RS Aggarwal Solutions for Class 10 Maths Chapter 5 Arithmetic Progression

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Question 1:
Show that each of the progressions given below is an AP. Find the first term, common difference and next term of each.
(i) 9, 15, 21, 27, …..
(ii) 11, 6, 1, -4, ……
(iii) -1, -5 /6 , -2 /3 , -1 /2 , …………
(iv) √2, √8, √18, √32, ……..
(v) √20, √45, √80, √125, ……

Solution:
(i) 9, 15, 21, 27, …
Here, 15 – 9 = 21 – 15 = 27 – 21 = 6 (which is constant)

Common difference is 6

Or d = 6

Next term = 27 + d = 27 + 6 = 33

(ii) 11, 6, 1, -4, …
Here, 6 – 11 = 1 – 6 = -5 -4 – 1 = -5 (which is constant)

d (common difference) = -5
Next term = -4 – 5 = -9

(iii) -1, -5 /6 , -2 /3 , -1 /2 , …………

-5/6 - (-1) = 1/6 and

-2/3 - (-5/6) = 1/6

d (common difference) = 1/6
Next term = -1/2 + 1/6 = -1/3

(iv) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \ldots$

$\sqrt{8} - \sqrt{2} = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$

$\sqrt{32} - \sqrt{18} = 4\sqrt{2} - 3\sqrt{2} = \sqrt{2}$

d (common difference) $= \sqrt{2}$

Next term $= \sqrt{32} + \sqrt{2} = 4\sqrt{2} + \sqrt{2} = 5\sqrt{2} = \sqrt{50}$

(v) $\sqrt{20}, \sqrt{45}, \sqrt{80}, \sqrt{125}, \ldots$

$\sqrt{20}, \sqrt{45}, \sqrt{80}, \sqrt{125}, \ldots$

$\sqrt{45} - \sqrt{20} = 3\sqrt{5} - 2\sqrt{5} = \sqrt{5}$

$\sqrt{125} - \sqrt{80} = 5\sqrt{5} - 4\sqrt{5} = \sqrt{5}$

d (common difference) $= \sqrt{5}$

Next term $= \sqrt{125} + \sqrt{5} = 5\sqrt{5} + \sqrt{5} = 6\sqrt{5}$ or $\sqrt{180}$

Question 2:
Find:
(i) the 20th term of the AP 9, 13, 17, 21, ……
(ii) the 35th term of the AP 20, 17, 14, 11, ……
(iii) the 18th term of the AP $\sqrt{2}, \sqrt{18}, \sqrt{50}, \sqrt{98}, \ldots$
(iv) the 9th term of the AP $3/4, 5/4, 7/4, 9/4, \ldots$
(v) the 15th term of the AP -40, -15, 10, 35, ……

Solution:
(i)Given: AP is 9, 13, 17, 21, ……

Here, first term = a = 9
Common difference = d = 13 – 9 = 4

$a_n = a + (n-1)d$

a_{20}= 9 + (20-1)4

= 85

(ii) the 35th term of the AP 20, 17, 14, 11, ……

Given: AP is 20, 17, 14, 11, ……

Here, first term = a = 20
Common difference = d = 17 - 20 = -3

n = 35

a_n = a + (n-1)d

a_{35} = 20 + (35-1)(-3) =

-82

(iii) the 18th term of the AP √2, √18 , √50, √98, ………

Given: AP is √2, √18 , √50, √98, ………

or √2, 3√2 , 5√2, 7√2,....

Here, first term = a = √2
Common difference = d = 3√2 - √2 = 2√2

n = 18

a_n = a + (n-1)d

a_{18} = √2 + 34√2 =

35√2

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(iv) the 9th term of the AP 3 /4 , 5 /4 , 7 /4 , 9 /4 , …….

Given: AP is 3 /4 , 5 /4 , 7 /4 , 9 /4 , …….

Here, first term = a = 3/4
Common difference = d = 5/4 - 3/4 = 1/2

n = 9

$a_n = a + (n-1)d$

$a_9 = 3/4 + (9 - 1)1/2$

= 19/4

(v) the 15th term of the AP -40, -15, 10, 35, ……

Given: AP is -40, -15, 10, 35, ……

Here, first term = a = -40
Common difference = d = -15 - (-40) = -15 + 40 = 25

n = 15

$a_n = a + (n-1)d$

$a_{15} = -40 + (15 - 1)25 =$

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Question 3:
(i)Find the 37th term of the AP 6, 7 3 /4 , 9_1 /2 , 11_1 /4 , ………..
(ii)Find the 25th term of the AP 5, 4 1 /2 , 4, 3_1 /2 , 3, ……….

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Solution:
(i)Given AP is

$$6, 7\frac{3}{4}, 9\frac{1}{2}, 11\frac{1}{4}, \ldots$$

Here, first term = a = 6

Common difference:

$$d = 7\frac{3}{4} - 6 = 1\frac{3}{4} = \frac{7}{4}$$

Here, first term = a = 6
Common difference = d = -15 - (-40) = -15 + 40 = 25

n = 37

$a_n = a + (n-1)d$

$a_{37} = 6 + (37-1)(7/4)$

= 69

(ii) Given AP is 5, 4_1 /2 , 4, 3_1 /2 , 3, ………
Given AP is

$$5, 4\frac{1}{2}, 4, 3\frac{1}{2}, 3, \ldots$$

Here, first term = a = 5

Common difference:

$$d = 4\frac{1}{2} - 5 = \frac{-1}{2}$$

n = 25

$a_n = a + (n-1)d$

$a_{25} = 5 + (25-1)(-1/2)$

$= -7$

Question 4:
Find the value of p for which the numbers 2p – 1, 3p + 1, 11 are in AP. Hence, find the numbers.
Solution:

If 2p – 1, 3p + 1, 11 are terms in AP, then

$a_2 – a_1 = a_3 – a_2$ …..(1)

From given:

$a_1 = 2p – 1$
$a_2 = 3p + 1$
$a_3 = 11$

From (1), we get
(3p + 1) – (2p – 1) = 11 – (3p + 1)
3p + 1 – 2p + 1 = 11 – 3p – 1
p + 2 = 10 – 3p
4p = 8
p = 2
For p = 2, these terms are in AP.

Question 5:
Find the nth term of each of the following APs:
(i) 5, 11, 17, 23, ……
(ii) 16, 9, 2, -5, ……

Solution:

(i)AP is 5, 11, 17, 23, ……

Here, first term = a = 5

Common difference = d = 11 – 5 = 6

Now,
$a_n = a (n – 1)d$

= 5 + (n – 1) 6
= 5 + 6n – 6
= (6n – 1)

(ii) AP is 16, 9, 2, -5, ……

Here, first term = a = 16

Common difference = d = 9 – 16 = -7

$a_n = a + (n – 1)d$

= 16 + (n – 1) (-7)

= 16 – 7n + 7

= (23 – 7n)

Question 6:
If the nth term of a progression is (4n – 10) show that it is an AP. Find its
(i) first term,
(ii) common difference, and
(iii) 16th term.

Solution:

n^{th} term of AP is 4n – 10 (Given)

Putting n = 1, 2, 3, 4, …, we get

At n = 1: 4n – 10 = 4 x 1 – 10 = 4 – 10 = -6
At n = 2: 4n – 10 = 4 x 2 – 10 = 8 – 10 = -2
At n = 3: 4n – 10 = 4 x 3 – 10 = 12 – 10 = 2
At n = 1: 4n – 10 = 4 x 4 – 10 = 16 – 10 = 6

We see that -6, -2, 2, 6,… are in AP

(i)first term = -6
(ii) Common difference = -2 – (-6) = 4
(iii) 16th term:

Using formula: $a_n = a + (n – 1)d$

Here n = 16

$a_{16} = -6 + (16 – 1)4 = 54$

Question 7:
How many terms are there in the AP 6, 10, 14, 18, …. 174?

Solution:

Given: AP is 6, 10, 14, 18,…, 174

Here, first term = a = 6

Common difference = d= 10 – 6 = 4

To find: the number of terms (n)

Last term = a + (n – 1)d
174 = 6 + (n – 1) 4
174 – 6 = (n – 1) 4
n – 1 = 168 /4 = 42
n = 42 + 1 = 43

There are 43 terms.

Question 8:
How many terms are there in the AP 41, 38, 35, …, 8?
Solution:
Given: AP is 41, 38, 35,…, 8

Here, first term = a = 41

Last term = 8

Common difference = d = 38 – 41 = -3

To find: the number of terms (n)

Last term = a + (n – 1)d
8 = 41 + (n – 1)(-3)
8 – 41 = (n – 1)(-3)
n – 1 = 11
n = 11 + 1 = 12
There are 12 terms.

Question 9:
How many terms are there in the AP is 18, 15 1/2 , 13, …, -47 ?
Solution:
Given: AP is 18, 31/2 , 13, …, -47

Here, first term = a = 18

Last term = -47

Common difference = d = -5/2

To find: the number of terms (n)

Last term = a + (n – 1)d

-47 = 18 + (n –
1)(-5/2)

-47 - 18 = (n - 1)(-5/2)

n = 27
There are 27 terms.

Question 10:
Which term of the AP 3, 8, 13, 18, … is 88?

Solution:

Let nth term is 88.

AP is 3, 8, 13, 18, …
Here,
First term = a = 3
Common difference = d = 8 – 3 = 5

nth term of AP is $a_n = a + (n – 1) d$

Now,
88 = 3 + (n – 1)(5)
88 – 3 = (n – 1) x 5
n – 1 = 88 /5
or n = 17 + 1 = 18

Therefore: 88 is the 18th term.

Question 11:
Which term of the AP 72, 68, 64, 60, ….. is 0?

Solution:

AP is 72, 68, 64, 60, …..

Let nth term is 0.

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Here,

First term = a = 72

Common difference = d = 68 – 72 = -4

$a_n = a + (n – 1)d$

0 = 72 + (n – 1)(-4)
-72 = -4(n – 1)
n – 1 = 18
n = 18 + 1 = 19

Therefore: 0 is the 19th term.

Question 12:
Which term of the AP 5 /6 , 1, 1 1 /6 , 1 1 /3 , ….. is 3?
Solution:

$\frac{5}{6}, 1, 1\frac{1}{6}, 1\frac{1}{3}, \ldots$

Here,

First term = a = 5/6

Common difference = d = 1 – 5/6 = 1/6

Now: $a_n = a + (n – 1)d$

3 = 5/6 + (n – 1)1/6

Let nth term is 3

Now, $a_n = a + (n-1)d$

3 = 5/6 + (n-1)1/6

n -1 = 13

n = 13 + 1 = 14

Therefore, 3 is the 14th term.

Question 13:

Which term of the AP 21, 18, 15,… is -81?
Solution:

AP is 21, 18, 15, ……

Let nth term -81

Here, a = 21, d = 18 – 21 = -3
$a_n = a + (n – 1)d$
-81 = 21 + (n – 1)(-3)
-81 – 21 = (n – 1)(-3)
-102 = (n – 1)(-3)
n = 34 + 1 = 35

Therefore, -81 is the 35th term

Question 14:
Which term of the AP 3, 8, 13, 18, …will be 55 more than its 20th term?
Solution:
Given AP is 3, 8, 13, 18,…
First term = a = 3
Common difference = d = 8 – 3 = 5
And n = 20 and a_{20} be the 20^{th} term, then

$a_{20} = a + (n – 1)d$
= 3 + (20 – 1) 5
= 3 + 95
= 98
The required term = 98 + 55 = 153

Now, 153 be the nth term, then
$a_n = a + (n – 1)d$
$153 = 3 + (n – 1) \times 5$
$153 – 3 = 5(n – 1)$
$150 = 5(n – 1)$
$n – 1 = 30$
$n = 31$
Required term will be 31st term.

Question 15:
Which term of the AP 5, 15, 25,… will be 130 more than its 31st term?
Solution:
AP is 5, 15, 25,…
First term = a = 5
Common difference = d = 15 – 5 = 10
Find 31[st] term:
$a_{31} = a + (n – 1)d$
$= 5 + (31 – 1)\ 10$
$= 5 + 30 \times 10$
$= 305$
Required term = 305 + 130 = 435
Now, say 435 be the nth term, then
$a_n = a + (n – 1)d$
$435 = 5 + (n – 1)10$
$435 – 5 = (n – 1)10$
$n – 1 = 43$
$n = 44$
The required term will be 44th term.

Question 16:
If the 10th term of an AP is 52 and 17th term is 20 more than its 13 th term, find the AP
Solution:
Let a be the first term and d be the common difference, then

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$T_{10} = a + (n-1)d = a + 9d = 52$
$T_{17} = a + 16d$ and $T_{13} = a + 12d$
$\therefore T_{17} - T_{13} = 20$
$(a + 16d) - (a + 12d) = 20$
$a + 16d - a - 12d = 20$
$\Rightarrow 4d = 20$

$$d = \frac{20}{4} = 5$$

$52 = a + 9d = a + 9 \times 5$
$52 = a + 45$
$\therefore a = 52 - 45 = 7$
Now, AP will be 7, 12, 17, 22, ...

Question 17:
Find the middle term of the AP 6, 13, 20,…, 216
Solution:
AP is 6, 13, 20,…, 216
a = 6, d = 7 and l = 216
Let $T_n = l = a + (n-1)d$

$$216 - 6 = (n-1)7 \Rightarrow \frac{210}{7} = n - 1$$

$n - 1 = 30 \Rightarrow n - 30 + 1 = 31$
This AP has 31 terms

$$\text{Middle term} = \frac{31+1}{2} = 16\text{th term}$$

$a_{16} = 6 + (16 - 1)7 = 6 + 105 = 111$
Therefore, midterm of the AP id 111.

Question 18:
Find the middle term of the AP 10, 7, 4, ………. (-62)
Solution:
AP is 10, 7, 4, ….…., (-62)
a = 10,

d = 7 – 10 = -3,
and
l = -62
Now, a_n = l = a + (n – 1)d
-62 = 10 + (n – 1) x (-3)
-62 – 10 = -3(n- 1)
-72 = -3(n – 1)
Or n = 24 + 1 = 25

Middle term = (25 + 1) /2 th = 13th term

Find the 13th term using formula, we get

a_{13} = 10 + (13 – 1)(-3) = 10 – 36 = -26

Question 19:
Find the sum of two middle most terms of the AP -4 /3 , -1, -2 /3 , …, 4 1 /3 .
Solution:
Given AP is -4 /3 , -1, -2 /3 , …, 13 /3

Here, $a = -\frac{4}{3}, d = -1 - \left(-\frac{4}{3}\right) = -1 + \frac{4}{3}$

$\frac{1}{3}, l = 4\frac{1}{3} = \frac{13}{3}$

$T_n = l = \frac{13}{3} = a + (n-1)d$

$\frac{13}{3} = \frac{-4}{3} + (n-1)\frac{1}{3}$

$\frac{13}{3} + \frac{4}{3} = (n-1)\frac{1}{3}$

$\frac{17}{3} = (n-1)\frac{1}{3} \Rightarrow n-1 = \frac{17}{3} \times \frac{3}{1} = 17$

$n = 17 + 1 = 18$

$\frac{13}{3}$ is the 18th term

Middle terms will be: (18/2)th + (18/2 + 1)th
= 9th + 10th term

Now,
$a_9 + a_{10} = a + 8d + a + 9d$

= 2a + 17d

= 2(-4/3) + 17(1/3)

= 3

Question 20:
Find the 8th term from the end of the AP 7, 10, 13, ……, 184.
Solution:
Given: AP is 7, 10, 13,…, 184
a = 7, d = 10 – 7 = 3 and l = 184
n^{th} term from the end = l – (n – 1)d
Now,
8th term from the end be
184 – (8 – 1)3 = 184 – 21 = 163

Question 21:
Find the 6th term from the end of the AP 17, 14, 11, …, (-40).
Solution:
Given: AP is 17, 14, 11, …,(-40)
a = 17, d = 14 – 17 = -3, l = -40
6th term from the end = l – (n – 1)d
= -40 - (6 - 1) (-3)
= -40 – (5 x (-3))
= -40 + 15
= -25

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Question 22:
Is 184 a term of the AP 3, 7, 11, 15, …..?
Solution:
Given AP is 3, 7, 11, 15, …
$a = 3$, $d = 7 - 3 = 4$
Let 184 be the nth term of the AP
$a_n = a + (n - 1)d$
$184 = 3 + (n - 1) \times 4$
$184 - 3 = (n - 1) \times 4$
$181/4 = n - 1$
$n = 181/4 + 1 = 185/4$ (Which is in fraction)

Therefore, 184 is not a term of the given AP.

Question 23:
Is -150 a term of the AP 11, 8, 5, 2,…?
Solution:
Given AP is AP 11, 8, 5, 2,…
Here $a = 11$, $d = 8-11= -3$
Let -150 be the nth term of the AP
$a_n = a + (n - 1)d$
$-150 = 11 +(n-1)(-3)$
or $n = 164/3$
Which is a fraction.
Therefore, -150 is not a term of the given AP.

Question 24:
Which term of the AP 121, 117, 113,… is its first negative term?
Solution:
Let nth of the AP 121, 117, 113,… is negative. Let T_n be the nth term then

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$T_n < 0$

Here, $a = 121$, $d = 117 - 121 = -4$

$T_n = 121 + (n-1)(-4)$

$121 - 4n + 4 < 0$

$125 - 44 < 0$

$125 < 4n \Rightarrow 4n > 125$

$n > \frac{125}{4} \Rightarrow n > 31\frac{1}{4}$

Therefore, 32nd term will be the 1st negative term.

Question 25:
Which term of the AP 20, 19 1 /4 , 18 1 /2 , 17 3 /4 , ………… is its first negative term?
Solution:

AP is $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \ldots$

a = 20, d = -3/4

Let nth term be the 1st negative term of the AP

$a_n < 0$

$a_n = a + (n-1)d$

$20 + (n-1)\left(\frac{-3}{4}\right) < 0$

$20 - \frac{3}{4}n + \frac{3}{4} < 0 \Rightarrow 20\frac{3}{4} - \frac{3}{4}n < 0$

$20\frac{3}{4} < \frac{3}{4}n \Rightarrow \frac{83}{4} \times \frac{4}{3} < n$

$\frac{83}{3} < n \Rightarrow n > \frac{83}{3} \Rightarrow n > 27\frac{2}{3}$

Therefore, 28^{th} term will be the 1st negative term.

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Question 26:
The 7th term of an AP is -4 and its 13th term is -16. Find the AP.

Solution:
Let us say a be the first term and d be the common difference of an AP
$a_n = a + (n - 1)d$
$a_7 = a + (7 - 1)d$
$= a + 6d = -4$ …………(1)

And $a_{13} = a + 12d = -16$ ….……(2)

Subtracting equation (1) from (2), we get
$6d = -16 - (-4) = -12$
From (1), $a + 6d = -4$
$a + (-12) = -4$
$a = -4 + 12 = 8$
$a = 8, d = -2$
AP will be 8, 6, 4, 2, 0, ……

Question 27:
The 4th term of an AP is zero, Prove that its 25th term is triple its 11th term.

Solution:
Let a be the first term and d be the common difference of an AP.

$a_4 = a + (n- 1)d$
$= a + (4 - 1)d$
$= a + 3d$
Since 4th term of an AP is zero.
$a + 3d = 0$
or $a = -3d$ ….(1)

Similarly,

$a_{25} = a + 24d = -3d + 24d = 21d$ …(2)

$a_{11} = a + 10d = -3d + 10d = 7d$ ….(3)

From (2) and (3), we have
$a_{25} = 3 \times a_{11}$
Hence proved.

Question 28:
If the sixth term of an AP is zero then show that its 33rd term is three times its 15th term.

Solution:
Sixth term of an AP is zero
that is $a_6 = 0$
$a + 5d = 0$
$a = -5d$
Now, $a_{15} = a + (n - 1)d$
$a + (15 - 1)d = -5d + 14d = 9d$

and $a_{33} = a + (n - 1)d = a + (33 - 1)d = -5d + 32d = 27d$

Now, $a_{33} : a_{12}$
$27d : 9d$
$3 : 1$
Which shows that $a_{33} = 3(a_{15})$
Hence proved.

Question 29:
The 4th term of an AP is 11. The sum of the 5th and 7th terms of this AP is 34. Find its common difference.
Solution:
Let a be the first term and d be the common difference of an AP.

$a_n = a + (n - 1)d$
$a_4 = a + (4 - 1)d = a + 3d$
$a + 3d = 11$ ....(1)
Now, $a_5 = a + 4d$ and $a_7 = a + 6d$

Now, $a_5 + a_7 = a + 4d + a + 6d = 2a + 10d$
$2a + 10d = 34$
$a + 5d = 17$ ……..(2)

Subtracting (1) from (2), we get
$2d = 17 – 11 = 6$
$d = 3$

The common difference = 3

Question 30:
The 9th term of an AP is -32 and the sum of its 11th and 13th terms is -94. Find the common difference of the AP.
Solution:
Let a be the first term and d be the common difference of an AP.

nth term = $a_n = a + (n – 1)d$
Given: 9th term of an AP is -32 and the sum of its 11th and 13th terms is -94

Now,
$a_9 = a + 8d = -32$ ...(1)

$a_{11} = a + 10d$

$a_{13} = a + 12d$

Sum of 11th and 13th terms:

$a_{11} + a_{13} = a + 10d + a + 12d$
$-94 = 2a + 22d$
or $a + 11d = -47$ ...(2)

Subtracting (1) from (2) , we have
$3d = -47 + 32 = -15$
or $d = -5$
Common difference is -5.

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Exercise 5B

Question 1:
Determine k so that (3k – 2), (4k – 6) and (k + 2) are three consecutive terms of an AP.
Solution:
Given: (3k – 2), (4k – 6) and (k + 2) are three consecutive terms of an AP.
So, (4k – 6) – (3k – 2) = (k + 2) – (4k – 6)
2(4k – 6) = (k + 2) + (3k – 2)
8k – 12 = 4k + 0
8k – 4k = 0 + 12
or k = 3

Question 2:
Find the value of x for which the numbers (5x + 2), (4x – 1) and (x + 2) are in AP.
Solution:
Given: (5x + 2), (4x – 1) and (x + 2) are terms in AP.
So, d = (4x – 1) – (5x + 2) = (x + 2) – (4x – 1)
2(4x – 1) = (x + 2) + (5x + 2)
8x – 2 = 6x + 2 + 2
8x – 2 = 6x + 4
8x – 6x = 4 + 2
or x = 3
The value of x is 3.

Question 3:
If (3y – 1), (3y + 5) and (5y + 1) are three consecutive terms of an AP then find the value of y.
Solution:
Given: (3y – 1), (3y + 5) and (5y + 1) are 3 consecutive terms of an AP.
So, (3y + 5) – (3y – 1) – (5y + 1) – (3y + 5)
2(3y + 5) = 5y + 1 + 3y – 1
6y + 10 = 8y
8y – 6y = 10
2y = 10
Or y = 5
The value of y is 5.

Question 4:
Find the value of x for which (x + 2), 2x, (2x + 3) are three consecutive terms of an AP.
Solution:
Given: (x + 2), 2x, (2x + 3) are three consecutive terms of an AP.
So, 2x – (x + 2) = (2x + 3) – 2x

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$2x - x - 2 = 2x + 3 - 2x$
$x - 2 = 3$
$x = 2 + 3 = 5$
The value of x is 5.

Question 5:
Show that $(a - b)^2$, $(a^2 + b^2)$ and $(a + b)^2$ are in AP.
Solution:
Assume that $(a - b)^2$, $(a^2 + b^2)$ and $(a + b)^2$ are in AP.
So, $(a^2 + b^2) - (a - b)^2 = (a + b)^2 - (a^2 + b^2)$
$(a^2 + b^2) - (a^2 + b^2 - 2ab) = a^2 + b^2 + 2ab - a^2 - b^2$
$2ab = 2ab$
Which is true.
Hence given terms are in AP.

Question 6:
Find three numbers in AP whose sum is 15 and product is 80.
Solution:
Let a – d, a, a + d are three numbers in AP.

So, their sum = $a - d + a + a + d = 15$

$3a = 15$
or $a = 5$
Again,
Their Product = $(a - d) \times a \times (a + d) = 80$
$a(a^2 - d^2) = 80$
$5(5^2 - d^2) = 80$
$25 - d^2 = 16$
$d^2 = 25 - 16 = 9 = (\pm 3)^2$
or $d = \pm 3$
d = 3 or d = -2
We have 2 conditions here:
At a = 5, d = 3
Numbers are: 2, 5 and 8

At a = 5 and d = -3
Numbers are : 8, 5, 2

Question 7:
The sum of three numbers in AP is 3 and their product is -35. Find the numbers.

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Solution:

Let a – d, a, a + d are three numbers in AP.

their sum = a – d + a + a + d = 3

3a = 3
or a = 1

Again,
Their Product = (a – d) x a x (a + d) = -35
$a(a^2 – d^2) = -35$
$(1^2 – d^2) = -35$
or d = ±6
d = 6 or d = -6
We have 2 conditions here:
At a = 1, d = 6
Numbers are: -5, 1 and 7

At a = 1 and d = -6
Numbers are : 7, 1, -5

Question 8:
Divide 24 in three parts such that they are in AP and their product is 440.
Solution:
Let a – d, a, a + d are three numbers in AP.

their sum = a – d + a + a + d = 24

3a = 24
or a = 8

Again,
Their Product = (a – d) x a x (a + d) = 440
$a(a^2 – d^2) = 440$
$8(8^2 – d^2) = 440$
or d = ±3

Numbers are : (5, 8, 11) or (11, 8, 5)

Question 9:
The sum of three consecutive terms of an AP is 21 and the sum of the squares of these terms is 165. Find these terms.

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Solution:
Let a – d, a, a + d are three numbers in AP.

their sum = a – d + a + a + d = 21

3a = 21
or a = 7

Again,
Sum of squares = (a – d)^2 + a^2 +(a + d)^2 = 165

$$a^2 + d^2 - 2ad + a^2 + a^2 + d^2 + 2ad = 165$$
$$3a^2 + 2d^2 = 165$$
$$3(7)^2 + 2d^2 = 165 \Rightarrow 3 \times 49 + 2d^2 = 165$$
$$147 + 2d^2 = 165 \Rightarrow 2d^2 = 165 - 147 = 18$$
$$d^2 = \frac{18}{2} = 9 = (\pm 3)^2$$
$$d = \pm 3$$

Numbers are : (4, 7, 10) or (10, 7, 4)

Question 10:
The angles of a quadrilateral are in AP whose common difference is 10°. Find the angles.

Solution:
Sum of angles of a quadrilateral = 360°
Common difference = 10 = d (say)
If the first number be a, then the next four numbers will be
a, a + 10, a + 20, a + 30

As per definition:
a + a + 10 + a + 20 + a + 30 = 360°
4a + 60 = 360
4a = 300
or a = 75°
Other angles:
a + 10 = 75 + 10 = 85
a + 20 = 75 + 20 = 95
a + 30 = 75 + 30 = 105

Therefore, Angles are 75°, 85°, 95°, 105°

Question 11:
Find four numbers in AP whose sum is 28 and the sum of whose squares is 216.
Solution:
Let a – 3d, a – d, a + d, a + 3d are the four numbers in AP.
Their sum = a – 3d + a – d + a + d + a + 3d = 28

Sum of their square = (a – 3d)^2 + (a – d)^2 + (a + d)^2 + (a + 3d)^2 = 216

$$a^2 + 9d^2 - 6ad + a^2 + d^2 - 2ad + a^2 + d^2 + 2ad + a^2 + 9d^2 + 6ad = 216$$

$$4a^2 + 20d^2 = 216$$

$$a^2 + 5d^2 = 54$$

$$(7)^2 + 5d^2 = 54 \Rightarrow 5d^2 + 49 = 54$$

$$5d^2 = 54 - 49 = 5 \Rightarrow d^2 = \frac{5}{5} = 1$$

$$d^2 = (\pm 1)^2$$

$$d = \pm 1$$

Numbers will be: (4, 6, 8,10) or (10, 8, 6, 4)

Question 12: Divide 32 into four parts which are the four terms of an AP such that the product of the first and the fourth terms is to the product of the second and the third terms as 7 : 15.

Solution:

Let a – 3d, a – d, a + d, a + 3d are the four numbers in AP.
Therefore:

$$-3d + a - d + a + d + a + 3d = 32$$

or a = 8

Again,

$$(a - 3d)(a + 3d) : (a - d)(a + d)$$

= 7:15

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$(a^2 - 9d^2) : (a^2 - d^2) = 7 : 15$

$\frac{a^2 - 9d^2}{a^2 - d^2} = \frac{7}{15}$

$15a^2 - 135d^2 = 7d^2 - 7d^2$

$15a^2 - 7a = 135d^2 - 7d^2$

$8a^2 = 128d^2$

$d^2 = \frac{8a^2}{128} = \frac{8 \times 8^2}{128} = \frac{8 \times 64}{128} = 4 = (\pm 2)^2$

d = ±2

four parts are: a - 6, a - 2, a+2 amd a + 6

which implies,
Number are: (2, 6, 8, 10, 14) or (14, 10, 6, 2)

Question 13: The sum of first three terms of an AP is 48. If the product of first and second terms exceeds 4 times the third term by 12. Find the AP.
Solution:
Let a – d, a, a + d are the three terms in AP
So,
Sum = a – d + a + a + d = 48
3a = 48
Or a = 16

And,
a(a – d) = 4 (a + d) + 12
16 (16 – d) = 4(16 + d) + 12
256 – 16d = 64 + 4d + 12 = 4d + 76
256 – 76 = 4d + 16d
180 = 20d
d = 9
Which implies:
Numbers are : (7, 16, 25)

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Exercise 5C

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Question 1: Find the sum of each of the following APs:
(i) 2, 7, 12, 17,..... to 19 terms.
(ii) 9, 7, 5, 3,.....to 14 terms.
(iii) -37, -33, -29,....12 terms.
(iv) 1/15 ,1/12,1/10,...to 11 terms
(v) 0.6, 1.7, 2.8,.... to 100 terms.

Solution:

Sum of n terms of AP formula:

$S_n = \frac{n}{2}[2a + (n-1)d]$ ......(1)

Where
First term = a
Common difference = d
Number of terms = n

(i) 2, 7, 12, 17,..... to 19 terms.

a = 2
d = 7 - 2 = 5
Using (1)

S_{19} = 19/2(2(2) + (19 - 1)5)

= (19)(4 + 90)/2
= (19 × 94)/2
= 893
Sum of 19 terms of this AP is 893.

(ii) 9, 7, 5, 3,.....to 14 terms.
a = 9
d = 7 - 9 - -2
Using (1)

S_{14} = 14/2 [2(9) + (14 - 1)(-2)] =
(7)(18 - 26)
= - 56

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Sum of 14 terms of this AP is - 56.

(iii) -37, -33, -29,....12 terms.

a = -37
d = (-33) - (-37) = 4
Using (1)

S_{12} = 12/2 [2(-37) + (12 - 1)(4)]
= (6)(-74 + 44)
= 6 × (-30)
= - 180
Sum of 12 terms of this AP is - 180.

(iv) 1/15 ,1/12,1/10,...to 11 terms

a = 1/15
d = (1/12) - (1/15) = 1/60
Using (1)

S_{11} = 11/2 [2(1/15) + (11 - 1)(1/60)] =
(11/2) × [(2/15) + (1/6)]
=33/20
Sum of 11 terms of this AP is 33/20.

(v) 0.6, 1.7, 2.8,.... to 100 terms.

a = 0.6
d = 1.7 - 0.6 = 1.1
Using (1)

S_{100} = 100/2 [2(0.6) + (100 - 1)(1.1)]
= (50) × [1.2 + (99 × 1.1)]
= 50 × 110.1
= 5505
Sum of 100 terms of this AP is 5505.

Question 2: Find the sum of each of the following arithmetic series:
(i) 7 + 10 1/2 + 14 + ... + 84
(ii) 34 + 32 + 30 + ... + 10
(iii) (-5) + (-8) + (-11) + ... + (-230)

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(iv) 5 + (-41) + 9 + (-39) + 13 + (-37) + 17 + ... + (-5) + 81 + (-3)

Solution:

(i) 7 + 10 1/2 + 14 + ... + 84
First term = a = 7
Common difference = d = (21/2) - 7 = (7/2)
Last term = l = 84
Now, using formula:
84 = a + (n - 1)d
84 = 7 + (n - 1)(7/2)
77 = (n - 1)(7/2)
154 = 7n - 7
7n = 161
n = 23
Thus, there are 23 terms in AP.
Now,
Find Sum of these 23 terms:
S_{23} = 23/2 [2(7) + (23 - 1)(7/2)]
= (23/2) [14 + (22)(7/2)]
= (23/2) [91]
= 2093/2

Sum of 23 terms of this AP is 2093/2.

(ii) 34 + 32 + 30 + ... + 10
First term = a = 34
Common difference = d = 34 - 32 = - 2
Last term = l = 10
Now, using formula:
10 = a + (n - 1)d
10 = 34 + (n - 1)(-2)
10 - 34 = (n - 1)(-2)
n = 13
Thus, there are 13 terms in AP.
Now,
Find Sum of these 13 terms:
S_{13} = 13/2 [2(34) + (13 - 1)(-2)]
= (13/2) [68 + (12)(-2)]
= (13/2) x 44
= 286

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Sum of 13 terms of this AP is 286.

(iii) (-5) + (-8) + (-11) + ... + (-230)
First term = a = -5
Common difference = d = - 8 - (-5) = - 3
Last term = l = -230
Now, using formula:
-230 = a + (n - 1)d
-230 = -5 + (n - 1)(-3)
-230 + 5 = (n - 1)(-3)
n = 76
Thus, there are 76 terms in AP.
Now,
Find Sum of these 76 terms:
S_{76} = 76/2 [2(-5) + (76 - 1)(-3)]
= 38 × [(-10) + (75)(-3)]
= - 8930

Sum of 76 terms of this AP is -8930.

(iv) 5 + (-41) + 9 + (-39) + 13 + (-37) + 17 + ... + (-5) + 81 + (-3)

The given series is combination of two AP's.
Let A_1 = 5 + 9 +13 + ...+ 77 + 81
and A_2 = -41 - 39 - 37 - ......(-3)

For A_1:

First term = a = 5
Common difference = d = 2
Last term = l = 81
Now, using formula:
81 = 5 + (n - 1)4
n = 20
Thus, there are 20 terms in AP.
Now,
Find Sum of these 76 terms:
S_{20} = n/2 [a + l]
= 20/2 (5 + 81)
= 860

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Sum of 20 terms of this AP is 860.

For A_2:

First term = a = -41
Common difference = d = 2
Last term = l = -3
Now, using formula:
-3 = -41 + (n - 1)2
n = 20
Thus, there are 20 terms in AP.
Now,
Find Sum of these 76 terms:
S_{20} = n/2 [a + l]
= 20/2 (-41- 3)
=-440

Sum of 20 terms of this AP is -440.

Therefore, Sum of total terms: 860 - 440 = 420.

Question 3: Find the sum of first n terms of an AP whose nth term is (5 – 6n). Hence, find the sum of its first 20 terms.
Solution:

Given: a_n = 5 - 6n
Find some of the terms of AP:
Put n = 1, we get a_1 = - 1 = first term
Put n = 2, we get a_2 = - 7 = second term

Common difference = d = a_2 - a_1 = - 7 - (-1) = - 6
Sum of first n terms:
S_n= n/2 [2a + (n - 1)d]
= n/2[- 2 + (n - 1)(-6)]
= n(2 - 3n)
sum of first 20 terms:
S_{20} = 20/2[2(-1) + (20 - 1)(-6)]
= 10 [- 2 - 114]
= - 1160

Sum of its first 20 terms of AP is -1160.

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Question 4:
The sum of the first n terms of an AP is $(3n^2 + 6n)$. Find the nth term and the 15th term of this AP.
Solution:
Given: $S_n = 3n^2 + 6n$

$S_1 = 3(1)^2 + 6 \times 1 = 3 + 6 = 9$
$S_2 = 3(2)^2 + 6 \times 2 = 12 + 12 = 24$
$T_2 = S_2 - S_1 = 24 - 9 = 15$
$d = 15 - 9 = 6$
$a = 9$
$T_n = a + (n - 1)d = 9 + (n - 1) \times 6$
$= 9 + 6n - 6 = 3 + 6n$
$= 6n + 3$
$T_{15} = 6 \times 15 + 3 = 90 + 3 = 93$

Question 5: The sum of the first n terms of an AP is given by $S_n = (3n^2 - n)$. Find its
(i) nth term,
(ii) first term and
(iii) common difference.

Solution:
$S_n = 3n^2 - n$
$S_1 = 3(1)^2 - 1 = 3 - 1 = 2$
$S_2 = 3(2)^2 - 2 = 12 - 2 = 10$
$a_1 = 2$
$a_2 = 10 - 2 = 8$

(i) $a_n = a + (n - 1) d$
$= 2 + (n - 1) \times 6$
$= 2 + 6n - 6$
$= 6n - 4$
(ii) First term = 2
(iii) Common difference = 8 – 2 = 6

Question 6: (i) The sum of the first n terms of an AP is $(5n^2/2 + 3n/2)$. Find the nth term and the 20th term of this AP.
(ii) The sum of the first n terms of an AP is $(3n^2/2 + 5n/2)$. Find its nth term and the 25th term.
Solution:

(i) $S_n = \frac{5n^2}{2} + \frac{3n}{2}$

$S_1 = \frac{5(1)^2}{2} + \frac{3(1)}{2} = \frac{5}{2} + \frac{3}{2} = \frac{8}{2} = 4$

$S_2 = \frac{5(2)^2}{2} + \frac{3(2)}{2} = \frac{20}{2} + \frac{6}{2}$

$= 10 + 3 = 13$

$T_2 = S_2 - S_1 = 13 - 4 = 9$

$T_1 = 4$

$d = 9 - 4 = 5$

Now, $T_n = a + (n-1)d$

$= 4 + (n-1) \times 5$

$= 4 + 5n - 5 = (5n - 1)$

$T_{20} = 5 \times 20 - 1 = 100 - 1 = 99$

(ii)

$S_n = \frac{3n^2}{2} + \frac{5n}{2}$

$S_1 = \frac{3(1)^2}{2} + \frac{5 \times 1}{2} = \frac{3}{2} + \frac{5}{2} = \frac{8}{2} = 4$

$S_2 = \frac{3(2)^2}{2} + \frac{5(2)}{2} = \frac{3 \times 4}{2} + \frac{5 \times 2}{2}$

$= \frac{12}{2} + \frac{10}{2} = 6 + 5 = 11$

$T_2 = S_2 - S_1 = 11 - 4 = 7$

$T_1 = 4$ or $a = 4$

$d = T_2 - T_1 = 7 - 4 = 3$

Now, $T_n = a + (n-1)d$

$= 4 + (n-1) \times 3 = 4 + 3n - 3$

$= 3n + 1$

$T_{25} = 3 \times 25 + 1 = 75 + 1 = 76$

Question 7: If mth term of an AP is 1/n and nth term is 1/m then find the sum of its first mn terms.

Solution:

Let a be first term and d be the common difference of an AP.

mth term = 1/n

so,

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$a_m = a + (m - 1) d = 1/n$ ....(1)

nth term = 1/m

$a_n = a + (n - 1) d = 1/m$ ....(2)

Subtract (2) from (1)

$$(m-1)d - (n-1)d = \frac{1}{n} - \frac{1}{m}$$

$$d(n-n) = \frac{m-n}{mn}$$

d = 1/mn ....(3)

From (3) and (1), we get

$$a + (m-1) \times \frac{1}{mn} = \frac{1}{n} \Rightarrow a = \frac{1}{n} - \frac{m-1}{mn}$$

$$a = \frac{1}{mn}$$

Sum of first mn terms:

S_{mn} = mn/2 [2(1/mn) + (mn-1) (1/mn)]

= mn/2 [1/mn + 1]

= (1+mn)/2

Question 8:
How many terms of the AP 21, 18, 15, … must be added to get die sum 0?
Solution:
AP is 21, 18, 15,…
a = 21,
d = 18 – 21 = -3
Sum of terms = S_n = 0
(n/2) [2a + (n - 1)d] = 0
(n/2) [2(21) + (n - 1)(-3)] = 0
(n/2) [45 - 3n] = 0
[45 - 3n] = 0

n = 15 (number of terms)
Thus, 15 terms of the given AP sums to zero.

Question 9: How many terms of the AP 9, 17, 25, … must be taken so that their sum is 636?
Solution:
AP is 9, 17, 25,…
a = 9, d = 17 – 9 = 8

Sum of terms = S_n = 636

(n/2) [2a + (n - 1)d] = 636
(n/2) [2(9) + (n - 1)(8)] = 636
(n/2)[10 + 8n] = 636
$4n^2 + 5n - 636 = 0$ (which is a quadratic equation)

(n- 12)(4n + 53) = 0

Either (n- 12) = 0 or (4n + 53) = 0

n = 12 or n = - 53/4
Since n can't be negative and fraction, so
n = 12
Number of terms = 12 terms.

Question 10: How many terms of the AP 63, 60, 57, 54, … must be taken so that their sum is 693? Explain the double answer.
Solution:
AP is 63, 60, 57, 54,…
Here, a = 63, d = 60 – 63 = -3 and sum = S_n = 693

$$693 = \frac{n}{2}[2 \times 63 + (n-1)(-3)]$$
$$693 \times 2 = n(126 - 3n + 3)$$
$$1386 = n(129 - 3n)$$
$$1386 = 129n - 3n^2$$
$$3n^2 - 129n + 1386 = 0$$
$$n^2 - 43n + 462 = 0$$
Which is a quadratic equation

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$n^2 - 21n - 22n + 462 = 0$
$n(n - 21) - 22(n - 21) = 0$
$(n - 21)(n - 22) = 0$
Either, $n - 21 = 0$, then $n = 21$
or $n - 22 = 0$, then $n = 22$
Number of terms = 21 or 22
$T_{22} = a + (n - 1)d$
$= 63 + (22 - 1)(-3)$
$= 63 + 21 \times (-3) = 63 - 63 = 0$
Which shows that, 22th term of AP is zero.

Number of terms are 21 or 22. So there will be no effect on the sum.

Question 11:
How many terms of the AP 20, 19 1/3 , 18 2/3 , … must be taken so that their sum is 300? Explain the double answer.
Solution:

AP is 20, $19\frac{1}{3}$, $18\frac{2}{3}$, ...

Here, a = 20, d = -2/3 and sum = s_n = 300 (for n number of terms)

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$300 = \frac{n}{2}\left[2 \times 20 + (n-1)\left(\frac{-2}{3}\right)\right]$$

$$600 = n\left[40 - \frac{2}{3}n + \frac{2}{3}\right]$$

$$600 = n\left[\frac{122}{3} - \frac{2}{3}n\right]$$

$$600 = \frac{n(122 - 2n)}{3}$$

$1800 = 122n - 2n^2$
$2n^2 - 122n + 1800 = 0$
$n^2 - 61n + 900 = 0$
$n^2 - 25n - 36n + 900 = 0$

n(n-25) - 36(n-25) = 0

(n - 25)(n - 36) = 0
Either (n - 25) = 0 or (n - 36) = 0

n = 25 or n = 36

For n = 25

$$300 = \frac{25}{2}\left[2\times 20+(25-1)\left(\frac{-2}{3}\right)\right]$$

$$= \frac{25}{2}\left[40+24\times\left(\frac{-2}{3}\right)\right]$$

$$= \frac{25}{2}[40-16] = \frac{25}{2}\times 24 = 300$$

Which is true.

For n = 36

$$300 = \frac{36}{2}\left[2\times 20+(36-1)\left(\frac{-2}{3}\right)\right]$$

$$= 18\left[40+35\times\frac{-2}{3}\right]$$

$$= 18\left[40-\frac{70}{3}\right] = 18\times\frac{120-70}{3}$$

= 300
Which is true.
Result is true for both the values of n

So both the numbers are correct.
Therefore, Sum of 11 terms is zero. (36-25 = 11)

Question 12: Find the sum of all odd numbers between 0 and 50.
Solution:
Odd numbers between 0 and 50 are 1, 3, 5, 7, 9, …, 49
Here, a = 1, d = 3 – 1 = 2, l = 49

$$49 = 1 + (n-1) \times 2$$
$$49 - 1 = 2(n-1) \Rightarrow 2(n-1) = 48$$
$$n - 1 = \frac{48}{2} = 24$$
$$n = 24 + 1 = 25$$

Sum of odd numbers:
= n/2(a + l)
= 25/2 (l + 49)
= 25/2(50)
= 625

Question 13: Find the sum of all natural numbers between 200 and 400 which are divisible by 7.
Solution:
Natural numbers between 200 and 400 which are divisible by 7 are 203, 210, 217,......,399.

Here,AP is 203, 210, 217,......,399

First term = a = 203,
Common difference = d = 7 and
Last term = l = 399

We know that, l = a + (n-1)d

$$399 = 203 + (n-1) \times 7$$
$$399 - 203 = 7(n-1) \Rightarrow 7(n-1) = 196$$
$$n - 1 = \frac{196}{7} = 28$$
$$n = 28 + 1 = 29$$
$$n = 29$$

There are total 29 terms.
Now, find the sum of all 29 terms:

$$S_{29} = \frac{n}{2}[a + l]$$

$$= \frac{29}{2}[203 + 399]$$

$$= \frac{29}{2} \times 602 = 8729$$

Sum of all 29 terms is 8729.

Question 14: Find the sum of first forty positive integers divisible by 6.
Solution:
First forty positive integers which are divisible by 6 are
6, 12, 18, 24,...... to 40 terms
Here, a = 6, d = 12 – 6 = 6, and n = 40.

Sum of 40 terms:

$$S_{40} = \frac{n}{2}[2a + (n - 1)d]$$

$$= \frac{40}{2}[2 \times 6 + (40 - 1) \times 6]$$

$$= 20[12 + 39 + 6] = 20[12 + 234]$$

$$= 20 \times 246 = 4920$$

Question 15: Find the sum of the first 15 multiples of 8.
Solution:
First 15 multiples of 8 are as given below:
8, 16, 24, 32,..... to 15 terms
Here, a = 8, d = 16 – 8 = 8, and n = 15

Sum of 15 terms:

$$S_{15} = \frac{15}{2}[2 \times 8 + (15 - 1) \times 8]$$

$$= \frac{15}{2}[16 + 14 \times 8] = \frac{15}{2}[16 + 112]$$

$$= \frac{15}{2} \times 128 = 960$$

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Question 16: Find the sum of all multiples of 9 lying between 300 and 700.
Solution:
Multiples of 9 lying between 300 and 700 = 306, 315, 324, 333, …, 693

Here, a = 306, d = 9, and l = 693

We know that, l = a + (n-1)d

$693 = 306 + (n-1) \times 9$
$(n-1) \times 9 = 693 - 306 = 387$

$n - 1 = \frac{387}{9} = 43$

$n = 43 + 1 = 44$

There are 44 terms.

Find the sum of 44 terms:

$S_{44} = \frac{n}{2}[a + l]$

$= \frac{44}{2}[306 + 693]$

$= 22 \times 999 = 21978$

Question 17: Find the sum of all three-digit natural numbers which are divisible by 13.
Solution:
3-digit natural numbers: 100, 101, 102,......., 999.

3-digit natural numbers divisible by 13:
104, 117, 130,........., 988
Which is an AP.

Here, a = 104, d = 13, l = 988
l = a + (n – 1) d

988 = 104 + (n-1)13

n = 69
There are 69 terms.

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Find the sum of 69 terms:

$S_n = n/2(a + l)$

= 69/2(104 + 988)

= 37674

Question 18: Find the sum of first 100 even natural numbers which are divisible by 5.

Solution:
Even natural numbers: 2, 4, 6, 8, 10, …
Even natural numbers divisible by 5: 10, 20, 30, 40, … to 100 terms

Here, a = 10, d = 20 – 10 = 10, and n = 100

$S_n = n/2(2a + (n-1)d)$

= 100/2 [2 x 10 + (100-1)10]

= 50500

Question 19: Find the sum of the following.
(4 – 1/n) + (4-2/n) + (4– 3/n) + ….

Solution:

Given sum can be written as (4 + 4 + 4 + 4+....) - (1/n, 2/n, 3/n, ...)

Now, We have two series:
First series: = 4 + 4 + 4 + ... up to n terms
= 4n

Second series: 1/n, 2/n, 3/n, ...
Here, first term = a = 1/n
Common difference = d = (2/n) - (1/n) = (1/n)

Sum of n terms formula:
$S_n =$ n/2[2a + (n - 1)d]

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Sum of n terms of second series:
S_n = n/2 [2(1/n) + (n - 1)(1/n)]
= n/2 [(2/n) + 1 - (1/n)]
= (n + 1)/2

Hence,
Sum of n terms of the given series = Sum of n terms of first series - Sum of n terms of second series
= 4n - (n + 1)/2
= (8n - n - 1)/2
= 1/2 (7n - 1)

Question 20: In an AP, it is given that $S_5 + S_7 = 167$ and $S_{10} = 235$, then find the AP, where S_n denotes the sum of its first n terms.
Solution:
Let a be the first term and d be the common difference of the AP.

$$S_5 = \frac{n}{2}[2a + (n - 1)d]$$

$$= \frac{5}{2}[2a + (5 - 1)d]$$

$$= \frac{5}{2}[2a + 4d]$$

Similarly,

$$S_7 = \frac{7}{2}[2a + 6d]$$

and $S_{10} = \frac{10}{2}[2a + 9d] = 5(2a + 9d)$

Now, $\frac{5}{2}(2a + 4d) + \frac{7}{2}(2a + 6d) = 167$

$5a + 10d + 7a + 21d = 167$
$12a + 31d = 167$ ...(i)
and $5(2a + 9d) = 235$
$2a + 9d = 47$ ...(ii)
Multiply (i) by (i) and (ii) by 6,

$$12a + 31d = 167$$
$$12a + 54d = 282$$
$$- \quad - \quad -$$

$$-23d = -115 \quad \Rightarrow d = \frac{-115}{-23} = 5$$

From (i), $12a + 31 \times 5 = 167$

$12a + 155 = 16 \Rightarrow 12a = 167 - 155$

$$12a = 12 \Rightarrow a = \frac{12}{12} = 1$$

Which implies: a = 1 and d = 5

Therefore, the AP is 1, 6, 11, 16,……

Question 21: In an AP, the first term is 2, the last term is 29 and the sum of all the terms is 155. Find the common difference.

Solution:
Let d be the common difference.
Given:
first term = a = 2
last term = l = 29
Sum of all the terms = S_n = 155

S_n = n/2[a + l]
155 = n/2[2 + 29]
n = 10

There are 10 terms in total.

Therefore, 29 is the 10th term of the AP.

Now, 29 = a + (10 - 1)d
29 = 2 + 9d
27 = 9d
d = 3

The common difference is 3.

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Question 22: In an AP, the first term is -4, the last term is 29 and the sum of all its terms is 150. Find its common difference.

Solution:
Let d be the common difference.
Given:
first term = a = -4
last term = l = 29
Sum of all the terms = S_n = 150

S_n = n/2[a + l]
150 = n/2[-4 + 29]
n = 12

There are 12 terms in total.

Therefore, 29 is the 12th term of the AP.

Now, 29 = -4 + (12 - 1)d
29 = -4 + 11d
d = 3

The common difference is 3.

Question 23: The first and the last terms of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Solution:
Let n be the total number of terms.
Given:
First term = a = 17
Last term = l = 350
Common difference = d = 9

l = a + (n-1)d

350 = 17 + (n-1)9

n = 38
Again,
S_n = n/2[a + l]

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= 38/2[17 + 350]
= 6973

There are 38 terms in total and their sum is 6973.

Question 24: The first and the last terms of an AP are 5 and 45 respectively. If the sum of all its terms is 400, find the common difference and the number of terms.

Solution:
Let n be the total number of terms and d be the common difference.
Given:
first term = a = 5
last term = l = 45
Sum of all terms = S_n = 400

S_n = n/2[a + l]

400 = n/2[5 + 45]

n/2 = 400/50
n = 16

There are 16 terms in the AP.

Therefore, 45 is the 16th term of the AP.
45 = a + (16 - 1)d
45 = 5 + 15d
40 = 15d
15d = 40
d = 8/3
Common difference = d = 8/3

Common difference is 8/3 and the number of terms are 16.

Question 25: In an AP, the first term is 22, nth term is -11 and sum of first n terms is 66. Find n and hence find the common difference.

Solution:
Let n be the total number of terms and d be the common difference.
Given:
first term = a = 22

nth term = -11
Sum of all terms = S_n = 66

$S_n = n/2[a + l]$

66 = n/2[22 + (-11)]

n = 12

There are 12 terms in the AP.

Since nth term is -11, so
$a_n = a + (n - 1)d$
-11 = 22 + (12-1)d
d = -3

Therefore, Common difference is -3 and the number of terms are 12.

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Question 1: The first three terms of an AP are respectively (3y – 1), (3y + 5) and (5y + 1), find the value of y.
Solution:
Given: (3y – 1), (3y + 5) and (5y+ 1) are in AP
So, (3y + 5) – (3y – 1) = (5y + 1) – (3y + 5)
2 (3y + 5) = (5y + 1) + (3y – 1)
6y + 10 = 8y
8y – 6y = 10
2y = 10
Or y = 5
The value of y is 5.

Question 2: If k, (2k – 1) and (2k + 1) are the three successive terms of an AP, find the value of k.

Solution:
Given: k, (2k – 1) and (2k + 1) are the three successive terms of an AP.
So, (2k – 1) – k = (2k + 1) – (2k – 1)
2 (2k – 1) = 2k + 1 + k
4k – 2 = 3k + 1
4k – 3k = 1 + 2
or k = 3
The value of k is 3.

Question 3: If 18, a, (b – 3) are in AP, then find the value of (2a – b).
Solution:
Given: 18, a, (b – 3) are in AP
a – 18 = b – 3 – a
a + a – b = -3 + 18
2a – b = 15

Question 4: If the numbers a, 9, b, 25 form an AP, find a and b.

Solution:
Given: a, 9, b, 25 are in AP.
So, 9 – a = b – 9 = 25 – b
b – 9 = 25 – b
b + b = 22 + 9 = 34
or b = 17
And,

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a – b = a – 9
9 + 9 = a + b
a + b = 18
a + 17 = 18
or a = 1
Answer: a = 18, b= 17

Question 5: If the numbers (2n – 1), (3n + 2) and (6n – 1) are in AP, find the value of n and the numbers.

Solution:
Given: (2n – 1), (3n + 2) and (6n – 1) are in AP
So, (3n + 2) – (2n – 1) = (6n – 1) – (3n + 2)
(3n + 2) + (3n + 2) = 6n – 1 + 2n – 1
6n + 4 = 8n – 2
8n – 6n = 4 + 2
Or n = 3

Numbers are:
2 x 3 – 1 = 5
3 x 3 + 2 = 11
6 x 3 – 1 = 17
Answer: (5, 11, 17) are required numbers.

Question 6: How many three-digit natural numbers are divisible by 7?
Solution:
3-digit natural numbers: 100, 101,…….. 990 and

3-digit natural numbers divisible by 7: 105, 112, 119, 126, …, 994

Here, a = 105, d= 7, l = 994

a_n = (l) = a + (n – 1) d
994 = 105 + (n – 1) x 7
994 – 105 = (n – 1) 7
n – 1 = 127
n = 128
Answer: There are 128 required numbers.

Question 7: How many three-digit natural numbers are divisible by 9?

Solution:

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3-digit numbers: 100, 101,…….,999

3-digit numbers divisible by 9 : 108, 117, 126, 135, …, 999

Here, a = 108, d= 9, l = 999
a_n (l) = a + (n – 1) d
999 = 108 + (n – 1) x 9
(n – 1) x 9 = 999 – 108 = 891
n – 1 = 99
n = 100

Question 8: If the sum of first m terms of an AP is ($2m^2 + 3m$) then what is its second term?
Solution:
Sum of first m terms of an AP = $2m^2 + 3m$ (given)
$S_m = 2m^2 + 3m$
Sum of one term = $S_1 = 2(1)^2 + 3 \times 1 = 2 + 3 = 5$ = first term
Sum of first two terms = $S_2 = 2(2)^2 + 3 \times 2 = 8 + 6 = 14$
Sum of first three terms = $S_3 = 2(3)^2 + 3 \times 3 = 18 + 9 = 27$
Now,
Second term = $a_2 = S_2 - S_1 = 14 - 5 = 9$

Question 9: What is the sum of first n terms of the AP, a, 3a, 5a,………….

Solution:
AP is a, 3a, 5a,…..
Here, a = a, d = 2a

Sum = $S_n = n/2\ [2a + (n-1)d]$

$= n/2[2a + 2an - 2a]$
$=an^2$

Question 10.: What is the 5th term from the end of the AP 2, 7, 12, ……… 47?

Solution:

Given AP is 2, 7, 12, 17, …… 47
Here, a = 2, d = 7 – 2 = 5, l = 47
nth term from the end = l – (n – 1)d
5th term from the end = 47 – (5 – 1) 5 = 47 – 4 x 5 = 27

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Question 11: If an denotes the nth term of the AP 2, 7, 12, 17, …, find the value of ($a_{30} – a_{20}$).

Solution:
Given AP is 2, 7, 12, 17,……..
Here, a = 2, d = 7 – 2 = 5
Now,
$a_n = a + (n – 1) d = 2 + (n – 1)\ 5 = 5n – 3$

$a_{30} = 2 + (30 – 1) 5 = 2 + 145 = 147$ and

$a_{20} = 2 + (20 – 1) 5 = 2 + 95 = 97$

$a_{30} – a_{20} = 147 – 97 = 50$

Question 12: The nth term of an AP is (3n + 5). Find its common difference.
Solution:
Nth term = $a_n = 3n + 5$ (given)
$a_{(n-1)} = 3 (n – 1) + 5 = 3n + 2$
Common difference = $d = a_n – a_{(n-1)}$
= (3n + 5) – (3n + 2)
= 3n + 5 – 3n – 2
= 3
Therefore, common difference is 3.

Question 13: The nth term of an AP is (7 – 4n). Find its common difference.
Solution:
Nth term = $a_n = 7 – 4n$
$a_{(n-1)} = 7 – 4(n – 1) = 11 – 4n$
Common difference = $d = a_n – a_{(n-1)}$
= (7 – 4n) – (11 – 4n)
= 7 – 4n – 11 + 4n
= -4
Therefore, common difference is -4.

Question 14: Write the next term of the AP √8, √18, √32, ……..

Solution:
Given AP is √8, √18, √32,……..
Above AP can be written as:
2√2 , 3√2, 4 √2, ……

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Here a = 2√2 and d = √2

Next term = 4√2 + √2 = 5√2 = √50

Question 15: Write the next term of the AP √2, √8, √18,....

Solution:
Given AP is √2, √8, √18,....

Can be written as:

√2, 2√2, 3√2,.....

First term = √2

Common difference = 2√2 - √2 = √2

Next term = 3√2 + √2 = 4√2 = √32

Question 16: Which term of the AP 21, 18, 15,.... is zero?

Solution:
Given AP is 21, 18, 15,....

First term = a = 21
Common difference = d = 18-21 = -3
Last term = l = 0
l = a + (n – 1) d
0 = 21 + (n – 1)(-3)
0 = 21 – 3n + 3
24 – 3n = 0
Or n = 8
Answer: Zero is the 8th term.

Question 17: Find the sum of first n natural numbers.
Solution:
First n natural numbers: 1, 2, 3, 4, 5, …, n
Here, a = 1, d = 1

Sum = S_n = n/2 [2a + (n-1)d]

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= n/2 [2(1) + (n-1)(1)]

= n(n+1)/2

Question 18: Find the sum of first n even natural numbers.

Solution:

First n even natural numbers: 2, 4, 6, 8, 10,....,n
Here, a = 2, d = 4 – 2 = 2

Sum = S_n = n/2 [2a + (n-1)d]

= n/2 [2(2) + (n-1)(2)]

= n(n+1)

Question 19: The first term of an AP is p and its common difference is q. Find its 10th term.

Solution:
Given:
First term =a = p and
Common difference = d =q
Now,
a_{10} = a + (n – 1) d
= p + (10 – 1)q
= (p + 9q)

Question 20: If 4/5, a, 2 are in AP, find the value of a.

Solution:

AP terms: 4/5, a, 2 (given)
Then,

a - 4/5 = 2 - a

a = 7/5

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Question 21: If (2p + 1), 13, (5p – 3) are in AP, find the value of p.
Solution:
Given, 2p + 1, 13, 5p – 3 are in AP
Then,
13 – (2p + 1) = (5p – 3) – 13
13 – 2p – 1 = 5p – 3 – 13
12 – 2p = 5p – 16
p = 4
The value of p is 4.

Question 22: If (2p – 1), 7, 3p are in AP, find the value of p.
Solution:
Given, (2p – 1), 7, 3p are in AP
Then,
7 – (2p – 1) = 3p – 7
7 – 2p + 1 = 3p – 7
5p = 15
p = 3
The value of p is 3.

Question 23: If the sum of first p terms of an AP is (ap^2 + bp), find its common difference.

Solution:

Sum of first p terms = S_p = (ap^2 + bp)

Sum of one term = $S_1 = a(1)^2 + b(1) = a+b$ = first term
Sum of first two terms = $S_2 = a(2)^2 + b \times 2 = 4a + 2b$

We know that, second term = $a_2 = S_2 - S_1$

= (4a + 2b) - (a + b)
= 3a + b

Now, $d = a_2 - a_1$

= 3a + b - (a+b)
= 2a

Answer: Common difference is 2a.

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Question 24: If the sum of first n terms is $(3n^2 + 5n)$, find its common difference.

Solution:

Sum of first n terms = $S_n = (3n^2 + 5n)$

Sum of one term = $S_1 = 3(1)^2 + 5(1) = 8$ = first term
Sum of first two terms = $S_2 = 3(2)^2 + 5(2) = 22$

We know that, second term = $a_2 = S_2 - S_1$

$= 22 - 8$
$= 14$

=> a_2 = 14

Now, $d = a_2 - a_1$

$= 14 - 8 = 6$

Answer: Common difference is 6.

Question 25: Find an AP whose 4th term is 9 and the sum of its 6th and 13th terms is 40.

Solution:

Let a be the first term and d be the common difference.
Given:
4th term = $a_4 = 9$

Sum of 6th and 13th terms = $a_6 + a_{13} = 40$

Now,

$a_4 = a + (4-1)d$
$9 = a + 3d$

$a = 9 - 3d$ ....(1)

And
$a_6 + a_{13} = 40$

a + 5d + a + 12d = 40
2a +17d = 40

2(9 - 3d) + 17d = 40 (using (1))

d = 2

From (1): a = 9 - 6 = 3

Required AP = 3,5,7,9,.....

Question 26: What is the common difference of an AP in which $a_{27} - a_7 = 84$?

Solution:
Given: $a_{27} - a_7 = 84$

[a + 26d] - [a + 6d] = 84

20d = 84
d = 4.2

Question 27: If 1 + 4 + 7 + 10 + ... + x = 287, find the value of x.

Solution:
Given: 1 + 4 + 7 + 10 + ... + x = 287

Here a = 1, d = 3 and S_n = 287

Sum = S_n = n/2 [2a + (n-1)d]

287 = n/2 [2 + (n-1)3]

574 = $3n^2$ - n
Which is a quadratic equation.

Solve $3n^2 - n - 574 = 0$

$3n^2 - 42n + 41n - 574 = 0$

3n(n - 14) + 41(n-14) = 0

(3n + 41)(n-14) = 0

Either (3n + 41) = 0   or (n-14) = 0

n = -41/3 or n = 14

Since number of terms cannot be negative, so result is n = 14.

=> Total number of terms in AP are 14.

Which shows, x = a_{14}

or x = a + 13d

or x = 1 + 39

or x = 40

The value of x is 40.