

Exercise 6A

Question 1: Find the distance between the points:

(i) A (9, 3) and B (15, 11)

(ii) A (7, -4) and B (-5, 1)

(iii) A (-6, -4) and B (9, -12)

(iv) A (1, -3) and B (4, -6)

(v) P (a + b, a - b) and Q (a - b, a + b)

(vi) P (a sin α , a cos α) and Q (a cos α , -a sin α)

Solution:

Distance formula:
$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

(i) A (9, 3) and B (15, 11)

AB =
$$\sqrt{(15-9)^2 + (11-3)^2}$$

= $\sqrt{(6)^2 + 8^2}$
= $\sqrt{100}$
=10 units

(ii)A (7, -4) and B (-5, 1)

AB =
$$\sqrt{(-5-7)^2 + (1-(-4))^2}$$

= $\sqrt{(-12)^2 + 5^2}$
= $\sqrt{169}$
=13 units

(iii)A (-6, -4) and B (9, -12)

AB =
$$\sqrt{(9 - (-6))^2 + ((-12) - (-4))^2}$$

= $\sqrt{(15)^2 + (-8)^2}$
= $\sqrt{289}$
= 17 units

(iv) A (1, -3) and B (4, -6)

$$AB = \sqrt{(4-1)^2 + ((-6) - (-3))^2}$$

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$$=\sqrt{(3)^2+(-3)^2}$$

$$=\sqrt{18}$$

=3
$$\sqrt{2}$$
units

(v) P
$$(a + b, a - b)$$
 and Q $(a - b, a + b)$

$$PQ = \sqrt{((a-b)-(a+b))^2 + ((a+b)-(a-b))^2}$$

$$=\sqrt{(2b)^2+(2b)^2}$$

$$=\sqrt{8b^2}$$

=
$$2b\sqrt{2}$$
 units

(vi) P (a sin
$$\alpha$$
, a cos α) and Q (a cos α , -a sin α)

$$PQ = \sqrt{(a\cos\alpha - a\sin\alpha)^2 + (-a\sin\alpha - a\cos\alpha)^2}$$
$$= \sqrt{a^2(\cos\alpha - \sin\alpha)^2 + a^2(\sin\alpha - \cos\alpha)^2}$$

$$= a\sqrt{2}\sin^2\alpha + 2\cos^2\alpha$$

= a
$$\sqrt{2}$$
 units

Question 2: Find the distance of each of the following points from the origin:

Solution:

Distance formula:
$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Distance from origin O(0, 0) and the given points (x, y) is

Distance formula: $\sqrt{x^2 + y^2}$

$$OA = \sqrt{(5)^2 + (-12)^2}$$

$$=\sqrt{25+144}$$

OB =
$$\sqrt{(-5)^2 + (5)^2}$$



$$=\sqrt{25+25}$$

= 5√2 units

$$OC = \sqrt{(-4)^2 + (-6)^2}$$

$$=\sqrt{16+36}$$

= **√**52

= 21/13 units

Question 3: Find all possible values of x for which the distance between the points A (x, -1) and B (5, 3) is 5 units.

Solution:

Given: Points A (x, -1), B (5, 3) and AB = 5 units

Distance formula:
$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$5 = \sqrt{(5-x)^2 + (3+1)^2}$$

Squaring both sides:

$$25 - 10x + x^2 + 16 = 25$$

$$x^2 - 10x + 16 = 0$$

$$x^2 - 2x - 8x + 16 = 0$$

$$x(x-2) - 8(x-2) = 0$$

$$(x-2)(x-8)=0$$

Either
$$(x-2) = 0$$
 or $(x-8) - 0$

$$x = 2$$
 or $x = 8$

Question 4: Find all possible values of y for which the distance between the points A (2, -3) and B (10, y) is 10 units.

Solution:

Given: Points A (2, -3), B (10, y) and AB = 10

Distance formula: $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



AB^2 =
$$(x_2 - x_1)^2 + (y_2 - y_1)^2$$

 $(10)^2 = (10 - 2)^2 + (y + 3)^2$
 $100 = (8)^2 + y^2 + 6y + 9$
 $y^2 + 6y + 9 + 64 = 100$
 $y^2 + 6y + 73 - 100 = 0$
 $y^2 + 6y - 27 = 0$
 $y^2 + 9y - 3y - 27 = 0$
 $y(y + 9) - 3(y + 9) = 0$
 $(y + 9)(y - 3) = 0$
Either, $y + 9 = 0$, then $y = -9$
or $y - 3 = 0$, then $y = 3$
 $y = 3, -9$

Question 5: Find the values of x for which the distance between the points P (x, 4) and Q (9, 10) is 10 units.

Solution:

Given: Points P (x, 4), Q (9, 10) and PQ = 10

Distance formula:
$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$PQ^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$100 = (9 - x)^2 + (10 - 4)^2$$

$$= 81 + x^2 - 18x + 36$$

= $117 + x^2 - 18x$

$$100 = 117 + x^2 - 18x$$

$$x^2 - 18x + 17x = 0$$
 (Solve this equation)

$$(x-1)(x-17)$$

$$x = 1 \text{ or } x = 17$$

Question 6: If the point A (x, 2) is equidistant from the points B (8, -2) and C (2, -2), find the value of x. Also, find the length of AB.

Solution:

Given: Point A (x, 2) is equidistant from B (8, -2) and C (2, -2)

Which implies:



AB = AC Squaring both sides AB^2 = AC^2

Using distance formula:

Distance formula: $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

We have,

$$(8-x)^2 + (-2-2)^2 = (2-x)^2 + (-2-2)^2$$

$$(8-x)^2 + (-4)^2 = (2-x)^2 + (-4)^2$$

$$64 - 16x + x^2 = 4 - 4x + x^2$$

$$64 - 4 = -4x + 16x$$

$$12x = 60 \Rightarrow x = \frac{60}{12} = 5$$

$$AB = \sqrt{(8-5)^2 + (-4)^2}$$

$$= \sqrt{3^2 + (-4)^2} = \sqrt{9 + 16}$$

$$=\sqrt{25}=5$$
 units

$$x = 5$$
, AB = 5 units

Question 7: If the point A (0, 2) is equidistant from the points B (3, p) and C (p, 5), find the value of p. Also, find the length of AB.

Solution:

Given: A (0, 2) is equidistant from B (3, p) and C(p, 5)

which implies: AB = AC

or $AB^2 = AC^2$

Using distance formula:

Distance formula: $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

We have,



$$(3-0)^{2} + (p-2)^{2} = (p-0)^{2} + (5-2)^{2}$$

$$(3)^{2} + (p-2)^{2} = p^{2} + (3)^{2}$$

$$p^{2} = p^{2} - 4p + 4$$

$$4p = 4 \Rightarrow p = \frac{4}{4} = 1$$
and AB = $\sqrt{(3-0)^{2} + (1-2)^{2}}$

Question 8: Find the point on the x-axis which is equidistant from the points (2, -5) and (-2, 9).

Solution:

Let point P(x, 0) is on x-axis and equidistant from A(2, -5) and B(-2, 9)

 $= \sqrt{9+1} = \sqrt{10} \text{ units}$

$$(2-x)^2 + (-5-0)^2 = (-2-x)^2 + (9-0)^2$$

$$(2-x)^2 + (-5)^2 = (-2-x)^2 + (9)^2$$

$$29 + x^2 - 4x = 85 + x^2 + 4x$$

$$56 = -8x$$

or
$$x = -7$$

The point on \times -axis is (-7, 0)

Question 9: Find points on the x-axis, each of which is at a distance of 10 units from the point A (11, -8).

Solution:

Let the points on x-axis be P(x, 0) and Q(y, 0) which are at distance of 10 units from point A(11, -8). Which implies:

$$PA = QA$$

or
$$PA^2 = QA^2$$



$$(11 - x_1)^2 + (-8)^2 = (11 - x_2)^2 + (-8)^2 = 10^2$$

$$(11 - x)^2 + (-8)^2 = 10^2$$

$$121 - 22x + x^2 + 64 = 100$$

$$x^2 - 22x + 85 = 0$$

$$x^2 - 12x - 5x + 85 = 0$$

$$x(x - 17) - 5(x - 17) = 0$$

$$(x - 17)(x - 5) = 0$$
Either $(x - 17) = 0$ or $(x - 5) = 0$

$$x = 17 \text{ or } x = 5$$

So, the points are: (17, 0) and (5, 0)

Question 10: Find the point on the y-axis which is equidistant from the points A (6, 5) and B (-4, 3).

Solution:

Let point P(0, y) is on the y-axis, then

PA = PB
or PA^2 = PB^2

$$(6 - 0)^2 + (5 - y)^2 = (-4 - 0)^2 + (3 - y)^2$$

 $36 + 25 - 10y + y^2 = 16 + 9 - 6y + y^2$
 $61 - 10y = 25 - 6y$
 $61 - 25 = -6y + 10y$
 $36 = 4y$
or $y = 9$

The required point is (0,9).

Question 11.

If the points P (x, y) is equidistant from the points A (5, 1) and B (-1, 5), prove that 3x = 2y.

Solution:

Since P (x, y) is equidistant from A (5, 1) and B (-1, 5), then PA = PB

or $PA^2 = PB^2$

$$(5-x)^2 + (1-y)^2 = (-1-x)^2 + (5-y)^2$$

(25 + x^2 - 10x) + (1 + y^2 - 2y) = (1 + x^2 + 2x + 25 + y^2 - 10y)



$$26 + x^2 - 10x + y^2$$

 $-2y = (26 + x^2 + 2x + y^2 - 10y)$
 $12x = 8y$
 $3x = 2y$
Hence proved.

Question 12: If P (x, y) is a point equidistant from the points A(6, -1) and B(2, 3), show that x - y = 3.

Solution:

Since P (x, y) is equidistant from A(6, -1) and B(2, 3), then PA = PB

or $PA^2 = PB^2$

$$(6-x)^2 + (-1-y)^2 = (2-x)^2 + (3-y)^2$$

 $(36+x^2-12x) + (1+y^2+2y) = (4+x^2-4x+9+y^2-6y)$
 $37-12x+2y=13-4x-6y$
 $8x = 8y + 24$
 $x-y=3$
Hence proved.

Question 13: Find the coordinates of the point equidistant from three given points A (5, 3), B (5, -5) and C (1, -5).

Solution:

Let the coordinates of the point be O(x, y), then

and
$$(5-x)^2 + (-5-y)^2$$

= $(1-x)^2 + (-5-y)^2$
25 - $10x + x^2 = 1 - 2x + x^2$
- $10x + 2x = 1 - 25$



-8x = -24

or x = 3

So, coordinates of the point is (3,1).

Question 14: If the points A (4, 3) and B (x, 5) lie on a circle with the centre O (2, 3), find the value of x.

Solution:

Given: Points A (4, 3) and B (x, 5) lie on a circle with centre O (2, 3)

To find: value of x

or $OA^2 = OB^2$

$$OA = OB \Rightarrow OA^2 = OB^2$$

$$OA^2 = (2-4)^2 + (3-3)^2$$
 and

$$OB^2 = (2 - x)^2 + (3 - 5)^2$$

$$(2-4)^2 + (3-3)^2 = (2-x)^2 + (3-5)^2$$

$$(-2)^2 + 0^2 = 4 - 4x + x^2 + (-2)^2$$

$$4 = 4 - 4x + x^2 + 4$$

$$x^2 - 4x + 4 = 0 \Rightarrow (x - 2)^2 = 0$$

$$x = 2$$

The value of x is 2.

Question 15: If the point C (-2, 3) is equidistant from the points A (3, -1) and B (x, 8), find the values of x. Also, find the distance BC.

Solution:

Given: Point C(-2, 3) is equidistant from points A(3, -1) and B(x,8).

Then

CA = CB

or $CA^2 = CB^2$



$$CB^{2} = (x + 2)^{2} + (8 - 3)^{2}$$

$$CA^{2} = (3 + 2)^{2} + (-1 - 3)^{2}$$

$$(x + 2)^{2} + (8 - 3)^{2} = (3 + 2)^{2} + (-1 - 3)^{2}$$

$$(x + 2)^{2} + 5^{2} = 5^{2} + (-4)^{2}$$

$$x^{2} + 4x + 4 + 25 = 25 + 16$$

$$x^{2} + 4x + 29 - 41 = 0$$

$$x^{2} + 4x - 12 = 0$$

$$x^{2} + 6x - 2x - 12 = 0$$

$$x(x + 6) - 2(x + 6) = 0$$

$$(x + 6)(x - 2) = 0$$
This implies: $x = 2$ or $x = -6$

NOW: AC =
$$\sqrt{5^2 + (-4)^2} = \sqrt{41}$$

Therefore: AC = $\sqrt{41}$ units

Question 16: If the point P(2, 2) is equidistant from the points A (-2, k) and B(-2k, -3), find k, Also, find the length of AP.

Solution:

Given: Point P(2, 2) is equidistant from the two points A(-2, k) and B(-2k, -3)

$$PA = PB \text{ or } PA^2 = PB^2$$

$$(2+2)^{2} + (2-k)^{2} = (2+2k)^{2} (2+3)^{2}$$

$$4^{2} + 4 - 4k + k^{2} = 4 + 8k + 4k^{2} + 5^{2}$$

$$16 + 4 - 4k + k^{2} = 4 + 8k + 4k^{2} + 25$$

$$4k^{2} + 8k + 29 - 20 + 4k - k^{2} = 0$$

$$3k^{2} + 12k + 9 = 0$$

$$k^{2} + 4k + 3 = 0$$

$$k^{2} + k + 3k + 3 = 0$$

$$k(k+1) + 3(k+1) = 0$$

$$(k+1)(k+3) = 0$$

If
$$k = -1$$

 $AP^2 = 20 - 4k + k^2$
 $= 20 + 4 + 1$
 $= 25$

thus, k = -1 or k = -3



AP = 5 units If k = -3 AP^2 = 20 - 4k + k^2 = 20 + 12 + 9 = 41 AP = $\sqrt{41}$ units

Question 17:

- (i) If the point (x, y) is equidistant from the points (a + b, b a) and (a b, a + b), prove that bx = ay.
- (ii) If the distance of P(x, y) from A(5, 1) and B(-1, 5) are equal then prove that 3x = 2y.

Solution:

(i)

Let point P(x, y) is equidistant from A(a + b, b - a) and B(a - b, a + b), then

$$AP = BP \text{ or } AP^2 = BP^2$$

$$((a + b) - x)^2 + ((a - b) - y)^2 = ((a - b) - x)^2 + ((a + b) - y)^2$$

$$(a + b)^2 + x^2 - 2(a + b)x + (a - b)^2 + y^2 - 2(a - b)y = (a - b)^2 + x^2 - 2(a - b)x + (a + b)^2 + y^2 - 2(a + b)y$$

$$(a^2 + b^2 + 2ab + x^2 - 2(a + b)x + b^2 + a^2 - 2ab + y^2 - 2(a - b)y = (a^2 + b^2 - 2ab + x^2 - 2(a - b)x + b^2 + a^2 + 2ab + y^2 - 2(a + b)y$$

=> $-2(a + b)x - 2(a - b)y = -2(a - b)x - 2(a + b)y$
=> $ax + bx + ay - by = ax - bx + ay + by$

$$=>$$
 bx $=$ ay

(ii)

Point P(x, y) is equidistant from the points A(5, 1) and B(-1, 5), means PA = PB or PA^2 = PB^2

$$(5-x)^2 + (1-y)^2 = (-1-x)^2 + (5-y)^2$$

$$(25 + x^2 - 10x) + (1 + y^2 - 2y) = (1 + x^2 + 2x + 25 + y^2 - 10y)$$

$$26 + x^2 - 10x + y^2 - 2y = (26 + x^2 + 2x + y^2 - 10y)$$

$$12x = 8y$$



3x = 2y

Hence proved.

Question 18: Using the distance formula, show that the given points are collinear:

- (i) (1, -1), (5, 2) and (9, 5)
- (ii) (6, 9), (0, 1) and (-6, -7)
- (iii) (-1, -1), (2, 3) and (8, 11)
- (iv) (-2, 5), (0, 1) and (2, -3)

Solution:

Points are collinear if sum of any two of distances is equal to the distance of the third.

A, B and C are collinear if AB + BC = AC

AB =
$$\sqrt{(5-1)^2 + (2+1)^2}$$
 = $\sqrt{4^2 + 3^2}$
= 16 + 9 = $\sqrt{25}$ = 5 units

BC =
$$\sqrt{(9-5)^2 + (5-2)^2} = \sqrt{4^2 + 3^2}$$

$$=\sqrt{16+9} = \sqrt{25} = 5$$
 units

AC
$$\sqrt{(9-1)^2+(5+1)^2}$$

$$= \sqrt{8^2 + 6^2} = \sqrt{64 + 36} = \sqrt{100}$$

= 10 units

From above, we can see that

$$AB + BC = 10 = AC$$

Therefore, A, B and C are collinear

A, B and C are collinear if AB + BC = AC



AB =
$$\sqrt{(0-6)^2 + (1-9)^2}$$
 = $\sqrt{(-6)^2 + (-8)^2}$
= $\sqrt{36+64}$ = $\sqrt{100}$ = 10 units
BC = $\sqrt{(-6-0)^2 + (-7-1)^2}$
= $\sqrt{(-6)^2 + (-8)^2}$ = $\sqrt{36+64}$
= $\sqrt{100}$ = 10 units
CA = $\sqrt{(6+6)^2 + (9+7)^2}$ = $\sqrt{12^2 + 16^2}$
= $\sqrt{144+256}$ = $\sqrt{400}$ = 20 units

From above, we can see that AB + BC = 10 + 10 = 20 = CA

Therefore, A, B and C are collinear.

A, B and C are collinear if AB + BC = AC

AB =
$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

= $\sqrt{(2+1)^2 + (3+1)^2} = \sqrt{3^2 + 4^2}$
= $\sqrt{9+16} = \sqrt{25} = 5$ units
BC = $\sqrt{(8-2)^2 + (11-3)^2} = \sqrt{6^2 + 8^2}$
= $\sqrt{36+64} = \sqrt{100} = 10$ units
CA = $\sqrt{(8+1)^2 + (11+1)^2} = \sqrt{9^2 + 12^2}$
= $\sqrt{81+144} = \sqrt{225} = 15$ units

From above, we can see that AB + BC = 5 + 10 = 15 = AC

Therefore, A, B and C are collinear.



(iv) Let A(-2, 5), B(0, 1) and C(2, -3).

A, B and C are collinear if AB + BC = AC

AB =
$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

= $\sqrt{(0+2)^2 + (1-5)^2} = \sqrt{2^2 + (-4)^2}$
= $\sqrt{4+16} = \sqrt{20}$
BC = $\sqrt{(2-0)^2 + (-3-1)^2} = \sqrt{2^2 + (-4)^2}$
= $\sqrt{4+16} = \sqrt{20}$
= $\sqrt{4\times5} = 2\sqrt{5}$ untis
CA= $\sqrt{(2+2)^2 + (-3-5)^2} = \sqrt{4^2 + (-8)^2}$
= $\sqrt{16+64} = \sqrt{80} = \sqrt{16\times5} = 4\sqrt{5}$

From above, we can see that AB + BC = $\sqrt{20}$ + $2\sqrt{5}$ = $2\sqrt{5}$ + $2\sqrt{5}$ = $4\sqrt{5}$ = AC

Therefore, A, B and C are collinear.

Question 19: Show that the points A(7, 10), B(-2, 5) and C(3, -4) are the vertices of an isosceles right triangle.

Solution:

Given points are A(7, 10), B(-2, 5) and C(3, -4) AB² = $(x_2 - x_1)^2 + (y_2 - y_1)^2$ = $(-2 - 7)^2 + (5 - 10)^2 = (-9)^2 + (-5)^2$ = 81 + 25 = 106BC² = $(3 + 2)^2 + (-4 - 5)^2 = (5)^2 + (-9)^2$ = 25 + 81 = 106CA² = $(7 - 3)^2 + (10 + 4)^2 = (4)^2 + (14)^2$ = 16 + 196 = 212AB² + BC² = $106 \Rightarrow$ AB = BC

From above, two of the sides are of equal length, so triangle ABC is an isosceles triangle.



Check for: Isosceles right triangle

Sum of square of two sides = Square of third side

$$AB^2 + BC^2 = 106 + 106 = 212 = CA^2$$

Hence given points are vertices of an isosceles right triangle.

Question 20: Show that the points A (3, 0), B (6, 4) and C (-1, 3) are the vertices of an isosceles right triangle.

Solution:

Given points are A (3, 0), B (6, 4) and C (-1, 3)

$$AB^{2} = (x_{2} - x_{1})^{2} + (y_{2} - y_{1})^{2}$$

$$= (6 - 3)^{2} + (4 - 0)^{2} = 3^{2} + 4^{2}$$

$$= 9 + 16 = 25$$

$$BC^{2} = (-1 - 6)^{2} + (3 - 4)^{2} = (-7)^{2} + (-1)^{2}$$

$$= 49 + 1 = 50$$

$$CA^{2} = (3 + 1)^{2} + (0 - 3)^{2} = 4^{2} + 3^{2}$$

$$= 16 + 9 = 25$$

$$AB^{2} = CA^{2} = 25$$

$$AB = CA$$

From above, two of the sides are of equal length, so triangle ABC is an isosceles triangle.

Check for: Isosceles right triangle

Sum of square of two sides = Square of third side

$$AB^2 + AC^2 = 25 + 25 = 50 = CB^2$$

Hence given points are vertices of an isosceles right triangle.



Exercise 6B

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Question 1:

- (i) Find the coordinates of the point which divides the join of A (-1, 7) and B (4, -3) in the ratio 2:3.
- (ii) Find the coordinates of the point which divides the join of A (-5, 11) and B (4, -7) in the ratio 7:2.

Solution:

If a point P(x,y) divides a line segment having end points coordinates (x_1, y_1) and (x_2, y_2) , then coordinates of the point P can be find using below formula:

$$X = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$$

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

(i) Let P(x, y) be the point which divides the line joining the points A (-1, 7) and B (4, -3) in the ratio 2 : 3. then

$$x = (2 \times 4 + 3 \times (-1))/(2 + 3)$$

$$= (8-3)/5$$

$$x = 1$$
.

$$y = (2 \times -3 + 3 \times 7)/5$$

$$=(-6+21)/5$$

$$y = 1$$

Therefore, required point is (1, 3).

$$x = (7 \times 4 + 2 \times (-5))/(7 + 2)$$

$$= (28 - 10)/9$$

$$= 18/9$$

$$y = (7 \times (-7) + 2 \times 11)/9$$

$$=(-49 + 22)/9$$

$$= -27/9$$

$$= -3$$

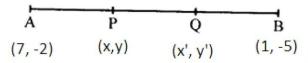
Therefore, required point is (2, -3)



Question 2: Find the coordinates of the points of trisection of the line segment joining the points A (7, -2) and B (1, -5).

Solution:

Let A(7, -2) and B(1, -5) be the given points and P(x, y) and Q(x', y') are the points of trisection.



Step 1: Find the coordinate of P

Point P divides AB internally in the ratio 1:2

$$(x, y) = \left[\frac{1(1) + 2(7)}{1 + 2}, \frac{1(-5) + 1(-2)}{1 + 2}\right]$$
$$= \left(\frac{1 + 14}{3}, \frac{-5 - 4}{3}\right) = \left(\frac{15}{3}, \frac{-9}{3}\right) = (5, -3)$$

Step 2: Find the coordinate of Q

Point Q is the mid-point PB.

$$(x', y') = ((5+1)/2, (-3-5)/2) = (3, -4)$$

Therefore, the coordinates of the points of trisection are (5, -3) and (3, -4)

Question 3: If the coordinates of points A and B are (-2, -2) respectively, find the coordinates of the point P such that AP = 3/7 AB, where P lies on the line segment AB.

Solution:

Coordinate of point P(x, y) can be calculated by using below formula:

$$X = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$$

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

Now,

$$x = ((3 \times 2) + 4(-2))/(3 + 4)$$



$$= (6-8)/7$$
$$= -2/7$$
$$y = (3(-4) + 4)$$

$$y = (3(-4) + 4(-2))/7$$

= $(-12-8)/7$
= $-20/7$

Point P is (-2/7, -20/7)

Question 4: Point A lies on the line segment PQ joining P (6, -6) and Q (-4, -1) in such a way that PA/PQ = 2/5. If the point A also lies on the line 3x + k(y + 1) = 0, find the value of k.

Solution:

Let the point A(x, y) which lies on line joining P(6, -6) and Q(-4, -1) such that PA/PQ = 2/5

Line segment PQ is divided by the point A in the ratio 2:3.

Step 1: Find coordinates of A(x, y)

$$X = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$$

$$x = (2(-4) + 3(6))/(2 + 3)$$

$$= (-8 + 18) / 5$$

$$= 10/5 = 2$$

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

$$y = (2(-1) + 3(-6))/5$$

$$=(-2-18)/5$$

Step 2: Point A also lies on the line 3x + k(y + 1) = 0

$$3(2) + k(-4 + 1) = 0$$

$$6 - 3k = 0$$

or
$$k = 2$$

Question 5: Points P, Q, R and S divide the line segment joining the points A (1, 2) and B (6, 7) in five equal parts. Find the coordinates of the points P, Q and R.



Solution:

Given: Points P, Q, R and S divides a line segment joining the points A (1, 2) and B (6, 7) in 5 equal parts. We know that:

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$$

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

Now,

Step 1: Find coordinates of P.

P(x, y) divides AB in the ratio 1:4

$$x = (1 \times 6 + 4 \times 1)/1 + 4$$

= $(6 + 4)/5$

$$= (6 + 4)$$

= 10/5

$$y = (1x7 + 4 \times 2)/5$$

$$=(7+8)/5$$

So,
$$P(x, y) = P(2, 3)$$

Step 2: Find coordinates of Q.

Q divides the segment AB in ratio 2:3

$$x = (2x 6 + 3x 1)/5$$

$$= (12 + 3) / 5$$

$$y = (2 \times 7 + 3 \times 2)/5$$

$$= (14 + 6)/5$$

$$= 20 / 5 = 4$$

So,
$$Q(x, y) = Q(3,4)$$

Step 3: Find coordinates of R.

R divides the segment AB in ratio 3:2

$$x = (3 \times 6 + 2 \times 1)/5$$

$$= (18 + 2)/5$$

$$= 20/5$$



= 4

$$y = (3 \times 7 + 2 \times 2)/5$$

= $(21 + 4)/5$
= $25/5$
= 5

So, R(x, y) = R(4,5)

Question 6: Points P, Q and R in that order are dividing a line segment joining A (1, 6) and B (5, -2) in four equal parts. Find the coordinates of P, Q and R.

Solution:

Given: Points P, Q and R in order divide a line segment joining the points A (1, 6) and B (5, -2) in four equal parts.

Using formulas:

$$X = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$$

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

Step 1: Find coordinates of P.

P(x, y) divides AB in the ratio of 1:3

$$x = (5+3)/4 = 8/4 = 2$$

$$y = (-2+18) = 16/4 = 4$$

So
$$P(x, y) = P(2, 4)$$

Step 2: Find coordinates of Q.

Q divides the segment AB in ratio 2:2 or 1:1. So Q ia midpoint of AB

So
$$Q((1+5)/2, (6-2)/2) = (3, 2)$$

So,
$$Q(x, y) = Q(3,2)$$

Step 3: Find coordinates of R.



R divides the segment AB in ratio 3:1

$$x = (3 \times 5 + 1 \times 1)/4$$

= 4

$$y = (3x(-2) + 1x6)/4$$

= 0

So,
$$R(x, y) = R(4,0)$$

Question 7: The line segment joining the points A (3, -4) and B (1, 2) is trisected at the points P(p, -2) and Q(1/2, q). Find the values of p and q.

Solution:

The line segment joining the point A(3, -4) and B(1, 2) is trisected by the points P(p, -2) and Q(1/2, q). (given)



Step 1: Find x coordinate of P which is p

P(p, -2) divides AB in the ratio of 1:2

$$p = (1+6)/3 = 7/3$$

Step 2: Find coordinates of Q

Q divides the segment AB in ratio 2:1

$$x = (2x1 + 1x3)/3$$

$$= (2 + 3)/3$$

= 5/3

$$y = (2 \times 2 + 1(-4))/3$$

$$=(4-4)/3$$

$$= 0/3$$

$$= q$$

Therefore, p = 7/3 and q = 0



Question 8: Find the coordinates of the midpoint of the line segment joining

(i) A (3, 0) and B (-5, 4)

(ii) P (-11, -8) and Q (8, -2)

Solution:

(i) Midpoint of the line segment joining A (3, 0) and B (-5, 4):

Midpoint =
$$((x_1 + x_2)/2, (y_1+y_2)/2)$$

$$= ((3-5)/2, (0+4)/2)$$

= $(-1, 2)$

(ii) Midpoint of the line segment joining P (-11, -8) and Q (8, -2)

PQ midpoint =
$$((-11+8)/2, (-8-2)/2)$$

$$=(-3/2,-5)$$

Question 9: If (2, p) is the midpoint of the line segment joining the points A (6, -5) and B (-2, 11), find the value of p.

Solution:

Given: (2, p) is the mid point of the line segment joining the points A (6, -5), B (-2, 11)

To find: the value of p

$$p = (-5+11)/2 = 6/2 = 3$$

Question 10: The midpoint of the line segment joining A (2a, 4) and B (-2, 3b) is C (1, 2a + 1). Find the values of a and b.

Solution:

Mid point of the line segment joining the points A(2a, 4) and B (-2, 3b) is C(1, 2a + 1)

Mid point of AB =
$$((2a-2)/2, (4+3b)/2)...(1)$$

Mid point of AB =
$$(1, 2a + 1)$$
 (2) (given)

Now, from (1) and (2)

$$1 = (2a-2)/2$$



=> a = 2and 2a + 1 = (4+3b)/210-4 = 3bor b = 2

Answer: a = 2 and b = 2

Question 11: The line segment joining A(-2, 9) and B(6, 3) is a diameter of a circle with centre C. Find the coordinates of C.

Solution:

The line segment joining the points A(-2, 9) and B(6, 3) is a diameter of a circle with centre C. Which means C is the midpoint of AB. let (x, y) be the coordinates of C, then

x = (-2 + 6)/2 = 2 and

y = (9+3)/2 = 6

So, coordinates of C are (2, 6).

Question 12: Find the coordinates of a point A, where AB is a diameter of a circle with centre C(2, -3) and the other end of the diameter is B (1, 4).

Solution:

Given:

AB is diameter of a circle with centre C. Coordinates of C(2, -3) and other point is B (1, 4)

Point C is the midpoint of AB.

Let (x, y) be the coordinates of A, then

2 = (x+1)/2

4 = x + 1

x = 3



and

$$-3 = (y+4)/2$$

$$-6 = y + 4$$

or
$$y = -10$$

So, coordinates of A are (3, -10).

Question 13: In what ratio does the point P(2, 5) divide the join of A(8, 2) and B(-6, 9)?

Solution:

Given: P (2, 5) divides the line segment joining the points A(8,2) and B(-6, 9).

Let P divides the AB in the ratio m:n

$$2 = \frac{mx_2 + nx_1}{m + n} = \frac{m(-6) + n \times 8}{m + n}$$

$$2m+2n=-6m+8n$$

$$2m + 6m = 8n - 2n \Rightarrow 8m = 6n$$

$$\frac{m}{m} = \frac{6}{9} = \frac{3}{4}$$

Question 14: Find the ratio in which the point P(3/4, 5/12) divides the line segment joining the points A(1/2, 3/5) and B (2, -5).

Solution:

Let P divides the line segment joining the points A and B in the ratio m:n.

$$\frac{3}{4} = \frac{mx_2 + nx_1}{m+n} = \frac{m \times 2 + n \times \frac{1}{2}}{m+n}$$

$$\frac{3}{4} = \frac{2m + \frac{n}{2}}{m+n}$$

$$3m + 3n = 8m + 2n$$

$$3n - 2n = 8m - 3n$$



or m/n = 1/5

Therefore, required ratio is 1:5.

Question 15: Find the ratio in which the point P (m, 6) divides the join of A (-4, 3) and B (2, 8). Also, find the value of m.

Solution:

Let P divides the join of A and B in the ratio k: 1, then

Step 1: Find coordinates of P:

$$6 = (k \times 8 + 1x3)/(k+1)$$

$$=> 6k + 6 = 8k + 3$$

or
$$k = 3/2$$

P divides the join of A and B in the ratio 3:2

Step 2: Find the value of m

$$m = (2k-4)/(k+1) = (2x3/2 - 4)/(3/2+1)$$

$$= -1/(5/2)$$

$$= -2/5$$

Question 16: Find the ratio in which the point (-3, k) divides the join of A (-5, -4) and B (-2, 3). Also, find the value of k.

Solution:

Let point P divides the join of A and B in the ratio m: n, then

$$-3 = \frac{m(-2) + n(-5)}{m+n}$$

$$-3m - 3n = -2m - 5n$$

$$-5n + 3n = -3m + 2m$$

$$-2n = -m \Rightarrow \frac{m}{n} = \frac{-2}{-1} = \frac{2}{1}$$



ratio = m:n = 2:1

Now,

$$k = \frac{m \times 3 + n \times (-4)}{m+n}$$
$$= \frac{2 \times 3 + 1 \times (-4)}{2+1}$$
$$= \frac{6-4}{3} = \frac{2}{3}$$

The value of k is 2/3.

Question 17: In what ratio is the segment joining A (2, -3) and B (5, 6) divided by the x-axis? Also, find the coordinates of the point of division.

Solution:

Let point P on the x-axis divides the line segment joining the points A and B the ratio m: n

Consider P lies on x-axis having coordinates (x, 0).

$$x = (m \times 5 + n \times 2) / (m + n)$$

 $x = (5m + 2n) / (m + n)$
 $5m + 2n = x(m + n)$

$$(5-x)m + (2-x)n = 0(1)$$

And,

$$y = 0 = (m \times 6 + n(-3))/(m+n)$$

 $0 = (6m - 3n)/(m+n)$
 $6m - 3n = 0$
 $6m = 3n$
or $m/n = 3/6 = 1/2$

P divides AB in the ratio 1:2.

$$=> m = 1 \text{ and } n = 2$$



From (2)

$$(5-x) + (2-x)(2) = 0$$

 $5-x+4-2x=0$
 $3x = 9$
 $x = 3$
Hence coordinates are (3,0)

Question 18: In what ratio is the line segment joining the points A (-2, -3) and B (3, 7) divided by the y-axis? Also, find the coordinates of the point of division.

Solution:

Given: A (-2, -3) and B (3, 7) divided by the y-axis

Let P lies on y-axis and dividing the line segment AB in the ratio m: n So, the coordinates of P be (0, y)

$$0 = \frac{m \times 3 + n \times (-2)}{m+n} \Rightarrow 0 = 3m - 2n$$

$$3m = 2n \Rightarrow \frac{m}{n} = \frac{2}{3}$$

Ratio = 2:3

and
$$y = \frac{2 \times 7 + 3 \times (-3)}{2 + 3} = \frac{14 - 9}{5} = \frac{5}{5} = 1$$

Therefore, the coordinates of P be (0, 1).

Question 19: In what ratio does the line x - y - 2 = 0 divide the line segment joining the points A (3, -1) and B (8, 9)?

Solution:

Given: A line segment joining the points A (3, -1) and B (8, 9) and another line x - y - 2 = 0.

Let a point P (x, y) on the given line x - y - 2 = 0 divides the line segment AB in the ratio m : n

To find: ratio m:n



$$x = \frac{mx_2 + nx_1}{m + n} = \frac{m \times 8 + n \times 3}{m + n} = \frac{8m + 3n}{m + n}$$

$$my_2 + ny_1 \qquad m \times 9 + n \times (-1)$$

and
$$y = \frac{my_2 + ny_1}{m+n} = \frac{m \times 9 + n \times (-1)}{m+n}$$

$$=\frac{9m-n}{m+n}$$

Since point P lies on x - y - 2 = 0, so

$$\frac{8m + 3n}{m + n} - \frac{9m - n}{m + n} - 2 = 0$$

$$\frac{8m+3n}{m+n}-\frac{9m-n}{m+n}=2$$

$$8m + 3n - 9m + n = 2m + 2n$$

$$-m+4n=2m+2n$$

$$-m-2m=+2n-4n$$

$$-3m = -2n$$

$$\frac{m}{n} = \frac{-2}{-3} = \frac{2}{3}$$

The required ratio is 2:3.

Question 20: Find the lengths of the medians of a \triangle ABC whose vertices are A(0, -1), B(2, 1) and C(0, 3).

Solution:

Given: Vertices of \triangle ABC are A(0, -1), B(2, 1) and C(0, 3) Let AD, BE and CF are the medians of sides BC, CA and AB respectively, then

Step 1: Find Coordinates of D, E and F

Coordinates of D:

$$=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{2+0}{2}, \frac{1+3}{2}\right)$$

$$=\left(\frac{2}{2},\frac{4}{2}\right)=(1,\,2)$$



Coordinates of E:

$$\left(0,\frac{2}{2}\right)=(0,\ 1)$$

Coordinates of F:

$$\left(\frac{2}{2},0\right)=(1,0)$$

Step 2: Find the length of AD, BE and CF Using

Distance formula =
$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

length of AD

$$= \sqrt{(1-0)^2 + (2+1)^2} = \sqrt{1^2 + 3^2}$$

$$= \sqrt{1+9} = \sqrt{10} \text{ units}$$

Length of BE

$$=\sqrt{(2-0)^2+(1-1)^2}$$

$$=\sqrt{2^2+(0)^2}$$

Length of CF

$$= \sqrt{(1-0)^2 + (0-3)^2}$$

$$= \sqrt{1+9} = \sqrt{10} \text{ units}$$



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Exercise 6C

Question 1: Find the area of $\triangle ABC$ whose vertices are:

(i) A (1, 2), B (-2, 3) and C (-3, -4)

(ii) A (-5, 7), B (-4, -5) and C (4, 5)

(iii) A (3, 8), B (-4, 2) and C (5, -1)

(iv) A (10, -6), B (2, 5) and C (-1, 3)

Solution:

Area of \triangle ABC whose vertices are is $(x_1,y_1),(x_2,y_2)$ and (x_3,y_3) are

Area of
$$\triangle ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

(i) In \triangle ABC, vertices are A (1, 2), B (-2, 3) and C (-3, -4) Area of triangle = 1/2(1(-2+3)-2(-4-2)-3(2-3))

$$= 1/2(1 + 12 + 3)$$

= 8 sq units

Area of triangle = 1/2(-5(-5-5)-4(5-7) + 4(7 + 5))

$$= 1/2(-50 + 8 + 48)$$

= 5 sq units

Area of triangle = 1/2(3(2 + 1)-4(-1-8) + 5(8-2))

- = 1/2(9 + 36 + 30)
- = 1/2(75)
- = 37.5 sq units

Area of triangle = 1/2(10(5-3) + 2(3+6)-1(-6-5))

- = 1/2(20 + 18 + 11)
- = 1/2(49)
- = 24.5 sq units

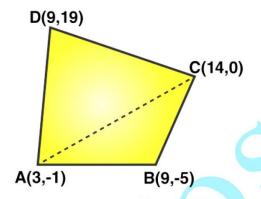


Question 2: Find the area of quadrilateral ABCD whose vertices are A (3, -1), B (9, -5), C (14, 0) and D (9, 19).

Solution: Vertices of quadrilateral ABCD are A(3,-1), B(9, -5), C (14, 0) and D(9, 19) Construction: Join diagonal AC.

We know that:

Area of
$$\triangle ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$



Area of triangle ABC:

$$= \frac{1}{2} [3(-5-0) + 9(0+1) + 14(-1+5)]$$

$$= \frac{1}{2} [-15+9+14(4)]$$

$$= \frac{1}{2} [-15+9+56]$$

Area of triangle ADC:



$$= \frac{1}{2} [3(0-19) + 14(19+1) + 9(-1+0)]$$

$$= \frac{1}{2} [3(-19) + 14 \times 20 + 9 \times (-1)]$$

$$= \frac{1}{2} [-57 + 280 - 9]$$

$$= 1/2 \times 214$$

Now, Area of quadrilateral ABCD = Area of triangle ABC + Area of triangle ADC

$$= 25 + 107$$

Question 3: Find the area of quadrilateral PQRS whose vertices are P (-5, -3), Q (-4, -6), R (2, -3) and S (1, 2).

Solution:

Given: PQRS is a quadrilateral whose vertices are P(-5, -3), Q(-4, -6), R(2, -3) and S(1, 2)

Construction: Join PR

We know that:

Area of
$$\triangle ABC = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

Area of triangle PQR:

$$= \frac{1}{2} [-5(-6+3) + (-4)(-3+3) + 2(-3+6)]$$

$$= \frac{1}{2} [-5(-3) + (-4)(0) + 2(3)]$$

$$= \frac{1}{2} [15+0+6] = \frac{21}{2} \text{ sq. units}$$

Area of triangle PSR.

$$= \frac{1}{2} [-5(-3-2) + 2(2+3) + 1(-3+3)]$$

$$= \frac{1}{2} [-5(-5) + 2 \times 5 + 1 \times 0]$$

$$= \frac{1}{2} [25 + 10 + 0] = \frac{35}{2} \text{ sq. units}$$



Now, Area of quadrilateral PQRS = Area of triangle PQR + Area of triangle PSR

$$= 21/2 + 35/2$$

= 28 sq. units

Question 4: Find the area of quadrilateral ABCD whose vertices are A (-3, -1), B (-2, -4), C (4, -1) and D (3, 4).

Solution:

Given: ABCD is a quadrilateral whose vertices are A (-3, -1), B (-2, -4), C (4, -1) and D (3, 4). By Joining AC, we get two triangles ABC and ADC

We know that:

Area of
$$\triangle ABC = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

Area of triangle ABC:

$$= \frac{1}{2} \left[-3(-4+1) + (-2)(-1+1) + 4(-1+4) \right]$$

$$= \frac{1}{2} [-3(-5) + (-2) \times 0 + 4 \times 3]$$

$$=\frac{1}{2}[15-0+12]=\frac{1}{2}\times 27=\frac{27}{2}$$
 sq. units

Area of triangle ADC.

$$= \frac{1}{2} \left[-3 \left(-4 \right) + 4 \left(4 + 1 \right) + \left(3 \right) \left(-1 + 1 \right) \right]$$

$$= \frac{1}{2} [-3(-3) + 4 \times 5 + 3 \times (0)]$$

$$=\frac{1}{2}[9+20-0]=\frac{1}{2}\times 29=\frac{29}{2}$$
 sq. units

Now, Area of quadrilateral PQRS = Area of triangle ABC + Area of triangle ADC

$$= 27/2 + 29/2$$

= 28 sq. units

Question 5: If A (-7, 5), B (-6, -7), C (-3, -8) and D (2, 3) are the vertices of a quadrilateral ABCD then find the area of the quadrilateral.

Solution:



Given: ABCD is a quadrilateral whose vertices are A (-7, 5), B (-6, -7), C (-3, -8) and D (2, 3). By Joining AC, we get two triangles ABC and ADC

We know that:

Area of
$$\triangle ABC = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

Area of triangle ABC:

$$= \frac{1}{2} [-7(-7 - (-8)] + (-6) [(-8) - 5] + (-3) [5 - (-7)]$$

$$= \frac{1}{2} [-7 \times (1) + (-6) \times (-13) + (-3) \times 12]$$

$$= \frac{1}{2} [-7 + 78 - 36]$$

$$= \frac{1}{2} \times 35 = \frac{35}{2} \text{ sq. units}$$

Area of triangle ADC.

$$= \frac{1}{2} [-7(-8 - 3) + (-3) (3 - 5) + (2) (-5 (-8)]$$

$$= \frac{1}{2} [(-7) \times (-11) + (-3) \times (-2) + 2 \times 13]$$

$$= \frac{1}{2} [77 + 6 + 26] = \frac{109}{2} \text{ sq. units}$$

Now, Area of quadrilateral PQRS = Area of triangle ABC + Area of triangle ADC

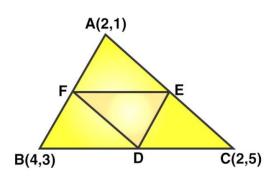
Question 6: Find the area of the triangle formed by joining the midpoints of the sides of the triangle whose vertices are A(2, 1), B(4, 3) and C(2, 5).

Solution:

Given: A triangle whose vertices are A(2, 1), B(4, 3) and C(2, 5)

Let D, E and F are the midpoints of the sides CB, CA and AB respectively of \triangle ABC, as shown in the below figure.





Find vertices of D, E and F:

Midpoint formula: $(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Vertices of D:

$$=\left(\frac{4+2}{2},\frac{3+5}{2}\right)$$

$$=\left(\frac{6}{2},\frac{8}{2}\right)=(3,4)$$

Vertices of E:

$$=\left(\frac{2+2}{2},\frac{5+1}{2}\right)$$

$$=\left(\frac{4}{2},\frac{6}{2}\right)=(2,3)$$

Vertices of F:

$$=\left(\frac{2+4}{2},\frac{1+3}{2}\right)$$

$$=\left(\frac{6}{2},\frac{4}{2}\right)=(3,2)$$

Area of triangle DEF:

We know that:

Area of
$$\triangle ABC = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$



Area of triangle DEF =

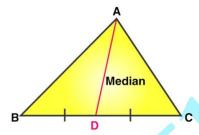
$$= \frac{1}{2} [3(3-2) + 2(2-4) + 3(4-3)]$$

$$= \frac{1}{2} [3 \times 1 + 2 \times (-2) + 3 \times 0]$$

$$= \frac{1}{2} \times 2 = 1 \text{ sq. units}$$

Question 7: A (7, -3), B (5, 3) and C (3, -1) are the vertices of a ΔABC and AD is its median. Prove that the median AD divides ΔABC into two triangles of equal areas.

Solution:



D is midpoint of BC, So find its coordinates using below:

Midpoint formula:
$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$D = ((3 + 5)/2, (3-1)/2) = (4, 1)$$

Find area of triangle ABD:

We know that:

Area of a triangle =
$$\frac{1}{2}$$
 [$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)$] So,

Area of triangle ABD:

$$= 1/2(7(3-1) + 5(1+3) + 4(-3-3))$$

$$= 1/2(14 + 20-24)$$

$$= 1/2(10)$$

$$= 5 \text{ sq. units } ...(1)$$

Area of triangle ACD:

$$= 1/2(7(-1-1) + 3(1+3) + 4(-3+1))$$



$$= 1/2(-14 + 12-8)$$

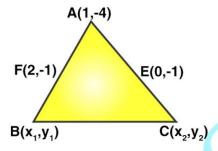
$$= 1/2(10)$$

From (1) and (2), we conclude that Area of triangle ABD and ACD is equal. Hence proved.

Question 8: Find the area of \triangle ABC with A (1, -4) and midpoints of sides through A being (2, -1) and (0, -1).

Solution:

Given: A \triangle ABC with A(1, -4)



Let F and E are the midpoints of AB and AC respectively Let Coordinates of F are (2, -1) and Coordinates of E are (0, -1)Let coordinates of B be (x_1, y_1) and Coordinates of C be (x_2, y_2)

Find coordinate of B:

using section formula:

$$2 = \frac{1 + x_1}{2} \implies x_1 = 4 - 1 = 3$$

$$-1 = \frac{4 + y_1}{2} \Rightarrow y_1 = -2 + 4 = 2$$

Coordinate of B are (3,2)

Find coordinate of C: using section formula:



$$0 = \frac{1 + x_2}{2} \Rightarrow 1 + x_2 = 0 \Rightarrow x_2 = -1$$

$$-1 = \frac{-4 + y_2}{2} \Rightarrow -4 + y_2 = -2$$

$$\Rightarrow y_2 = -2 + 4 = 2$$

Coordinate of C are (-1,2)

Now,

Area of
$$\triangle ABC = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

Area of triangle ABC:

$$= \frac{1}{2}[1(2-2) + 3(2+4) + (-1)(-4-2)]$$

$$= \frac{1}{2}[1 \times 0 + 3 \times 6 + (-1) \times (-6)]$$

$$= \frac{1}{2}[0 + 18 + 6] = \frac{24}{2} = 12 \text{ sq. units}$$

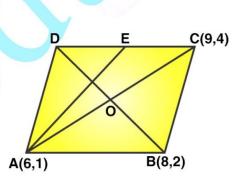
Question 9: A(6, 1), B(8, 2) and C(9, 4) are the vertices of a parallelogram ABCD. If E is the midpoint of DC, find the area of \triangle ADE.

Solution:

A(6, 1), B(8, 2) and C(9, 4) are the three vertices of a parallelogram ABCD.

E is the midpoint of DC.

Join AE, AC and BD which intersects at O, where O is midpoint of AC.



Midpoint formula:
$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$



Find coordinates of midpoints D and E:

$$\frac{x+8}{2} = \frac{15}{2} \Rightarrow x+8 = 15$$

$$x = 15 - 8 = 7$$

and

$$\frac{y+2}{2} = \frac{5}{2} \Rightarrow y+2 = 5$$

$$y = 5 - 2 = 3$$

Coordinate of D are (7, 3)

And

$$=\left(\frac{7+9}{2},\frac{3+4}{2}\right)$$

$$=\left(8,\frac{7}{2}\right)$$

Coordinate of E are (7, 3)

Now, We know that:

Area of
$$\triangle ABC = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

Area of **AADE**

$$= \frac{1}{2} \left[6 \left(3 - \frac{7}{2} \right) + 7 \left(\frac{7}{2} - 1 \right) + 8(1 - 3) \right]$$

$$= \frac{1}{2} \left[6 \times \left(\frac{-1}{2} \right) + 7 \times \frac{5}{2} + 8(-2) \right]$$

$$=\frac{5}{2}[-3+\frac{35}{2}-16]$$

$$=\frac{1}{2}\times\frac{3}{2}=\frac{3}{4}$$
 sq. units

Question 10: (i) If the vertices of \triangle ABC be A (1, -3), B (4, p) and C (-9, 7) and its area is 15 square units, find the values of p.



(ii) The area of a triangle is 5 sq units. Two of its vertices are (2, 1) and (3, -2). If the third vertex is

(7/2, y), find the value of y.

Solution:

We know that:

Area of
$$\triangle ABC = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

(i) Vertices of a \triangle ABC are A (1, -3), B (4, p) and C (-9, 7) and Area = 15 sq. units

$$15 = \frac{1}{2} [1(p-7) + 4(7+3) + (-9)(-3-p)]$$

$$30 = (p - 7 + 40 + 27 + 9p)$$

$$30 = 10p + 60$$

$$10p = 30 - 60 = -30 \Rightarrow p = \frac{-30}{10} = -3$$

$$5 = \frac{1}{2} [2(-2 - y) + 3(y - 1) + \frac{7}{2}(1 + 2)]$$

$$10 = \left[-4 - 2y + 3y - 3y + \frac{7}{2} + 7 \right]$$

$$10 = \left[y + \frac{7}{2}\right] \quad 10 - \frac{7}{2} = y \Rightarrow y = \frac{13}{2}$$

Question 11: Find the value of k so that the area of the triangle with vertices A (k + 1, 1), B (4, -3) and C (7, -k) is 6 square units.

Solution:

Area of
$$\triangle ABC = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$



$$6 = \frac{1}{2} [(k+1)(-3+k) + 4(-k-1) + 7(1+3)]$$

$$6 \times 2 = [-3k + k^2 - 3 + k - 4k - 4 + 28]$$

$$12 = [k^2 - 6k + 21]$$

$$k^2 - 6k + 21 - 12 = 0 \Rightarrow k^2 - 6k + 9 = 0$$

$$(k-3)^2 = 0 \Rightarrow k - 3 = 0$$

$$k = 3$$

The value of k is 3.

Question 12: For what value of k (k > 0) is the area of the triangle with vertices (-2, 5), (k, -4) and (2k + 1, 10) equal to 53 square units?

Solution:

Area of triangle = 53 square units

Area of
$$\triangle ABC = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$53 = \frac{1}{2} \left[-2(-4 - 10) + k(10 - 5) + (2k + 1)(5 + 4) \right]$$

$$53 = \frac{1}{2} \left[-2 \times (-14) + k \times 5 + (2k+1) \times 9 \right]$$

$$106 = 23k + 37$$

$$k = \frac{69}{23} = 3$$

The value of k is 3.

Question 13: Show that the following points are collinear.

Solution:

Points are collinear if the area of a triangle is equal to zero.

Area of
$$\triangle ABC = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$\Delta = 1/2\{2(8-4) + (-3)(4+2) - 1(2-8)\}$$

$$\Delta = 1/2 \{8-18 + 10\}$$

$$\Delta = 0$$



Hence points are collinear.

(ii) A (-5, 1), B (5, 5) and C (10, 7)
$$\Delta = 1/2\{-5(5-7) + 5 (7-1) + 10 (1-5)\}$$

$$\Delta = 1/2\{10 + 30-40\}$$

$$\Delta = 0$$

Hence points are collinear.

$$\Delta = \frac{1}{2}\{5(-1-4) + 1(4-1) + 11(1+1)\}$$

= \frac{1}{2}\{-25 + 3 + 22\}
= 0

Hence points are collinear.

$$\Delta = 1/2\{8(-4+5) + 3 (-5-1) + 2 (1+4)\}$$

= 1/2{8-18 + 10}
= 0

Hence points are collinear.

Question 14: Find the value of x for which the points A (x, 2), B (-3, -4) and C (7, -5) are collinear.

Solution:

Points are A (x, 2), B (-3, -4) and C (7, -5) are collinear.

Which means area of triangle ABC = 0

Area of
$$\triangle ABC = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

= $\frac{1}{2} [x(-4 + 5) + (-3)(-5 - 2) + 7(2 + 4)]$

$$= \frac{1}{2} [x \times 1 + (-3) \times (-7) + 7 \times 6]$$

$$= \frac{1}{2}[x+21+42] = \frac{1}{2}(x+63)$$

Since points are collinear:

$$\frac{1}{2}(x + 63) = 0$$

Or
$$x = -63$$



Question 15: For what value of x are the points A (-3, 12), B (7, 6) and C (x, 9) collinear?

Solution:

Points are A (-3, 12), B (7, 6) and C (x, 9) are collinear. Which means area of triangle ABC = 0

Area of a triangle = $1/2[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$

$$= \frac{1}{2} [-3(6-9) + 7(9-12) + x(12-6)]$$

$$= \frac{1}{2} [-3 \times (-3) + 7 \times (-3) + x \times 6]$$

$$= \frac{1}{2}[9 - 21 + 6x]$$

$$=\frac{1}{2}[6x-12]$$

Since points are collinear:

$$\frac{1}{2}(6x-12)=0$$

Or x = 2

Question 16: For what value of y are points P (1, 4), Q (3, y) and R (-3, 16) are collinear?

Solution:

Points are P (1, 4), Q (3, y) and R (-3, 16) are collinear.

Which means area of triangle PQR = 0

Area of a triangle = $1/2[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$

$$= \frac{1}{2}[1(y-16)+3(16-4)+(-3)(4-y)]$$

$$= \frac{1}{2} [y - 16 + 3 \times 12 - 12 + 3y]$$

$$= \frac{1}{2} [4y - 16 + 36 - 12] = \frac{1}{2} [4y + 8]$$

Since points are collinear:

$$\frac{1}{2}(4y + 8) = 0$$

Or
$$y = -2$$



Question 17: Find the value of y for which the points A (-3, 9), B (2, y) and C (4, -5) are collinear.

Solution:

Points are A (-3, 9), B (2, y) and C (4, -5) are collinear. Which means area of triangle ABC = 0

Area of a triangle = $1/2[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$

$$= \frac{1}{2} [-3(y+5) + 2(-5-9) + 4(9-y)]$$

$$= \frac{1}{2} [-3y - 15 + 2 \times (-14) + 36 - 4y]$$

$$= \frac{1}{2} \left[-7y - 15 - 28 + 36 \right]$$

$$=\frac{1}{2}[-7y-7]$$

Since points are collinear:

$$\frac{1}{2}(-7y - 7) = 0$$

Or
$$y = -1$$

Question 18: For what values of k are the points A (8, 1), B (3, -2k) and C (k, -5) collinear.

Solution:

Points are A (8, 1), B (3, -2k) and C (k, -5) are collinear.

Which means area of triangle ABC = 0

Area of a triangle = $1/2[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$

$$= \frac{1}{2} [8(-2k+5) + 3(-5-1) + k(1+2k)]$$

$$= \frac{1}{2} \left[-16k + 40 + 3(-6) + k + 2k^2 \right]$$

$$= \frac{1}{2} \left[-16k + 40 - 18 + k + 2k^2 \right]$$

$$= \frac{1}{2} [22 - 15k + 2k^2]$$

Since points are collinear:

$$\frac{1}{2}(2k^2 - 15k + 22) = 0$$



$$Or 2k^2 - 15k + 22 = 0$$

$$2k^2 - 11k - 4k + 22 = 0$$

$$k(2k - 11) - 2(2k - 11) = 0$$

$$(k-2)(2k-11) = 0$$

k = 2 or k = 11/2. Answer.

Question 19: Find a relation between x and y, if the points A(2, 1), B(x, y) and C(7, 5) are collinear.

Solution:

Points are A(2, 1), B(x, y) and C(7, 5) are collinear. Which means area of triangle ABC = 0

Area of a triangle = $1/2[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$

$$= \frac{1}{2} [2(y-5) + x(5-1) + 7(1-y)]$$

$$= \frac{1}{2} [2y - 10 + 4x + 7 - 7y]$$

$$= \frac{1}{2} [4x - 5y - 3]$$

Since points are collinear:

$$\frac{1}{2}(4x - 5y - 3) = 0$$

$$4x - 5y - 3 = 0$$

Relationship between x and y.

Question 20: Find a relation between x and y, if the points A(x, y), B(-5, 7) and C(-4, 5) are collinear.

Solution:

Points are A(x, y), B(-5, 7) and C(-4, 5) are collinear. Which means area of triangle ABC = 0

Area of a triangle = $1/2[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$



$$= \frac{1}{2} [x(7-5) + (-5)(5-y) + (-4)(y-7)]$$

$$= \frac{1}{2} [x \times 2 - 25 + 5y - 4y + 28]$$

$$= \frac{1}{2} [2x + y + 3]$$

Since points are collinear:

$$\frac{1}{2}(2x + y + 3) = 0$$

$$2x + y + 3 = 0$$

Relationship between x and y.

Question 21: Prove that the points A(a, 0), B(0, b) and C(1, 1) are collinear, if 1/a + 1/b = 1.

Solution:

Points are A(a, 0), B(0, b) and C(1, 1) are collinear.

Which means area of triangle ABC = 0

Area of
$$\triangle ABC = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2} [a(b-1) + 0(1-0) + 1(0-b)]$$

$$= \frac{1}{2} [ab - a + 0 - b]$$

Since points are collinear:

$$\frac{1}{2}(ab - a - b) = 0$$

$$ab - a - b = 0$$

Divide each term by "ab", we get

$$1 - 1/b - 1/a = 0$$

or 1/a + 1/b = 1. hence proved.

Question 22: If the points P(-3, 9), Q(a, b) and R(4, -5) are collinear and a + b = 1, find the values of a and b.

Solution:



Points are P(-3, 9), Q(a, b) and R(4, -5) are collinear. Which means area of triangle = 0

Area of
$$\triangle ABC = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2} [-3(b+5) + a(-5-9) + 4(9-b)]$$

$$= \frac{1}{2} [-3b - 15 - 5a - 9a + 36 - 4b]$$

$$= \frac{1}{2} [-14a - 7b + 21]$$

Since points are collinear, we have

$$\frac{1}{2}(-14a - 7b + 21) = 0$$

$$-14a - 7b + 21 = 0$$

$$2a + b = 3 \dots (1)$$

$$a + b = 1(2)$$
 (given)

From (1) and (2)

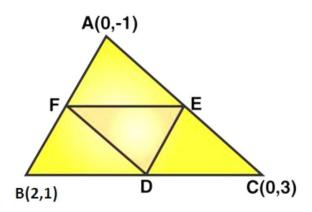
$$a = 2$$
 and $b = -1$

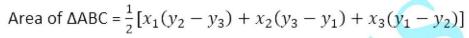
Question 23: Find the area of \triangle ABC with vertices A (0, -1), B (2, 1) and C (0, 3). Also, find the area of the triangle formed by joining the midpoints of its sides. Show that the ratio of the areas of two triangles is 4 : 1.

Solution:

Vertices of ΔABC are A (0, -1), B (2, 1) and C (0, 3)







Area of triangle ABC:

$$=\frac{1}{2}[0(1-3)+2(3+1)+0(-1-1)]$$

$$= \frac{1}{2} [0 + 2 \times 4 + 0] = \frac{1}{2} \times 8 = 4 \text{ sq. units}$$

From figure: Points D, E and F are midpoints of sides BC, CA and AB respectively.

Find the coordinates of D, E and F

coordinates of D

$$=\left(\frac{2+0}{2},\frac{1+3}{2}\right)$$

$$=(1,2)$$

coordinates of E

$$=\left(\frac{0+0}{2},\frac{3-1}{2}\right)$$

$$=(0, 1)$$

coordinates of F

$$=\left(\frac{0+2}{2},\frac{-1+1}{2}\right)$$



=(1,0)

Area of triangle DEF:

$$= 1/2[1+0+1]$$

Therefore,

Ratio in the area of triangles ABC and DEF = 4/1 = 4:1.

Question 24: If $a \neq b \neq c$, prove that (a, a^2) , (b, b^2) , (0, 0) will not be collinear.

Solution:

Let A (a, a^2), B (b, b^2) and C (0, 0) are the vertices of a triangle.

Let us assume that that points are collinear, then area of ΔABC must be zero. Now, area of ΔABC

$$= \frac{1}{2} [a(b^2 - 0) + b(0 - a^2) + 0(a^2 - b^2)]$$

$$= \frac{1}{2} (ab^2 - bc^2)$$

$$= \frac{ab}{2} (b - a)$$

$$\neq 0$$

Which is contraction to our assumption.

This implies points are not be collinear. Hence proved.



Exercise 6D

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Question 1: Points A(-1, y) and B(5, 7) lie on a circle with centre O(2, -3y). Find the values of y.

Solution:

Points A(-1, y) and B(5, 7) lie on a circle with centre O(2, -3y).

Which means: OA = OB or OA^2 = OB^2

using distance formula, we get

$$(-1-2)^2 + (y-(-3y))^2 = (5-2)^2 + (7-(-3y))^2$$

$$9 + 16y^2 = 9 + (7 + 3y)^2$$

$$16y^2 = 49 + 42y + 9y^2$$

$$7y^2 - 42y - 49 = 0$$

$$7(y^2-6y-7) = 0$$

$$y^2-7y + y-7 = 0$$

$$y(y-7) + 1(y-7) = 0$$

$$(y + 1)(y-7) = 0$$

Therefore, y = 7 or y = -1

Possible values of y are 7 or -1.

Question 2: If the point A (0, 2) is equidistant from the points B (3, p) and C (p, 5) find p.

Solution:

A (0, 2) is equidistant from the points B (3, p) and C (p, 5)

Which means: AB = AC or AB^2 = AC^2

using distance formula, we get

$$(0-3)^2 + (2-p)^2 = (0-p)^2 + (2-5)^2$$

$$9 + 4 + p^2 - 4p = p^2 + 9$$

$$4p-4=0$$

p = 1

Therefore, the value of p is 1.

Question 3: ABCD is a rectangle whose three vertices are B (4, 0), C (4, 3) and D (0, 3). Find the length of one of its diagonal.

Solution:



ABCD is a rectangle whose three vertices are B (4, 0), C (4, 3) and D (0, 3).

Find length of one of its diagonal, say BD: using distance formula, we get

BD =
$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

= $\sqrt{(4 - 0)^2 + (0 - 3)^2}$
= $\sqrt{(4)^2 + (-3)^2}$ = $\sqrt{16 + 9}$
= $\sqrt{25}$ = 5 units

Therefore, length of one of its diagonal is 1.

Question 4: If the point P (k - 1, 2) is equidistant from the points A (3, k) and B (k, 5), find the values of k.

Solution:

Point P (k-1, 2) is equidistant from the points A (3, k) and B (k, 5).

$$PA = PB \text{ or } PA^2 = PB^2$$

$$(3-k+1)^2 + (k-2)^2 = (k-k+1)^2 + (5-2)^2$$

$$(4-k)^2 + (k-2)^2 = 1^2 + 3^2$$

$$16 - 8k + k^2 + k^2 - 4k + 4 = 1 + 9$$

$$2k^2 - 12k + 20 = 10$$

$$2k^2 - 12k + 20 - 10 = 0$$

$$2k^2 - 12k + 10 = 0$$

$$k^2 - 6k + 5 = 0$$

$$k^2 - k - 5k + 5 = 0$$

$$k(k-1)-5(k-1)=0$$

$$(k-5)(k-1) = 0$$

$$k=1$$
 or $k=5$

Question 5: Find the ratio in which the point P (x, 2) divides the join of A (12, 5) and B (4, -3).

Solution:

If point P (x, 2) divides the join of A (12, 5) and B (4, -3), then



using section formula, we get

$$2 = \frac{m \times (-3) + n \times (5)}{m + n}$$

2m + 2n = -3m + 5n

5m = 3n

m/n = m:n = 3:5

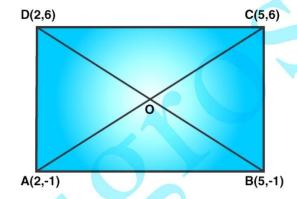
The required ratio is 3:5.

Question 6: Prove that the diagonals of a rectangle ABCD with vertices A(2, -1), B(5, -1), C(5, 6) and D(2, 6) are equal and bisect each other.

Solution:

Vertices f a rectangle ABCD are A(2, -1), B(5, -1), C(5, 6) and D(2, 6)

To prove: Diagonals of the rectangle are equal and bisect each other.



Diagonal AC

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(5-2)^2 + (6+1)^2}$$

$$=\sqrt{9+49}=\sqrt{58}$$



Diagonal BD

$$\sqrt{(5-2)^2 + (-1-6)^2}$$
= $\sqrt{3^2 + (-7)^2}$
= $\sqrt{58}$

AC and BD are equal in length. Thus, Diagonals are equal.

Now,

Consider that O is the midpoint of AC then its coordinate are

Midpoint formula:

$$(x, y) = (\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$$

$$=((2+5)/2, (-1+6)/2)$$

$$=(7/2,5/2)$$

If point O divides AC in the ratio m:n, then

$$\frac{7}{2} = \frac{mx_2 + nx_1}{m+n} = \frac{m \times 2 + n \times 5}{m+n}$$

$$=\frac{2m+5n}{}$$

$$7m + 7n = 4m + 10n$$

$$7m - 4m = 10n - 7n \Rightarrow 3m = 3n$$

$$m = n$$

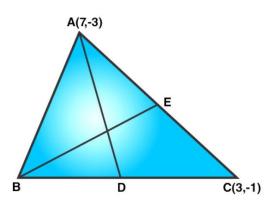
Which shows, O is the midpoint of diagonals.

Question 7: Find the lengths of the medians AD and BE of \triangle ABC whose vertices are A(7, -3), B(5, 3) and C(3, -1).

Solution:

Vertices of \triangle ABC are A(7, -3), B(5, 3) and C(3, -1)





From figure: BE and AD are the medians of triangle.

Find Coordinates of E and D:

Coordinates of E =

$$\left(\frac{3+7}{2},\frac{-1-3}{2}\right)$$

$$=(5,-2)$$

Coordinates of D=

$$\left(\frac{3+5}{2}, \frac{-1+3}{2}\right) = (4, 1)$$

Find AD and BE using distance formula:

AD =
$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

= $\sqrt{(7 - 4)^2 + (3 + 1)^2}$ = $\sqrt{3^2 + 4^2}$
= $\sqrt{9 + 16}$ = $\sqrt{25}$ = 5 units
and BE = $\sqrt{(5 - 5)^2 + (3 + 2)^2}$ = $\sqrt{0^2 + 5^2}$
= $\sqrt{25}$ = 5 units

Therefore, BE = 5 units and AD = 5 units (both are equal)

Question 8: If the points C (k, 4) divides the join of A (2, 6) and B (5, 1) in the ratio 2: 3 then find the value of k.



Solution:

C(k, 4) divides the join of A(2, 6) and B(5, 1) in the ratio 2:3.

Using Section Formula:

$$k = \frac{\frac{2}{3} \times (5) + 1 \times (2)}{\frac{2}{3} + 1}$$

After simplifying, we get k = 16/5The value of k is 16/5.

Question 9: Find the point on x-axis which is equidistant from points A (-1, 0) and B (5, 0).

Solution:

Since point lies on x-axis, y-coordinate of the point will be zero. Let P (x, 0) be on x-axis which is equidistant from A (-1, 0) and B (5, 0)

Using section formula:

$$x = \frac{1 \times (5) + 1 \times (-1)}{1 + 1}$$

Or x = 2

Thus, the required point is (2, 0).

Question 10: Find the distance between the points (-8/2, 2) and (2/5, 2).

Solution:

Using distance formula, we have

$$=\sqrt{\left(\frac{2}{5}+\frac{8}{5}\right)^2+(2-2)^2}$$

$$=\sqrt{\left(\frac{10}{5}\right)^2+0^2}$$

$$=\sqrt{2^2+0^2}$$

$$=\sqrt{4}=2$$
 units



Question 11: Find the value of a, so that the point (3, a) lies on the line represented by 2x - 3y = 5.

Solution:

The points (3, a) lies on the line 2x - 3y = 5.

Put value of x = 3 and y = a in given equation,

$$2 \times 3 - 3 \times a = 5$$

$$6 - 3a = 5$$

$$3a = 6 - 5$$

$$a = 1/3$$

Question 12: If the points A (4, 3) and B (x, 5) lie on the circle with centre O(2, 3), find the value of x.

Solution:

Points A (4, 3) and B (x, 5) lie on the circle with centre O(2, 3)

Which means: OA = OB

$$(2-4)^2 + (3-3)^2 = (2-x)^2 + (3-5)^2$$

$$(-2)^2 + 0^2 = (2 - x)^2 + (-2)^2$$

$$(2-x)^2=0$$

$$2-x = 0$$

$$x = 2$$

The value of x is 2.

Question 13: If P (x, y) is equidistant from the points A (7, 1) and B (3, 5), find the relation between x and y.

Solution:

P(x, y) is equidistant from the point A(7, 1) and B(3, 5)

$$PA = PB$$

$$(x-7)^2 + (y-1)^2 = (x-3)^2 + (y-5)^2$$

$$x^2 - 14x + 49 + y^2 - 2y + 1 = x^2 - 6x + 9 + y^2 - 10y + 25$$

$$-8x + 8y = -16$$

$$x - y = 2$$

Relation between x and y is x - y = 2



Question 14: If the centroid of $\triangle ABC$ having vertices A (a, b), B (b, c) and C (c, a) is the origin, then find the value of (a + b + c).

Solution:

Centroid of \triangle ABC having vertices A (a, b), B (b, c) and C (c, a) is the origin.

Let O (0, 0) is the centroid of \triangle ABC. a + b + c = 0

And

Centroid =
$$\frac{x_1 + x_2 + x_3}{3}$$
, $\frac{y_1 + y_2 + y_3}{3}$

Question 15: Find the centroid of \triangle ABC whose vertices are A(2, 2), B(-4, -4) and C(5, -8).

Solution:

Centroid of \triangle ABC whose vertices are A(2, 2), B(-4, -4) and C(5, -8).

Centroid =
$$\frac{x_1 + x_2 + x_3}{3}$$
, $\frac{y_1 + y_2 + y_3}{3}$

$$=\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$$

$$=\left(\frac{2-4+5}{3},\frac{2-4-8}{3}\right)$$

$$=\left(\frac{3}{3},\frac{-10}{3}\right)$$

$$=\left(1,\frac{-10}{3}\right)$$

Question 16: In what ratio does the point C(4, 5) divide the join of A(2, 3) and B(7, 8)?

Solution:

Point C(4, 5) divide the join of A(2, 3) and B(7, 8)



Let point C(4, 5) divides the AB in the ratio m:n

Using section formula:

$$x = \frac{mx_2 + nx_1}{m + n}$$

$$4 = \frac{m(7) + n(2)}{m + n}$$

$$4m + 4n = 7m + 2n$$

3m = 2n

m:n = 2:3

The required ratio is 2:3.

Question 17: If the points A(2, 3), B(4, k) and C(6, -3) are collinear, find the value of k.

Solution:

Points A(2, 3), B(4, k) and C(6, -3) are collinear. Area of triangle having vertices A, B and C = 0

Area of a triangle =
$$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

Area of given \triangle ABC = 0

$$= \frac{1}{2} [(2(k - (-3)) + 4(-3 - 3) + 6(3 - k))] = 0$$

$$2k + 6 - 24 + 18 - 6k = 0$$

$$-4k = 0$$

or k = 0

The value of k is zero.