q. 24. If A(1, 4), B(2, - 3) and C(- 1, - 2) are the vertices of a ΔABC, find the equation of

(i) the median through A
(ii) the altitude through A
(iii) the perpendicular bisector of BC

Solution : **<u>Construction:</u>** Draw a line segment from vertex A intersecting BC at the midpoint (D).



(i) Equation of median AD, we will find the midpoint of side BC

$$\text{For side BC(midpoint D)}: (x, y) = \frac{2 + (-1)}{2}, \frac{-3 + (-2)}{2}$$

$$(x, y) = \left(\frac{1}{2}, \frac{-5}{2}\right)$$

Now using two point form of the equation of the line, we have

Equation of side AD:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 4 = \frac{\frac{-5}{2} - 4}{\frac{1}{2} - 1}(x - 1) \Rightarrow y - 4 = \frac{\frac{-5-8}{2}}{\frac{1-2}{2}}(x - 1)$$

$$y - 4 = \frac{-13}{-1}(x - 1) \Rightarrow y - 4 = 13x - 13$$

13x - y - 13 + 4 = 0

13x - y - 9 = 0

So, required equation of altitude is 3x - y - 9 = 0.

(ii) For the equation of altitude, we will need slope as we have a point through which line passes (A).

Now we will find the slope of side BC and using the relation between the slopes of perpendicular lines, i.e. $m_1 . m_2 = -1$ we will find the slopes of altitude.

$$\text{Slope of BC}: \ m_1 = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \frac{-2 + (-3)}{-1 - 2} = \frac{-5}{-3}$$

$$m_1 = \frac{5}{3}$$

$$\text{Slope of AM}: \ m_1 . m_1' = -1 \Rightarrow \frac{5}{3} . m_1' = -1$$

$$m_1' = \frac{-3}{5}$$

Using slope intercept form, we will first calculate intercept,

y = mx + c.............................(1)

$$4 = \frac{-3}{5}(1) + c \Rightarrow c = 4 + \frac{3}{5}$$

$$c = \frac{20+3}{5} \Rightarrow c = \frac{23}{5}$$

Putting in equation (1)

$$y = \frac{-3}{5}x + \frac{23}{5} \Rightarrow 3x + 5y - 23 = 0$$

So, required equation of altitude is 3x + 5y - 23 = 0.

(iii) We have a slope of perpendicular and a mid point from the previous solution

$$m_1' = \frac{-3}{5}, \text{ midpoint of BC(point D) } (x, y) = \left(\frac{1}{2}, \frac{-5}{2}\right)$$

Now for perpendicular bisector, it passes through the midpoint of BC, i.e. we have a slope of the equation and a point through which it passes so we can use the slope - intercept form and calculate intercept,

y = mx + c........................ (i)

$$\frac{-5}{2} = \frac{-3}{5}\left(\frac{1}{2}\right) + c \Rightarrow c = \frac{-5}{2} + \frac{3}{10}$$

$$c = \frac{-25+3}{10} \Rightarrow c = \frac{-22}{10}$$

$$c = \frac{-11}{5}$$

Putting in equation (i) value of c,

$$y = \frac{-3}{5}x + \frac{-11}{5} \Rightarrow 3x + y + 11 = 0$$

So, the required equation of perpendicular bisector is 3x + y + 11 = 0.

EXERCISE 20D PAGE NO: 677

Q. 1. Find the equation of the line whose

(i) slope = 3 and y - intercept = 5
(ii) slope = - 1 and y - intercept = 4
(iii) slope = $-\frac{2}{5}$ and y - intercept = - 3

Solution : (i) Formula to be used: y = mx + c where m is the slope of the line and c is the y - intercept.

Here, m = 3 and c = 5.

Hence, y = (3)x + (5)

i.e. y = 3x + 5

(ii) Formula to be used: y = mx + c where m is the slope of the line and c is the y - intercept.

Here, m = - 1 and c = 4.

Hence, y = (- 1)x + (4)

i.e. x + y = 4

(iii) Formula to be used: y = mx + c where m is the slope of the line and c is the y - intercept.

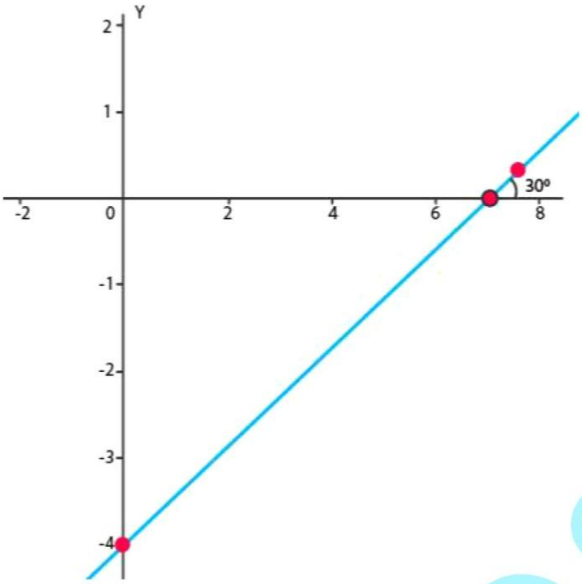
Here, m = $-\frac{2}{5}$ and c = - 3.

Hence, y = $(-\frac{2}{5})$x + (- 3)

Or, 5y = - 2x - 3 i.e. 2x + 5y + 3 = 0

Q. 2. Find the equation of the line which makes an angle of 30^0 with the positive direction of the x - axis and cuts off an intercept of 4 units with the negative direction of the y - axis.

Solution :



Given : The given line makes an angle of 30° with the x - axis. The y - intercept = - 4.

So, the slope of the line is $m = \tan\theta = \tan 30° = {}^{1}/_{\sqrt{3}}$.

Formula to be used: y = mx + c where m is the slope of the line and c is the y - intercept.

The equation of the line is $y = \frac{1}{\sqrt{3}}x - 4$

Or, $\sqrt{3}y = x - 4\sqrt{3}$ i.e. $x - \sqrt{3}y = 4\sqrt{3}$

Q. 3. Find the equation of the line whose inclination is $\frac{5\pi}{6}$ and which makes an intercept of 6 units on the negative direction of the y - axis.

Solution : Given:

$$\theta = \frac{5\pi}{6}$$

$$\therefore \text{slope, } m = \tan\theta = \tan\frac{5\pi}{6} = -\frac{1}{\sqrt{3}}$$

The y - intercept is 6 units.

Formula to be used: y = mx + c where m is the slope of the line and c is the y - intercept.

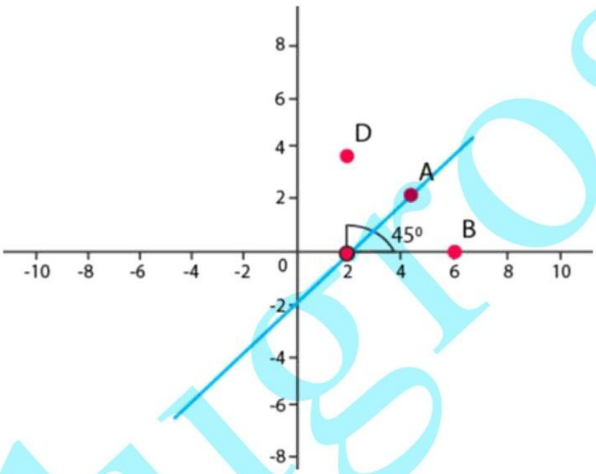
The equation of the line is

$$y = -\frac{1}{\sqrt{3}}x - 6$$

i.e. $\sqrt{3}y + x + 6\sqrt{3} = 0$

Q. 4. Find the equation of the line cutting off an intercept - 2 from the y - axis and equally inclined to the axes.

Solution :



Given: The line is equally inclined to both the axes.

The angle between the coordinate axes = 90°

If the inclination to both the axes is θ then θ+θ = 90°

i.e. θ = 45 θ°

∴ slope of the line, m = tan θ = tan45° = 1

The y - intercept = - 2 units

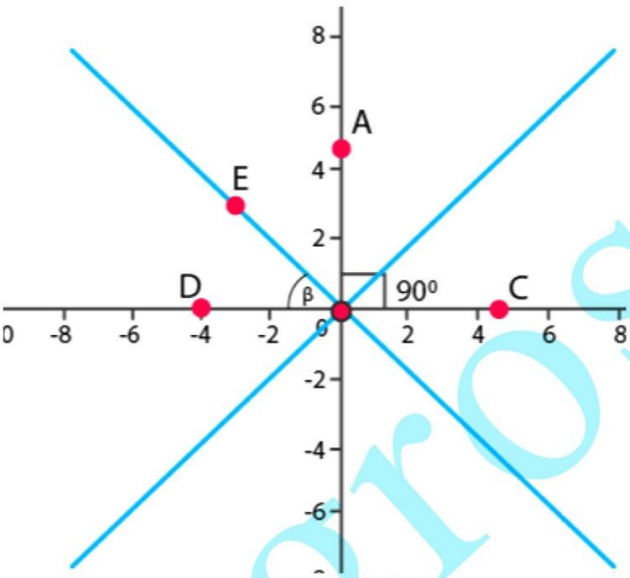
Formula to be used: y = mx + c where m is the slope of the line and c is the y - intercept.

The equation of the line is y = 1.x + (-2) = x - 2

i.e. x - y = 2

Q. 5. Find the equation of the bisectors of the angles between the coordinate axes.

Solution :



Given: The straight lines are x = 0 and y = 0.

Formula to be used: If θ is the angle between two straight lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ then the equation of their angle bisector is

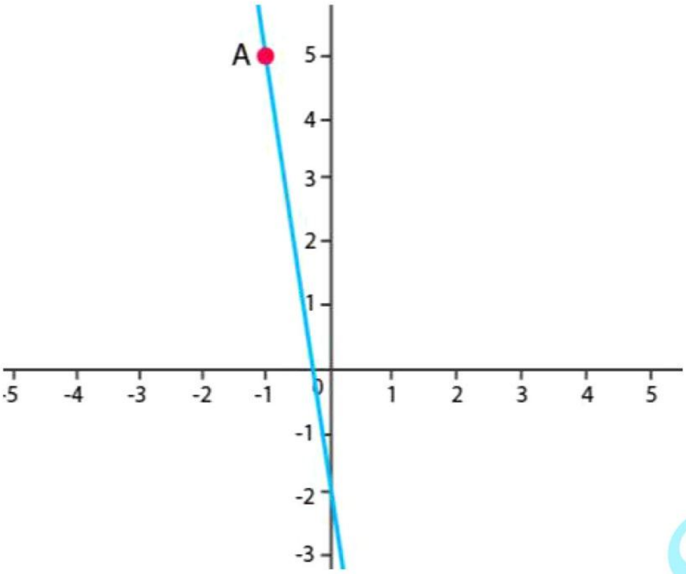
$$\left|\frac{a_1x + b_1y + c_1}{\sqrt{{a_1}^2 + {b_1}^2}}\right| = \left|\frac{a_2x + b_2y + c_2}{\sqrt{{a_2}^2 + {b_2}^2}}\right|$$

∴the equation of the angle bisectors is $\left|\frac{x}{\sqrt{1^2}}\right| = \left|\frac{y}{\sqrt{1^2}}\right|$

i.e. x = ±y

Q. 6. Find the equation of the line through the point (- 1, 5) and making an intercept of - 2 on the y - axis.

Solution :



Given: The y - intercept = - 2.

The line passes through (- 1,5).

Formula to be used: y = mx + c where m is the slope of the line and c is the y - intercept.

The equation of the line is y = mx + (- 2) = mx - 2.

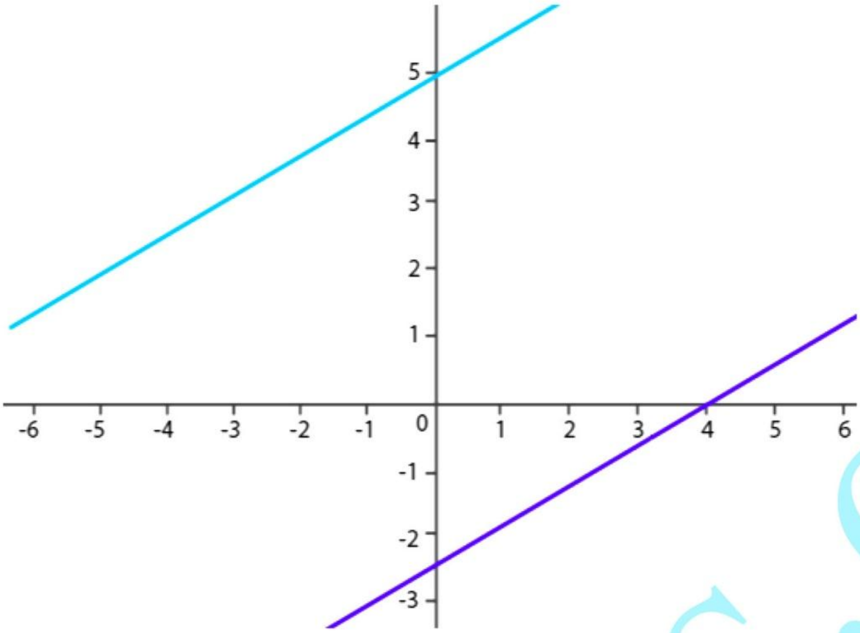
Now, this line passes through (- 1,5).

∴ 5 = m(- 1) - 2 = - m - 2 i.e. m = - (5 + 2) = - 7

∴ y = (- 7)x + (- 2) = - 7x - 2 i.e.7x + y + 2 = 0

Q. 7. Find the equation of the line which is parallel to the line 2x – 3y = 8 and whose y - intercept is 5 units.

Solution :



Given: The given line is 2x - 3y = 8. The line parallel to this line has a y - intercept of 5units.

Formula to be used: If ax + by = c is a straight line then the line parallel to the given line is of the form ax + by = d, where a,b,c,d are arbitrary real constants.

A line parallel to the given line has a slope of $\frac{2}{3}$ and is of the form 2x - 3y = k , where k is any arbitrary real constant.

Now, 2x - 3y = k

or, 3y = 2x - k

$$\text{or, } y = \left(\frac{2}{3}\right)x + \left(-\frac{k}{3}\right)$$

which is of the form y = mx + c , where c is the y - intercept.

$$\therefore c = -\frac{k}{3} = 5$$

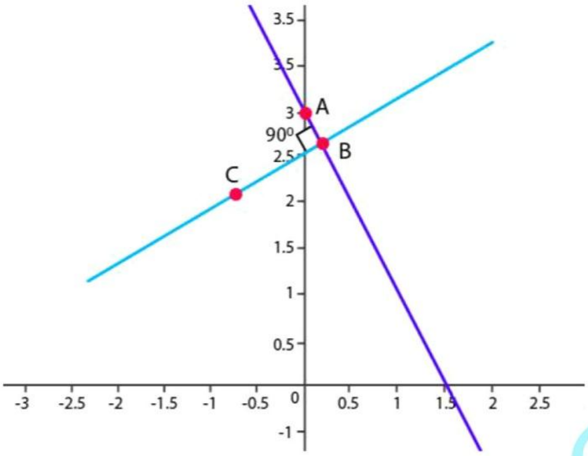
So, k = (- 3)x5 = - 15

Equation of the required line is 2x - 3y = - 15

i.e. 2x - 3y + 15 = 0

Q. 8. Find the equation of the line passing through the point (0, 3) and perpendicular to the line x – 2y + 5 = 0

Solution :



Given: The given line is x - 2y + 5 = 0. The line perpendicular to this given line passes through (0,3)

Formula to be used: The product of slopes of two perpendicular lines = - 1.

The slope of this line is 1/2 .

$\therefore$ the slope of the perpendicular line $= \frac{-1}{1/2} = -2.$

The equation of the line can be written in the form y = (- 2)x + c

(c is the y - intercept)

This line passes through (0,3) so the point will satisfy the equation of the line.

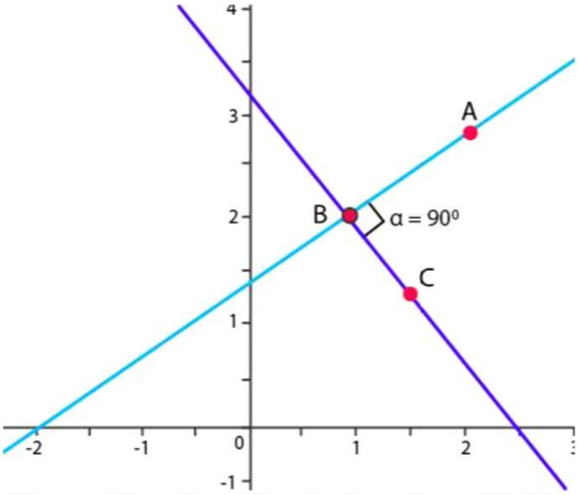
$\therefore$ 3 = (- 2)x0 + c i.e. c = 3

The required equation is y = - 2x + 3

i.e. 2x + y = 3

Q. 9. Find the equation of the line passing through the point (2, 3) and perpendicular to the line 4x + 3y = 10

Solution :



Given: The given line is 4x + 3y = 10. The line perpendicular to this line passes through (2,3).

Formula to be used: The product of slopes of two perpendicular lines = - 1

Slope of this line is $-{}^{4}/_{3}$.

∴the slope of the perpendicular line = $\frac{-1}{-{}^{4}/_{3}} = {}^{3}/_{4}$.

The equation of the line can be written in the form $y = \left(\frac{3}{4}\right)x + c$

(c is the y - intercept)

This line passes through (2,3), so the point will satisfy the equation of the line.

$\therefore 3 = \left(\frac{3}{4}\right)x2 + c \text{ i.e. } c = 3 - \frac{3}{2} = \frac{3}{2}$

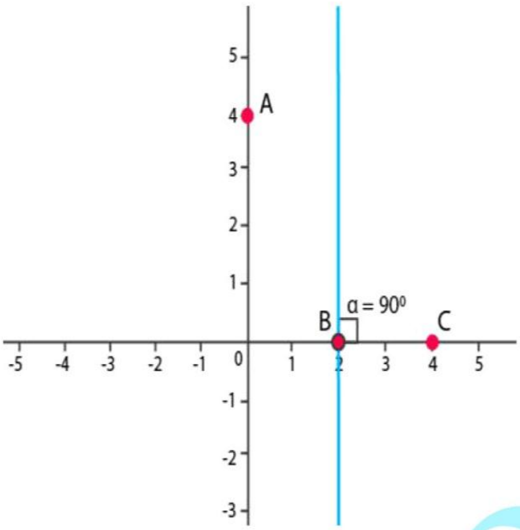
The required equation is

$y = \frac{3}{4}x + \frac{3}{2}$

or, 4y = 3x + 6 i.e. 3x - 4y + 6 = 0.

Q. 10. Find the equation of the line passing through the point (2, 4) and perpendicular to the x - axis.

Solution :



Given: The line is perpendicular to x - axis and passes through (2,4)

The equation of the line perpendicular to the x - axis (y = 0) can be represented as x = c, where c is a real constant.

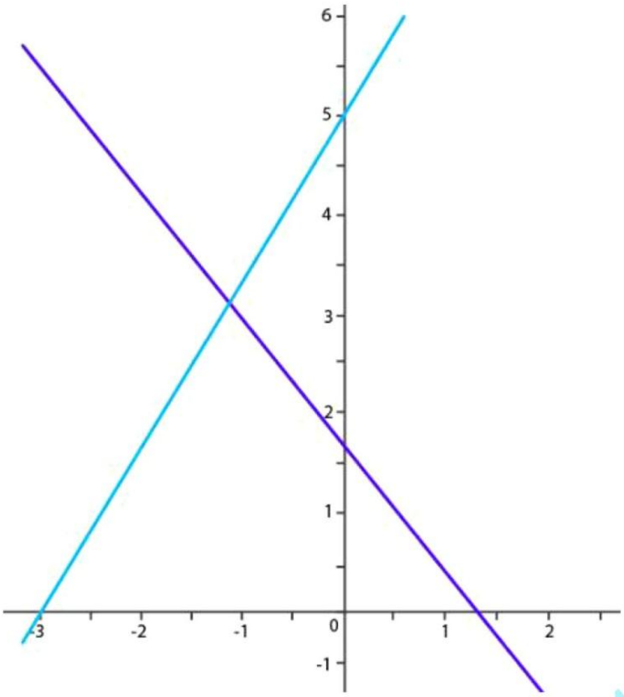
Now, this line passes through (2,4).

∴ c = 2

The required equation is x = 2

Q. 11. Find the equation of the line that has x - intercept - 3 and which is perpendicular to the line 3x + 5y = 4

Solution :



Given: The given line is 3x + 5y = 4. The perpendicular line has an x - intercept of - 3.

Formula to be used: The product of slopes of two perpendicular lines = - 1.

The slope of this line is $-{}^{3}/_{5}$.

∴ the slope of the perpendicular line =

$$\frac{-1}{-{}^{3}/_{5}} = {}^{5}/_{3}.$$

The equation of the line can be written in the form

$$y = \left(\frac{5}{3}\right)x + c$$

(c is the y - intercept)

This line intercepts the x - axis when y = 0.

So, the x - intercept:

$$0 = \left(\frac{5}{3}\right)x + c \text{ i.e. } x = -\frac{3c}{5}$$

Now, it is given that the x - intercept is - 3.

$$\therefore -\frac{3c}{5} = -3 \text{ i.e. } c = 5$$

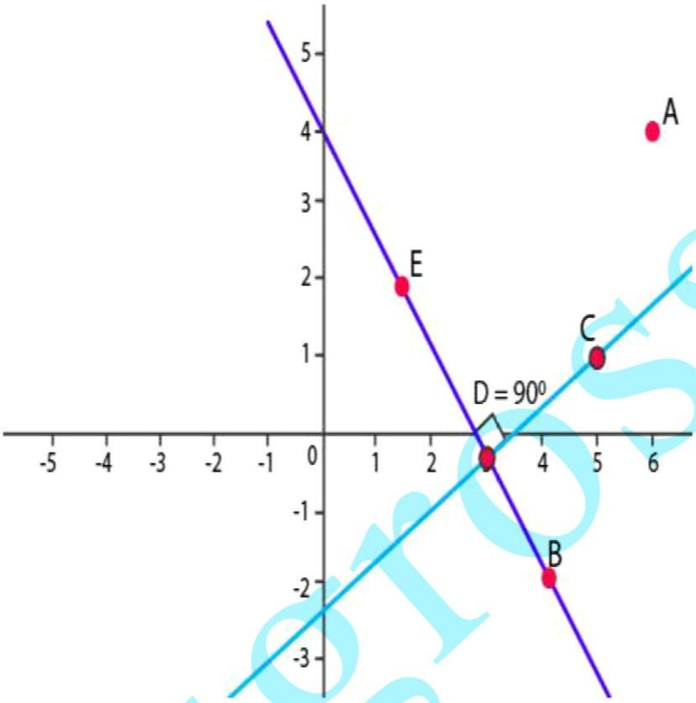
The required equation of the line is

$y = \left(\frac{5}{3}\right)x + 5$

i.e. 5x - 3y + 15 = 0

Q. 12. Find the equation of the line which is perpendicular to the line 3x + 2y = 8 and passes through the midpoint of the line joining the points (6, 4) and (4, - 2).

Solution :



Given: The given line is 3x + 2y = 8. The perpendicular line passes through the midpoint of (6,4) and (4, - 2).

Formulae to be used: The product of slopes of two perpendicular lines = - 1.

If (a,b) and (c,d) be two points, then their midpoint is given by

$\left(\frac{a+c}{2}, \frac{b+d}{2}\right)$

The slope of this line is $-{}^{3}/_{2}$.

∴ the slope of the perpendicular line =

$\frac{-1}{-3/2} = 2/3.$

The equation of the line can be written in the form

$$y = \left(\frac{2}{3}\right)x + c$$

(c is the y - intercept)

This line passes through the midpoint of (6,4) and (4, - 2).

The co - ordinates of the midpoint of the line joining the given points is

$$\left(\frac{6+4}{2}, \frac{4+(-2)}{2}\right) = (5,1)$$

(5,1) satisfies the equation

$$y = \left(\frac{2}{3}\right)x + c$$

$$\therefore 1 = \left(\frac{2}{3}\right)x5 + c \text{ or}, c = 1 - \frac{10}{3} = -\frac{7}{3}$$

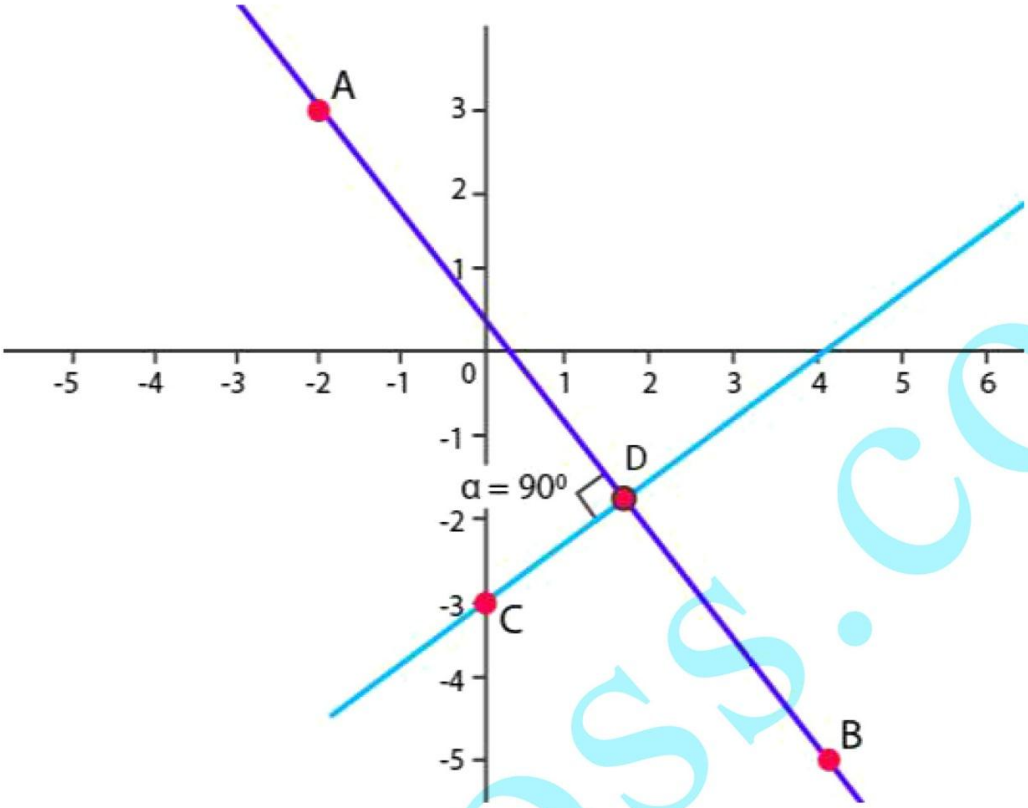
The required equation is

$$y = \left(\frac{2}{3}\right)x + \left(-\frac{7}{3}\right)$$

i.e. 2x - 3y = 7

Q. 13. Find the equation of the line whose y - intercept is - 3 and which is perpendicular to the line joining the points (- 2, 3) and (4, - 5).

Solution :



Given: The line perpendicular to the line passing through (- 2,3) and (4, - 5) has the y - intercept of - 3.

Formula to be used: If (a,b) and (c,d) are two points then the equation of the line passing through them is

$$\frac{y-d}{x-c}=\frac{d-b}{c-a}$$

Product of slopes of two perpendicular lines = - 1

The equation of the line joining points (- 2,3) and (4, - 5) is

$$\frac{y-(-5)}{x-4}=\frac{(-5)-3}{4-(-2)}$$

$$\text{or,}\frac{y+5}{x-4}=\frac{-8}{6}=-\frac{4}{3}$$

or,3y + 15 = - 4x + 16 or, 4x + 3y = 1

Slope of this line is $-{}^{4}/_{3}$.

∴ the slope of the perpendicular line =

$\frac{-1}{-{}^{4}/_{3}} = {}^{3}/_{4}$.

The equation of the line can be written in the form

$y = \left(\frac{3}{4}\right)x + c$

(c is the y - intercept)

But, the y - intercept is - 3.

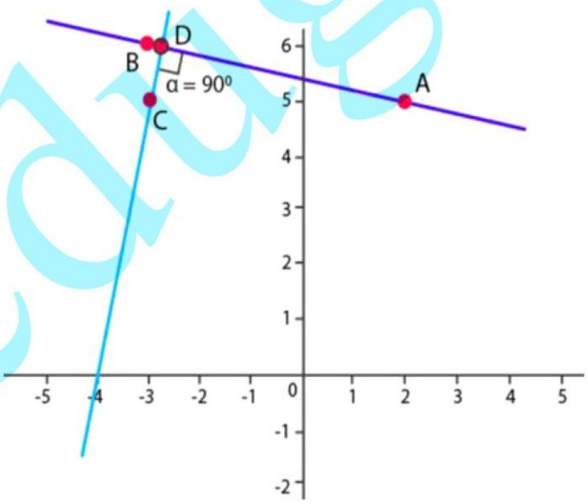
The required line is

$y = \frac{3}{4}x + (-3)$

i.e. 3x - 4y = 12

Q. 14. Find the equation of the line passing through (- 3, 5) and perpendicular to the line through the points (2, 5) and (- 3, 6).

Solution :



Given: The line perpendicular to the line passing through (2,5) and (- 3,6) passes through (- 3,5).

Formula to be used: If (a,b) and (c,d) are two points then the equation of the line passing through them is

$$\frac{y-b}{x-a}=\frac{b-d}{a-c}$$

Product of slopes of two perpendicular lines = - 1

The equation of the line joining points (2,5) and (- 3,6) is

$$\frac{y-5}{x-2}=\frac{5-6}{2-(-3)}$$

$$\text{or,}\frac{y-5}{x-2}=\frac{-1}{5}$$

Or, 5y - 25 = - x + 2

i.e. the given line is x + 5y = 27.

The slope of this line is $-{}^{1}/_{5}$.

∴ the slope of the perpendicular line =

$$\frac{-1}{-{}^{1}/_{5}}=5.$$

The equation of the line can be written in the form y = 5x + c.

(c is the y - intercept)

This line passes through (- 3,5).

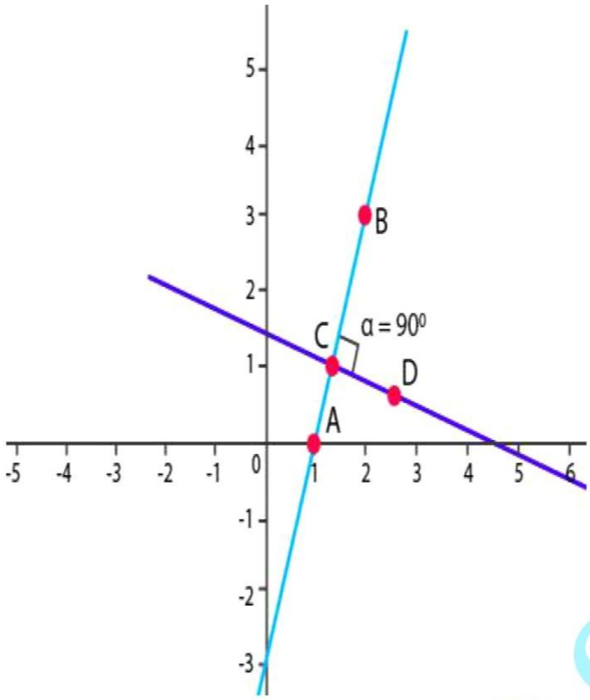
Hence, 5 = 5x(- 3) + c or, c = 20

The required equation of the line will be y = 5x + 20

i.e. 5x - y + 20 = 0

Q. 15. A line perpendicular to the line segment joining the points (1, 0) and (2, 3) divides it in the ratio 1 : 2. Find the equation of the line.

Solution :



Given: A line perpendicular to the line segment joining the points (1, 0) and (2, 3) divides it in the ratio 1 : 2.

Formula to be used: If (a,b) and (c,d) are two points then the equation of the line passing through them is

$$\frac{y-b}{x-a}=\frac{b-d}{a-c}$$

If (a_1,b_1) and (a_2,b_2) be two points , then the co - ordinates of the point dividing their join in the ratio a:b is given by

$$x-\text{co ordinate}=\left(\frac{a_1Xb\ +\ a_2Xa}{a\ +\ b}\right)$$

$$y-\text{co ordinate}=\left(\frac{b_1Xb\ +\ b_2Xa}{a\ +\ b}\right)$$

The equation of the line joining points (1,0) and (2,3) is

$$\frac{y-0}{x-1}=\frac{0-3}{1-2}$$

or, $\frac{y}{x-1} = \frac{-3}{-1} = 3$

or,

$y = 3x - 3$ or, $3x - y = 3$

i.e. the given line is 3x - y = 3.

Accordingly, the required co - ordinates of the point dividing the join of (1,0) and (2,3) in the ratio 1:2 are

$$\left(\left(\frac{1X2 + 2X1}{1 + 2}\right), \left(\frac{0X2 + 3X1}{1 + 2}\right)\right) = \left(\frac{4}{3}, 1\right)$$

The given line is 3x - y = 3.

The slope of this line is 3.

∴the slope of the perpendicular line $= \frac{-1}{3} = -\frac{1}{3}$.

The equation of the line can be written in the form $y = -\frac{1}{3}x + c$

(c is the y - intercept)

This line will pass through $\left(\frac{4}{3}, 1\right)$.

$$\therefore 1 = -\frac{1}{3}X\frac{4}{3} + c \text{ or, } c = 1 + \frac{4}{9} = \frac{13}{9}$$

The required equation is $y = -\frac{1}{3}x + \frac{13}{9}$

i.e. 3x + 9y = 13

EXERCISE 20E

Q. 1. Find the equation of the line which cuts off intercepts -3 and 5 on the x-axis and y-axis respectively.

Solution : To Find: The equation of a line with intercepts -3 and 5 on the x-axis and y-axis respectively.

Given :Let a and b be the intercepts on x-axis and y-axis respectively.

Then, the x-intercept is a = -3

y-intercept is b = 5

Formula used:

we know that intercept form of a line is given by:

$$\frac{x}{a}+\frac{y}{b}=1$$

$$\frac{x}{-3}+\frac{y}{5}=1$$

5x-3y = -15

5x - 3y + 15 = 0

Hence 5x - 3y + 15 = 0 is the required equation of the given line.

Q. 2. Find the equation of the line which cuts off intercepts 4 and -6 on the x-axis and y-axis respectively.

Solution : To Find:The equation of the line with intercepts 4 and -6 on the x-axis and y-axis respectively.

Given : Let a and b be the intercepts on x-axis and y-axis respectively.

Then,x-intercept be a = 4

y-intercept be b = -6

Formula used:

we know that intercept form of a line is given by:

$$\frac{x}{a}+\frac{y}{b}=1$$

$$\frac{x}{4}+\frac{y}{-6}=1$$

-3x + 2y = -12

3x – 2y -12 = 0

Hence 3x – 2y -12 = 0 is the required equation of the given line.

Q. 3. Find the equation of the line and cuts off equal intercepts on the coordinate axes and passes through the point (4,7).

Solution : To Find: The equation of the line with equal intercepts on the coordinate axes and that passes through the point (4,7).

Given : Let a and b be two intercepts of x-axis and y-axis respectively.

Also,given that two intercepts are equal, i.e., a=b

And (4, 7) passes through the point (x, y)

Formula used:

Now since intercept form of a line is given:

$$\frac{x}{a}+\frac{y}{b}=1$$

$$\frac{4}{a}+\frac{7}{b}=1$$

$$\frac{4+7}{a}=1$$

a= 11 = b

Therefore, The required Equation of the line is $\frac{x}{11}+\frac{y}{11}=1$

$\Rightarrow$ x + y = 11

Q. 4. Find the equation of the line which passes through the point (3, -5) and cuts off intercepts on the axes which are equal in magnitude but opposite in sign.

Solution : To Find: The equation of the line passing through (3, -5) and cuts off intercepts on the axes which are equal in magnitude but opposite in sign.

Given : Let a and b be two intercepts of x-axis and y-axis respectively.

According to the question a = -b or b = -a

And (3 , -5) passes through the point (x, y), thus satisfies the equation

Formula used:

Now since intercept form of the line is given by ,

$$\frac{x}{a}+\frac{y}{b}=1$$

$$\frac{3}{a}+\frac{-5}{-a}=1$$

$$\frac{3+5}{a}=1$$

a = 8 and b= -8

Equation of the line is $\frac{x}{8}+\frac{y}{-8}=1$

$\Rightarrow$Hence ,the required equation of the line is $\frac{x}{8}-\frac{y}{8}=1 \Rightarrow x-y=8$

Q. 5. Find the equation of the line passing through the point (2, 2) and cutting off intercepts on the axes, whose sum is 9.

Solution : To Find: The equation of the line passing through the point (2, 2) and cutting off intercepts on the axes, whose sum is 9.

Given : Let a and b be two intercepts of x-axis and y-axis respectively.

sum of the intercepts is 9,i.e,a+b = 9

$\Rightarrow$ a = 9 – b or b = 9 – a

Formula used:

The equation of a line is given by:

$$\frac{x}{a}+\frac{y}{b}=1$$

The given point (2, 2) passing through the line and satisfies the equation of the line.

$$\frac{2}{a}+\frac{2}{9-a}=1$$

$2(9-a)+2a=9a-a^2$

$18-2a+2a=9a-a^2$

$a^2-9a+18=0$

$a^2-6a-3a+18=0$

$a(a-6)-3(a-6)=0$

$(a-3)(a-6)=0$

a = 3, a = 6

when a = 3, b=6 and a=6, b=3

case 1 : when a=3 and b=6

Equation of the line : $\frac{x}{a}+\frac{y}{b}=1$

$$\frac{x}{3}+\frac{y}{6}$$

Hence, 2x + y = 6 is the required equation of the line.

case 2 : when a=6 and b=3

Equation of the line : $\frac{x}{a}+\frac{y}{b}=1$

$$\frac{x}{6}+\frac{y}{3}=1$$

Hence , x + 2y = 6 is the required equation of the line.

Therefore, 2x + y = 6 is the required equation of the line when a=3 and b=6.And , x + 2y = 6 is the required equation of the line when a=6 and b=3.

Q. 6. Find the equation of the line which passes through the point (22, -6) and whose intercept on the x-axis exceeds the intercept on the y-axis by 5.

Solution : To Find:The equation of the line that passes through the point (22, -6) and intercepts on the x-axis exceeds the intercept on the y-axis by 5.

Given : let x-intercept be a and y-intercept be b.

According to the question : a = b + 5

Formula used: And the given point satisfies the equation of the line, so

$$\frac{x}{a}+\frac{y}{b}=1$$

$$\frac{22}{b+5}+\frac{-6}{b}=1$$

$22b - 6b - 30 = b^2 + 5b$

$11b - 30 = b^2$

$b^2 - 11b + 30 = 0$

$b^2 - 6b - 5b + 30 = 0$

$b(b-6) - 5(b-6) = 0$

$(b-5)(b-6) = 0$

The values are b=5 ,b=6

When b=5 then a=10

and b=6 then a=11

case 1 : when b=5 and a=10

Equation of the line : $\frac{x}{a} + \frac{y}{b} = 1$

$$\frac{x}{10} + \frac{y}{5} = 1$$

$$\frac{x + 2y}{10} = 1$$

Hence, x + 2y = 10 is the required equation of the line.

case 2 : when b=6 and a=11

Equation of the line : $\frac{x}{a} + \frac{y}{b} = 1$

$$\frac{x}{11} + \frac{y}{6} = 1$$

$$\frac{6x + 11y}{66} = 1$$

Hence, 6x + 11y = 66 is the required equation of the line.

Therefore, x + 2y = 10 is the required equation of the line when b=5 and a=10 .And 6x + 11y = 66 is the required equation of the line when b=6 and a=11.

Q. 7. Find the equation of the line whose portion intercepted between the axes is bisected at the point (3, -2).

Solution : To Find: The equation of the line whose portion intercepted between the axes is bisected at the point (3, -2).

Formula used:

Let the equation of the line be

$$\frac{x}{a}+\frac{y}{b}=1$$

Since it is given that this equation , whose portion is intercepted between the axes is bisected i.e.; is divided into ratio 1:1 .

Let A(a,0) and B(0,b) be the points foring the coordinate axis.

⟹ a and b are intercepts of x and y-axis respectively.

By using mid-point formula (m:n = 1:1)

$$(x, y) = \left(\frac{y_1+x_1}{2}, \frac{y_2+x_2}{2}\right) = \left(\frac{a}{2}, \frac{b}{2}\right)$$

Since given point (3 , -2) divides coordinate axis in 1:1 ratio

(x , y) = (3 , -2)

$$\Rightarrow \frac{a}{2} = 3 \text{ and } \frac{b}{2} = -2$$

a=6 b=-4

equation of the line : $\frac{x}{a}+\frac{y}{b}=1$

$$\frac{x}{6}+\frac{y}{-4}=1$$

-4x +6y = -24

-2x +3y = -12

Hence the required equation of the line is 2x -3y = 12.

Q. 8. Find the equation of the line whose portion intercepted between the coordinate axes is divided at the point (5, 6) in the ratio 3 : 1.

Solution : To Find: The equation of the line whose portion intercepted between the coordinate axes is divided at the point (5, 6) in the ratio 3 : 1.

Given : The coordinate axes is divided in the ratio 3 : 1

(x_1 , y_1) = A(a,0)

(x_2 , y_2) = B(0, b)

Where a and b are intercepts of the line.

Formula used:

The equation of the line is :

The equation of the line is : $\frac{x}{a}+\frac{y}{b}=1$

And the co-ordinate axis is divided at (5,6) , thus by using Section formula

$$(x ,y) = \left(\frac{my_1+nx_1}{m+n}, \frac{my_2+nx_2}{m+n}\right)$$

$$= \left(\frac{3*0+a}{4}, \frac{3b}{4}\right) = \left(\frac{a}{4}, \frac{3b}{4}\right)$$

(5,6) divides the co-ordinate axis, thus (x,y)= (5,6).

$$\frac{a}{4} = 5 \Rightarrow a = 20, \frac{3b}{4} = 6 \Rightarrow b= 8$$

Equation of the line becomes $\frac{x}{20} + \frac{y}{8} = 1$

8x +20y = 160

2x +5y = 40

Hence the required equation of the line is 2x +5y = 40.

Q. 9. A straight line passes through the point (5, -2) and the portion of the line intercepted between the axes is divided at this point in the ratio 2 : 3. Find the equation of the line.

Solution : Given : The ratio of the line intercepted between the axes is

2 :3 Let (x_1, y_1) = A(a,0)

And (x_2, y_2) = B(0, b)

Where a and b are intercepts of the line.

Formula used:

The equation of the line is : $\frac{x}{a} + \frac{y}{b} = 1$

And the co-ordinate axis is divided at (5,-2) , thus by using Section formula

$$(x, y) = \left(\frac{my_1 + nx_1}{m+n}, \frac{my_2 + nx_2}{m+n}\right)$$

$$= \left(\frac{2*0+3a}{5}, \frac{2b+3*0}{5}\right) = \left(\frac{3a}{5}, \frac{2b}{5}\right)$$

(5,-2) divides the co-ordinate axis, thus (x,y)= (5,-2).

$\frac{3a}{5} = 5 \Rightarrow a = 25/3$, $\frac{2b}{4} = -2 \Rightarrow b = -5$

Equation of the line becomes $\frac{x}{a} + \frac{y}{b} = 1$

$$\frac{x}{25/3} + \frac{y}{-5} = 1$$

$$\frac{3x}{25} - \frac{y}{5} = 1$$

$$\frac{3x - 5y}{25} = 1$$

Hence ,3x – 5y = 25 is the required equation of the line.

Q. 10. If the straight line $\frac{x}{a} + \frac{y}{b} = 1$ passes through the points (8, -9) and (12, -15), find the values of a and b.

Solution : To Find: The values of a and b when the line $\frac{x}{a} + \frac{y}{b} = 1$ passes through the points (8, -9) and (12, -15).

Given : the equation of the line : $\frac{x}{a} + \frac{y}{b} = 1$ equation 1

Also (8, -9) passes through equation 1

$$\frac{8}{a} - \frac{9}{b} = 1$$

8b - 9a = ab equation 2

And (12, -15) passes through equation 1

$$\frac{12}{a} - \frac{15}{b} = 1$$

12b - 15a = ab equation 3

Solving equation 2 and 3

a= 2.

Put a=2 in equation 2

8b- 9a = ab

8b -18 = 2b

6b =18 $\Rightarrow$ b=3

Hence the values of a and b are 2 and 3 respectively.

EXERCISE 20F

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Q. 1 A. Find the equation of the line for which

p = 3 and ∝ = 450

Solution : To Find:The equation of the line.

Given: p = 3 and ∝ = 450

Here p is the perpendicular that makes an angle ∝ with positive direction of x-axis , hence the equation of the straight line is given by:

Formula used:

X cos ∝ + y sin ∝ =p

X cos 450 + y sin 450 =3

i.e; cos 450 = cos (360 +90) = cos 90 [∵ cos(360+ x) = cos x]

similarly, sin 450 = sin (360 +90) = sin 90 [∵ sin(360+ x) = sin x]

hence, x cos 90 + y sin 90 =3

x × (0)+ y ×1=3

Hence the required equation of the line is y=3.

Q. 1 B. Find the equation of the line for which

p = 5 and ∝ = 1350

Solution : Given: p = 5 and ∝ = 1350

Here p is the perpendicular that makes an angle ∝ with positive direction of x-axis , hence the equation of the straight line is given by:

Formula used:

x cos ∝ + y sin ∝ =p

x cos 1350 + y sin 1350 =5

i.e; cos 1350 = cos ((4 × 360) -90) = cos((4×2π)- 90)= cos 90

similarly, sin 1350 = sin ((4 × 360) - 90) = sin((4×2π) - 90)= -sin 90

hence, x cos 90 + y (–sin 90) =5

x × (0)- y ×1=5

Hence The required equation of the line is y=-5.

Q. 1 C. Find the equation of the line for which

p = 8 and ∝ = 1500

Solution : Given: p = 8 and ∝ = 1500

Here p is the perpendicular that makes an angle ∝ with positive direction of x-axis , hence the equation of the straight line is given by:

Formula used:

x cos ∝ + y sin ∝ =p

x cos 1500 + y sin 1500 =8

i.e; cos 1500 = cos ((4×360) +60) = cos((4×2π)+60)= cos 60

similarly, sin 1500 = sin ((4×360) +60) = sin((4×2π)+60)= sin 60

x × (1/2)+ y × (√3/2)=8

Hence The Required equation of the line is x + √3 y = 16.

Q. 1 D. Find the equation of the line for which

p = 3 and ∝ = 2250

Solution : Given: p = 3 and ∝ = 2250

Here p is the perpendicular that makes an angle ∝ with positive direction of x-axis , hence the equation of the straight line is given by:

Formula used:

x cos ∝ + y sin ∝ =p

x cos 2250 + y sin 2250 =3

i.e; cos 2250 = cos ((6×360) +90) = cos((6×2π)+ 90)= cos 90

similarly, sin 2250 = sin ((6×60) + 90) = sin((6×2π) +90)= sin 90

hence, x cos 90 + y sin 90 =3

x × (0)+y ×1=3

Hence The required equation of the line is y=3.

Q. 1 E. Find the equation of the line for which

p = 2 and ∝ = 3000

Solution : Given: p = 2 and ∝ = 3000

Here p is the perpendicular that makes an angle ∝ with positive direction of x-axis , hence the equation of the straight line is given by:

Formula used:

X cos ∝ + y sin ∝ =p

X cos 3000 + y sin 3000 =2

i.e; cos 3000 = cos ((8×360) +120) = cos((8×2π)+120)= cos 120 =cos(180-60)=cos60

similarly, sin 3000 = sin ((8×360) +120) = sin((8×2π)+120)= sin 120

= sin(180-60) = -sin 60

hence, x cos 60 + y (–sin 60) =2

x × (1/2)- y × (√3/2)=2

Hence The required equation of the line is x - √3y = 4

Q. 1 F. Find the equation of the line for which

p = 4 and ∝ = 1800

Solution : Given: p = 4 and ∝ = 1800

Here p is the perpendicular that makes an angle ∝ with positive direction of x-axis , hence the equation of the straight line is given by:

Formula used:

x cos ∝ + y sin ∝ =p

x cos 1800 + y sin 1800 =4

i.e; cos 1800 = cos (5 × 360) = cos(5 × 2π)= cos 360 = 1

similarly, sin 1800 = sin (5 × 360) = sin(5 × 2π)= sin 360 =0

hence, x × 1+ y ×0=4

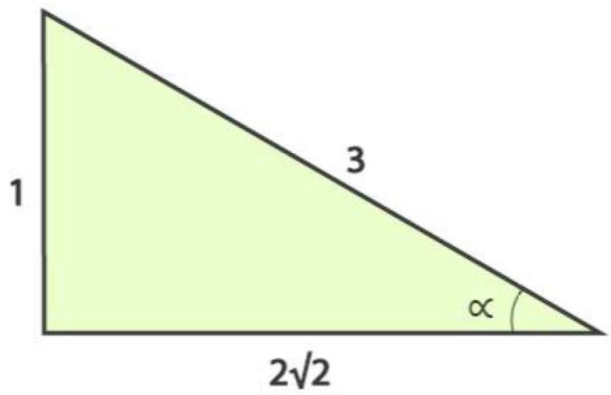
Hence The required equation of the line is x=4.

Q. 2. The length of the perpendicular segment from the origin to a line is 2 units and the inclination of this perpendicular is ∝ such that sin $\propto = \frac{1}{3}$ **and ∝ is acute. Find the equation of the line.**

Solution : To Find: The equation of the line .

Given : p=2 units and $\sin \propto = \frac{1}{3}$.

Since $\sin \propto = \frac{\text{opp}}{\text{hyp}} = \frac{1}{3}$



Using Pythagoras theorem:

adj = $\sqrt{9-1} = \sqrt{8} = 2\sqrt{2}$ units.

i.e; $\cos \propto = \frac{\text{adj}}{\text{hyp}} = \frac{2\sqrt{2}}{3}$

Formula used:

equation of the line: x cos ∝ +y sin ∝ =p

$x \times \left(\frac{2\sqrt{2}}{3}\right) + y \times \left(\frac{1}{3}\right) = 2$

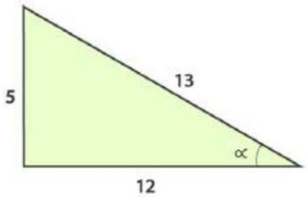
Hence ,$2\sqrt{2}$ x + y = 6 Or $\sqrt{8}$ x +y =6 is the required equation of the line.

Q. 3. Find the equation of the line which is at a distance of 3 units from the origin such that $\tan \propto = \frac{5}{12}$, where ∝ is the acute angle which this perpendicular makes with the positive direction of the x-axis.

Solution : To Find:The equation of the line.

Given : $\propto = \frac{5}{12}$ and p =3 units.

Since $\tan\propto = \frac{\text{opp}}{\text{adj}} = \frac{5}{12}$



Using Pythagoras theorem :

hyp = $\sqrt{25+144} = \sqrt{169}$ = 13 units.

From the figure: $\cos\propto = \frac{\text{adj}}{\text{hyp}} = \frac{12}{13}$ and $\sin\propto = \frac{\text{opp}}{\text{hyp}} = \frac{5}{13}$

Formula used:

equation of the line: $x\cos\propto + y\sin\propto = p$

$x \times (\frac{12}{13}) + y \times (\frac{5}{13}) = 5$

Hence ,12x + 5y = 65 is the required equation of the line.

EXERCISE 20G

Q. 1. Reduce the equation 2x – 3y – 5 = 0 to slope-intercept form, and find from it the slope and y-intercept.

Solution : Given equation is 2x – 3y – 5 = 0

We can rewrite it as 2x – 5 = 3y

$$\Rightarrow 3y = 2x - 5$$

$$\Rightarrow y = \frac{2}{3}x - \frac{5}{3}$$

This equation is in the slope-intercept form i.e. it is the form of

$y = m \times x + c$, where m is the slope of the line and c is y-intercept of the line

Therefore, $m = \frac{2}{3}$ and $c = -\frac{5}{3}$

Conclusion:

Slope is $\frac{2}{3}$ and y − intercept is $-\frac{5}{3}$

Q. 2. Reduce the equation 5x + 7y – 35 = 0 to slope-intercept form, and hence find the slope and the y-intercept of the line

Solution : Given equation is 5x + 7y -35 = 0

We can rewrite it as 7y = 35 - 5x

⇒7y = - 5x + 35

$$\Rightarrow y = -\frac{5}{7}x + 5$$

This equation is in the slope-intercept form i.e. it is the form of

$y = m \times x + c$, where m is the slope of the line and c is y-intercept of the line

Therefore, $m = -\frac{5}{7}$ and c = 5

Conclusion: Slope is $-\frac{5}{7}$ and y-intercept is 5

Q. 3. Reduce the equation y + 5 = 0 to slope-intercept form, and hence find the slope and the y-intercept of the line.

Solution : Given equation is y + 5 = 0

We can rewrite it as y = -5

This equation is in the slope-intercept form, i.e. it is the form of

$y = m \times x + c$, where m is the slope of the line and c is y-intercept of the line

Therefore, m = 0 and c = - 5

Conclusion: Slope is 0 and y-intercept is -5

Q. 4. Reduce the equation 3x – 4y + 12 = 0 to intercepts form. Hence, find the length of the portion of the line intercepted between the axes

Solution : Given equation is 3x - 4y +12 = 0

We can rewrite it as 3x - 4y = -12

$$\Rightarrow \frac{3}{-12}x + \frac{4}{12}y = 1$$

$$\Rightarrow \frac{x}{-4} + \frac{y}{3} = 1$$

This equation is in the slope intercept form i.e. in the form

$\frac{x}{a} + \frac{y}{b} = 1$

Where, x-intercept = -4 and y-intercept = 3

Two points are: (-4, 0) on the x-axis and (0, 3) on y-axis

We know distance between two points $(x_1, y_1), (x_2, y_2)$ is

$$= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Length of the line

$$=\sqrt{(-4-0)^2+(0-3)^2}$$

$$=\sqrt{16+9}$$

$$=\sqrt{25}$$

= 5

Q. 5. Reduce the equation 5x – 12y = 60 to intercepts form. Hence, find the length of the portion of the line intercepted between the axes

Solution : Given equation is 5x - 12y = 60

We can rewrite it as

$$\frac{5}{60}x-\frac{12}{60}y=1$$

$$\Rightarrow \frac{x}{12}-\frac{y}{5}=1$$

$$\Rightarrow \frac{x}{12}+\frac{y}{-5}=1$$

This equation is in the slope intercept form i.e. in the form

$$\frac{x}{a}+\frac{y}{b}=1$$

Where, x-intercept = 12 and y-intercept = -5

Two points are: (12, 0) on the x-axis and (0,-5) on y-axis

We know the distance between two points $(x_1, y_1),(x_2, y_2)$ is

$$=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

Length of the line

$$=\sqrt{(12-0)^2+(0+5)^2}$$

$$=\sqrt{144+25}$$

$$=\sqrt{169}$$

= 13

Q. 6. Find the inclination of the line:

(i) x + $\sqrt{3}$ y + 6 =0
(ii) 3x + 3y + 8 = 0
(iii) $\sqrt{3}x-y-4=0$

Solution :

(i) Given equation is x + $\sqrt{3}$ y + 6 =0

We can rewrite it as $\sqrt{3}y=-x-6$

$$\Rightarrow y=\frac{-1}{\sqrt{3}}x+\frac{-6}{\sqrt{3}}$$

It is in the form of $y=x\times\tan\alpha+c$

Where $\tan\alpha=-\frac{1}{\sqrt{3}}$ and $c=-\frac{6}{\sqrt{3}}$

The inclination of the line is α

Therefore $\alpha=\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)$

$=\frac{5\pi}{6}$ $3x+3y=8$

Conclusion: Inclination $x+\sqrt{3}y+6=0$ of the line is $\frac{5\pi}{6}$

$3y=8-3x$

(ii) Given equation is

We can rewrite it as

$\Rightarrow y=-x+\frac{-3}{8}$

It is in the form of $y=x\times\tan\alpha+c$

Where $\tan\alpha=-1$ and $c=-\frac{3}{8}$

Therefore $\alpha=\tan^{-1}(-1)$

$= \frac{3\pi}{4}$

Conclusion: Inclination of line 3x + 3y + 8 = 0 is $\frac{3\pi}{4}$

(iii) Given equation is $\sqrt{3}\,x - y - 4 = 0$

We can rewrite it as $y = \sqrt{3}x - 4$

It is in the form of $y = x \times \tan\alpha + c$

Where $\tan\alpha = \sqrt{3}$ and c = -4

$\Rightarrow \alpha = \tan^{-1}(\sqrt{3})$

$= \frac{\pi}{3}$

Conclusion: Inclination of the line is $\frac{\pi}{3}$

Q. 7. Reduce the equation x + y - √2 = 0 to the normal form x cos ∝ + y sin ∝ = p, and hence find the values of ∝ and p.

Solution :

Given equation is $x + y - \sqrt{2} = 0$

If the equation is in the form of ax + by = c, to get into the normal form, we should divide it by $\sqrt{a^2+b^2}$, so now

Divide by $\sqrt{1+1}=\sqrt{2}$

$$\Rightarrow \frac{x}{\sqrt{2}}+\frac{y}{\sqrt{2}}=1$$

This is in the form of $x\cos\alpha + y\sin\alpha = p$

$$\cos\alpha = \frac{1}{\sqrt{2}} \Rightarrow \alpha = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$\Rightarrow \alpha = \frac{\pi}{4}$ And

$$\Rightarrow p = 1$$

Conclusion: $\alpha = \frac{\pi}{4}$ and $p = 1$

Q. 8. Reduce the equation $x+\sqrt{3}y-4=0$ **to the normal form x cos ∝ + y sin ∝ = p, and hence find the values of ∝ and p.**

Solution : Given equation is

$$x+\sqrt{3}y-4=0$$

If the equation is in the form of ax + by = c, to get into the normal form, we should divide it by $\sqrt{a^2+b^2}$, so now

Divide by $\sqrt{\sqrt{3}^2+1^2}=2$

Now we get $\Rightarrow \frac{x}{2}+\frac{\sqrt{3}y}{2}=1$

This is in the form of $x\cos\alpha+y\sin\alpha=p$

Q. 8. Reduce the equation $x+\sqrt{3}y-4=0$ **to the normal form x cos ∝ + y sin ∝ = p, and hence find the values of ∝ and p.**

Solution : Given equation is $x+\sqrt{3}y-4=0$

If the equation is in the form of ax + by = c, to get into the normal form, we should divide it by $\sqrt{a^2+b^2}$, so now

Divide by

$$\sqrt{\sqrt{3}^2+1^2}=2$$

Now we get

$$\Rightarrow \frac{x}{2}+\frac{\sqrt{3}y}{2}=1$$

This is in the form of

$$x\cos\alpha+y\sin\alpha=p$$

Where

$$\cos\alpha=\frac{1}{2}\Rightarrow \alpha=\frac{\pi}{3}$$

And p = 1

Conclusion: $\alpha = \frac{\pi}{3}$ and p =1

Q. 9. Reduce each of the following equations to normal form :

(i) x + y - 2 = 0
(ii) $x + y + \sqrt{2} = 0$
(iii) x + 5 = 0
(iv) 2y – 3 =0
(v) 4x + 3y - 9 = 0

Solution :

$\Rightarrow x + y = 2$

If the equation is in the form of ax + by = c, to get into the normal form we should divide it by $\sqrt{a^2 + b^2}$, so now

Divide by $\sqrt{1^2 + 1^2} = \sqrt{2}$

$$\Rightarrow \frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = \frac{2}{\sqrt{2}}$$

$$\Rightarrow \frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = \sqrt{2}$$

This is in the form of $x\cos\alpha + y\sin\alpha = p$, where $\alpha = \frac{\pi}{4}$ and $p = \sqrt{2}$

Conclusion: $\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = \sqrt{2}$ is the normal form of x + y - 2 = 0

(ii) $x + y + \sqrt{2} = 0$

$\Rightarrow x + y = -\sqrt{2}$

If the equation is in the form of ax + by = c, to get into the normal form, we should divide it by $\sqrt{a^2 + b^2}$, so now

Divide by $\sqrt{1^2 + 1^2} = \sqrt{2}$

Our new equation is $\frac{x}{-\sqrt{2}} + \frac{y}{-\sqrt{2}} = 1$

This is in the form of $x\cos\alpha + y\sin\alpha = p$, where $\alpha = \frac{5\pi}{4}$ and p =1

Conclusion: $\frac{x}{-\sqrt{2}} + \frac{y}{-\sqrt{2}} = 1$ is the normal form of $x + y + \sqrt{2} = 0$

(iii) $\Rightarrow -x = 5$

If the equation is in the form of ax + by = c, to get into the normal form, we should divide it by $\sqrt{a^2 + b^2}$, so now

Divide the equation by $\sqrt{1^2 + 0^2} = 1$

Our new equation is $-x = 5$

This is in the form of $x\cos\alpha + y\sin\alpha = p$, where $\alpha = \pi$ and p = 5

Conclusion: $-x = 5$ is the normal form of $x + 5 = 0$

(iv) $\Rightarrow 2y = 3$

If the equation is in the form of ax + by = c, to get into the normal form, we should divide it by $\sqrt{a^2 + b^2}$, so now

Divide by $\sqrt{2^2 + 0^2} = 2$

Our new equation is $y = \frac{3}{2}$

This is in the form of $x\cos\alpha + y\sin\alpha = p$, where $\alpha = \frac{\pi}{2}$ and $p = \frac{3}{2}$

Conclusion: $y = \frac{3}{2}$ is the normal form of $2y = 3$

(v) $\Rightarrow 4x + 3y - 9 = 0$

If the equation is in the form of ax + by = c, to get into the normal form, we should divide it by $\sqrt{a^2 + b^2}$, so now

Divide by $\sqrt{4^2 + 3^2} = 5$

Our new equation is $\frac{4}{5}x+\frac{3}{5}y=\frac{9}{5}$

This is in the form of $x\cos\alpha+y\sin\alpha=p$, where

$\alpha=\sin^{-1}\left(\frac{3}{5}\right)$ or $\alpha=\cos^{-1}\left(\frac{4}{5}\right)$ and $p=\frac{9}{5}$

Conclusion: $\frac{4}{5}x+\frac{3}{5}y=\frac{9}{5}$ is the normal form of $4x+3y-9=0$

EXERCISE 20H

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Q. 1. Find the distance of the point (3, -5) from the line 3x – 4y = 27

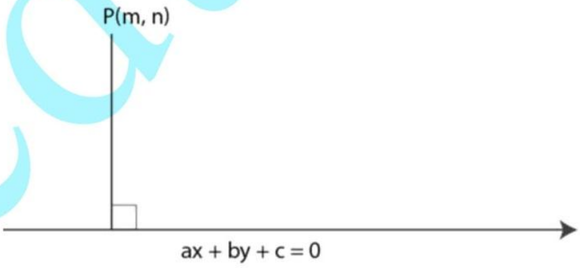
Solution : Given: Point (3,-5) and line 3x – 4y = 27

To find: The distance of the point (3, -5) from the line 3x – 4y = 27

Formula used:

We know that the distance between a point P(m,n) and a line ax + by + c = 0 is given by,

$$D=\frac{|am+bn+c|}{\sqrt{a^2+b^2}}$$



The equation of the line is 3x – 4y - 27=0

Here m= 3 and n = -5 , a = 3 , b = -4 , c = -27

$D=\frac{|3(3)-4(-5)-27|}{\sqrt{3^2+4^2}}$

$D=\frac{|9+20-27|}{\sqrt{9+16}}=\frac{|29-27|}{\sqrt{25}}=\frac{|2|}{5}$

$D=\frac{2}{5}$

The distance of the point (3, -5) from the line 3x – 4y = 27 is $\frac{2}{5}$ units

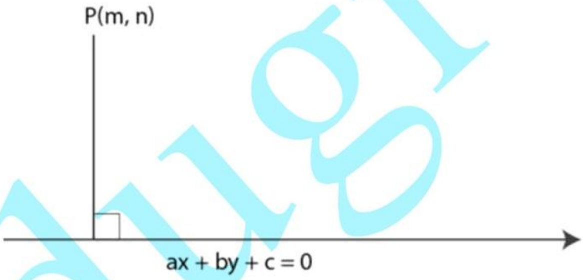
Q. 2. Find the distance of the point (-2, 3) from the line 12x = 5y + 13.

Solution : Given: Point (-2,3) and line 12x – 5y = 13

To find: The distance of the point (-2, 3) from the line 12x – 5y = 13

Formula used: We know that the distance between a point P (m,n) and a line ax + by + c = 0 is given by,

$D=\frac{|am+bn+c|}{\sqrt{a^2+b^2}}$



The given equation of the line is 12x – 5y - 13=0

Here m= -2 and n = 3 , a = 12 , b = -5 , c = -13

$D=\frac{|12(-2)-5(3)-13|}{\sqrt{12^2+5^2}}$

$D=\frac{|-24-15-13|}{\sqrt{144+25}}=\frac{|-52|}{\sqrt{169}}=\frac{|-52|}{13}=\frac{52}{13}=4$

D = 4

The distance of the point (-2, 3) from the line 12x = 5y + 13 is 4 units

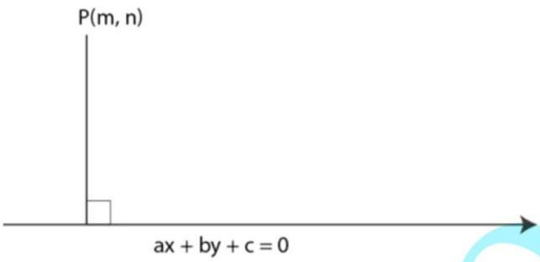
Q. 3. Find the distance of the point (-4, 3) from the line 4(x + 5) = 3(y – 6).

Solution : Given: Point (-4,3) and line 4(x + 5) = 3(y – 6)

To find: The distance of the point (-4, 3) from the line 4(x + 5) = 3(y – 6)

Formula used: We know that the distance between a point P (m,n) and a line ax + by + c = 0 is given by,

$$D=\frac{|am+bn+c|}{\sqrt{a^2+b^2}}$$



The equation of the line is 4x + 20 = 3y – 18

4x – 3y + 38 = 0

Here m= -4 and n = 3 , a = 4 , b = -3 , c = 38

$$D=\frac{|4(-4)-3(3)+38|}{\sqrt{4^2+3^2}}$$

$$D=\frac{|-16-9+38|}{\sqrt{16+9}}=\frac{|-25+38|}{\sqrt{25}}=\frac{|13|}{5}$$

$$D=\frac{13}{5}$$

The distance of the point (-4, 3) from the line 4(x + 5) = 3(y – 6) is $\frac{13}{5}$ units

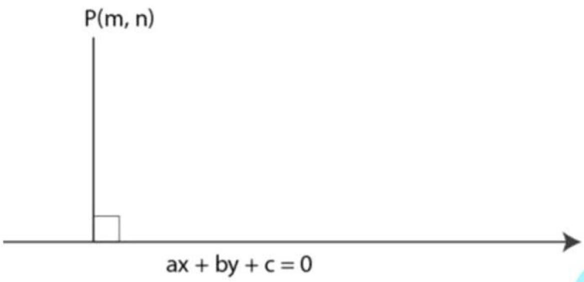
Q. 4. Find the distance of the point (2, 3) from the line y = 4.

Solution : Given: Point (2,3) and line y = 4

To find: The distance of the point (2, 3) from the line y = 4

Formula used: We know that the distance between a point P (m,n) and a line ax + by + c = 0 is given by,

$$D=\frac{|am+bn+c|}{\sqrt{a^2+b^2}}$$



The equation of the line is y - 4 = 0

Here m= 2 and n = 3 , a = 0 , b = 1 , c = -4

$$D=\frac{|1(3)-4|}{\sqrt{0^2+1^2}}$$

$$D=\frac{|3-4|}{\sqrt{0+1}}=\frac{|-1|}{\sqrt{1}}=1$$

D = 1

The distance of the point (2, 3) from the line y = 4 is 1 units

Q. 5. Find the distance of the point (4, 2) from the line joining the points (4, 1) and (2, 3)

Solution : Given: Point (4,2) and the line joining the points (4, 1) and (2, 3)

To find: The distance of the point (4,2) from the line joining the points (4, 1) and (2, 3)

Formula used: The equation of the line joining the points (x_1,y_1) and (x_2,y_2) is given by

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

Here $x_1 = 4$ $y_1 = 1$ and $x_2 = 2$ $y_2 = 3$

$$\frac{y-1}{x-4} = \frac{3-1}{2-4} = \frac{2}{-2} = -1$$

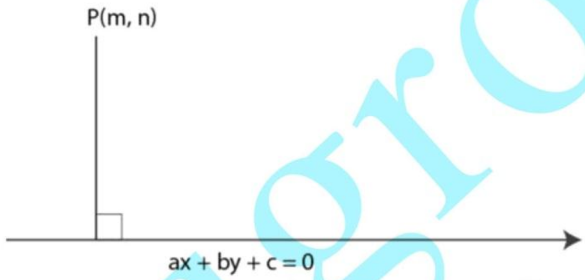
$$y - 1 = -x + 4$$

$$x + y - 5 = 0$$

The equation of the line is $x + y - 5 = 0$

Formula used: We know that the distance between a point P (m,n) and a line ax + by + c = 0 is given by,

$$D = \frac{|am+bn+c|}{\sqrt{a^2+b^2}}$$



The equation of the line is $x + y - 5 = 0$
Here m = 4 and n = 2 , a = 1 , b = 1 , c = -5

$$D = \frac{|1(4)+1(2)-5|}{\sqrt{1^2+1^2}}$$

$$D = \frac{|4+2-5|}{\sqrt{1+1}} = \frac{|6-5|}{\sqrt{2}} = \frac{|1|}{\sqrt{2}}$$

$$D = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

The distance of the point (4,2) from the line joining the points (4, 1) and (2, 3) is $\frac{\sqrt{2}}{2}$ units

Q. 6. Find the length of the perpendicular from the origin to each of the following lines :

(i) 7x + 24y = 50
(ii) 4x + 3y = 9
(iii) x = 4

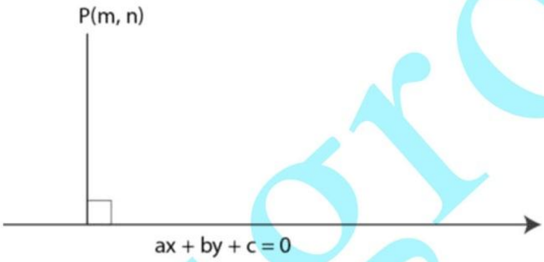
Solution : Given: Point (0,0) and line 7x + 24y = 50

To find: The length of the perpendicular from the origin to the line 7x + 24y = 50

Formula used:

We know that the length of the perpendicular from P (m,n) to the line ax + by + c = 0 is given by,

$$D=\frac{|am+bn+c|}{\sqrt{a^2+b^2}}$$



The given equation of the line is 7x + 24y - 50=0

Here m= 0 and n = 0 , a = 7 , b = 24 , c = -50

$$D=\frac{|7(0)+24(0)-50|}{\sqrt{7^2+24^2}}$$

$$D=\frac{|0+0-50|}{\sqrt{49+576}}=\frac{|-50|}{\sqrt{625}}=\frac{|-50|}{25}=\frac{50}{25}=2$$

D = 2

The length of perpendicular from the origin to the line 7x + 24y = 50 is 2 units

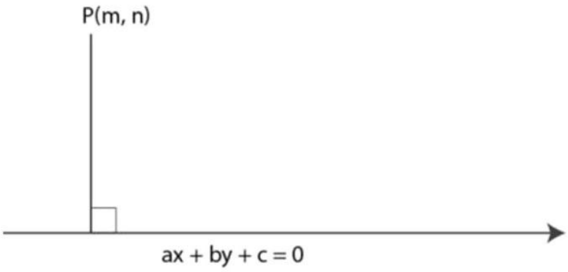
(ii) Given: Point (0,0) and line 4x + 3y = 9

To find: The length of perpendicular from the origin to the line 4x + 3y = 9

Formula used:

We know that the length of perpendicular from P (m,n) to the line ax + by + c = 0 is given by,

$$D=\frac{|am+bn+c|}{\sqrt{a^2+b^2}}$$



The given equation of the line is 4x + 3y - 9=0

Here m= 0 and n = 0 , a = 4 , b = 3 , c = -9

$$D=\frac{|4(0)+3(0)-9|}{\sqrt{4^2+3^2}}$$

$$D=\frac{|0+0-9|}{\sqrt{16+9}}=\frac{|-9|}{\sqrt{25}}=\frac{|-9|}{5}=\frac{9}{5}$$

$$D=\frac{9}{5}$$

The length of perpendicular from the origin to the line 4x + 3y = 9 is $\frac{9}{5}$ units

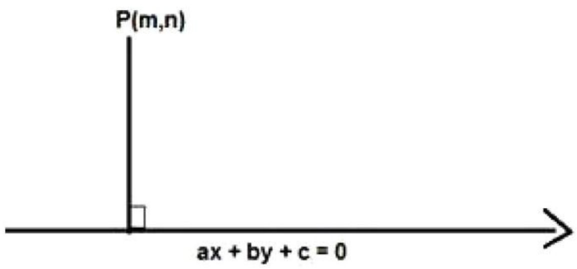
(iii) Given: Point (0,0) and line x = 4

To find: The length of perpendicular from the origin to the line x = 4

Formula used: We know that the length of perpendicular from (m,n) to the line ax + by + c = 0 is given by,

$$D=\frac{|am+bn+c|}{\sqrt{a^2+b^2}}$$

$D = \frac{|am+bn+c|}{\sqrt{a^2+b^2}}$



The given equation of the line is x - 4=0

Here m= 0 and n = 0 , a = 1 , b = 0 , c = -4

$D = \frac{|1(0)+0(0)-4|}{\sqrt{1^2+0^2}}$

$D = \frac{|0+0-4|}{\sqrt{1+0}} = \frac{|-4|}{\sqrt{1}} = \frac{|-4|}{1} = 4$

D = 4

The length of perpendicular from the origin to the line x = 4 is 4 units

Q. 7. Prove that the product of the lengths of perpendiculars drawn from the points

$A(\sqrt{a^2-b^2}, 0)$ **and** $B(-\sqrt{a^2-b^2}, 0)$ **to the line** $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$**, is** b^2

Solution :

Given: Point $A(\sqrt{a^2-b^2}, 0)$, $B(-\sqrt{a^2-b^2}, 0)$ and line $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$

To Prove: The product of the lengths of perpendiculars drawn from the points

$A(\sqrt{a^2-b^2},0)$ and $B(-\sqrt{a^2-b^2},0)$ to the line $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$, is b^2

Formula used:

We know that the length of the perpendicular from (m,n) to the line ax + by + c = 0 is given by,

$D=\frac{|am+bn+c|}{\sqrt{a^2+b^2}}$

The equation of the line is $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta-1=0$

At point A, m= $\sqrt{a^2-b^2}$ and n= 0, $a=\frac{\cos\theta}{a}$ $b=\frac{\sin\theta}{b}$ $c=-1$

$D_1=\frac{\left|\frac{\cos\theta}{a}\left(\sqrt{a^2-b^2}\right)+\frac{\sin\theta}{b}(0)-1\right|}{\sqrt{\left(\frac{\cos\theta}{a}\right)^2+\left(\frac{\sin\theta}{b}\right)^2}}$

$D_1=\frac{\left|\frac{\cos\theta}{a}\left(\sqrt{a^2-b^2}\right)-1\right|}{\sqrt{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}}}$

At point B, m= $-\sqrt{a^2-b^2}$ and n= 0, $a=\frac{\cos\theta}{a}$ $b=\frac{\sin\theta}{b}$ $c=-1$

$D_2=\frac{\left|\frac{\cos\theta}{a}\left(-\sqrt{a^2-b^2}\right)+\frac{\sin\theta}{b}(0)-1\right|}{\sqrt{\left(\frac{\cos\theta}{a}\right)^2+\left(\frac{\sin\theta}{b}\right)^2}}$

$D_2=\frac{\left|\frac{\cos\theta}{a}\left(-\sqrt{a^2-b^2}\right)-1\right|}{\sqrt{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}}}=\frac{\left|\frac{\cos\theta}{a}\left(\sqrt{a^2-b^2}\right)+1\right|}{\sqrt{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}}}$

Product of the lengths of perpendiculars drawn from the points A and B is $D_1\times D_2$

$$D_1\times D_2=\frac{\left|\frac{\cos\theta}{a}\left(\sqrt{a^2-b^2}\right)-1\right|}{\sqrt{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}}}\times\frac{\left|\frac{\cos\theta}{a}\left(\sqrt{a^2-b^2}\right)+1\right|}{\sqrt{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}}}=\frac{\left|\frac{\cos^2\theta}{a^2}(a^2-b^2)-1\right|}{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}}$$

(In the numerator we have $(x-y)\times(x+y)=x^2+y^2$ and $\sin^2\theta+\cos^2\theta$)

$$D_1\times D_2=\frac{\left|\frac{\cos^2\theta\times a^2}{a^2}+\frac{\cos^2\theta\times(-b^2)}{a^2}-\cos^2\theta-\sin^2\theta\right|}{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}}=\frac{\left|\cos^2\theta+\frac{\cos^2\theta\times(-b^2)}{a^2}-\cos^2\theta-\sin^2\theta\right|}{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}}$$

$$D_1\times D_2=\frac{\left|\frac{\cos^2\theta\times(-b^2)}{a^2}-\sin^2\theta\right|}{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}}=b^2\times\frac{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}}{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}}=b^2$$

$$D_1\times D_2=b^2$$

Product of the lengths of perpendiculars drawn from the points A and B is b^2

Q. 8. Find the values of k for which the length of the perpendicular from the point (4, 1) on the line 3x – 4y + k = 0 is 2 units

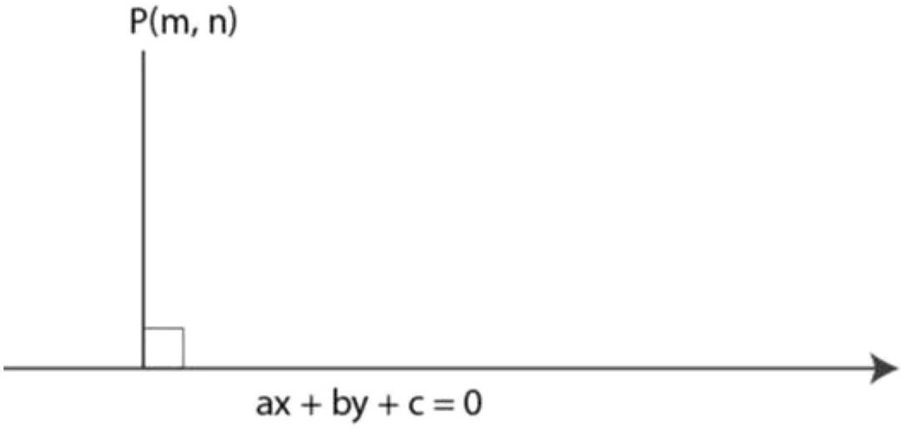
Solution : Given: Point (4,1) , line 3x – 4y + k = 0 and length of perpendicular is 2

units To find: The values of k

Formula used:

We know that the length of the perpendicular from (m,n) to the line ax + by + c = 0 is given by,

$$D=\frac{|am+bn+c|}{\sqrt{a^2+b^2}}$$



The equation of the line is 3x – 4y + k =0

Here m= 4 and n= 1 , a = 3 , b = -4 , c = k and D=2 units

$$D=\frac{|3(4)-4(1)+k|}{\sqrt{3^2+4^2}}=2$$

$$D=\frac{|12-4+k|}{\sqrt{9+16}}=\frac{|8+k|}{\sqrt{25}}=\frac{|8+k|}{5}=2$$

$$|8+k|=2\times5=10$$

$8+k=10$ or $8+k=-10$

$k=10-8$ or $k=-10-8$

$k=2$ or $k=-18$

The values of k are 2 and -18

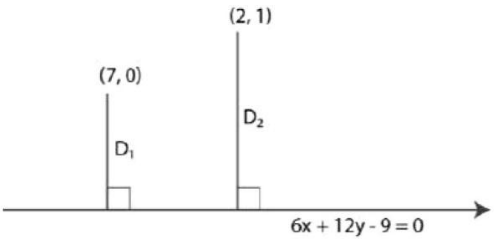
Q. 9. Show that the length of the perpendicular from the point (7, 0) to the line 5x + 12y – 9 = 0 is double the length of perpendicular to it from the point (2, 1)

Solution : Given: Points (7,0) and (2,1) , line 5x + 12y – 9 = 0

To Prove : length of the perpendicular from the point (7, 0) to the line 5x + 12y – 9 = 0 is double the length of perpendicular to it from the point (2, 1)

Formula used: We know that the length of the perpendicular from (m,n) to the line ax + by + c = 0 is given by,

$$D = \frac{|am + bn + c|}{\sqrt{a^2 + b^2}}$$



Let D_1 be the length of perpendicular from the point (7, 0) to the line 5x + 12y – 9 = 0

The given equation of the line is 5x + 12y – 9 = 0

Here at point (7,0) m= 7 and n= 0 , a = 5 , b = 12 , c = -9

$$D_1 = \frac{|5(7) + 12(0) - 9|}{\sqrt{5^2 + 12^2}}$$

$$D_1 = \frac{|35 + 0 - 9|}{\sqrt{25 + 144}} = \frac{26}{\sqrt{169}} = \frac{26}{13} = 2$$

D_1=2

Let D_2 be the length of perpendicular from the point (2, 1) to the line 5x + 12y – 9 = 0

The given equation of the line is 5x + 12y – 9 = 0

Here at point (2,1) m= 2 and n= 1 , a = 5 , b = 12 , c = -9

$$D_2 = \frac{|5(2) + 12(1) - 9|}{\sqrt{5^2 + 12^2}}$$

$$D_2 = \frac{|10 + 12 - 9|}{\sqrt{25 + 144}} = \frac{22 - 9}{\sqrt{169}} = \frac{13}{13} = 1$$

$D_2 = 1$

$D_1 = 2D_2 = 2$

Thus the length of the perpendicular from the point (7, 0) to the line 5x + 12y – 9 = 0 is double the length of perpendicular to it from the point (2, 1)

Q. 10. The points A(2, 3), B(4, -1) and C(-1, 2) are the vertices of ΔABC. Find the length of the perpendicular from C on AB and hence find the area of ΔABC

Solution : Given: points A(2, 3), B(4, -1) and C(-1, 2) are the vertices of

ΔABC To find : length of the perpendicular from C on AB and the area of

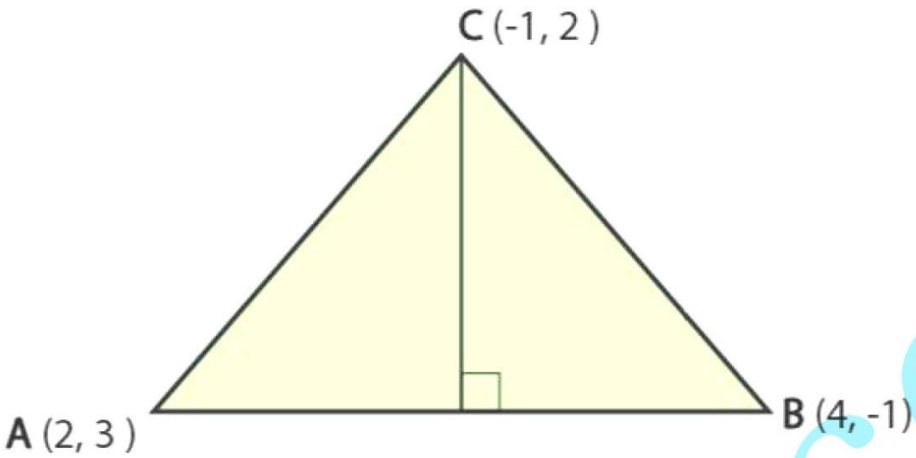
ΔABC **Formula used:**

We know that the length of the perpendicular from (m,n) to the line ax + by + c = 0 is given by,

$$D = \frac{|am + bn + c|}{\sqrt{a^2 + b^2}}$$

The equation of the line joining the points (x_1,y_1) and (x_2,y_2) is given by

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$



The equation of the line joining the points A(2,3) and B(4,-1) is

Here x_1 =2 y_1 =3 and x_2=4 y_2=-1

$$\frac{y-3}{x-2}=\frac{-1-3}{4-2}=\frac{-4}{2}=-2$$

$$y-3=-2x+4$$

$$2x+y-7=0$$

The equation of the line is 2x + y – 7 =0

The length of perpendicular from C(-1, 2) to the line AB

The given equation of the line is 2x + y – 7 =0

Here m= -1 and n= 2 , a = 2 , b = 1 , c = -7

$$D=\frac{|2(-1)+1(2)-7|}{\sqrt{2^2+1^2}}$$

$$D=\frac{-2+2-7}{\sqrt{4+1}}=\frac{|-7|}{\sqrt{5}}=\frac{|-7|}{\sqrt{5}}=\frac{7}{\sqrt{5}}$$

$$D=\frac{7}{\sqrt{5}}$$

The length of the perpendicular from C on AB is $\frac{7}{\sqrt{5}}$ units.

Height of the triangle is $\frac{7}{\sqrt{5}}$ units

The distance between points A(x_1, y_1) and B(x_2, y_2) is given by

AB = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Here x1=2 and y1=3 ,x2=4 and y2=-1

AB = $\sqrt{(4-2)^2+(-1-3)^2}=\sqrt{2^2+(-4)^2}=\sqrt{4+16}=\sqrt{20}=2\sqrt{5}$

Base AB = $2\sqrt{5}$ units

Area of the triangle = $\frac{1}{2}\times \text{BASE}\times \text{HEIGHT}$

Area of the triangle ABC = $\frac{1}{2}\times \text{AB}\times \text{HEIGHT}=\frac{1}{2}\times 2\sqrt{5}\times\frac{7}{\sqrt{5}}=7$

Area of the triangle ABC = 7 square units

Q. 11. What are the points on the x-axis whose perpendicular distance from the line $\frac{x}{3}+\frac{y}{4}=1$ is 4 units?

Solution : Given: perpendicular distance is 4 units and line

$$\frac{x}{3}+\frac{y}{4}=1$$

To find : points on the x-axis

Formula used:

We know that the length of the perpendicular from (m,n) to the line ax + by + c = 0 is given by,

$$D=\frac{|am+bn+c|}{\sqrt{a^2+b^2}}$$

The equation of the line is 4x + 3y - 12=0

Any point on the x-axis is given by (x,0)

Here m= x and n= 0 , a = 4 , b = 3 , c = -12 and D=4 units

$$D=\frac{|4(x)+3(0)-12|}{\sqrt{4^2+3^2}}=4$$

$$D=\frac{|4x-12|}{\sqrt{16+9}}=\frac{|4x-12|}{\sqrt{25}}=\frac{|4x-12|}{5}=4$$

|4x-12| = 4 × 5 = 20

4x - 12 = 20 or 4x - 12 = -20

4x = 20 + 12 or 4x = -20 + 12

4x = 32 or 4x = -8

x = 32/4 = 8 or x = (-8)/4 = -2

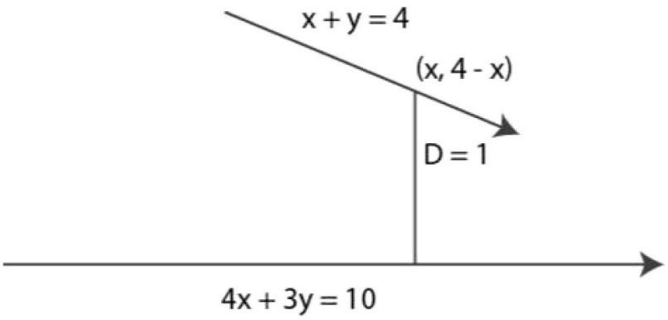
(8,0) and (2,0)are the points on the x-axis whose perpendicular distance from the line is 4 units

Q. 12. Find all the points on the line x + y = 4 that lie at a unit distance from the line 4x+3y=10.

Solution : Given: points lie on the line x + y = 4 , perpendicular distance = 1

units To find : points on the line x + y = 4

Formula used: We know that the distance between a point (m,n) and a line ax + by + c = 0 is given by, $D=\frac{|am+bn+c|}{\sqrt{a^2+b^2}}$



The equation of the line is 4x + 3y – 10 = 0 and D=1 units

Here m= x and n= 4 - x (from the equation x + y = 4),a = 4 , b = 3 , c = -10

$$D=\frac{|4(x)+3(4-x)-10|}{\sqrt{4^2+3^2}}=1$$

$$D=\frac{|4x+12-3x-10|}{\sqrt{16+9}}=\frac{|x-2|}{\sqrt{25}}=\frac{|x-2|}{5}=1$$

$$|x-2|=1\times 5=5$$

$x-2=5$ or $x-2=-5$

$x=5+2$ or $x=-5+2$

$x=7$ or $x=-3$

We know that the points lie on the line x + y = 4

y = 4 – 7 = -3 or y=4-(-3) =7

(7,-3) and (-3,7) are the points on the line x + y = 4 that lie at a unit distance from

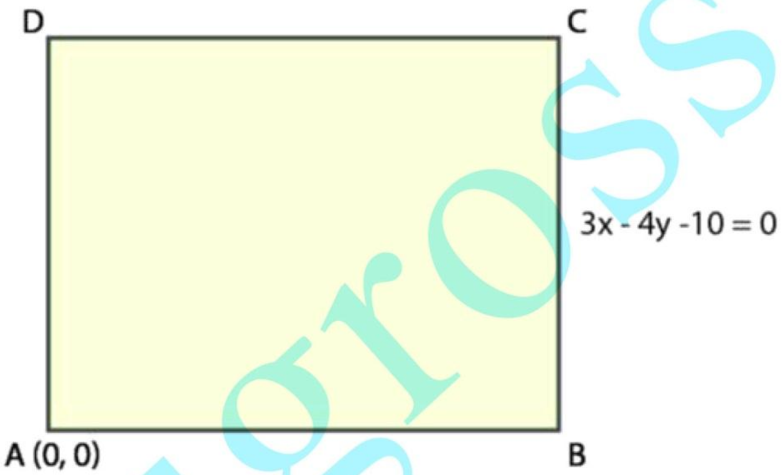
4x + 3y = 10.

Q. 13. A vertex of a square is at the origin and its one side lies along the line 3x – 4y – 10 = 0.

Find the area of the square.

Solution : Given: ABCD is a square and equation of BC is 3x – 4y – 10 =

0 To find : Area of the square



Formula used:

We know that the length of perpendicular from (m,n) to the line ax + by + c = 0 is given by,

$$D = \frac{|am + bn + c|}{\sqrt{a^2 + b^2}}$$

The given equation of the line is 3x – 4y – 10 = 0

Here m= 0 and n= 0 , a = 3 , b = -4 , c = -10

The given equation of the line is 3x – 4y – 10 = 0

Here m= 0 and n= 0 , a = 3 , b = -4 , c = -10

$$D = \frac{|3(0) - 4(0) - 10|}{\sqrt{3^2 + 4^2}}$$

$$D = \frac{|0 + 0 - 10|}{\sqrt{9 + 16}} = \frac{|-10|}{\sqrt{25}} = \frac{|-10|}{5} = \frac{10}{5} = 2$$

D = 2

Side of the square=D=2

Area of the square = 2×2 = 4 square units

Area of the square = 4 square units

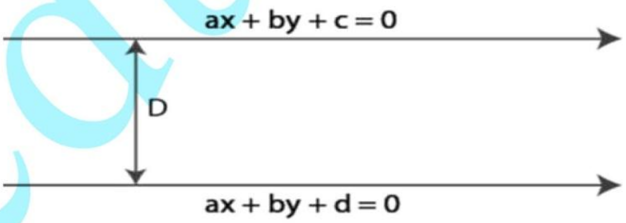
Q. 14. Find the distance between the parallel lines 4x – 3y + 5 = 0 and 4x – 3y + 7 = 0

Solution : Given: parallel lines 4x – 3y + 5 = 0 and 4x – 3y + 7 =

0 To find : distance between the parallel lines

Formula used : The distance between the parallel lines ax + by + c =0 and ax + by + d =0 is,

$$D = \frac{|d - c|}{\sqrt{a^2 + b^2}}$$



Here a = 4 ,b = -3 ,c = 5 ,d = 7

$$D=\frac{|7-5|}{\sqrt{4^2+(-3)^2}}=\frac{|2|}{\sqrt{16+9}}=\frac{2}{\sqrt{25}}=\frac{2}{5}$$

The distance between the parallel lines 4x – 3y + 5 = 0 and 4x – 3y + 7 = 0 is $\frac{2}{5}$ units

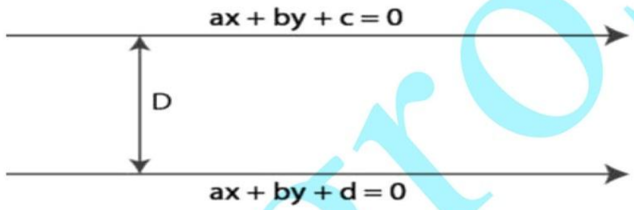
Q. 15. Find the distance between the parallel lines 8x + 15y – 36 = 0 and 8x + 15y + 32 = 0.

Solution : Given: parallel lines 8x + 15y – 36 = 0 and 8x + 15y + 32 =

0. To find : distance between the parallel lines

Formula used : The distance between the parallel lines ax + by + c =0 and ax + by + d =0 is,

$$D=\frac{|d-c|}{\sqrt{a^2+b^2}}$$



Here a = 8 ,b = 15 ,c = -36 ,d = 32

$$D=\frac{|32-(-36)|}{\sqrt{8^2+15^2}}=\frac{|32+36|}{\sqrt{64+225}}=\frac{68}{\sqrt{289}}=\frac{68}{17}=4$$

The distance between the parallel lines 8x + 15y – 36 = 0 and 8x + 15y + 32 =0 is
4 Units

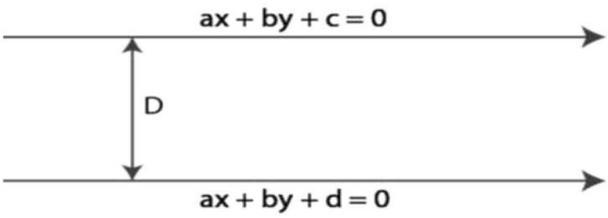
Q. 16. Find the distance between the parallel lines y = mx + c and y = mx + d

Solution : Given: parallel lines y = mx + c and y = mx + d

To find : distance between the parallel lines

Formula used : The distance between the parallel lines ax + by + c =0 and ax + by + d =0 is,

$$D=\frac{|d-c|}{\sqrt{a^2+b^2}}$$



The parallel lines are mx – y + c = 0 and mx – y + d=0

Here a = m ,b = -1 ,c = c ,d = d

$$D=\frac{|d-c|}{\sqrt{m^2+1^2}}=\frac{|d-c|}{\sqrt{m^2+1}}$$

The distance between the parallel lines y = mx + c and y = mx + d is $\frac{|d-c|}{\sqrt{m^2+1}}$ units

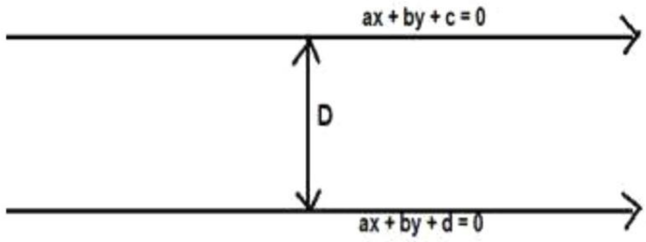
Q. 17. Find the distance between the parallel lines p(x + y) = q = 0 and p(x + y) – r =0

Solution : Given: parallel lines p(x + y) = q = 0 and p(x + y) – r =0

To find : distance between the parallel lines p(x + y) - q = 0 and p(x + y) – r =0

Formula used : The distance between the parallel lines ax + by + c =0 and ax + by + d =0 is,

$$D=\frac{|d-c|}{\sqrt{a^2+b^2}}$$



The parallel lines are p(x + y) - q = 0 and p(x + y) – r =0

The parallel lines are px + py - q = 0 and px + py – r = 0

Here a = p ,b = p ,c = -q ,d = -r

$$D=\frac{|-r-(-q)|}{\sqrt{p^2+p^2}}=\frac{|-r+q|}{\sqrt{2p^2}}=\frac{|q-r|}{p\sqrt{2}}$$

The distance between the parallel lines p(x + y) =q = 0 and p(x + y) – r = 0 is $\frac{|q-r|}{p\sqrt{2}}$ units

Q. 18. Prove that the line 12x – 5y – 3 = 0 is mid-parallel to the lines 12x – 5y + 7 = 0 and 12x – 5y – 13 = 0

Solution : Given: parallel lines 12x – 5y – 3 = 0, 12x – 5y + 7 = 0, 12x – 5y – 13 = 0

To Prove : line 12x – 5y – 3 = 0 is mid-parallel to the lines 12x – 5y + 7 = 0 and 12x – 5y – 13 = 0

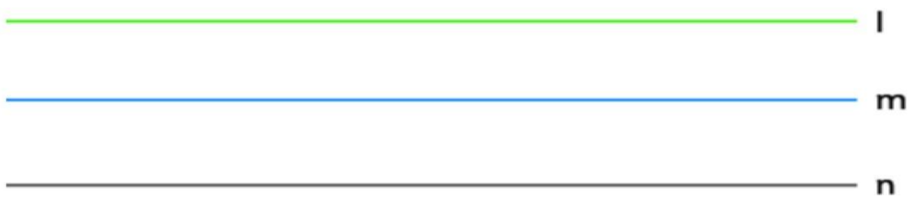
Formula used : The distance between the parallel lines ax + by + c =0 and ax + by + d =0 is,

$$D=\frac{|d-c|}{\sqrt{a^2+b^2}}$$

The equation of line l is 12x – 5y + 7 = 0

The equation of line m is 12x – 5y – 3 = 0

The equation of line n is 12x – 5y – 13 = 0



Let D_1 be the distance between the lines l and m .

Here a = 12 ,b = -5 ,c = 7 ,d = -3

$$D_1 = \frac{|-3-7|}{\sqrt{12^2+(-5)^2}} = \frac{|-10|}{\sqrt{144+25}} = \frac{10}{\sqrt{169}} = \frac{10}{13}.$$

The distance between the parallel lines l and m is $\frac{10}{13}$ units

Let D_2 be the distance between the lines m and n .

Here a = 12 ,b = -5 ,c = 7 ,d = -3

$$D_2 = \frac{|-13-(-3)|}{\sqrt{12^2+(-5)^2}} = \frac{|-13+3|}{\sqrt{144+25}} = \frac{|-10|}{\sqrt{169}} = \frac{10}{13}$$

The distance between the parallel lines m and n is $\frac{10}{13}$ units

Distance between the parallel lines l and m = Distance between the parallel lines m and n

Thus the line 12x – 5y – 3 = 0 is mid-parallel to the lines 12x – 5y + 7 = 0 and 12x – 5y – 13 = 0

Q. 19. The perpendicular distance of a line from the origin is 5 units, and its slope is -1. Find the equation of the line.

Solution :

Given: perpendicular distance from orgin is 5 units, and the slope is -1

To find : the equation of the line

Formula used :

We know that the perpendicular distance from a point (x_0, y_0) to the line ax + by + c = 0 is given by

$$D = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$$

The equation of a straight line is given by y=mx+c where m denotes the slope of the line.

The equation of the line is mx – y + c =0

Here $x_0 = 0$ and $y_0 = 0$, a = m , b = -y , c = c and D=5 units

$$D = \frac{|m(0) - 1(0) + c|}{\sqrt{m^2 + 1^2}} = \frac{|c|}{\sqrt{m^2 + 1}} = \frac{c}{\sqrt{m^2 + 1}} = 5$$

Slope of the line = m = -1 ,Substituting in the above equation we get,

$$\frac{c}{\sqrt{(-1)^2 + 1^2}} = 5$$

$$\frac{c}{\sqrt{1+1}} = \frac{c}{\sqrt{2}} = 5$$

$$c = 5\sqrt{2}$$

Thus the equation of the straight line is $y = -x + 5\sqrt{2}$ or $x + y - 5\sqrt{2} = 0$

EXERCISE 20I

Q. 1. Find the points of intersection of the lines 4x + 3y = 5 and x = 2y – 7.

Solution : Suppose the given two lines intersect at a point $P(x_1, y_1)$. Then, (x_1, y_1) satisfies each of the given equations.

∴ 4x + 3y = 5

or 4x + 3y – 5 = 0 …(i)

and x = 2y – 7

or x – 2y + 7 = 0 …(ii)

Now, we find the point of intersection of eq. (i) and (ii)

Multiply the eq. (ii) by 4, we get

4x – 8y + 28 = 0 …(iii)

On subtracting eq. (iii) from (i), we get

4x – 8y + 28 – 4x – 3y + 5 = 0

⇒ -11y + 33 = 0

⇒ -11y = -33

$$\Rightarrow y = \frac{33}{11} = 3$$

Putting the value of y in eq. (i), we get

4x + 3(3) – 5 = 0

⇒ 4x + 9 – 5 = 0

⇒ 4x + 4 = 0

⇒ 4x = -4

⇒ x = -1

Hence, the point of intersection $P(x_1, y_1)$ is (-1, 3)

Q. 2. Show that the lines x + 7y = 23 and 5x + 2y = a 16 intersect at the point (2, 3).

Solution : Suppose the given two lines intersect at a point P(2, 3). Then, (2, 3) satisfies each of the given equations.

So, taking equation x + 7y = 23

Substituting x = 2 and y = 3

Lhs = x + 7y

= 2 + 7(3)

= 2 + 21

= 23

= RHS

Now, taking equation 5x + 2y = 16

Substituting x = 2 and y = 3

LHS = 5x + 2y

= 5(2) + 2(3)

= 10 + 6

= 16

= RHS

In both the equations pair (2, 3) for (x, y) satisfies the given equations, therefore both lines pass through (2, 3).

Q. 3. Show that the lines 3x – 4y + 5 = 0, 7x – 8y + 5 = 0 and 4x + 5y = 45 are concurrent. Also find their point of intersection.

Solution : Given: 3x – 4y + 5 =

0, 7x – 8y + 5 = 0

and 4x + 5y = 45

or 4x + 5y – 45 = 0

To show: Given lines are concurrent

The lines $a_1x + b_1y + c_1 = 0$, $a_1x + b_1y + c_1 = 0$ and $a_1x + b_1y + c_1 = 0$ are concurrent if

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

We know that,

We have,

$a_1 = 3$, $b_1 = -4$, $c_1 = 5$

$a_2 = 7$, $b_2 = -8$, $c_2 = 5$

$a_3 = 4$, $b_2 = 5$, $c_3 = -45$

$$\Rightarrow \begin{vmatrix} 3 & -4 & 5 \\ 7 & -8 & 5 \\ 4 & 5 & -45 \end{vmatrix}$$

Now, expanding along first row, we get

$\Rightarrow 3[(-8)(-45) – (5)(5)] – (-4)[(7)(-45) – (4)(5)] + 5[(7)(5) – (4)(-8)]$

$\Rightarrow 3[360 – 25] + 4[-315 – 20] + 5[35 + 32]$

$\Rightarrow 3[335] + 4[-335] + 5[67]$

$\Rightarrow 1005 – 1340 + 335$

$\Rightarrow 1340 – 1340$

$= 0$

So, the given lines are concurrent.

Now, we have to find the point of intersection of the given lines

$3x – 4y + 5 = 0,$

$7x – 8y + 5 = 0$

and 4x + 5y – 45 = 0 …(A)

We know that, if three lines are concurrent the point of intersection of two lines lies on the third line.

So, firstly, we find the point of intersection of two lines

3x – 4y + 5 = 0, …(i)

7x – 8y + 5 = 0 …(ii)

Multiply the eq. (i) by 2, we get

6x – 8y + 10 = 0 …(iii)

On subtracting eq. (iii) from (ii), we get

7x – 8y + 5 – 6x + 8y – 10 = 0

⇒ x – 5 = 0

⇒ x = 5

Putting the value of x in eq. (i), we get

3(5) – 4y + 5 = 0

⇒ 15 – 4y + 5 = 0

⇒ 20 – 4y = 0

⇒ -4y = -20

⇒ y = 5

Thus, the first two lines intersect at the point (5, 5). Putting x = 5 and y = 5 in eq. (A), we get

4(5) + 5(5) – 45

= 20 + 25 – 45

= 45 – 45

= 0

So, point (5, 5) lies on line $4x + 5y - 45 = 0$

Hence, the point of intersection is (5, 5)

Q. 4. Find the value of k so that the lines $3x - y - 2 = 0$, $5x + ky - 3 = 0$ and $2x + y - 3 = 0$ are concurrent.

Solution : Given that $3x - y - 2 = 0$,

$5x + ky - 3 = 0$

and $2x + y - 3 = 0$ are concurrent

We know that,

The lines $a_1x + b_1y + c_1 = 0$, $a_1x + b_1y + c_1 = 0$ and $a_1x + b_1y + c_1 = 0$ are concurrent if

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

It is given that the given lines are concurrent.

$$\Rightarrow \begin{vmatrix} 3 & -1 & -2 \\ 5 & k & -3 \\ 2 & 1 & -3 \end{vmatrix} = 0$$

Now, expanding along first row, we get

$\Rightarrow 3[(k)(-3) - (-3)(1)] - (-1)[(5)(-3) - (-3)(2)] + (-2)[5 - 2k] = 0$

$\Rightarrow 3[-3k + 3] + 1[-15 + 6] - 2[5 - 2k] = 0$

$\Rightarrow -9k + 9 - 9 - 10 + 4k = 0$

$\Rightarrow -5k - 10 = 0$

$\Rightarrow -5k = 10$

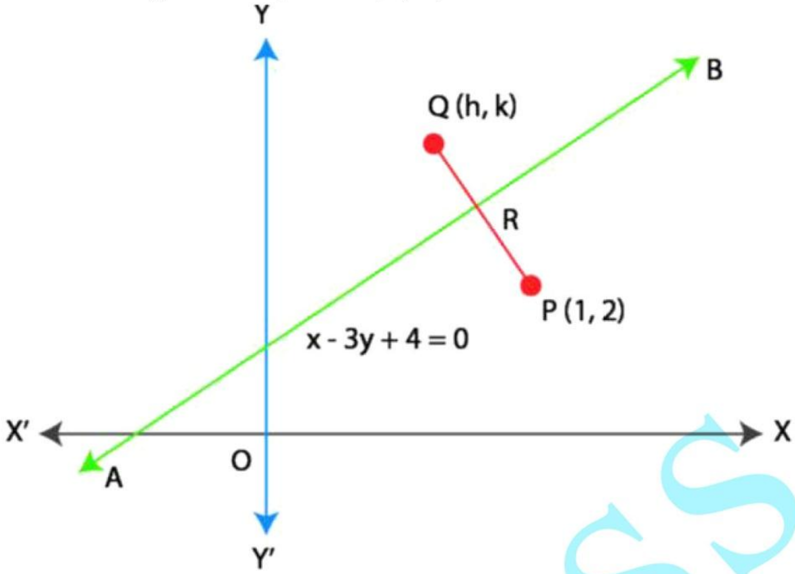
$\Rightarrow k = -2$

Hence, the value of $k = -2$

Q. 5. Find the image of the point P(1, 2) in the line x – 3y + 4 = 0.

Solution : Let line AB be x – 3y + 4 = 0 and point P be (1, 2)

Let the image of the point P(1, 2) in the line mirror AB be Q(h, k).



Since line AB is a mirror. Then PQ is perpendicularly bisected at R.

Since R is the midpoint of PQ.

We know that,

$$\text{Midpoint of a line joining } (x_1, y_1) \,\&\, (x_2, y_2) = \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}$$

$$\text{So, Midpoint of the line joining } (1, 2) \,\&\, (h, k) = \frac{1+h}{2}, \frac{2+k}{2}$$

Since point R lies on the line AB. So, it will satisfy the equation of line AB x – 3y + 4 = 0

Substituting the $x = \frac{1+h}{2}$ & $y = \frac{2+k}{2}$ in abthe ove equation, we get

$$\frac{1+h}{2} - 3\left(\frac{2+k}{2}\right) + 4 = 0$$

$$\Rightarrow \frac{1+h-6-3k+8}{2} = 0$$

⇒ 3 + h – 3k = 0

⇒ h – 3k = -3 …(i)

Also, PQ is perpendicular to AB

We know that, if two lines are perpendicular then the product of their slope is equal to -1

∴ Slope of AB × Slope of PQ = -1

$$\Rightarrow \text{Slope of PQ} = \frac{-1}{\text{Slope of AB}}$$

Now, we find the slope of line AB i.e. x – 3y + 4 = 0

We know that, the slope of an equation is

$$m = -\frac{a}{b}$$

and here, a = 1 & b = -3

$$\Rightarrow m = -\frac{1}{(-3)} = \frac{1}{3}$$

$= \frac{-1}{\frac{1}{3}}$

= - 3

Now, Equation of line PQ formed by joining the points P(1, 2) and Q(h, k) and having the slope – 3 is

$y_2 - y_1 = m(x_2 - x_1)$

⇒ k – 2 = (-3)(h – 1)

⇒ k – 2 = -3h + 3

⇒ 3h + k = 5 …(ii)

Now, we will solve the eq. (i) and (ii) to find the value of h and k

h – 3k = -3 …(i)

and 3h + k = 5 …(ii)

From eq. (i), we get

h = -3 + 3k

Putting the value of h in eq. (ii), we get

3(-3 + 3k) + k = 5

⇒ -9 + 9k + k = 5

⇒ -9 + 10k = 5

⇒ 10k = 5 + 9

⇒ 10k = 14

$\Rightarrow k = \frac{14}{10} = \frac{7}{5}$

Putting the value of k in eq. (i), we get

$$h - 3\left(\frac{7}{5}\right) = -3$$

⇒ 5h – 21 = -3 × 5

⇒ 5h – 21 = -15

⇒ 5h = -15 + 21

⇒ 5h = 6

$$\Rightarrow h = \frac{6}{5}$$

Q. 6. Find the area of the triangle formed by the lines x + y = 6, x – 3y = 2 and 5x – 3y + 2 = 0.

Solution : The given equations

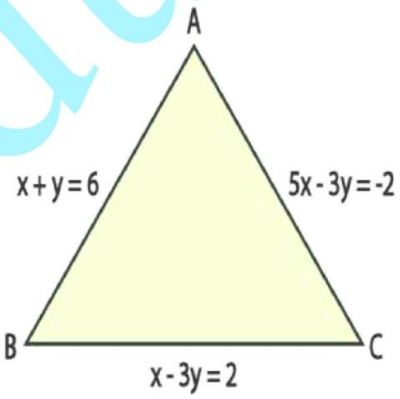
are x + y = 6 …(i)

x – 3y = 2 …(ii)

and 5x – 3y + 2 = 0

or 5x – 3y = -2 …(iii)

Let eq. (i), (ii) and (iii) represents the sides AB, BC and AC respectively of ΔABC



Firstly, we solve the equation (i) and (ii)

x + y = 6 …(i)

x – 3y = 2 …(ii)

Subtracting eq. (ii) from (i), we get

x + y – x + 3y = 6 – 2

⇒ 4y = 4

⇒ y = 1

Putting the value of y = 1 in eq. (i), we get

x + 1 = 6

⇒ x = 5

Thus, AB and BC intersect at (5, 1)

Now, we solve eq. (ii) and (iii)

x – 3y = 2 …(ii)

5x – 3y = -2 …(iii)

Subtracting eq. (ii) from (iii), we get

5x – 3y – x + 3y = – 2 – 2

⇒ 4x = - 4

⇒ x = -1

Putting the value of x = -1 in eq. (ii), we get

– 1 – 3y = 2

⇒ -3y = 2 + 1

⇒ -3y = 3

⇒ y = -1

Thus, BC and AC intersect at (-1, -1)

Now, we solve eq. (iii) and (i)

$5x - 3y = -2$ …(iii)

$x + y = 6$ …(i)

From eq. (i), we get

$x = 6 - y$

Putting the value of x in eq. (iii), we get

$5(6 - y) - 3y = -2$

$\Rightarrow 30 - 5y - 3y = -2$

$\Rightarrow 30 - 8y = -2$

$\Rightarrow -8y = -32$

$\Rightarrow y = 4$

Putting the value of y = 4 in eq. (i), we get

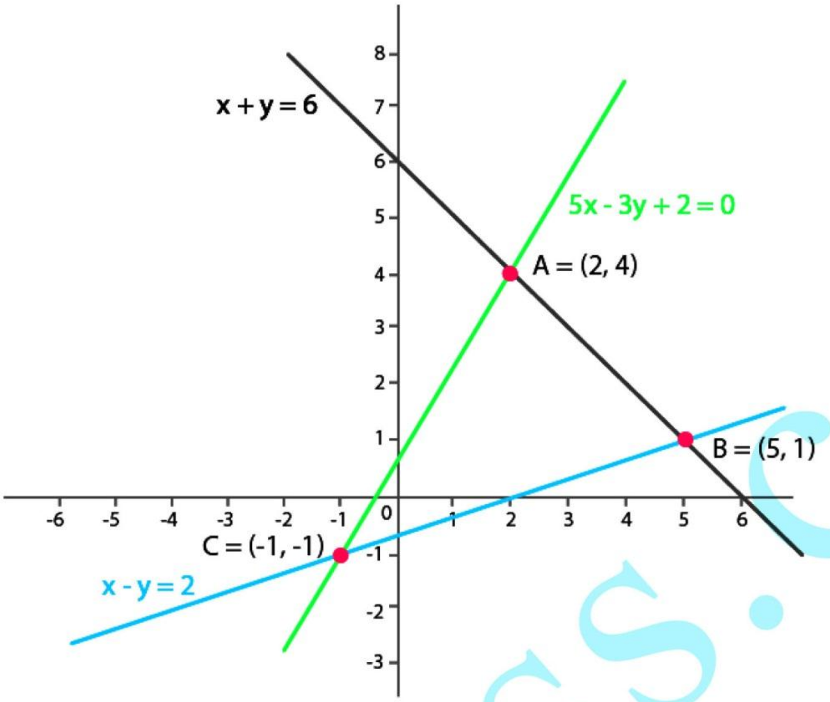
$x + 4 = 6$

$\Rightarrow x = 6 - 4$

$\Rightarrow x = 2$

Thus, AC and AB intersect at (2, 4)

So, vertices of triangle ABC are: (5, 1), (-1, -1) and (2, 4)



$$\therefore \text{Area of } \Delta ABC = \frac{1}{2}\begin{vmatrix} 2 & 4 & 1 \\ 5 & 1 & 1 \\ -1 & -1 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[2\{(1)(1)-(1)(-1)\}-4\{(5)(1)-(1)(-1)\}+1\{(5)(-1)-(1)(-1)\}\right]$$

$$= \frac{1}{2}\left[2\{1+1\}-4\{5+1\}+1\{-5+1\}\right]$$

$$= \frac{1}{2}[4-24-4]$$

$$= \frac{1}{2}[|-24|]$$

= 12 sq. units [∵, area can't be negative]

Q. 7. Find the area of the triangle formed by the lines x = 0, y = 1 and 2x + y = 2.

Solution : The given equations

are x = 0 …(i)

y = 1 …(ii)

and 2x + y = 2 …(iii)

Let eq. (i), (ii) and (iii) represents the sides AB, BC and AC respectively of ΔABC

From eq. (i) and (ii), we get x = 0 and y = 1

Thus, AB and BC intersect at (0, 1)

Solving eq. (ii) and (iii), we get

y = 1 …(ii)

and 2x + y = 2 …(iii)

Putting the value of y = 1 in eq. (iii), we get

2x + 1 = 2

⇒ 2x = 1

$$\Rightarrow x = \frac{1}{2}$$

Thus, BC and AC intersect at $\left(\frac{1}{2}, 1\right)$

Now, Solving eq. (iii) and (i), we get

2x + y = 2 …(iii)

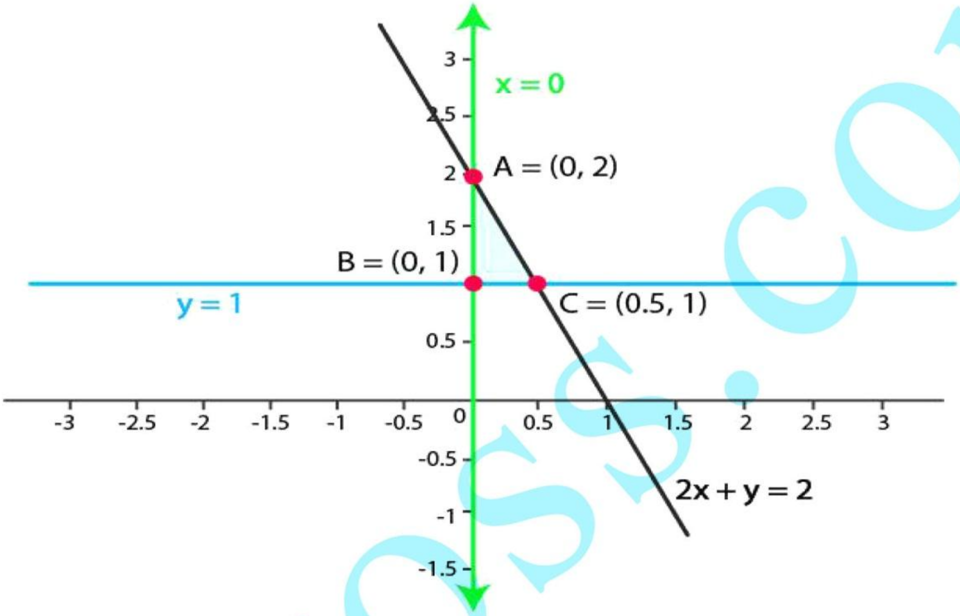
and x = 0 …(i)

Putting the value of x = 0 in eq. (iii), we get

y = 2

Thus, AC and AB intersect at (0, 2)

So, vertices of triangle ABC are : $(0,1), \left(\frac{1}{2},1\right)$ and $(0,2)$



$\therefore$ Area of $\Delta ABC = \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times \frac{1}{2} \times 1$

$= \frac{1}{4}$ sq. units

Q. 8. Find the area of the triangle, the equations of whose sides are y = x, y = 2x and y – 3x = 4.

Solution : The given equations

are y = x …(i)

y = 2x …(ii)

and $y - 3x = 4$ …(iii)

Let eq. (i), (ii) and (iii) represents the sides AB, BC and AC respectively of ΔABC

From eq. (i) and (ii), we get $x = 0$ and $y = 0$

Thus, AB and BC intersect at (0, 0)

Solving eq. (ii) and (iii), we get

$y = 2x$ …(ii)

and $y - 3x = 4$ …(iii)

Putting the value of $y = 2x$ in eq. (iii), we get

$2x - 3x = 4$

$\Rightarrow -x = 4$

$\Rightarrow x = -4$

Putting the value of $x = -4$ in eq. (ii), we get

$y = 2(-4)$

$\Rightarrow y = -8$

Thus, BC and AC intersect at (-4, -8)

Now, Solving eq. (iii) and (i), we get

$y - 3x = 4$ …(iii)

and $y = x$ …(i)

Putting the value of $y = x$ in eq. (iii), we get

$x - 3x = 4$

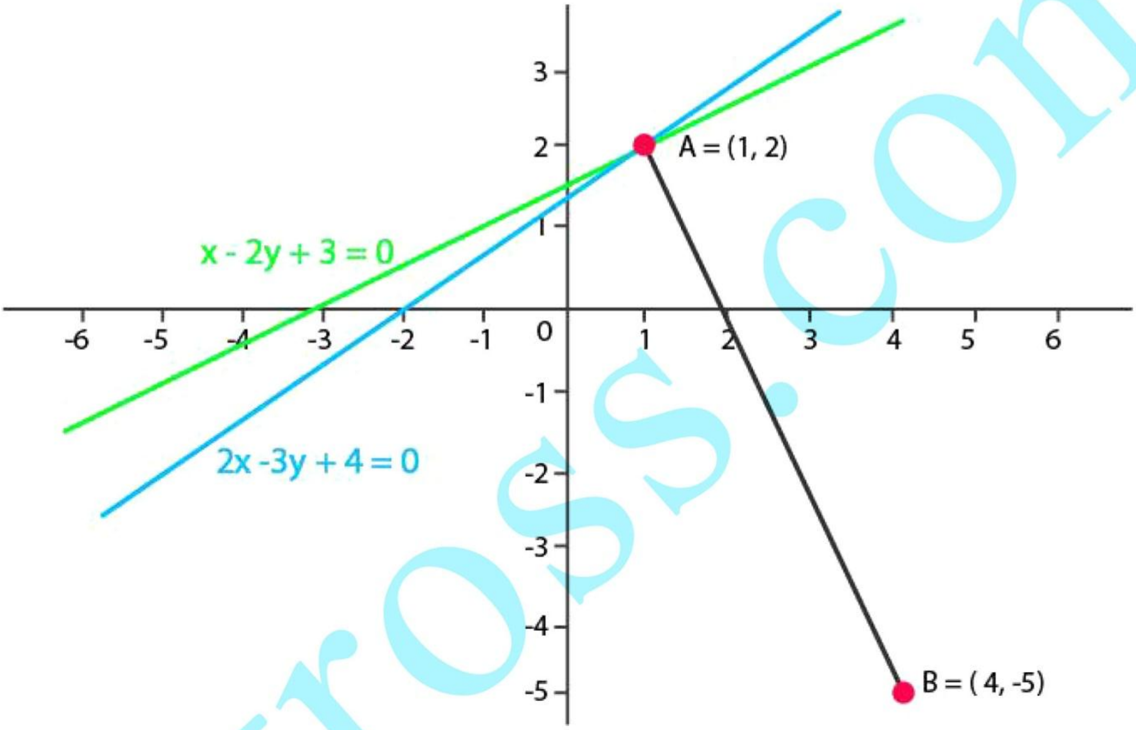
$\Rightarrow -2x = 4$

$\Rightarrow x = -2$

Putting the value of $x = -2$ in eq. (i), we get y = -2

Thus, AC and AB intersect at (-2, -2)

So, vertices of triangle ABC are: (0, 0), (-4, -8) and (-2, -2)



$$\therefore \text{Area of } \Delta ABC = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ -2 & -2 & 1 \\ -4 & -8 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[0 - 0 + 1\{(-2)(-8) - (-2)(-4)\}\right]$$

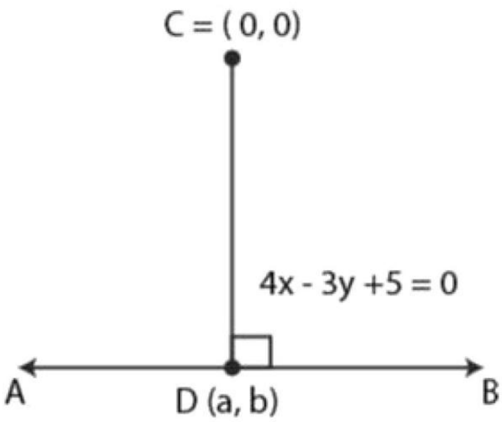
$$= \frac{1}{2}\left[1\{16 - 8\}\right]$$

$$= \frac{1}{2}[8]$$

= 4 sq. units

Q. 9. Find the equation of the perpendicular drawn from the origin to the line 4x – 3y + 5 = 0. Also, find the coordinates of the foot of the perpendicular.

Solution :



Let the equation of line AB be 4x – 3y + 5 = 0

and point C be (0, 0)

CD is perpendicular to the line AB, and we need to find:

1) Equation of Perpendicular drawn from point C

2) Coordinates of D

Let the coordinates of point D be (a, b)

Also, point D(a, b) lies on the line AB, i.e. point (a, b) satisfy the equation of line AB

Putting x = a and y = b, in equation, we get

4a – 3b + 5 = 0 …(i)

Also, the CD is perpendicular to the line AB

and we know that, if two lines are perpendicular then the product of their slope is equal to -1

∴ Slope of AB × Slope of CD = -1

$$\Rightarrow \text{Slope of CD} = \frac{-1}{\text{Slope of AB}}$$

$$= \frac{-1}{\frac{4}{3}}$$

$$\text{Slope of CD} = -\frac{3}{4}$$

Now, Equation of line CD formed by joining the points C(0, 0) and D(a, b) and having the slope $-\frac{3}{4}$ is

$y_2 - y_1 = m(x_2 - x_1)$

$$\Rightarrow b - 0 = -\frac{3}{4}(a - 0)$$

$$\Rightarrow b = -\frac{3}{4}a$$

⇒ 4b = -3a

⇒ 3a + 4b = 0 …(ii)

Now, our equations are

4a – 3b + 5 = 0 …(i)

and 3a + 4b = 0 …(ii)

Multiply the eq. (i) by 4 and (ii) by 3, we get

16a – 12b + 20 = 0 …(iii)

9a + 12b = 0 …(iv)

Adding eq. (iii) and (iv), we get

16a – 12b + 20 + 9a + 12b = 0

⇒ 25a + 20 = 0

⇒ 25a = -20

$$\Rightarrow a = -\frac{20}{25} = -\frac{4}{5}$$

Putting the value of a in eq. (ii), we get

$$3\left(-\frac{4}{5}\right) + 4b = 0$$

$$\Rightarrow -\frac{12}{5} + 4b = 0$$

⇒ -12 + 20b = 0

⇒ 20b = 12

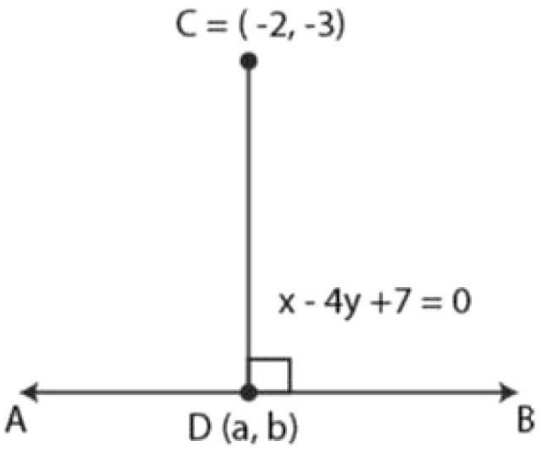
$$\Rightarrow b = \frac{12}{20}$$

$$\Rightarrow b = \frac{3}{5}$$

Hence, the coordinates of $D(a, b)$ is $\left(-\frac{4}{5}, \frac{3}{5}\right)$

Q. 10. Find the equation of the perpendicular drawn from the point P(-2, 3) to the line x– 4y + 7 = 0. Also, find the coordinates of the foot of the perpendicular.

Solution :



Let the equation of line AB be x – 4y + 7 = 0

and point C be (-2, 3)

CD is perpendicular to the line AB, and we need to find:

1) Equation of Perpendicular drawn from point C

2) Coordinates of D

Let the coordinates of point D be (a, b)

Also, point D(a, b) lies on the line AB, i.e. point (a, b) satisfy the equation of line AB

Putting x = a and y = b, in equation, we get

a – 4b + 7 = 0 …(i)

Also, the CD is perpendicular to the line AB

and we know that, if two lines are perpendicular then the product of their slope is equal to -1

∴ Slope of AB × Slope of CD = -1

$$\Rightarrow \text{Slope of CD} = \frac{-1}{\text{Slope of AB}}$$

$$= \frac{-1}{\frac{1}{4}}$$

Slope of CD = - 4

Now, Equation of line CD formed by joining the points C(-2, 3) and D(a, b) and having the slope – 4 is

$y_2 - y_1 = m(x_2 - x_1)$

⇒ b – 3 = (-4)[a – (-2)]

⇒ b – 3 = -4(a + 2)

⇒ b – 3 = -4a – 8

⇒ 4a + b + 5 = 0 …(ii)

Now, our equations are

a – 4b + 7 = 0 …(i)

and 4a + b + 5 = 0 …(ii)

Multiply the eq. (ii) by 4, we get

16a + 4b + 20 = 0 …(iii)

Adding eq. (i) and (iii), we get

a – 4b + 7 + 16a + 4b + 20 = 0

⇒ 17a + 27 = 0

⇒ 17a = -27

$$\Rightarrow a = -\frac{27}{17}$$

Putting the value of a in eq. (i), we get

$$-\frac{27}{17}-4b+7=0$$

$$\Rightarrow \frac{-27-68b+119}{17}=0$$

⇒ 92 – 68b = 0

⇒ -68b = -92

$$\Rightarrow b=\frac{92}{68}$$

$$\Rightarrow b=\frac{23}{17}$$

Hence, the coordinates of $D(a,\ b)$ is $\left(-\frac{27}{17},\frac{23}{17}\right)$

Q. 11. Find the equations of the medians of a triangle whose sides are given by the equations 3x + 2y + 6 = 0, 2x – 5y + 4 = 0 and x -3y – 6 = 0.

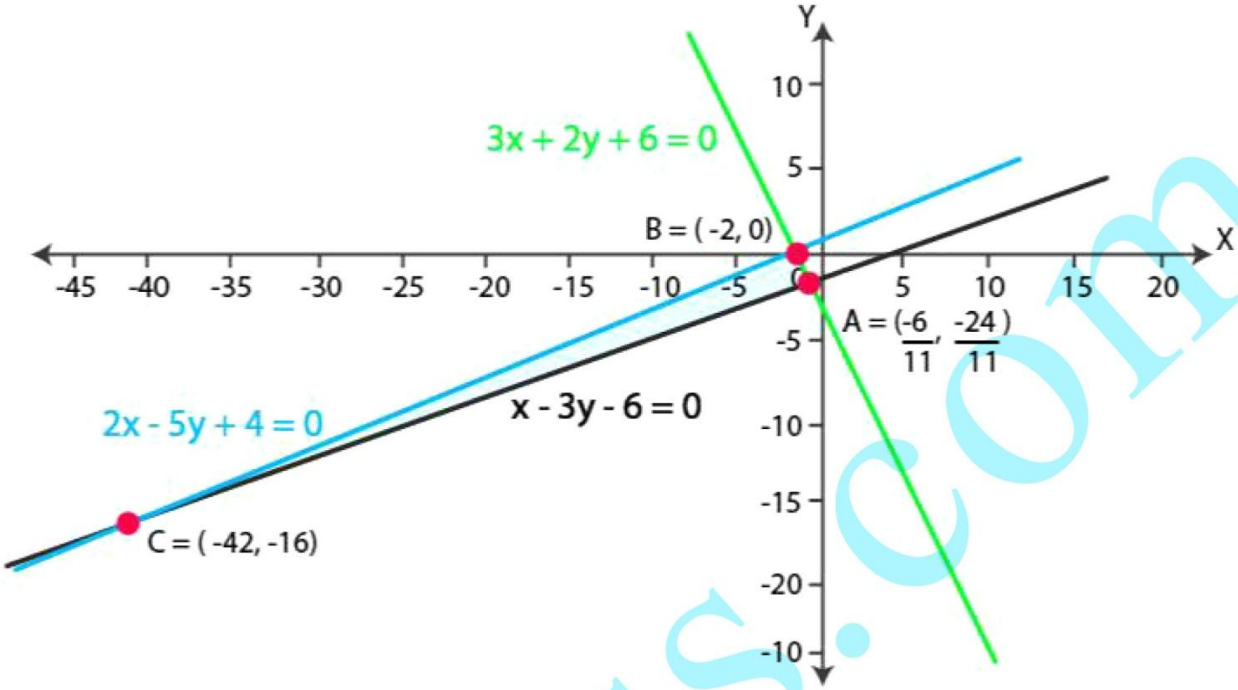
Solution : The given equations

are 3x + 2y + 6 = 0 …(i)

2x – 5y + 4 = 0 …(ii)

and x – 3y – 6 = 0 …(iii)

Let eq. (i), (ii) and (iii) represents the sides AB, BC and AC respectively of ΔABC



Firstly, we solve the equation (i) and (ii)

3x + 2y + 6 = 0 …(i)

2x – 5y + 4 = 0 …(ii)

Multiplying the eq. (i) by 2 and (ii) by 3, we get

6x + 4y + 12 = 0 …A

6x – 15y + 12 = 0 …B

Subtracting eq. (B) from (A), we get

6x + 4y + 12 – 6x + 15y – 12 = 0

⇒ 19y = 0

⇒ y = 0

Putting the value of y = 0 in eq. (i), we get

3x + 2(0) + 6 = 0

⇒ 3x + 6 = 0

⇒ 3x = -6

⇒ x = -2

Thus, AB and BC intersect at (-2, 0)

Now, we solve eq. (ii) and (iii)

2x – 5y + 4 = 0 …(ii)

and x – 3y – 6 = 0 …(iii)

Multiplying the eq. (iii) by 2, we get

2x – 6y – 12 = 0 …(iv)

Subtracting eq. (iv) from (ii), we get

2x – 5y + 4 – 2x + 6y + 12 = 0

⇒ y + 16 = 0

⇒ y = -16

Putting the value of y = -16 in eq. (ii), we get

2x – 5(-16) + 4 = 0

⇒ 2x + 80 + 4 = 0

⇒ 2x + 84 = 0

⇒ 2x = -84

⇒ x = -42

Thus, BC and AC intersect at (-42, -16)

Now, we solve eq. (iii) and (i)

x – 3y – 6 = 0 …(iii)

3x + 2y + 6 = 0 …(i)

Multiplying the eq. (iii) by 3, we get

3x – 9y – 18 = 0 …(v)

Subtracting eq. (v) from (i), we get

3x + 2y + 6 – 3x + 9y + 18 = 0

⇒ 11y + 24 = 0

⇒ 11y = -24

$$\Rightarrow y = -\frac{24}{11}$$

Putting the value of y in eq. (iii), we get

$$x - 3\left(-\frac{24}{11}\right) - 6 = 0$$

$$\Rightarrow x + \frac{72}{11} - 6 = 0$$

$$\Rightarrow x = 6 - \frac{72}{11}$$

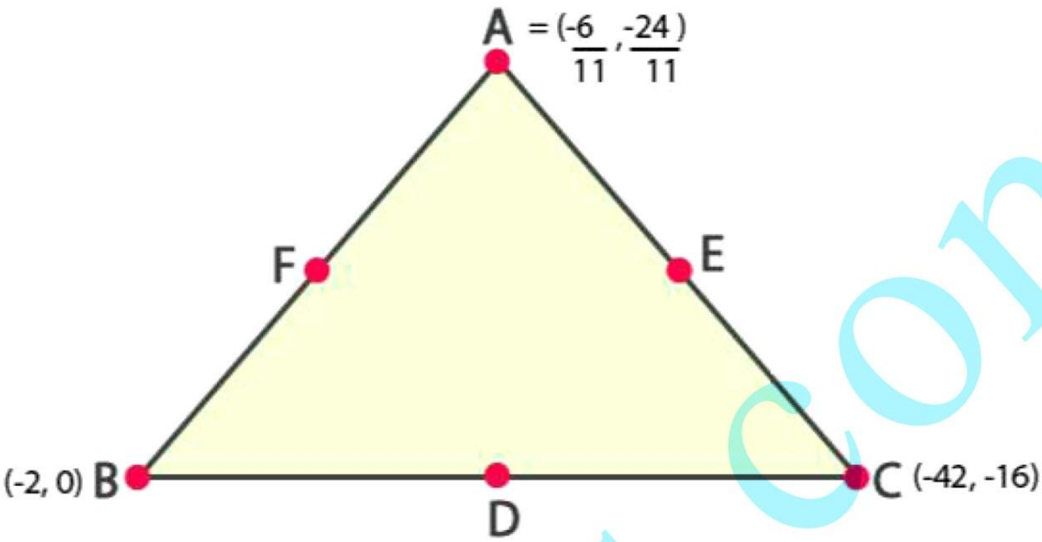
$$\Rightarrow x = \frac{66 - 72}{11}$$

$$\Rightarrow x = -\frac{6}{11}$$

$$\text{Thus, AC and AB intersect at} \left(-\frac{6}{11}, -\frac{24}{11}\right)$$

$$\text{So, vertices of triangle ABC are: } A\left(-\frac{6}{11}, -\frac{24}{11}\right), B(-2, 0) \;\&\; C(-42, -16)$$

Let D, E and F be the midpoints of sides BC, CA and AB respectively.



Then the coordinates of D, E and F are

$$\text{Coordinates of D} = \left(\frac{-42+(-2)}{2}, \frac{-16+0}{2}\right)$$

$$= \left(\frac{-42-2}{2}, -\frac{16}{2}\right)$$

$$= \left(-\frac{44}{2}, -8\right)$$

= (-22, -8)

$$\text{Coordinates of E} = \left(\frac{-42+\left(-\frac{6}{11}\right)}{2}, \frac{-16+\left(-\frac{24}{11}\right)}{2}\right)$$

$$= \left(\frac{-42-\frac{6}{11}}{2}, \frac{-16-\frac{24}{11}}{2}\right)$$

$$=\left(\frac{-462-6}{22},\frac{-176-24}{22}\right)$$

$$=\left(-\frac{468}{22},-\frac{200}{22}\right)$$

$$=\left(-\frac{234}{11},-\frac{100}{11}\right)$$

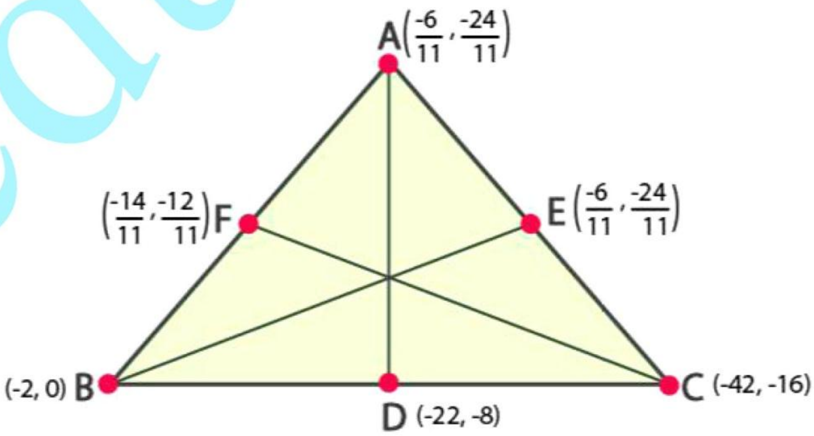
$$\text{Coordinates of F}=\left(\frac{-\frac{6}{11}+(-2)}{2},\frac{-\frac{24}{11}+0}{2}\right)$$

$$=\left(\frac{-6-22}{22},-\frac{24}{22}\right)$$

$$=\left(-\frac{28}{22},-\frac{12}{11}\right)$$

$$=\left(-\frac{14}{11},-\frac{12}{11}\right)$$

Now, we have to find the equations of Medians AD, BE and CF



The equation of median AD is

$$y-\left(-\frac{24}{11}\right)=\frac{-8-\left(-\frac{24}{11}\right)}{-22-\left(-\frac{6}{11}\right)}\left[x-\left(-\frac{6}{11}\right)\right]$$

$$\Rightarrow y+\frac{24}{11}=\frac{\frac{-88+24}{11}}{\frac{-222+6}{11}}\left(x+\frac{6}{11}\right)$$

$$\Rightarrow y+\frac{24}{11}=\frac{-64}{-216}\left(x+\frac{6}{11}\right)$$

$$\Rightarrow y+\frac{24}{11}=\frac{16}{59}\left(x+\frac{6}{11}\right)$$

$$\Rightarrow y+\frac{24}{11}=\frac{16}{59}x+\frac{96}{59\times11}$$

$$\Rightarrow \frac{16}{59}x-y=\frac{24}{11}-\frac{96}{59\times11}$$

$$\Rightarrow \frac{16x-59y}{59}=\frac{1416-96}{59\times11}$$

$$\Rightarrow 16x-59y=\frac{1320}{11}$$

$\Rightarrow 16x - 59y = 120$

The equation of the median BE is

$$y-(0)=\frac{-\frac{100}{11}-0}{-\frac{234}{11}-(-2)}\left[x-(-2)\right]$$

$$\Rightarrow y=\frac{-\frac{100}{11}}{\frac{-234+2}{11}}(x+2)$$

$$\Rightarrow y=\frac{-100}{-232}(x+2)$$

$$\Rightarrow y=\frac{25}{58}(x+2)$$

⇒ 58y = 25x + 50

⇒ 25x – 58y + 50 = 0

The equation of median AD is

$$y-(-16)=\frac{-16-\left(-\frac{12}{11}\right)}{-42-\left(-\frac{14}{11}\right)}\left[x-(-42)\right]$$

$$\Rightarrow y+16=\frac{\frac{-176+12}{11}}{\frac{-462+14}{11}}(x+42)$$

$$\Rightarrow y+16=\frac{-164}{-448}(x+42)$$

$$\Rightarrow y + 16 = \frac{41}{112}(x + 42)$$

$\Rightarrow$ 112y + 1792 = 41x + 1722

$\Rightarrow$ 41x – 112y + 1722 – 1792 = 0

$\Rightarrow$ 41x – 112y – 70 = 0

EXERCISE 20J

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Q. 1. If the origin is shifted to the point (1, 2) by a translation of the axes, find the new coordinates of the point (3, -4).

Solution : Let the new origin be (h, k) = (1, 2) and (x, y) = (3, -4) be the given point. Let the new coordinates be (X, Y)

We use the transformation formula:

x = X + h and y = Y + k

$\Rightarrow$ 3 = X + 1 and -4 = Y + 2

$\Rightarrow$ X = 2 and Y = -6

Thus, the new coordinates are (2, -6)

Q. 2. If the origin is shifted to the point (-3, -2) by a translation of the axes, find the new coordinates of the point (3, -5).

Solution : Let the new origin be (h, k) = (-3, -2) and (x, y) = (3, -5) be the given point. Let the new coordinates be (X, Y)

We use the transformation formula:

x = X + h and y = Y + k

$\Rightarrow$ 3 = X – 3 and -5 = Y – 2

$\Rightarrow$ X = 6 and Y = -3

Thus, the new coordinates are (6, -3)

Q. 3. If the origin is shifted to the point (0, -2) by a translation of the axes, the coordinates of a point become (3, 2). Find the original coordinates of the point.

Solution : Let the new origin be (h, k) = (0, -2) and (x, y) = (3, 2) be the given

point. Let the new coordinates be (X, Y)

We use the transformation formula:

$x = X + h$ and $y = Y + k$

$\Rightarrow 3 = X + 0$ and $2 = Y + (-2)$

$\Rightarrow X = 3$ and $Y = 4$

Thus, the new coordinates are (3, 4)

Q. 4. If the origin is shifted to the point (2, -1) by a translation of the axes, the coordinates of a point become (-3, 5). Find the origin coordinates of the point.

Solution : Let the new origin be (h, k) = (2, -1) and (x, y) = (-3, 5) be the given

point. Let the new coordinates be (X, Y)

We use the transformation formula:

$x = X + h$ and $y = Y + k$

$\Rightarrow -3 = X + 2$ and $5 = Y + (-1)$

$\Rightarrow X = -5$ and $Y = 6$

Thus, the new coordinates are (-5, 6)

Q. 5. At what point must the origin be shifted, if the coordinates of a point (-4,2) become (3, -2)?

Solution : Let (h, k) be the point to which the origin is shifted.

Then, $x = -4$, $y = 2$, $X = 3$ and $Y = -2$

$\therefore x = X + h$ and $y = Y + k$

$\Rightarrow -4 = 3 + h$ and $2 = -2 + k$

$\Rightarrow h = -7$ and $k = 4$

Hence, the origin must be shifted to (-7, 4)

Q. 6. Find what the given equation becomes when the origin is shifted to the point (1, 1).

$x^2 + xy - 3x - y + 2 = 0$

Solution : Let the new origin be (h, k) = (1,

1) Then, the transformation formula become:

$x = X + 1$ and $y = Y + 1$

Substituting the value of x and y in the given equation, we get

$x^2 + xy - 3x - y + 2 = 0$

Thus,

$(X + 1)^2 + (X + 1)(Y + 1) - 3(X + 1) - (Y + 1) + 2 = 0$

$\Rightarrow (X^2 + 1 + 2X) + XY + X + Y + 1 - 3X - 3 - Y - 1 + 2 = 0$

$\Rightarrow X^2 + 1 + 2X + XY - 2X - 1 = 0$

$\Rightarrow X^2 + XY = 0$

Hence, the transformed equation is $X^2 + XY = 0$

Q. 7. Find what the given equation becomes when the origin is shifted to the point (1, 1).

$xy - y^2 - x + y = 0$

Solution : Let the new origin be (h, k) = (1,

1) Then, the transformation formula become:

$x = X + 1$ and $y = Y + 1$

Substituting the value of x and y in the given equation, we get

$xy - y^2 - x + y = 0$

Thus,

$(X + 1)(Y + 1) - (Y + 1)^2 - (X + 1) + (Y + 1) = 0$

$\Rightarrow XY + X + Y + 1 - (Y^2 + 1 + 2Y) - X - 1 + Y + 1 = 0$

$\Rightarrow XY + X + Y + 1 - Y^2 - 1 - 2Y - X + Y = 0$

$\Rightarrow XY - Y^2 = 0$

Hence, the transformed equation is $XY - Y^2 = 0$

Q. 8. Find what the given equation becomes when the origin is shifted to the point (1, 1).

$x^2 - y^2 - 2x + 2y = 0$

Solution : Let the new origin be (h, k) = (1,

1) Then, the transformation formula become:

$x = X + 1$ and $y = Y + 1$

Substituting the value of x and y in the given equation, we get

$x^2 - y^2 - 2x + 2y = 0$

Thus,

$(X + 1)^2 - (Y + 1)^2 - 2(X + 1) + 2(Y + 1) = 0$

$\Rightarrow (X^2 + 1 + 2X) - (Y^2 + 1 + 2Y) - 2X - 2 + 2Y + 2 = 0$

$\Rightarrow X^2 + 1 + 2X - Y^2 - 1 - 2Y - 2X + 2Y = 0$

$\Rightarrow X^2 - Y^2 = 0$

Hence, the transformed equation is $X^2 - Y^2 = 0$

Q. 9. Find what the given equation becomes when the origin is shifted to the point (1, 1).

$xy - x - y + 1 = 0$

Solution :

Let the new origin be (h, k) = (1, 1)

Then, the transformation formula become:

x = X + 1 and y = Y + 1

Substituting the value of x and y in the given equation, we get

$xy - x - y + 1 = 0$

Thus,

$(X + 1)(Y + 1) - (X + 1) - (Y + 1) + 1 = 0$

$\Rightarrow XY + X + Y + 1 - X - 1 - Y - 1 + 1 = 0$

$\Rightarrow XY = 0$

Hence, the transformed equation is XY = 0

Q. 10. Transform the equation $2x^2 + y^2 - 4x + 4y = 0$ to parallel axes when the origin is shifted to the point (1, -2).

Solution :

Let the new origin be (h, k) = (1, -2)

Then, the transformation formula become:

x = X + 1 and y = Y + (-2) = Y – 2

Substituting the value of x and y in the given equation, we get

$2x^2 + y^2 - 4x + 4y = 0$

Thus,

$2(X + 1)^2 + (Y - 2)^2 - 4(X + 1) + 4(Y - 2) = 0$

$\Rightarrow 2(X^2 + 1 + 2X) + (Y^2 + 4 - 4Y) - 4X - 4 + 4Y - 8 = 0$

$\Rightarrow 2X^2 + 2 + 4X + Y^2 + 4 - 4Y - 4X + 4Y - 12 = 0$

$\Rightarrow 2X^2 + Y^2 - 6 = 0$

$\Rightarrow 2X^2 + Y^2 = 6$

Hence, the transformed equation is $2X^2 + Y^2 = 6$

EXERCISE 20K

Q. 1. Find the equation of the line drawn through the point of intersection of the lines x – 2y + 3 = 0 and 2x – 3y + 4 = 0 and passing through the point (4, -5).

Solution : Suppose the given two lines intersect at a point $P(x_1, y_1)$. Then, (x_1, y_1) satisfies each of the given equations.

$x - 2y + 3 = 0$ …(i)

$2x - 3y + 4 = 0$ …(ii)

Now, we find the point of intersection of eq. (i) and (ii)

Multiply the eq. (i) by 2, we get

$2x - 4y + 6 = 0$ …(iii)

On subtracting eq. (iii) from (ii), we get

$2x - 3y + 4 - 2x + 4y - 6 = 0$

$\Rightarrow y - 2 = 0$

$\Rightarrow y = 2$

Putting the value of y in eq. (i), we get

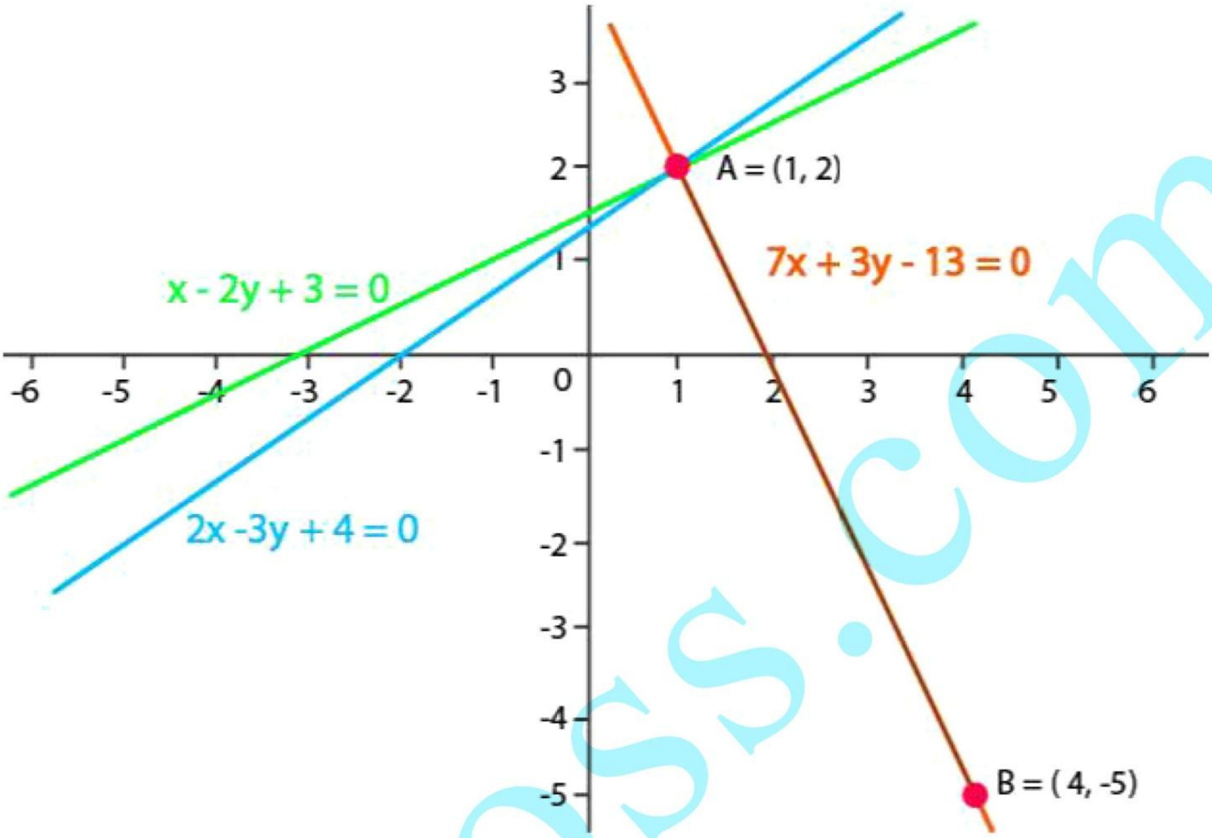
$x - 2(2) + 3 = 0$

$\Rightarrow x - 4 + 3 = 0$

$\Rightarrow x - 1 = 0$

$\Rightarrow x = 1$

Hence, the point of intersection $P(x_1, y_1)$ is (1, 2)



Let AB is the line drawn from the point of intersection (1, 2) and passing through the point (4, -5)

Firstly, we find the slope of the line joining the points (1, 2) and (4, -5)

$$\text{Slope of line joining two points} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\therefore m_{AB} = \frac{-5-2}{4-1} = \frac{-7}{3}$$

Now, we have to find the equation of line passing through point (4, -5)

Equation of line: $y - y_1 = m(x - x_1)$

$$\Rightarrow y - (-5) = -\frac{7}{3}(x - 4)$$

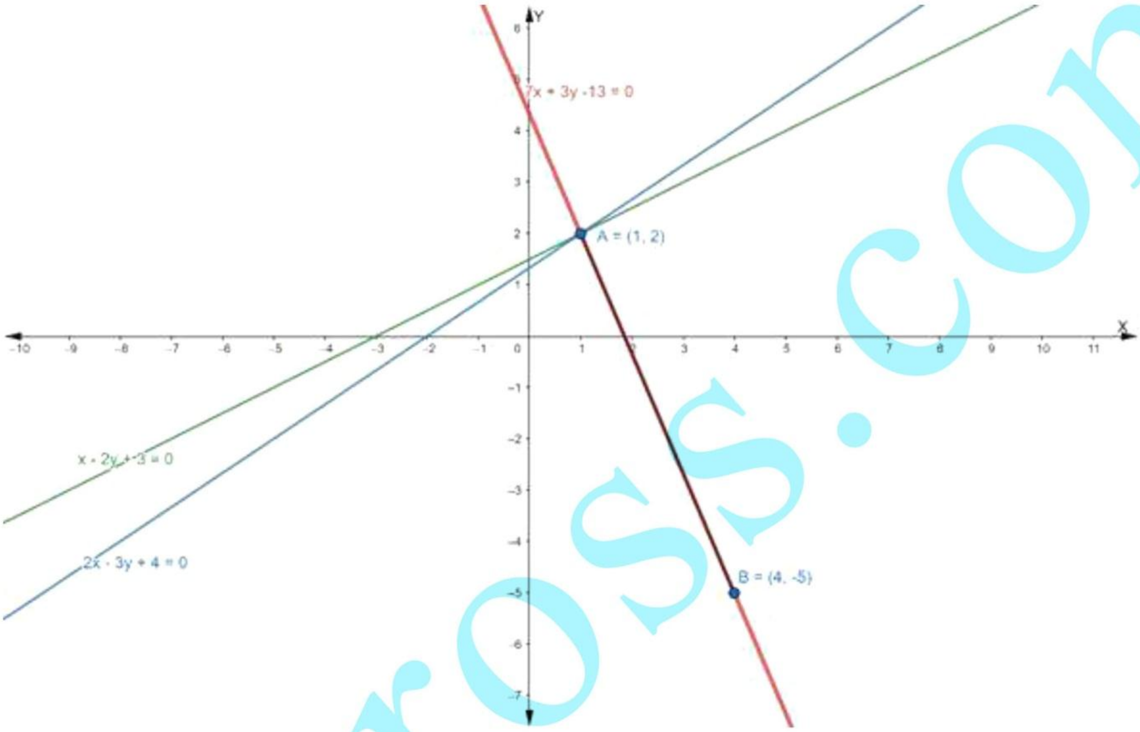
$$\Rightarrow y + 5 = -\frac{7}{3}(x - 4)$$

$\Rightarrow 3y + 15 = -7x + 28$

$\Rightarrow 7x + 3y + 15 - 28 = 0$

$\Rightarrow 7x + 3y - 13 = 0$

Hence, the equation of line passing through the point (4, -5) is $7x + 3y - 13 = 0$



Q. 2. Find the equation of the line drawn through the point of intersection of the lines x – y = 7 and 2x + y = 2 and passing through the origin.

Solution : Suppose the given two lines intersect at a point $P(x_1, y_1)$. Then, (x_1, y_1) satisfies each of the given equations.

$x - y = 7$ …(i)

$2x + y = 2$ …(ii)

Now, we find the point of intersection of eq. (i) and (ii)

Multiply the eq. (i) by 2, we get

$2x - 2y = 14$ …(iii)

On subtracting eq. (iii) from (ii), we get

$2x - 2y - 2x - y = 14 - 2$

⇒ – 3y = 12

⇒ y = -4

Putting the value of y in eq. (i), we get

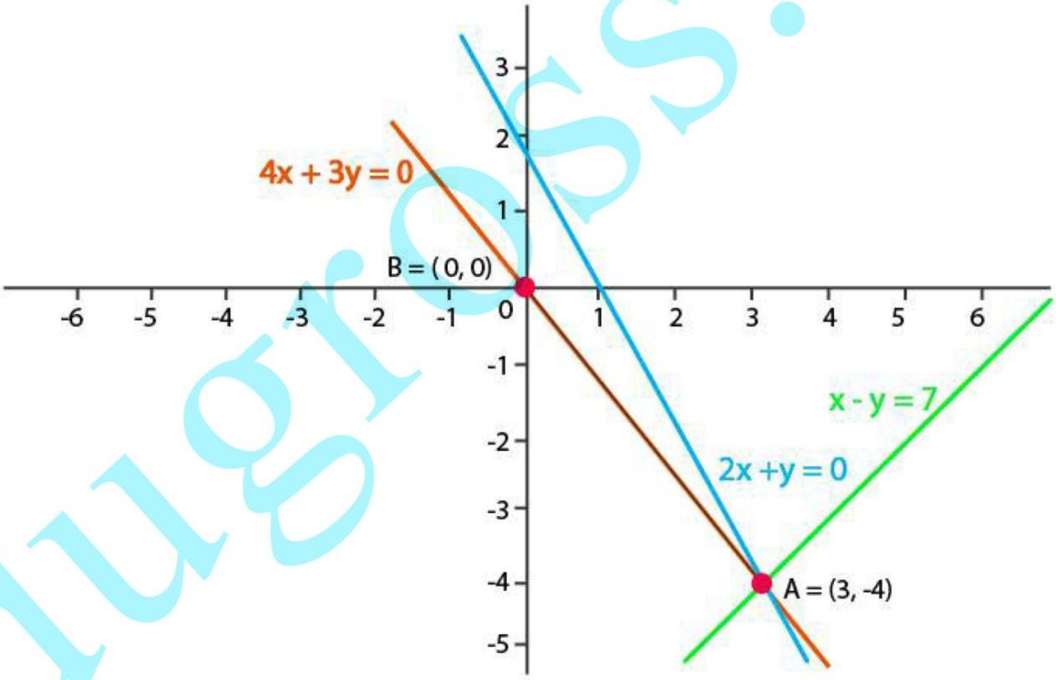
x – (-4) = 7

⇒ x + 4 = 7

⇒ x = 7 – 4

⇒ x = 3

Hence, the point of intersection P(x_1, y_1) is (3, -4)



Let AB is the line drawn from the point of intersection (3, -4) and passing through the origin.

Firstly, we find the slope of the line joining the points (3, -4) and (0, 0)

$$\text{Slope of line joining two points} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\therefore m_{AB} = \frac{0-(-4)}{0-3} = \frac{4}{-3}$$

Now, we have to find the equation of the line passing through the origin

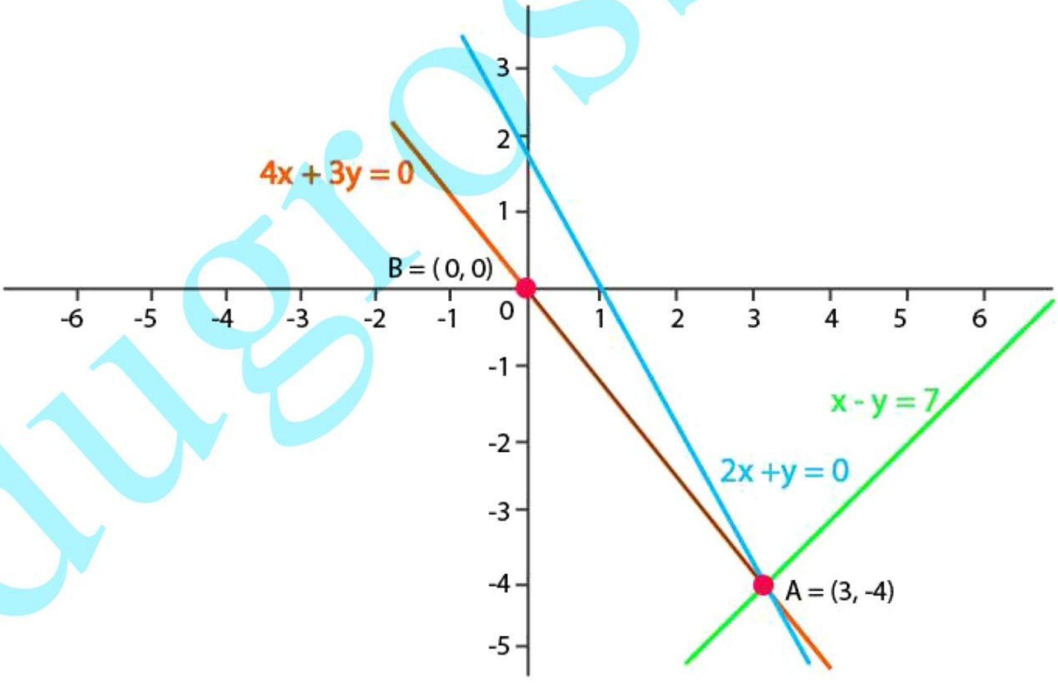
Equation of line: $y - y_1 = m(x - x_1)$

$$\Rightarrow y - 0 = -\frac{4}{3}(x - 0)$$

$$\Rightarrow 3y = -4x$$

$\Rightarrow 4x + 3y = 0$

Hence, the equation of the line passing through the origin is $4x + 3y = 0$



Q. 3. Find the equation of the line drawn through the point of intersection of the lines $x + y = 9$ and $2x - 3y + 7 = 0$ and whose slope is $\frac{-2}{3}$.

Solution : Suppose the given two lines intersect at a point $P(x_1, y_1)$. Then, (x_1, y_1) satisfies each of the given equations.

$x + y = 9$ …(i)

$2x – 3y + 7 = 0$ …(ii)

Now, we find the point of intersection of eq. (i) and (ii)

Multiply the eq. (i) by 2, we get

$2x + 2y = 18$

or $2x + 2y – 18 = 0$ …(iii)

On subtracting eq. (iii) from (ii), we get

$2x – 3y + 7 – 2x – 2y + 18 = 0$

$\Rightarrow -5y + 25 = 0$

$\Rightarrow -5y = -25$

$\Rightarrow y = 5$

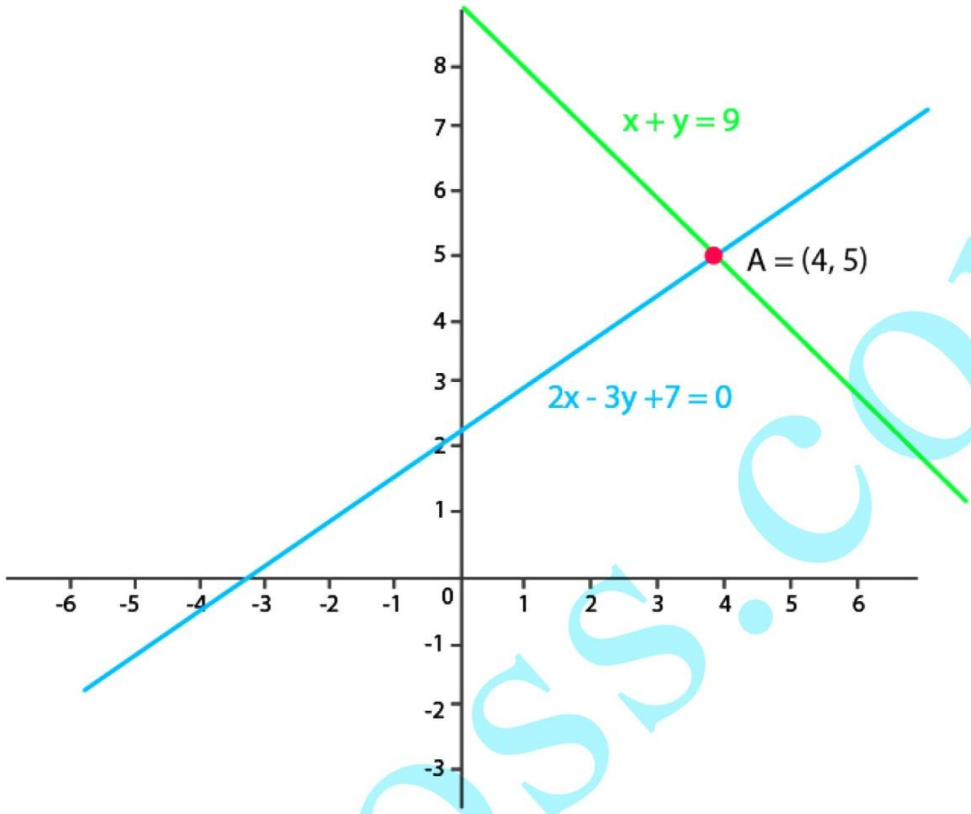
Putting the value of y in eq. (i), we get

$x + 5 = 9$

$\Rightarrow x = 9 – 5$

$\Rightarrow x = 4$

Hence, the point of intersection $P(x_1, y_1)$ is (4, 5)



Now, we have to find the equation of the line passing through the point (4, 5) and having slope $= -\frac{2}{3}$

Equation of line: $y - y_1 = m(x - x_1)$

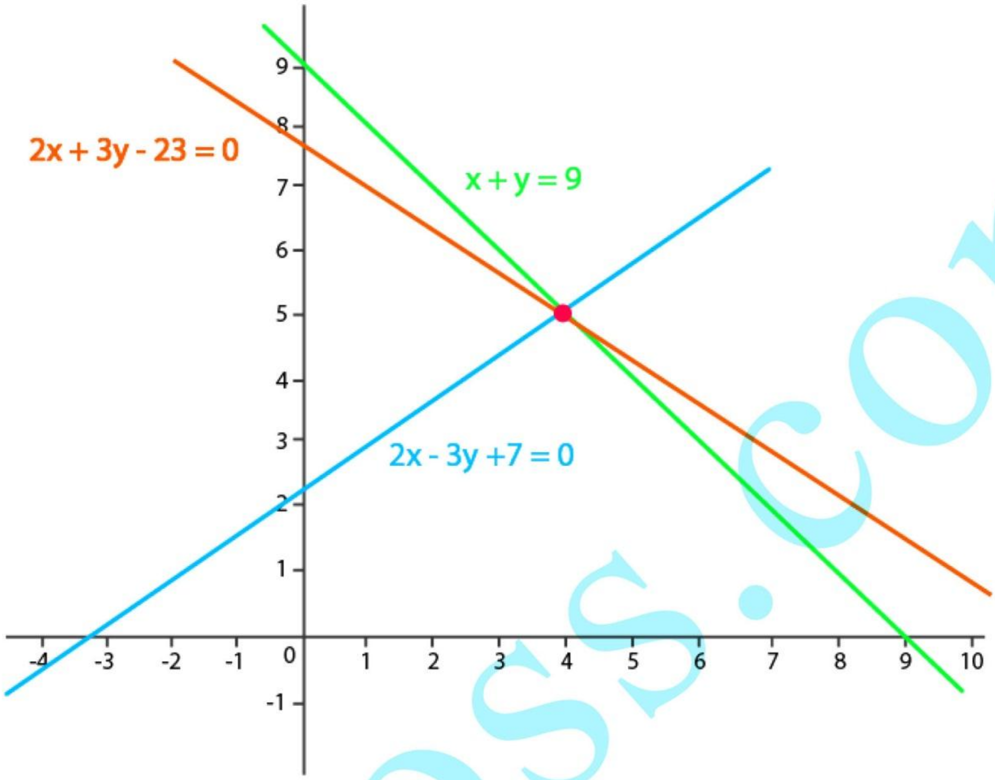
$$\Rightarrow y - 5 = -\frac{2}{3}(x - 4)$$

$$\Rightarrow 3y - 15 = -2x + 8$$

$\Rightarrow 2x + 3y - 15 - 8 = 0$

$\Rightarrow 2x + 3y - 23 = 0$

Hence, the equation of line having slope -2/3 is 2x + 3y – 23 = 0



Q. 4. Find the equation of the line drawn through the point of intersection of the lines x – y = 1 and 2x – 3y + 1 = 0 and which is parallel to the line 3x + 4y = 12.

Solution : Suppose the given two lines intersect at a point $P(x_1, y_1)$. Then, (x_1, y_1) satisfies each of the given equations.

x – y = 1 …(i)

2x – 3y + 1 = 0 …(ii)

Now, we find the point of intersection of eq. (i) and (ii)

Multiply the eq. (i) by 2, we get

2x – 2y = 2

or 2x – 2y – 2 = 0 …(iii)

On subtracting eq. (iii) from (ii), we get

2x – 3y + 1 – 2x + 2y + 2 = 0

⇒ -y + 3 = 0

⇒ y = 3

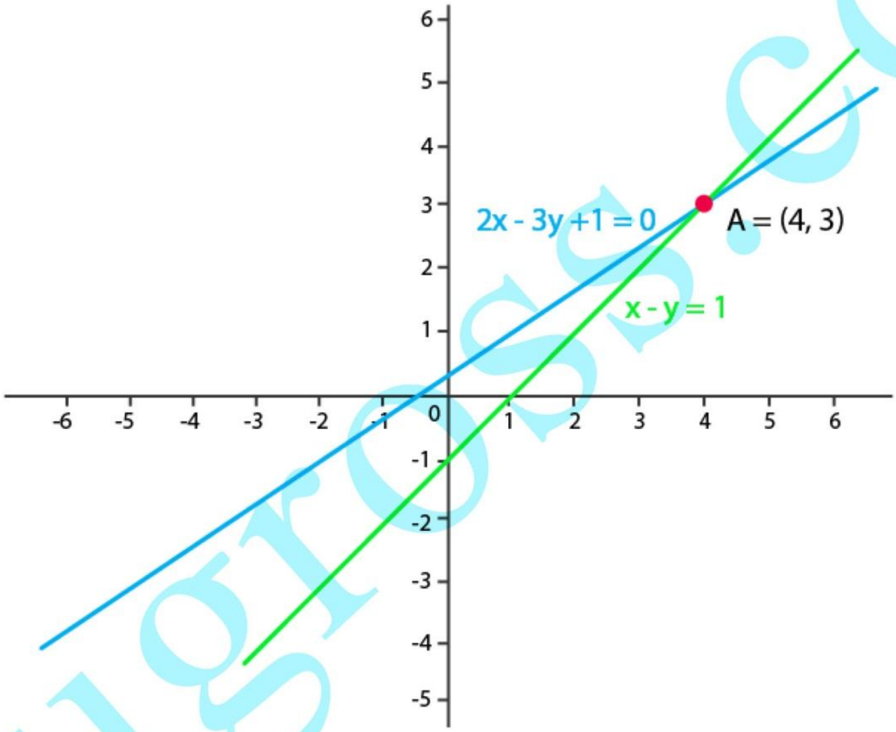
Putting the value of y in eq. (i), we get

$x - 3 = 1$

$\Rightarrow x = 1 + 3$

$\Rightarrow x = 4$

Hence, the point of intersection $P(x_1, y_1)$ is (4, 3)



Now, we find the slope of the given equation $3x + 4y = 12$

We know that the slope of an equation is

$$m = -\frac{a}{b}$$

$$\Rightarrow m = -\frac{3}{4}$$

So, the slope of a line which is parallel to this line is also $-\frac{3}{4}$

Then the equation of the line passing through the point (4, 3) having a slope $-\frac{3}{4}$ is:

$y - y_1 = m(x - x_1)$

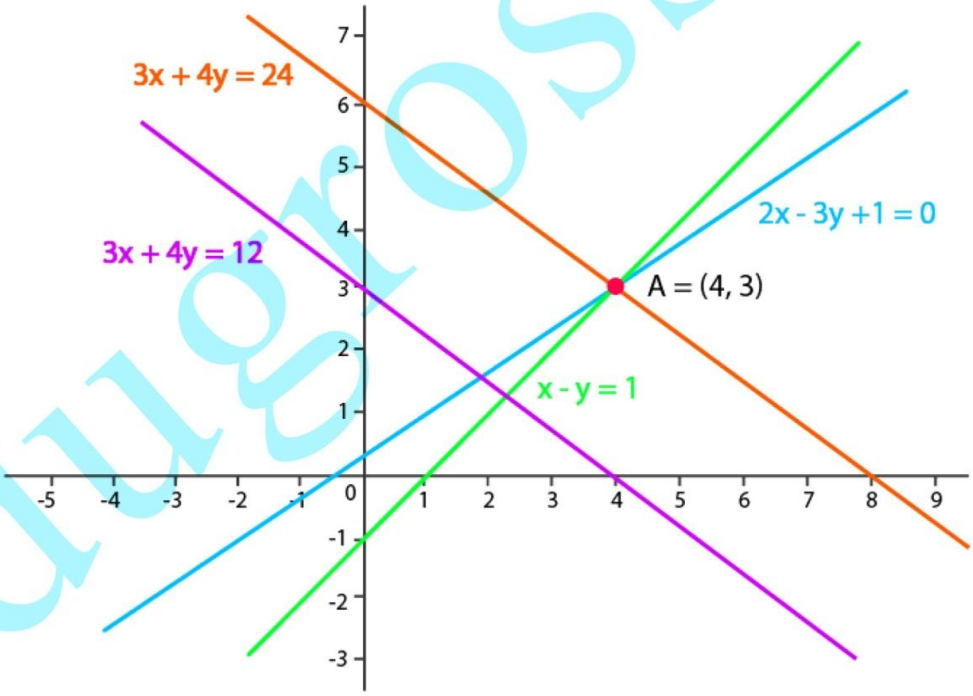
$$\Rightarrow y - (3) = -\frac{3}{4}(x - 4)$$

$\Rightarrow y - 3 = -3x + 12$

$\Rightarrow 4y - 12 = -3x + 12$

$\Rightarrow 3x + 4y - 12 - 12 = 0$

$\Rightarrow 3x + 4y - 24 = 0$



Q. 5. Find the equation of the line through the intersection of the lines 5x – 3y = 1 and 2x + 3y = 23 and which is perpendicular to the line 5x – 3y = 1.

Solution : Suppose the given two lines intersect at a point $P(x_1, y_1)$. Then, (x_1, y_1) satisfies each of the given equations.

5x – 3y = 1 …(i)

2x + 3y = 23 …(ii)

Now, we find the point of intersection of eq. (i) and (ii)

Adding eq. (i) and (ii) we get

5x – 3y + 2x + 3y = 1 + 23

⇒ 7x = 24

$$\Rightarrow x = \frac{24}{7}$$

Putting the value of x in eq. (i), we get

$$5\left(\frac{24}{7}\right) - 3y = 1$$

$$\Rightarrow \frac{120}{7} - 3y = 1$$

$$\Rightarrow -3y = 1 - \frac{120}{7}$$

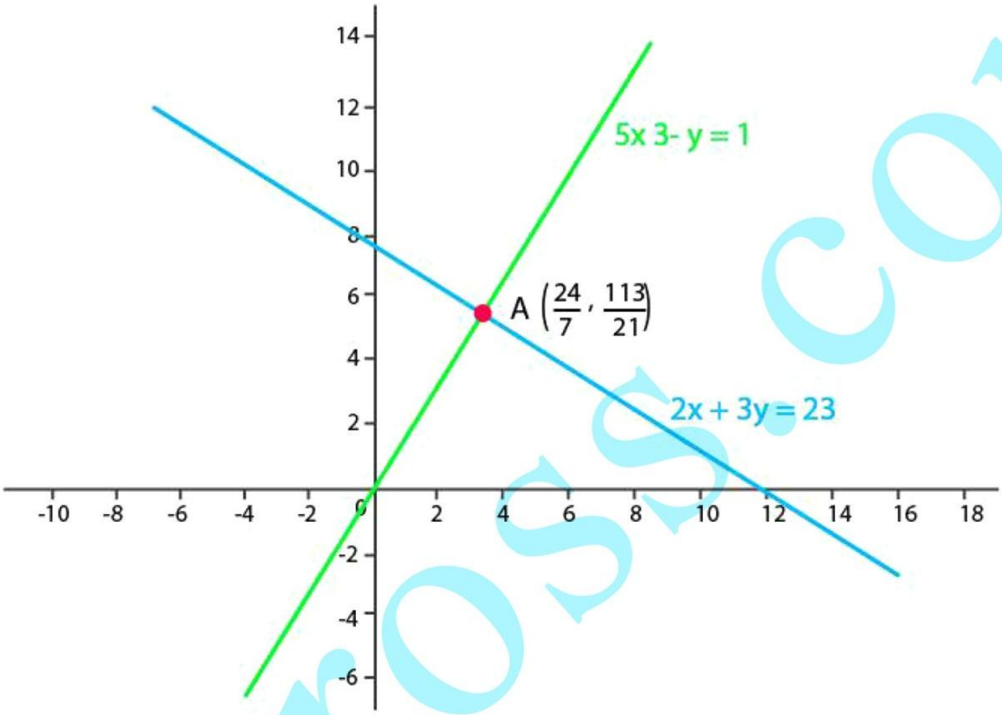
$$\Rightarrow -3y = \frac{7-120}{7}$$

$$\Rightarrow -3y = -\frac{113}{7}$$

$$\Rightarrow y = \frac{113}{21}$$

Hence, the point of intersection $P(x_1, y_1)$ is

$\left(\frac{24}{7}, \frac{113}{21}\right)$



Now, we know that, when two lines are perpendicular, then the product of their slope is equal to -1

$m_1 \times m_2 = -1$

⇒ Slope of the given line × Slope of the perpendicular line = -1

$\because \frac{5}{3} \times$ Slope of the perpendicular line = -1

⇒The slope of the perpendicular line $= -\frac{3}{5}$

So, the slope of a line which is perpendicular to the given line is $-\frac{3}{5}$

Then the equation of the line passing through the point $\left(\frac{24}{7}, \frac{113}{21}\right)$ having slope $-\frac{3}{5}$ is :

$y - y_1 = m(x - x_1)$

$$\Rightarrow y - \left(\frac{113}{21}\right) = -\frac{3}{5}\left(x - \frac{24}{7}\right)$$

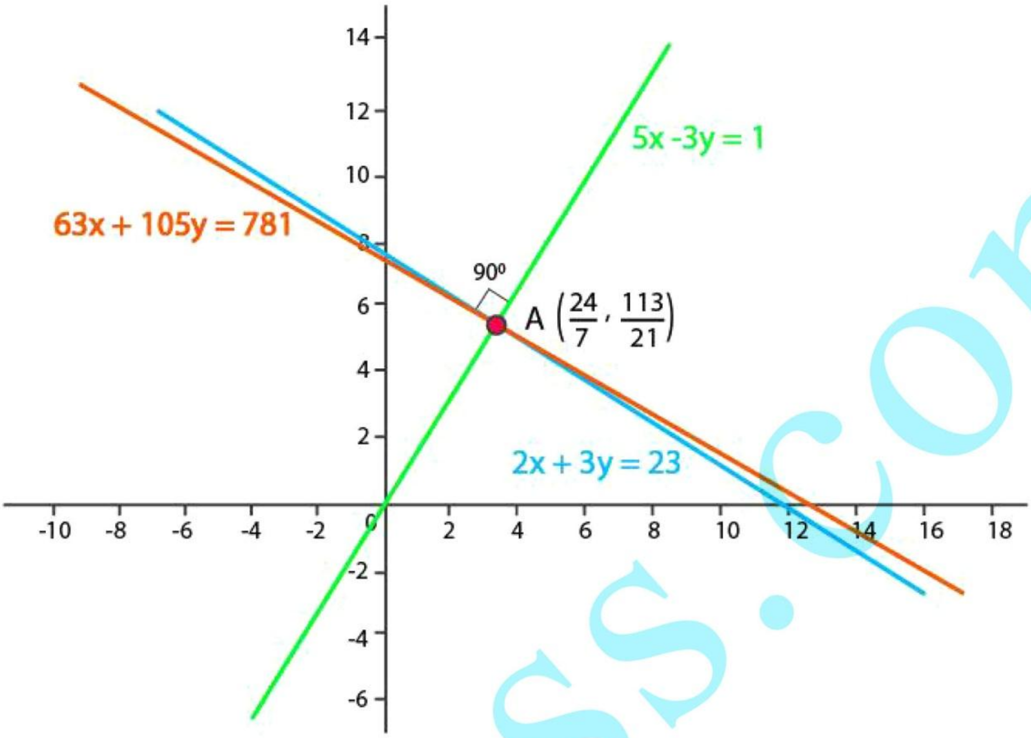
$$\Rightarrow 5y - 5 \times \frac{113}{21} = -3x + \frac{24}{7}$$

$$\Rightarrow 5y - \frac{565}{21} = -3x + \frac{72}{7}$$

$$\Rightarrow 3x + 5y - \frac{565}{21} - \frac{72}{7} = 0$$

$$\Rightarrow \frac{63x + 105y - 565 - 216}{21} = 0$$

$\Rightarrow 63x + 105y - 781 = 0$



Q. 6. Find the equation of the line through the intersection of the lines 2x – 3y = 0 and 4x – 5y = 2 and which is perpendicular to the line x + 2y + 1 = 0.

Solution : Suppose the given two lines intersect at a point $P(x_1, y_1)$. Then, (x_1, y_1) satisfies each of the given equations.

2x – 3y = 0 …(i)

4x – 5y = 2 …(ii)

Now, we find the point of intersection of eq. (i) and (ii)

Multiply the eq. (i) by 2, we get

4x – 6y = 0 …(iii)

On subtracting eq. (iii) from (ii), we get

4x – 5y – 4x + 6y = 2 – 0

⇒ y = 2

Putting the value of y in eq. (i), we get

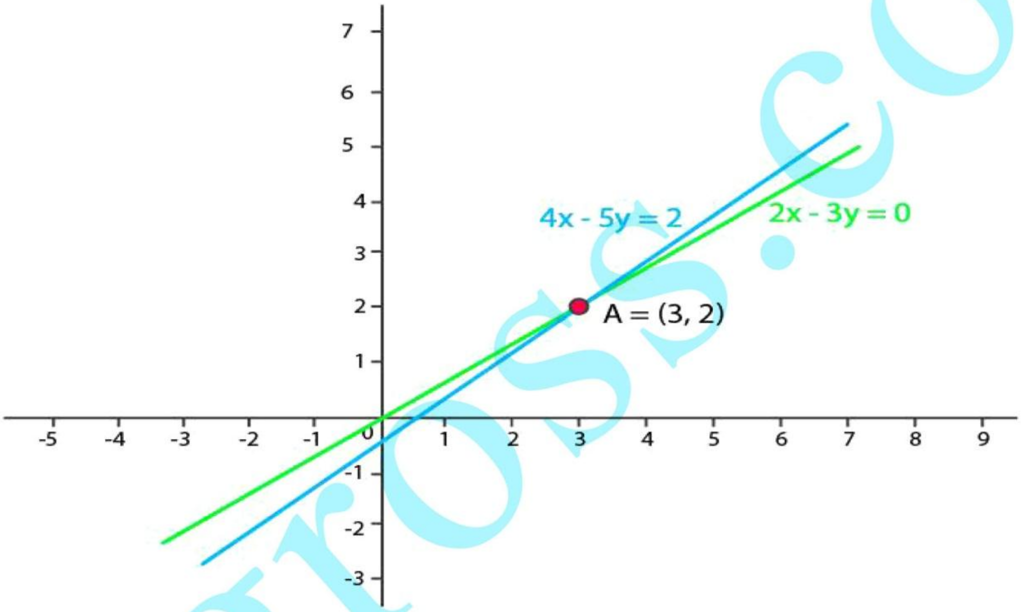
$2x - 3(2) = 0$

$\Rightarrow 2x - 6 = 0$

$\Rightarrow 2x = 6$

$\Rightarrow x = 3$

Hence, the point of intersection $P(x_1, y_1)$ is (3, 2)



Now, we know that, when two lines are perpendicular, then the product of their slope is equal to -1

$m_1 \times m_2 = -1$

$\Rightarrow$ Slope of the given line × Slope of the perpendicular line = -1

$$\therefore \left(-\frac{1}{2}\right) \times \text{Slope of the perpendicular line} = -1$$

$\Rightarrow$ The slope of the perpendicular line = 2

So, the slope of a line which is perpendicular to the given line is 2

Then the equation of the line passing through the point (3, 2) having slope 2 is:

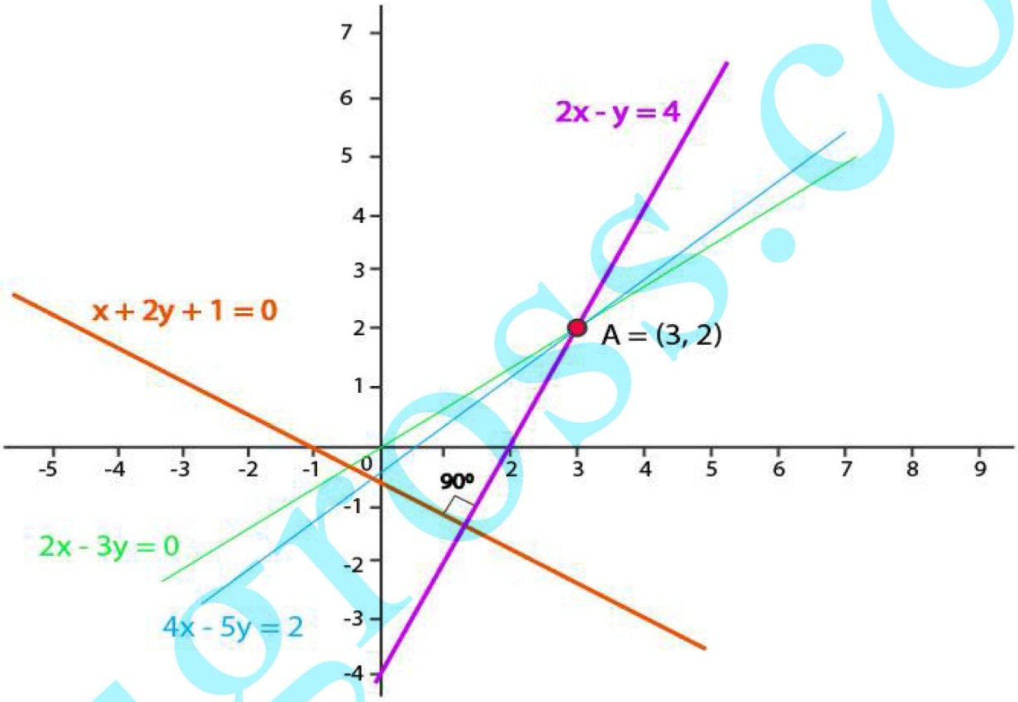
$y - y_1 = m(x - x_1)$

$\Rightarrow y - 2 = 2(x - 3)$

$\Rightarrow y - 2 = 2x - 6$

$\Rightarrow 2x - y - 6 + 2 = 0$

$\Rightarrow 2x - y - 4 = 0$



Q. 7. Find the equation of the line through the intersection of the lines x – 7y + 5 = 0 and 3x + y – 7 = 0 and which is parallel to x-axis.

Solution : Suppose the given two lines intersect at a point $P(x_1, y_1)$. Then, (x_1, y_1) satisfies each of the given equations.

$x - 7y + 5 = 0$ …(i)

$3x + y - 7 = 0$ …(ii)

Now, we find the point of intersection of eq. (i) and (ii)

Multiply the eq. (i) by 3, we get

$3x - 21y + 15 = 0$ …(iii)

On subtracting eq. (iii) from (ii), we get

$3x + y - 7 - 3x + 21y - 15 = 0$

$\Rightarrow 22y - 22 = 0$

$\Rightarrow 22y = 22$

$\Rightarrow y = 1$

Putting the value of y in eq. (i), we get

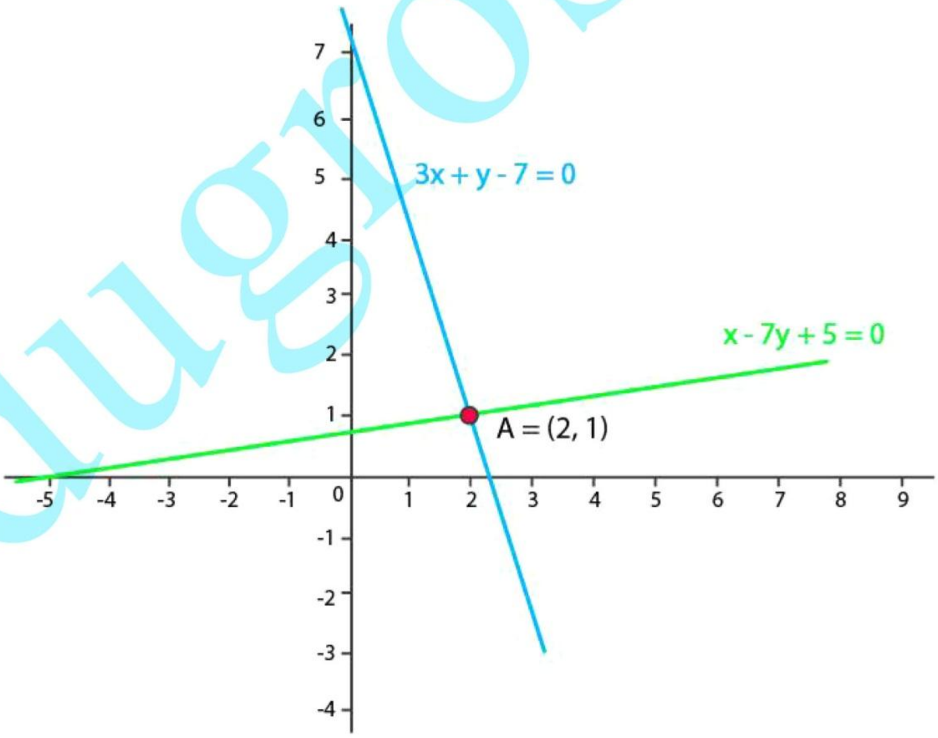
$x - 7(1) + 5 = 0$

$\Rightarrow x - 7 + 5 = 0$

$\Rightarrow x - 2 = 0$

$\Rightarrow x = 2$

Hence, the point of intersection $P(x_1, y_1)$ is (2, 1)



y = b where b is some constant

Given that this equation of the line passing through the point of intersection (2, 1)

Hence, point (2, 1) will satisfy the equation of a line.

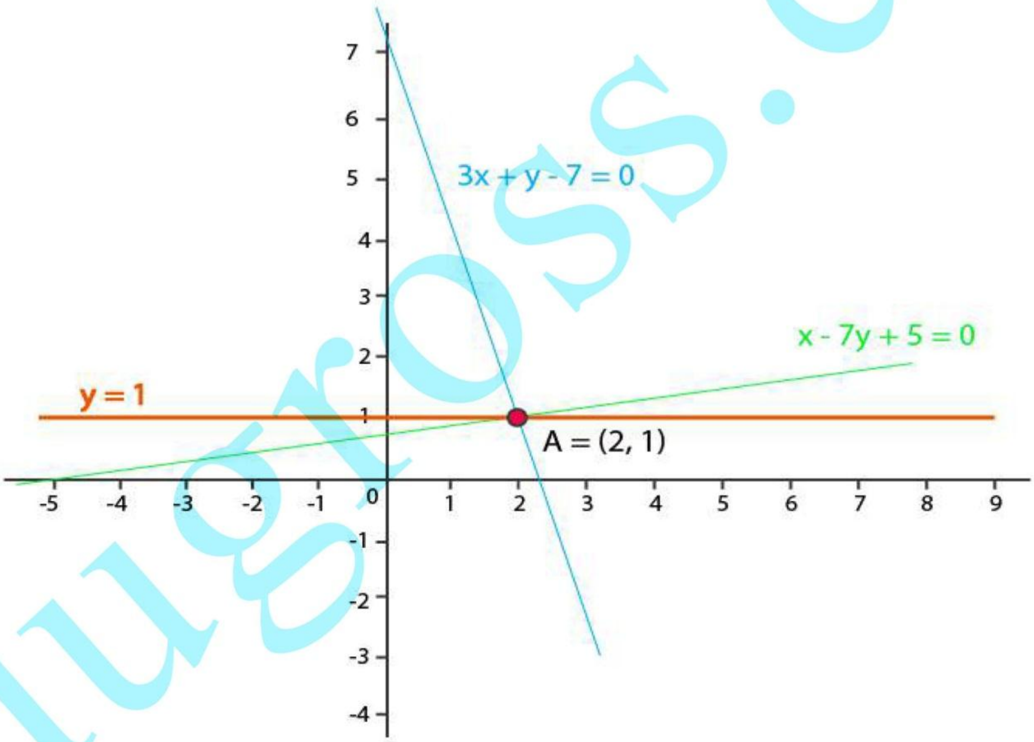
Putting y = 1 in the equation y = b, we get

y = b

⇒ 1 = b

or b = 1

Now, the required equation of a line is y = 1



Q. 8. Find the equation of the line through the intersection of the lines 2x – 3y + 1 = 0 and x + y – 2 = 0 and drawn parallel to y-axis.

Solution : Suppose the given two lines intersect at a point $P(x_1, y_1)$. Then, (x_1, y_1) satisfies each of the given equations.

$2x - 3y + 1 = 0$ …(i)

$x + y - 2 = 0$ …(ii)

Now, we find the point of intersection of eq. (i) and (ii)

Multiply the eq. (ii) by 2, we get

$2x + 2y - 4 = 0$ …(iii)

On subtracting eq. (iii) from (i), we get

$2x - 3y + 1 - 2x - 2y + 4 = 0$

$\Rightarrow -5y + 5 = 0$

$\Rightarrow -5y = -5$

$\Rightarrow y = 1$

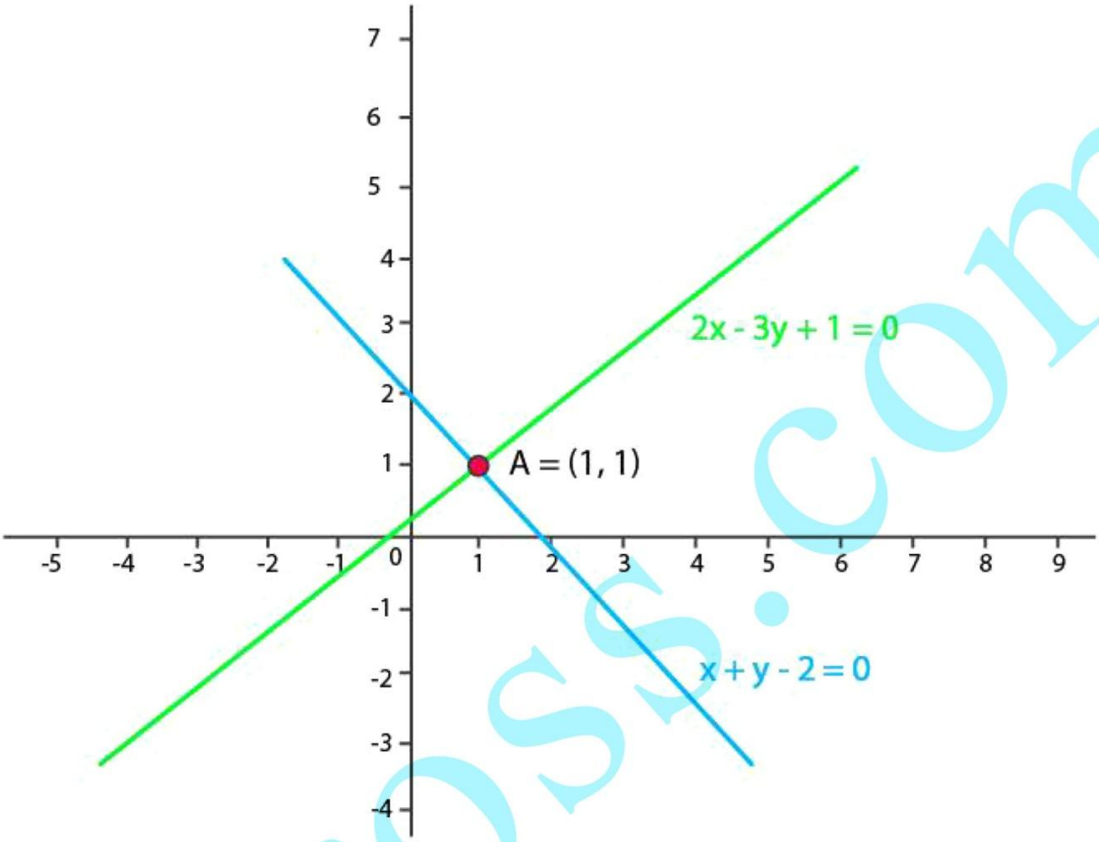
Putting the value of y in eq. (ii), we get

$x + 1 - 2 = 0$

$\Rightarrow x - 1 = 0$

$\Rightarrow x = 1$

Hence, the point of intersection $P(x_1, y_1)$ is (1, 1)



The equation of a line parallel to y – axis is of the form

x = a where a is some constant

Given that this equation of the line passing through the point of intersection (1, 1)

Hence, point (1, 1) will satisfy the equation of a line.

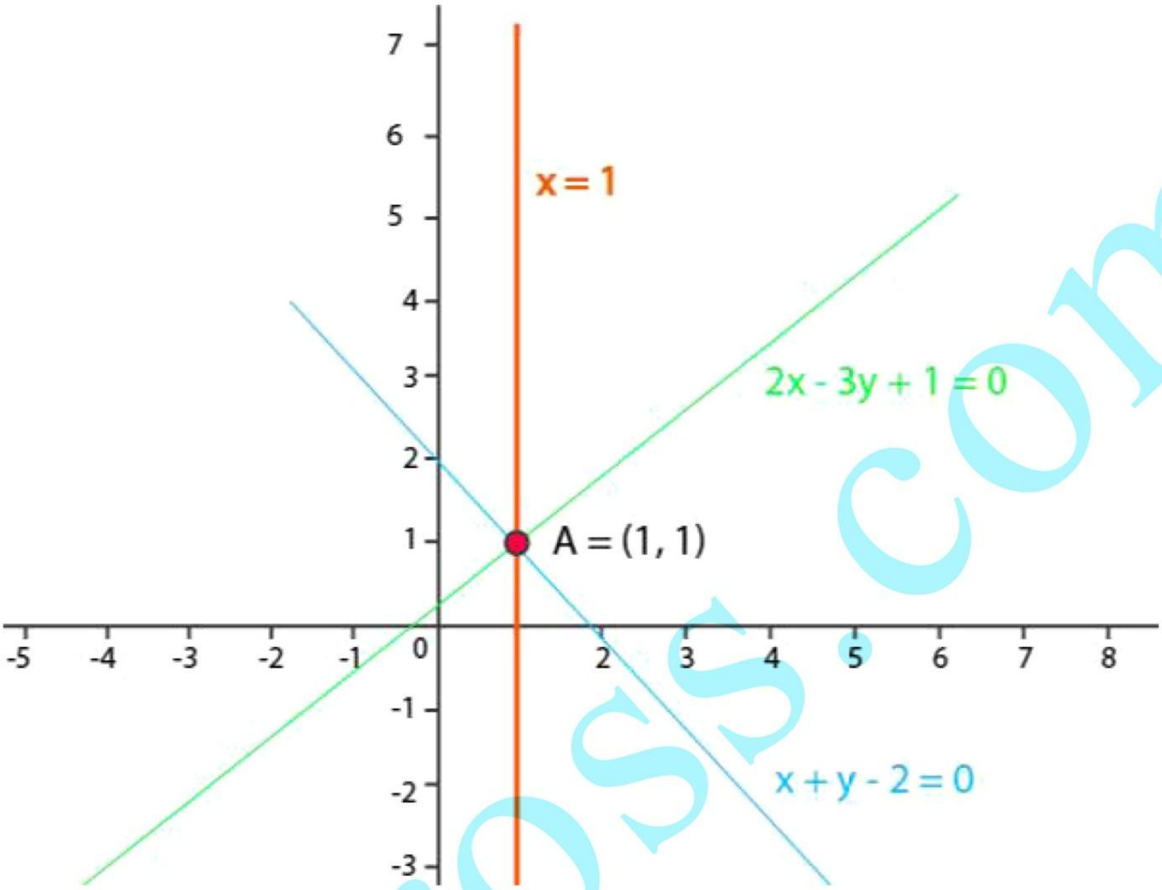
Putting x = 1 in the equation y = b, we get

$x = a$

$\Rightarrow 1 = a$

or $a = 1$

Now, required equation of line is $x = 1$



Q. 9. Find the equation of the line through the intersection of the lines 2x + 3y – 2 = 0 and x – 2y + 1 = 0 and having x-intercept equal to 3.

Solution : Suppose the given two lines intersect at a point $P(x_1, y_1)$. Then, (x_1, y_1) satisfies each of the given equations.

$2x + 3y – 2 = 0$ …(i)

$x – 2y + 1 = 0$ …(ii)

Now, we find the point of intersection of eq. (i) and (ii)

Multiply the eq. (ii) by 2, we get

$2x – 4y + 2 = 0$ …(iii)

On subtracting eq. (iii) from (i), we get

$2x + 3y – 2 – 2x + 4y – 2 = 0$

$\Rightarrow 7y – 4 = 0$

$\Rightarrow 7y = 4$

$\Rightarrow y = \frac{4}{7}$

Putting the value of y in eq. (ii), we get

$x - 2\left(\frac{4}{7}\right) + 1 = 0$

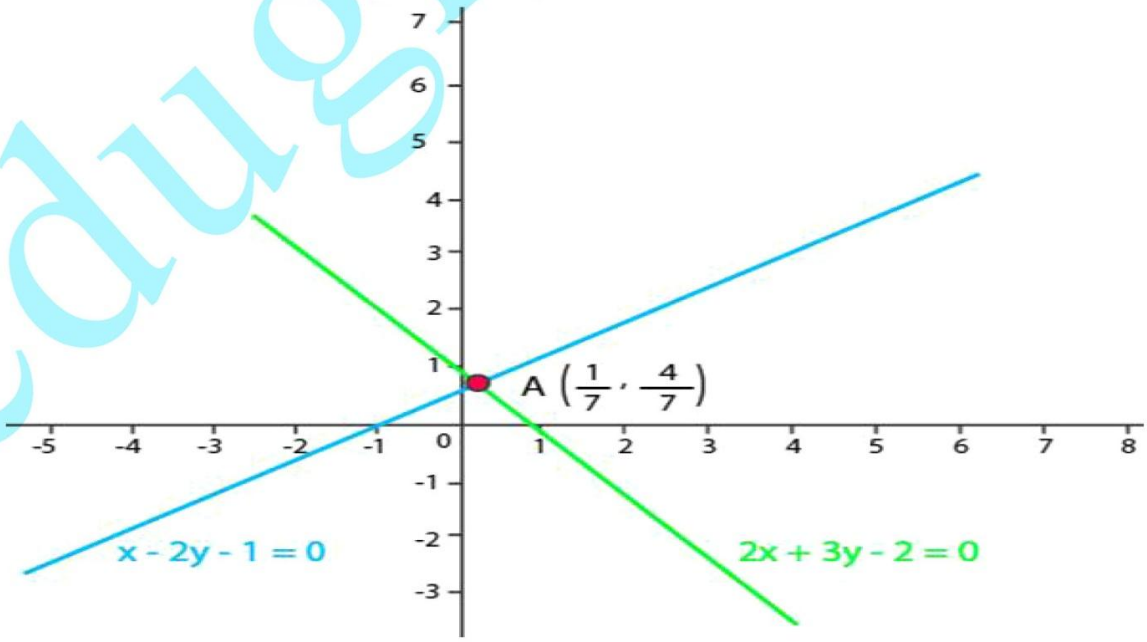
$\Rightarrow x - \frac{8}{7} + 1 = 0$

$\Rightarrow x = \frac{8}{7} - 1$

$\Rightarrow x = \frac{1}{7}$

Hence, the point of intersection $P(x_1, y_1)$ is

$\left(\frac{1}{7}, \frac{4}{7}\right)$



Now, the equation of a line in intercept form is:

$$\frac{x}{a}+\frac{y}{b}=1$$

where a and b are the intercepts on the axis.

Given that: a = 3

$$\Rightarrow \frac{x}{3}+\frac{y}{b}=1$$

$$\Rightarrow \frac{bx+3y}{3b}=1$$

⇒ bx + 3y = 3b …(i)

If eq. (i) passes through the point $\left(\frac{1}{7},\frac{4}{7}\right)$, we get

$$b\left(\frac{1}{7}\right)+3\left(\frac{4}{7}\right)=3b$$

$$\Rightarrow \frac{b+12}{7}=3b$$

⇒ b + 12 = 21b

⇒ b – 21b = - 12

⇒ 20b = 12

$$\Rightarrow b=\frac{12}{20}=\frac{3}{5}$$

Putting the value of ‘b’ in eq. (i), we get

$$\frac{3}{5}x + 3y = 3 \times \frac{3}{5}$$

$$\Rightarrow \frac{3}{5}x + 3y = \frac{9}{5}$$

$\Rightarrow 3x + 15y = 9$

$\Rightarrow x + 5y = 3$

Hence, the required equation of line is $x + 5y = 3$

Q. 10. Find the equation of the line passing through the intersection of the lines 3x – 4y + 1 = 0 and 5x + y – 1 = 0 and which cuts off equal intercepts from the axes.

Solution : Suppose the given two lines intersect at a point $P(x_1, y_1)$. Then, (x_1, y_1) satisfies each of the given equations.

$3x - 4y + 1 = 0$ …(i)

$5x + y - 1 = 0$ …(ii)

Now, we find the point of intersection of eq. (i) and (ii)

Multiply the eq. (ii) by 4, we get

$20x + 4y - 4 = 0$ …(iii)

On adding eq. (iii) and (i), we get

$20x + 4y - 4 + 3x - 4y + 1 = 0$

$\Rightarrow 23x - 3 = 0$

$\Rightarrow 23x = 3$

$$\Rightarrow x = \frac{3}{23}$$

Putting the value of x in eq. (ii), we get

$$5\left(\frac{3}{23}\right)+y-1=0$$

$$\Rightarrow \frac{15}{23}+y-1=0$$

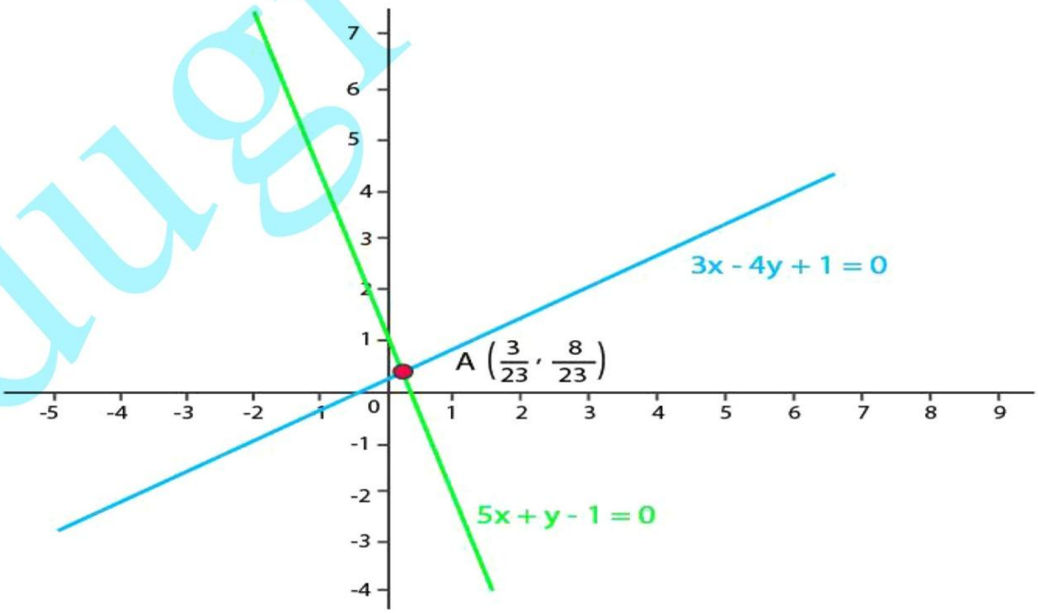
$$\Rightarrow y=1-\frac{15}{23}$$

$$\Rightarrow y=\frac{23-15}{23}$$

$$\Rightarrow y=\frac{8}{23}$$

Hence, the point of intersection P(x_1, y_1) is

$$\left(\frac{3}{23},\frac{8}{23}\right)$$



Now, the equation of line in intercept form is:

$$\frac{x}{a}+\frac{y}{b}=1$$

where a and b are the intercepts on the axis.

Given that: a = b

$$\Rightarrow \frac{x}{a}+\frac{y}{a}=1$$

$$\Rightarrow \frac{x+y}{a}=1$$

⇒ x + y = a …(i)

If eq. (i) passes through the point $\left(\frac{3}{23},\frac{8}{23}\right)$, we get

$$\frac{3}{23}+\frac{8}{23}=a$$

$$\Rightarrow \frac{11}{23}=a$$

$$\Rightarrow a=\frac{11}{23}$$

Putting the value of 'a' in eq. (i), we get

$$x+y=\frac{11}{23}$$

⇒ 23x + 23y = 11

Hence, the required line is 23x + 23y = 11