RS Aggarwal Solutions for Class 10 Maths Chapter 8 Circles

Exercise 8A

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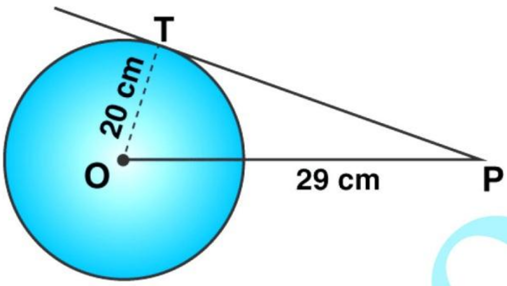
Question 1: A point P is at a distance of 29 cm from the centre of a circle of radius 20 cm. Find the length of the tangent drawn from P to the circle.

Solution:
Let O be the center of circle, OP = 29cm (given)
Radius of a circle = 20 cm
Let T be any point on the circumference of the circle, then PT is the tangent to the circle.



From Figure:

OT is radius and PT is the tangent
This implies, OT ⊥ PT

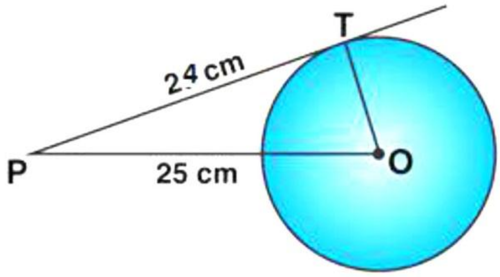
In right ΔOPT,
By Pythagoras Theorem:
$OP^2 = OT^2 + PT^2$
$(29)^2 = (20)^2 + PT^2$
$841 = 400 + PT^2$
$PT^2 = 441$
or PT = 21
Length of tangent PT is 21 cm

Question 2: A point P is 25 cm away from the centre of a circle and the length of tangent drawn from P to the circle is 24 cm. Find the radius of the circle.

Solution:

Let O be the center of circle, OP = 25 cm (given)
Let T be the any point on the circle, then PT = 24 cm

To Find: Radius of the circle



From Figure:

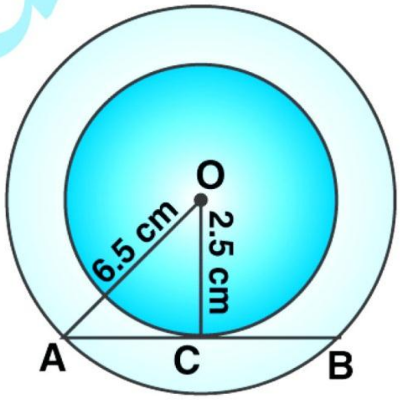
OT ⊥ PT

In right ΔOPT,
By Pythagoras Theorem:
$OP^2 = OT^2 + PT^2$
$25^2 = OT^2 + (24)^2$
$625 = OT^2 + 576$
OT = 7
Radius of the circle is 7 cm.

Question 3: Two concentric circles are of radii 6.5 cm and 2.5 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Solution:

Let O be the center of circle. Draw two concentric circles are of radii 6.5 cm and 2.5 cm, and
AB = Chord of the larger circle which touches the smaller circle at C.



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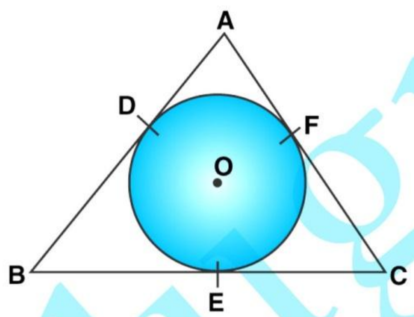
From Figure:
OC = radius = 2.5 cm
OA = 6.5 cm
AC = CB
OC ⊥ AB and OC bisects AB at C.

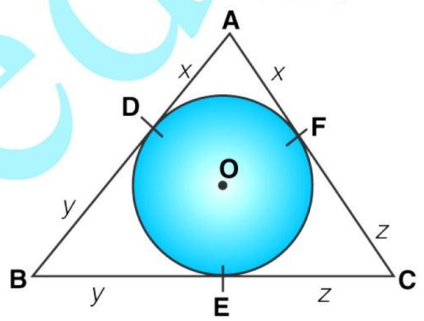
In right ΔOPT,
By Pythagoras Theorem:
$OA^2 = OC^2 + AC^2$
$6.5^2 = 2.5^2 + AC^2$
$42.25 = 6.25 + AC^2$
AC = 6

Length of chord of a circle = AB = 2xAC = 2 x 6 = 12 cm.

Question 4: In the given figure, a circle inscribed in a triangle ABC, touches the sides AB, BC and AC at points D, E and F respectively. If AB = 12 cm, BC = 8 cm and AC = 10 cm, find the lengths of AD, BE and CF.



Solution:



AB = 12 cm, BC = 8 cm and AC = 10 cm

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To Find: Lengths of AD, BE and CF

AD and AF are tangents to the circle from A.
AD = AF = x
Similarly, BD and BE are tangents to the circle.
BD = BE = y
and CE and CF are tangents to the circle
CE = CF = z
x + y + 12 ......(1)
y + z = 8 .....(2)
z + x = 10 .....(3)

Adding (1), (2) and (3), we get
2(x + y + z) = 12 + 8 + 10 = 30
x + y + z = 15 .....(4)
Subtract (1) from (4): z = 3
Subtract (2) from (4): x = 7
Subtract (3) from (4): y = 5
Therefore,
AD = 7 cm, BE = 5 cm and CF = 3 cm

Question 5: In the given figure, PA and PB are the tangent segments to a circle with centre O. Show that the points A, O, B and P are concyclic.

Solution:

PA and PB are tangents to a circle with center O (given)

To show: Points A, O, B and P are concyclic.

Since OB $\perp$ PB and OA $\perp$ AP

$\angle OBP = \angle OAP = 90°$

$\angle OBP + \angle OAP = 90 + 90 = 180°$

[Sum of opposite angles in a quadrilateral is 180°]

AOBP is a cyclic quadrilateral, thus A, O, B and P are concyclic.
Hence proved.

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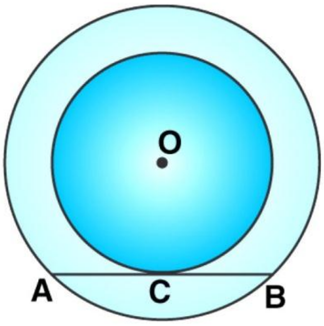
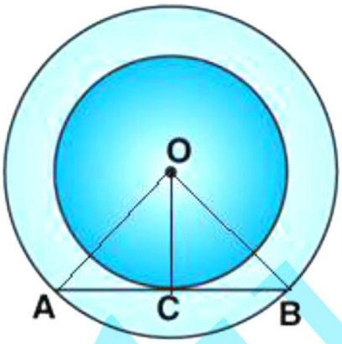
Question 6: In the given figure, the chord AB of the larger of the two concentric circles, with centre O, touches the smaller circle at C. Prove that AC = CB.



Solution:
In the given figure, chord AB of larger circle of the two concentric circles with centre O, touches the smaller circle at C.
To Prove: AC = CB

Join OC, OA and OB.



Now,
AB is tangent to the smaller circles and OC is the radius.
this implies, OC ⊥ AB.

We have two right angled triangles: ΔOAC and ΔOBC

Here OA = OB = radius of same circle
Side OC = OC = common among both the triangles
ΔOAC = ΔOBC (RHS axiom)

By c.p.c.t.

AC = CB
Hence proved.

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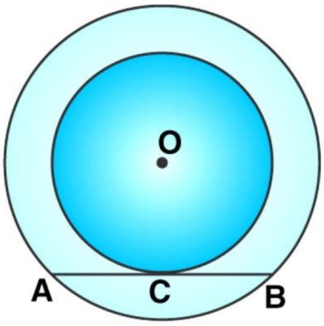
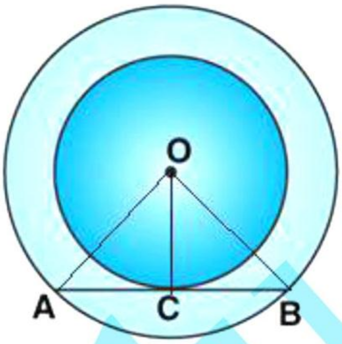
Question 6: In the given figure, the chord AB of the larger of the two concentric circles, with centre O, touches the smaller circle at C. Prove that AC = CB.



Solution:
In the given figure, chord AB of larger circle of the two concentric circles with centre O, touches the smaller circle at C.
To Prove: AC = CB

Join OC, OA and OB.



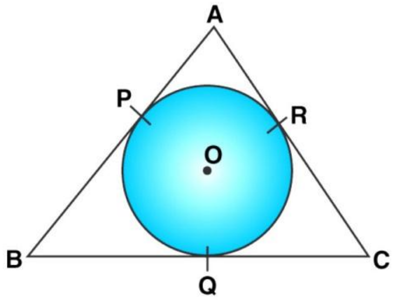
Now,
AB is tangent to the smaller circles and OC is the radius.
this implies, OC ⊥ AB.

We have two right angled triangles: ΔOAC and ΔOBC

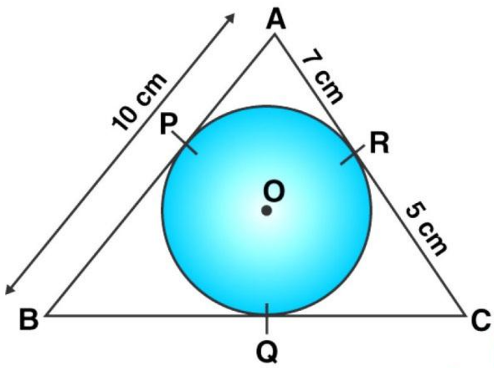
Here OA = OB = radius of same circle
Side OC = OC = common among both the triangles
ΔOAC = ΔOBC (RHS axiom)

By c.p.c.t.

AC = CB
Hence proved.



Solution:
AB = 10 cm, AR = 7 cm and CR = 5 cm



AP and AR are the tangents to the circle
AP = AR = 7 cm
From figure: BP = AB – AP = 10 – 7 = 3 cm
Again, BP and BQ are the tangents to the circle, we have
BQ = BP = 3 cm
Similarly, CQ = CR = 5 cm

Now,
BC = CQ + BQ = 5 + 3 = 8

The length of BC is 8 cm.

Question 9: In the given figure, a circle touches all the four sides of a quadrilateral ABCD whose three sides are AB = 6 cm, BC = 7 cm and CD = 4 cm. Find AD.

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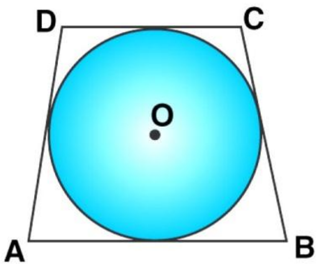
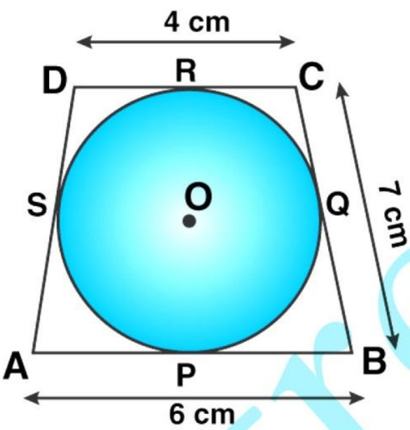


Solution:

Here: AB = 6 cm, BC = 7 cm and CD = 4 cm

Let P, Q, R and S are 4 points touches the sides of a quadrilateral.



We know, tangents from an external point to a circle are always equal.

Therefore, AP = AS, BP = BQ, CR = CQ and DR = DS

Now from figure, AP + BP + CR + DR = AS + BQ + CQ + DC

(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)

AB + CD = AD + BC

6 + 4 = AD + 7

AD = 3

AD = 3 cm. Answer!!

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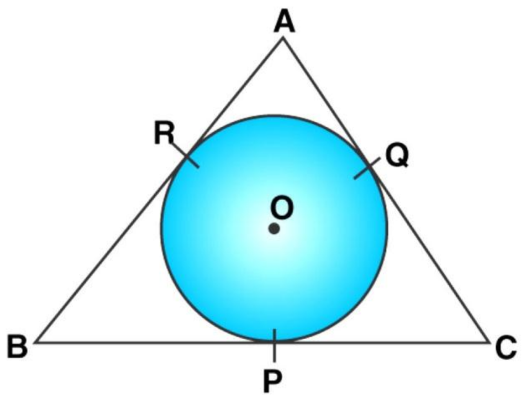
Question 10: In the given figure, an isosceles triangle ABC, with AB = AC, circumscribes a circle. Prove that the point of contact P bisects the base BC.



Solution:

ΔABC is an isosceles triangle where AB = AC and circumscribed a circle.
The circle touches its sides BC, CA and AB at P, Q and R respectively.

To Prove : P bisects the base BC, i.e. BP = PC

Now form figure,

BR and BP are tangents to the circle.
So, BR = BP .....(1)

AR and AQ are tangents to the circle.
AR = AQ But AB = AC
AB – AR = AC – AQ
BR = CQ .....(2)

Similarly, CP and CQ are tangents to the circle.

CP = CQ ......(3)

From (1), (2) and (3)

BP = PC
Hence Proved.

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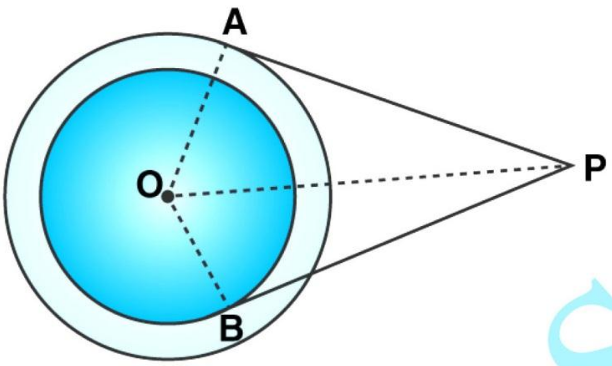
Question 11: In the given figure, O is the centre of two concentric circles of radii 4 cm and 6 cm respectively. PA and PB are tangents to the outer and inner circle respectively. If PA = 10 cm, find the length of PB up to one place of decimal.



Solution:
In the given figure,
PA and PB are the tangents drawn from P, to the outer circle and inner circle respectively.
PA = 10 cm
OA and OB are the radii and
PA and PB are two tangents to the circles respectively
So,
OA ⊥ PA and OB ⊥ PB

In right ΔOAP,
By Pythagoras Theorem:
$OP^2 = OA^2 + PA^2$
$= (6)^2 + (10)^2$
$OP^2 = 136$ ...(1)

From right ΔOBP,
$OP^2 = OB^2 + PB^2$
$136 = (4)^2 + PB^2$
$136 = 16 + PB^2$
[Using equation (1)]
We have $PB^2 = 136 – 16 = 120$
Or $PB = \sqrt{120}$ cm $= 2\sqrt{30}$ cm $= 2 \times 5.47 = 10.9$

Answer: Length of PB is 10.9 cm

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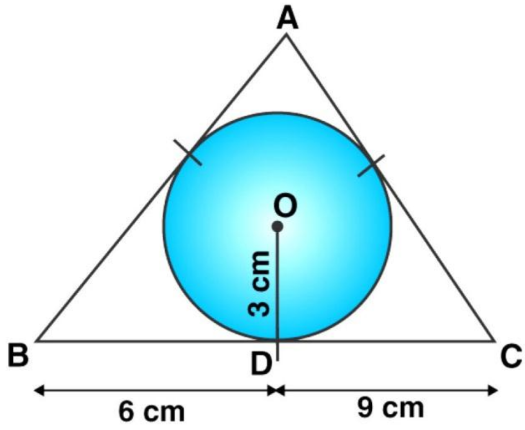
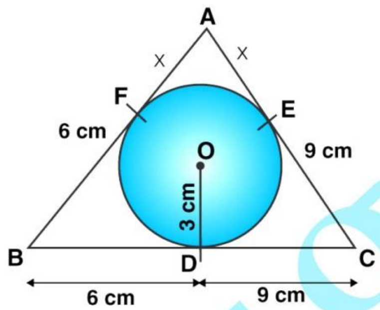
Question 12: In the given figure, a triangle ABC is drawn to circumscribe a circle of radius 3 cm such that the segments BD and DC into which BC is divided by the point of contact D, are of lengths 6 cm and 9 cm respectively. If the area of ΔABC = 54 cm² then find the lengths of sides AB and AC.



Solution:



In the given figure, ΔABC circumscribed the circle with centre O.
Radius OD = 3 cm
BD = 6 cm, DC = 9 cm
Area of ΔABC = 54 cm²
To find : Lengths of AB and AC.

AF and EA are tangents to the circle at point A.
Let AF = EA = x

BD and BF are tangents to the circle at point B.
BD = BF = 6 cm

CD and CE are tangents to the circle at point C.
CD = CE = 9 cm

Now, new sides of the triangle are:
AB = AF + FB = x + 6 cm

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AC = AE + EC = x + 9 cm
BC = BD + DC = 6 + 9 = 15 cm

Now, using Heron's formula:

Area of triangle ABC = $\sqrt{s(s-a)(s-b)(s-c)}$

Where S = $\frac{a+b+c}{2}$

S = 1/2(x + 6 + x + 9+15) = x + 15

Area of ABC =

$$\sqrt{(x+15)(x+15-(x+6))(x+15-(x+9))(x+15-15)}$$

Or

$$54 = \sqrt{(x+15)(9)(6)(x)}$$

Squaring both sides, we have

54^2 = 54x(x + 15)
x^2 + 15x - 54 = 0
Solve this quadratic equation and find the value of x.
x^2 + 18x - 3x - 54 = 0
x(x + 18) - 3(x + 18) = 0
(x - 3)(x + 18) = 0
Either x = 3 or x = - 18
But x cannot be negative.
So, x = 3

Answer:
AB = x + 6 = 3 + 6 = 9 cm
AC = x + 9 = 3 + 9 = 12 cm

Question 13: PQ is a chord of length 4.8 cm of a circle of radius 3 cm. The tangents at P and Q intersect at a point T as shown in the figure. Find the length of TP.

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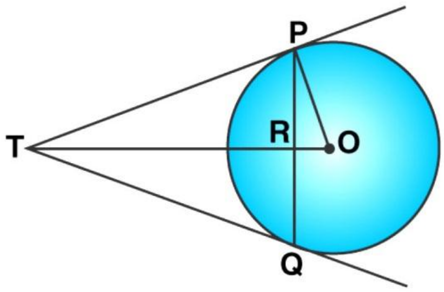

Solution:

Radius of the circle is 3 cm and PQ = 4.8 cm.
PQ is a chord of the circle with centre O.
The tangents at P and Q intersect at point T
So, TP and TQ are tangents
OP and OT are joined.

Join OQ

We have two right triangles: △OPT and △OQT

Here,
OT = OT (Common)

PT = QT (tangents of the circle)

OP = OQ (radius of the same circle)

By Side - Side - Side Criterion

△OPT ≅△OQT

Therefore, ∠POT = ∠OQT

Again, from triangles △OPR and △OQR

OR = OR (Common)

OP = OQ (radius of the same circle)

∠POR = ∠OQR (from above result)

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By Side - Angle - Side Criterion

△OPR ≅△OQT

Therefore, ∠ORP = ∠ORQ
Now,

∠ORP + ∠ORQ = 180°
(Sum of linear angles = 180 degrees)
∠ORP + ∠ORP = 180°
∠ORP = 90°
This implies, OR ⊥ PQ and RT ⊥ PQ

Also OR perpendicular from center to a chord bisects the chord,

PR = QR = PQ/2 = 4.8/2 = 2.4 cm

Applying Pythagoras Theorem on right triangle △OPR,

(OP)^2 = (OR)^2 + (PR)^2
(3)^2 = (OR)^2 + (2.4)^2
OR = 1.8 cm

Applying Pythagoras Theorem on right angled △TPR,

(PT)^2 = (PR)^2 + (TR)^2 ...(1)

Also, OP ⊥ OT

Applying Pythagoras Theorem on right △OPT,
(PT)^2 + (OP)^2 = (OT)^2
[(PR)^2 + (TR)^2] + (OP)^2 = (TR + OR)^2

(Using equation (1) and from figure)

(2.4)^2 + (TR)^2 + (3)^2 = (TR + 1.8)^2
4.76 + (TR)^2 + 9 = (TR)^2 + 2(1.8)TR + (1.8)^2
13.76 = 3.6 TR + 3.24
TR = 2.9 cm [approx.]

From (1) => PT^2 = (2.4)^2 + (2.9)^2
PT^2 = 4.76 + 8.41

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or PT = 3.63 cm [approx.]

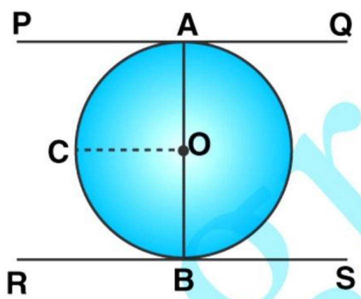
Answer: Length of PT is 3.63 cm.

Question 14: Prove that the line segment joining the points of contact of two parallel tangents of a circle, passes through its centre.

Solution:
Let O is the centre of the circle.
Let PQ and RS are two parallel tangents which touches the circle at points A and B.
OA and OB are joined.

To Prove : AB passes through point O.

Draw OC || RS ||PQ



Now,
OA = OB = Radius and
PQ is tangent of circle passing through A
OA ⊥ PQ
Which implies, ∠OAP = 90°

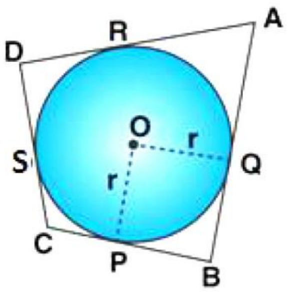
RS is the tangent of circle passing through B
OB ⊥ RS
Which implies, ∠OBR = 90°

Since PQ || OC
∠AOC + ∠OAP = 180° (Co-interior angles)
∠AOC + 90° = 180°
∠AOC = 180° – 90° = 90°
Similarly, ∠BOC = 90°

∠AOC + ∠BOC = 90° + 90° = 180°

AOB is a straight line. Therefore, AB passes through the centre of the circle.

Question 15: In the given figure, a circle with centre O, is inscribed in a quadrilateral ABCD such that it touches the side BC, AB, AD and CD at points P, Q, R and S respectively. If AB = 29 cm, AD = 23 cm, ∠B = 90° and DS = 5 cm then find the radius of the circle.



Solution:
In the given figure,
O be the centre of circle which is inscribed in a quadrilateral ABCD.
And OP = OQ = r = radius of circle
The circle touches the sides of quadrilateral at P, Q, R and S respectively.
AB = 29 cm, AD = 23 cm, ∠B = 90°
DS = 5 cm
Join OP and OQ.
Now,
OP = OQ = r and ∠B = 90°
So, PBQO is a square.
DR and DS are the tangents to the circle.
DR = DS = 5 cm
AQ and AR are tangents to the circle.
AR = AD – DR = 23 – 5 = 18 cm
AQ = AR = 18 cm
And BQ = AB – AQ = 29 – 18 = 11 cm
Since PBQO is a square.
OP = OQ = BQ = 11 cm
Hence, radius of the circle is 11 cm

Question 16: In the given figure, O is the centre of the circle and TP is the tangent to the circle from an external point T. If ∠PBT = 30°, prove that BA : AT = 2 : 1.

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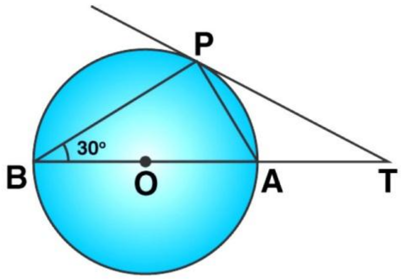

Solution:

In the given figure,
TP is the tangent from an external point T and
∠PBT = 30°

Now,
∠APB = 90° (Angle in a semicircle)
∠PBT = 30° (given)
So, ∠PAB = 90° – 30° = 60°
But, ∠PAT + ∠PAB = 180° (Linear pair)
∠PAT + 60° = 180°
∠PAT = 180° – 60° = 120°
Also, ∠APT = ∠PBA = 30° (Angles in the alternate segment)
In ΔPAT,
∠PTA = 180° – (120° + 30°) = 180° – 150° = 30°
PA = AT
In right ΔAPB,
sin 30^0 = AP/AB

1/2 = AP/AB

AB = 2 AP

Since AP = AT

AB = 2 AT

or AB/AT = 2/1

AB:AT = 2:1
or BA:AT = 2:1. Hence Proved.

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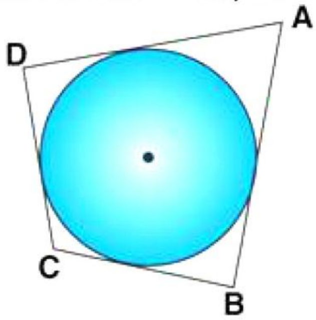
Question 1: In the adjoining figure, a circle touches all the four sides of a quadrilateral ABCD whose sides are AB = 6 cm, BC = 9 cm and CD = 8 cm. Find the length of side AD.

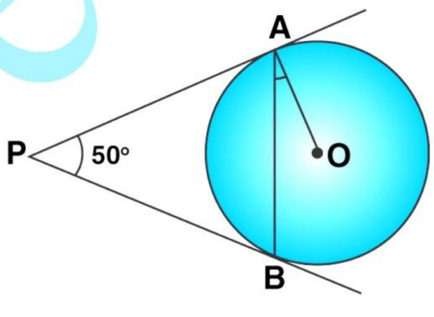


Solution:

In the given figure,
A circle touches all the four sides of a quadrilateral ABCD whose sides are AB = 6 cm, BC = 9 cm and CD = 8 cm
Let P, Q, R and S are points where circle touches the sides AB, BC, CD and DA respectively.
Therefore,
AB + CD = BC + AD
6 + 8 = 9 + AD
14 = 9 + AD
AD = 14 - 9 = 5

Length of AD is 5 cm.

Question 2: In the given figure, PA and PB are two tangents to the circle with centre O. If ∠APB = 50° then what is the measure of ∠OAB.



Solution:

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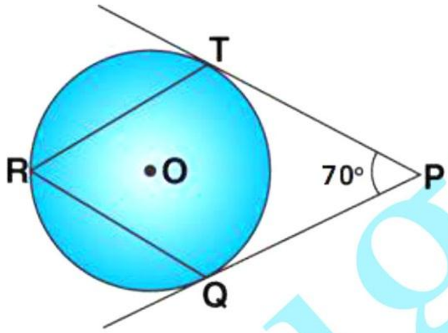
In the given figure,
∠APB = 50°
PA and PB are two tangents to the circle with centre O.
Join OB.

In ΔAPB,
PA = PB (tangents of the circle)
∠PAB = ∠PBA
But, ∠PAB + ∠PBA + ∠APB = 180° (Angles of a triangle)
∠PAB + ∠PAB + 50° = 180°
2∠PAB = 180° – 50° = 130°
Or ∠PAB = 65°
But OA is radius and PA is tangent, so ∠OAP = 90°
∠OAB = 90° – 65° = 25°

Question 3: In the given figure, O is the centre of a circle. PT and PQ are tangents to the circle from an external point P. If ∠TPQ = 70°, find ∠TRQ.



Solution:
Here, O is the centre of circle.
PQ and PT are tangents to the circle from a point P
R is any point on the circle. RT and RQ are joined.
∠TPQ = 70°

Now,

Join TO and QO.

∠TOQ = 180° – 70° = 110°
Here, OQ and OT are perpendicular on QP and TP.
∠TOQ is on the centre and ∠TRQ is on the rest part.
∠TRQ = 1/2∠TOQ = ½(110°) = 55°
Therefore, ∠TRQ = 55 degrees.

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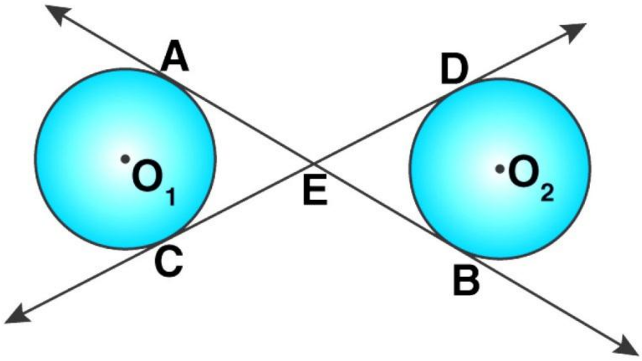
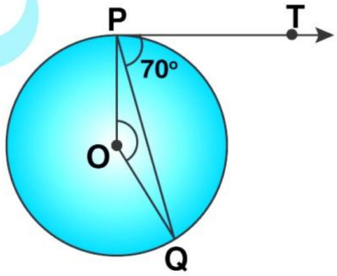
Question 4: In the given figure, common tangents AB and CD to the two circles with centres O_1 and O_2 intersect at E. Prove that AB = CD.



Solution:
In the given figure, common tangents
AB and CD are common tangents to the two circles with centres O_1 and O_2 intersecting at point E.

Circle O_1:
EA and EC are tangents
EA = EC ...(1)
Circle O2:
EB and ED are tangents
EB = ED ...(2)
Adding (1) and (2),
EA + EB = EC + ED
AB = CD
Hence proved.

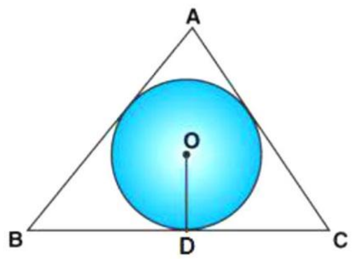
Question 5: If PT is a tangent to a circle with centre O and PQ is a chord of the circle such that ∠QPT = 70°, then find the measure of ∠POQ.



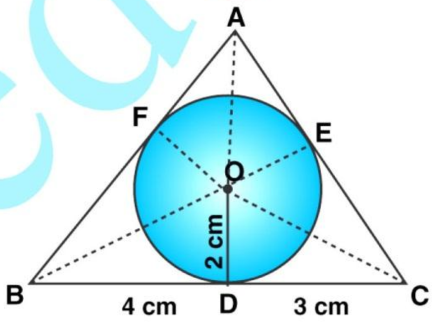
Solution:

OP is the radius and PT is tangent to the circle.
So, ∠OPT = 90°
But ∠QPT = 70°
∠OPQ = 90° – 70° = 20°
From right ΔOPQ,
OP = OQ (radius of circle)
∠OQP = ∠OPQ = 20°
and ∠POQ = 180° – (∠OPQ + ∠OQP)
= 180° – (20° + 20°)
= 180° – 40° = 140°
Measure of ∠POQ is 140 degrees.

Question 6: In the given figure, a triangle ABC is drawn to circumscribe a circle of radius 2 cm such that the segments BD and DC into which BC is divided by the point of contact D, are of lengths 4 cm and 3 cm respectively. If the area of ΔABC = 21 cm² then find the lengths of sides AB and AC.



Solution:
Given:
ΔABC is circumscribed a circle with centre O and radius 2 cm.
Point D divides BC as
BD = 4 cm, DC = 3 cm, OD = 2 cm
Area of ΔABC = 21 cm²
Join OA, OB, OC, OE and OF.



From figure:
BF and BD are tangents to the circle.
So, BF = BD = 4 cm

CD and CE are tangents to the circle.
So, CE = CD = 3 cm

AF and AE are tangents to the circle.
Let say, AE = AF = x cm

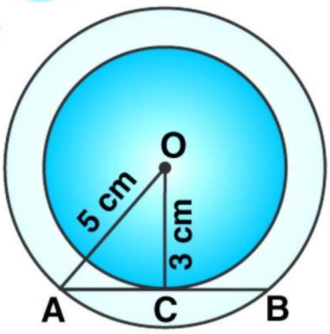
Now,
Area of ΔABC = x Perimeter of ΔABC x Radius
21 = ½[AB + BC + CA] OD
21 = 1/2[4 + 3 + 3 + x + x + 4) x 2
21 = 14 + 2x
x = 3.5
Therefore,
AB = AF + FB = 3.5 + 4 = 7.5 cm
AC = AE + CE = 3.5 + 3 = 6.5 cm

Question 7: Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle (in cm) which touches the smaller circle.

Solution:

Two concentric circles are of radii 5 cm and 3 cm.
Let O be the centre of circle.
Let AB is chord of larger circle which touches the smaller circle at C.

Join OA and OC.
Below is the figure:



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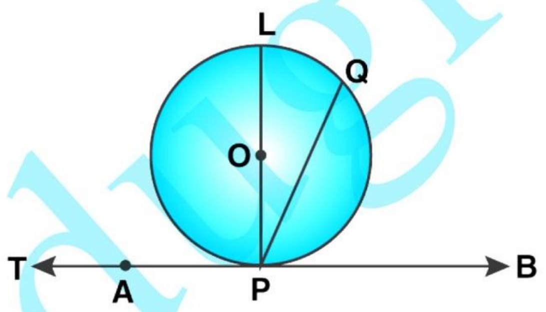
Here,
OC is radius of the smaller circle.
AB is tangent of the circle
OC ⊥ AB and OC bisects AB at C.
Since AB is chord and OC ⊥ AB.

From right ΔOAC,
By Pythagoras Theorem find the length of AC:
$OA^2 = OC^2 + AC^2$
$(5)^2 = (3)^2 + AC^2$
$25 = 9 + AC^2$
$AC^2 = 16$
Or AC = 4
and AB = 2 x AC = 2 x 4 = 8
The length of the chord is 8 cm.

Question 8: Prove that the perpendicular at the point of contact of the tangent to a circle passes through the centre.

Solution:
AB is the tangent to the circle at point P
Let O be the centre.
PL ⊥ AB



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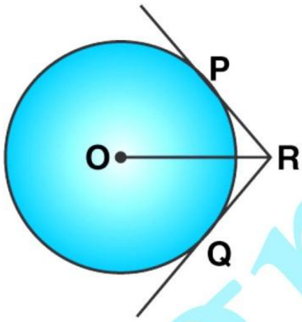
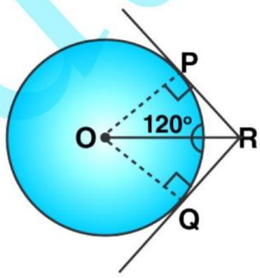
Let PQ ⊥PT where Q lies on the circle.
So, ∠QPT = 90°
Join PO and produce it to meet the circle at L.
Let PQ does not pass through the centre O.
PO is the radius of the circle.
OP ⊥ AB
This implies, ∠OPB = 90°
∠LPB = 90°

But, PQ ⊥ AB
So, ∠QPB = 90°
It is possible only if L and Q coincide each other.
Hence, PQ passes through the centre and is perpendicular from the point of contact.

Question 9: In the given figure, two tangents RQ and RP are drawn from an external point R to the circle with centre O. If ∠PRQ = 120°, then prove that OR = PR + RQ.



Solution:
Two tangents RQ and RP are drawn from the external point R to the circle with centre O.
∠PRQ = 120°
To Prove : OR = PR + RQ
Join OP and OQ and OR



∠PRQ = ∠QRO = 120°/2 = 60°

RQ and RP are the tangent to the circle.
OQ and OP are radii

OQ ⊥ QR and OP ⊥ PR

Form right ΔOPR,
∠POR = 180° - (90° + 60°) = 30°

and∠QOR = 30°

cos a = PR/OR (suppose 'a' be the angle)

cos 60° =PR/OR
1/2 = PR/OR

OR = 2 PR

Again from right ΔOQR,

OR = 2 QR

From both the results, we have

2 PR + 2 QR = 2OR

or OR = PR + RQ
Hence Proved.

Question 10: In the given figure, a circle inscribed in a triangle ABC touches the sides AB, BC and CA at points D, E and F respectively. If AB = 14 cm, BC = 8 cm and CA = 12 cm. Find the lengths AD, BE and CF.

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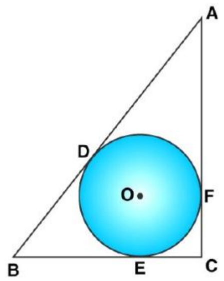
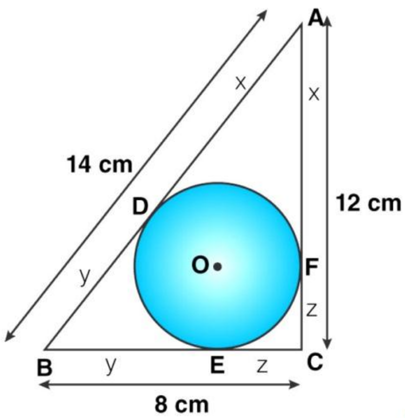


Solution:

From given statements, let us reframed the given figure:



AB = 14 cm, BC = 8 cm and CA = 12 cm.

Let AD = x, BE = y and CF = z
AD and AF are the tangents to the circle from A.
AD = AF = x
Again,
BE and BD are tangents
BD = BE = y
and CF and CE are the tangents
CE = CF = z
Now,
AB + BC + CA = 14 + 8 + 12 = 34
(x + y) + (y + z) + (z + x) = 34
2 (x + y + z) = 34
x + y + z = 17 ....(1)
But x + y = 14 cm ....(2)
y + z = 8 cm ....(3)

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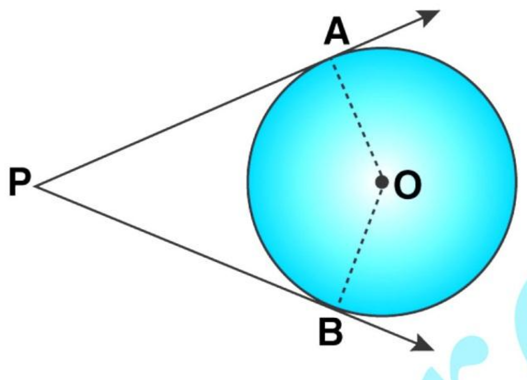
z + x = 12 cm …….. (4)
Subtracting (3), (4) and (2) from (1), we get
x = 17 – 8 = 9 cm
y = 17 – 12 = 5 cm
z = 17 – 14 = 3 cm
Answer: AD = 9 cm, BE = 5 cm and CF = 3 cm.

Question 11: In the given figure, O is the centre of the circle. PA and PB are tangents. Show that AOBP is a cyclic quadrilateral.



Solution:

From figure, PA and PB are the tangents.
O is the Centre of the circle.
To Prove: AOBP is a cyclic quadrilateral.
Now,
OA is radius and PA is tangent
OA ⊥ PA
So, ∠OAP = 90° ….. (1)
Similarly, OB is radius and PB is tangent.
OB ⊥ PB
So, ∠OBP = 90° ….. (2)
Add (1) and (2), we have
∠OAP + ∠OBP = 90° + 90° = 180°
But these are opposite angles of the quadrilateral AOBP.
Therefore, Quadrilateral AOBP is a cyclic.

Question 12: In two concentric circles, a chord of length 8 cm of the larger circle touches the smaller circle. If the radius of the larger circle is 5 cm then find the radius of the smaller circle.

Solution:

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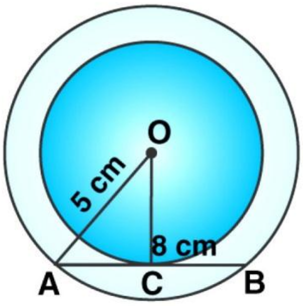
Draw a figure based on given instructions:

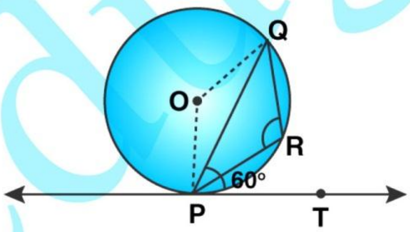


AC = CB = 8/2 = 4 cm
OA = 5 cm

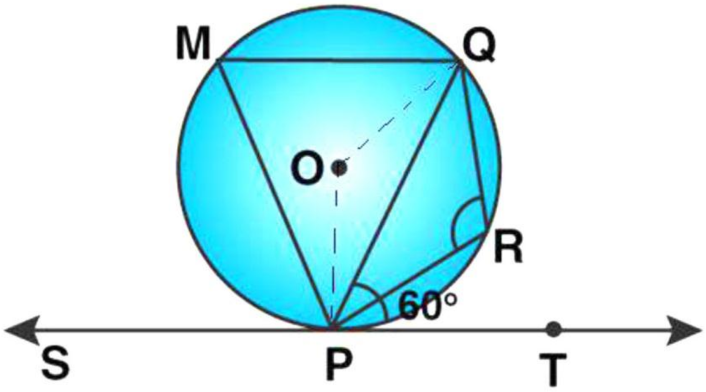
AB is the tangent and OC is the radius
OC ⊥ AB

From right ΔOCA,
By Pythagoras Theorem:
$OA^2 = OC^2 + AC^2$
$(5)^2 = OC^2 + (4)^2$
$OC^2 = (5)^2 - (4)^2 = 25 - 16 = 9 = (3)^2$
OC = 3
Therefore, Radius of smaller circle is 3 cm

Question 13: In the given figure, PQ is a chord of a circle with centre O and PT is a tangent. If ∠QPT = 60°, find ∠PRQ.

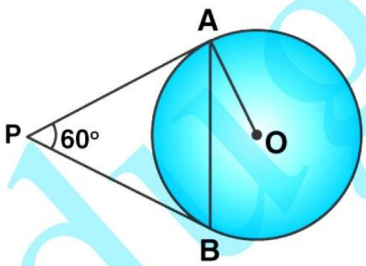


Solution:
From given figure: PQ is a chord of a circle with centre O.
PT is the tangent ∠QPT = 60°.
Reframed the given figure based on given instructions and our requirement to find the unknown angle.
Take a point M on the alternate segment.
Join MP and MQ.



∠MPQ = ∠QPT = 60°
[Angles in the alternate segment]
∠PMQ + ∠PRQ = 180°
[Opposite angles of a cyclic quadrilateral]
So,
60° + ∠PRQ = 180°
∠PRQ = 180° – 60° = 120°
Therefore measure of ∠PRQ is 120°.

Question 14: In the given figure, PA and PB are two tangents to the circle with centre O. If ∠APB = 60° then find the measure of ∠OAB.



Solution:
From given figure,
PA and PB are the two tangents to the circle.
O be the centre of circle. OA and AB are joined

∠APB = 60°

To Find :∠OAB

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Since PA and PB are tangents to the circle from P
PA = PB
∠PAB = ∠PBA
But ∠APB = 60°
∠PAB + ∠PBA = 180° – 60° = 120°
2 ∠PAB = 120°
∠PBA = 60°
OA is radius and PA is tangent.
OA ⊥ PA
∠OAP = 90°
∠OAB + ∠PAB = 90°

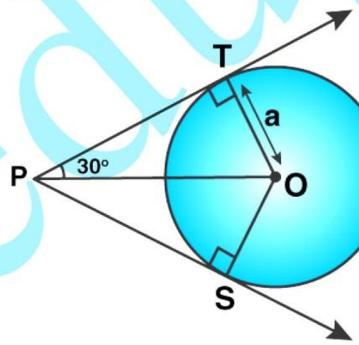
∠OAB + 60° = 90°

∠OAB = 90° – 60° = 30°

Therefore, the measure of ∠OAB is 30°.

Question 15: If the angle between two tangents drawn from an external point P to a circle of radius and centre O, is 60° then find the length of OP.

Solution:
Since, tangents drawn from an external point are equally inclined to the line joining centre to that point.
Draw an image based on given instructions:



Here, PT is the tangent of the circle

OT ⊥ PT

And, ∠TPS = 60° (given)
∠TPO = 60°/2 = 30°

Now, from right ΔTPO:
sin 30° = OT/OP

1/2 = a/OP

or OP = 2a