

Exercise 4A

Page No: 175

Question 1: Which of the following are quadratic equations in x?

(i)
$$x^2 - x + 3 = 0$$

(ii)
$$2x^2 + 5/2 x - \sqrt{3} = 0$$

(iii)
$$\sqrt{2x^2 + 7x + 5}\sqrt{2} = 0$$

(iv)
$$1/3x^2 + 1/5x - 2 = 0$$

(v)
$$x^2 - 3x - \sqrt{x} + 4 = 0$$

(vi)
$$x - 6/x = 3$$

(vii)
$$x + 2/x = x^2$$

(viii)
$$x^2 - 1/x^2 = 5$$

(ix)
$$(x + 2)^3 = x^3 - 8$$

(x)
$$(2x + 3)(3x + 2) = 6(x - 1)(x - 2)$$

(xi)
$$(x + 1/x)^2 = 2x + 1/x + 3$$

Solution:

A quadratic equation is an equation of the second degree.

$$(i)x^2 - x + 3 = 0$$

Highest Degree: 2

Quadratic equation.

(ii)
$$2x^2 + 5/2x - \sqrt{3} = 0$$

Highest Degree: 2

Quadratic equation.

(iii)
$$\sqrt{2}x^2+7x+5\sqrt{2}=0$$

Highest Degree: 2

Quadratic equation.



(iv)
$$1/3x^2 + 1/5x - 2 = 0$$

Above equation can be simplify as: $5x^2 + 3x - 2 = 0$

Highest Degree: 2

Quadratic equation.

(v)
$$x^2 - 3x - \sqrt{x} + 4 = 0$$

Equation has a fractional power.

Not a quadratic equation.

(vi)
$$x - 6/x = 3$$

Simply as $x^2 - 3x - 6 = 0$

Degree: 2

Quadratic equation.

(vii)
$$x + 2/x = x^2$$

Simplify above equation:

$$x^3 - x^2 - 2 = 0$$

Degree: 3

Not a quadratic equation.

(viii)
$$x^2 - 1/x^2 = 5$$

Simplify above equation

$$x^4 - 1 = 5 x^2$$



or $x^4-5x^2-1=0$

Degree: 4

Not a quadratic equation.

(ix)
$$(x + 2)^3 = x^3 - 8$$

$$x^3 + 8 + 6x^2 + 12x = x^3 - 8$$

$$-6x^2 + 12x + 16 = 0$$

Degree = 2

A quadratic equation.

$$(x)(2x + 3)(3x + 2) = 6(x - 1)(x - 2)$$

Simplify above equation:

$$6x^2 + 4x + 9x + 6 = 6x^2 - 12x - 6x + 12$$

$$31x - 6 = 0$$

Degree: 1

Not a quadratic equation

$$(xi)(x + 1/x)^2 = 2x + 1/x + 3$$

Simplify above equation:

$$(x^4 + 2x^2 + 1) / x^2 = (2x^2 + 2) / x + 3$$

$$(x^4 + 2x^2 + 1)x = x^2(2x^2 + 2) + 3$$



Not a quadratic equation.

Answer: (i), (ii), (iii), (iv), (vi) and (ix) are only quadratic equations.

Question 2:

Which of the following are the roots of $3x^2 + 2x - 1=0$?

- (i) -1
- (ii) 1/3
- (iii) -1/2

Solution:

Simplify given equation:

$$3x^2 + 2x - 1 = 3x^2 + 3x - x - 1$$

$$= 3x (x + 1) - 1 (x + 1)$$

$$= (x + 1) (3x - 1)$$

To find roots, put $3x^2 + 2x - 1 = 0$

Either,
$$x + 1 = 0$$
 or $3x - 1 = 0$

$$x = -1$$
 or $x = 1/3$

Therefore, (-1) and 1/3 are the required roots.

Question 3:

- (i) Find the value of k for which x = 1 is a root of the equation $x^2 + kx + 3 = 0$. Also, find the other root.
- (ii) Find the values of a and 6 for which x = 3/4 and x = -2 are the roots of the equation $ax^2 + bx 6 = 0$.

Solution:

(i) x = 1 is a solution of $x^2+kx+3=0$, which means it must satisfy the equation.

$$(1)^2 + k(1) + 3 = 0$$

$$k = -4$$

Hence the required value of k = -4



Find other root:

We have equation, $x^2 - 4x + 3 = 0$

$$x^2 - x - 3x + 3 = 0$$

$$x(x - 1) - 3(x - 1) = 0$$

$$(x - 1)(x - 3) = 0$$

either
$$x - 1 = 0$$
 or $x - 3 = 3$

$$x = 1 \text{ or } x = 3$$

Other root is 3.

(ii) given equation is $ax^2 + bx - 6 = 0$

As ¾ is its root, then must satisfy the equation

$$a(3/4)^2 + b(3/4) - 6 = 0$$

$$9a + 12b - 96 = 0 \dots (1)$$

Again, x = -2 is its root

$$a(-2)^2 + b(-2) - 6 = 0$$

$$4a - 2b - 6 = 0 \dots (2)$$

Solving (1) and (2), we get

$$a = 4$$
 and $b = 5$

Question 4: Show that x = -bc/ad is a solution of the quadratic equation

$$ad^2\left(\frac{ax}{b} + \frac{2c}{d}\right)x + bc^2 = 0$$

$$ad^2\left(\frac{ax}{b} + \frac{2c}{d}\right)x + bc^2 = 0$$
....(1)



x = -bc/ad is solution of equation (1), if it satisfy the equation.

$$ad^{2}\left(\frac{-bc}{bd} + \frac{2c}{d}\right) \frac{b^{2}c^{2}}{a^{2}d^{2}} + bc^{2} = 0$$

$$\left(\frac{-bcd + 2bcd}{bd^2}\right) \times \frac{b^2c^2}{a} + bc^2 = 0$$

$$\frac{bcd}{bd^2} \times \frac{b^2c^2}{a} + bc^2 = 0$$

$$\frac{b^2c^3}{a}+bc^2=0$$

Which is not true.

So x = -bc/ad is not a solution of given quadratic equation.

Solve each of the following quadratic equations.

Question 5:

$$(2x-3)(3x+1)=0$$

Solution:

$$(2x-3)(3x+1)=0$$

Either
$$2x - 3 = 0$$
 or $3x + 1 = 0$

$$x = 3/2$$
 or $x = -1/3$

Question 6:

$$4x^2 + 5x = 0$$

Solution:

$$4x^2 + 5x = 0$$

Or
$$x(4x + 5) = 0$$

Either x = 0 or 4x + 5 = 0, then



$$x = -5/4 \text{ or } 0$$

Question 7: $3x^2 - 243 = 0$

Solution:

$$3x^2 - 243 = 0$$

or
$$x^2 - 81 = 0$$

$$(x)^2 - (9)^2 = 0$$

 $(x + 9) (x - 9) = 0$

Either,
$$x + 9 = 0$$
 or $x - 9 = 0$

$$x = -9 \text{ or } 9$$

Question 8:

$$2x^2 + x - 6 = 0$$

Solution:

$$2x^2 + x - 6 = 0$$

$$2x^2 + 4x - 3x - 6 = 0$$

$$2x(x+2)-3(x+2)=0$$

$$(x+2)(2x-3)=0$$

Either
$$x + 2 = 0$$
 or $2x - 3 = 0$

$$x = -2 \text{ or } 3/2$$

Question 9:

$$x^2 + 6x + 5 = 0$$

$$x^2 + 6x + 5 = 0$$

$$x^2 + x + 5x + 5 = 0$$

$$x(x + 1) + 5(x + 1) = 0$$



$$(x + 5)(x + 1) = 0$$

either
$$x + 5 = 0$$
 or $x + 1 = 0$

$$x = -5 \text{ or } -1$$

Question 10:

$$9x^2 - 3x - 2 = 0$$

Solution:

$$9x^2 - 3x - 2 = 0$$

$$9x^2 - 6x + 3x - 2 = 0$$

$$3x(3x -2) + (3x - 2) = 0$$

$$(3x +1)(3x - 2) = 0$$

either
$$(3x +1) = 0$$
 or $(3x - 2) = 0$

$$x = -1/3$$
 or $2/3$

Question 11:

 $x^2 + 12x + 35 = 0$

Solution:

$$x^2 + 12x + 35 = 0$$

$$x^2 + 7x + 5x + 35 = 0$$

$$x(x + 7) + 5((x + 7) = 0$$

$$(x + 5)(x + 7) = 0$$

either
$$(x + 5) = 0$$
 or $(x + 7) = 0$

$$x = -5 \text{ or } -7$$

Question 12:

$$x^2 = 18x - 77$$

$$x^2 - 18x + 77 = 0$$

$$x^2 - 7x - 11x + 77 = 0$$



$$x(x-7) - 11(x-7)$$

= 0

$$(x - 11)(x - 7) = 0$$

either
$$(x - 11) = 0$$
 or $(x - 7) = 0$

$$x = 11 \text{ or } 7$$

Question 13:

 $6x^2 + 11x + 3 = 0$

Solution:

$$6x^2 + 11x + 3 = 0$$

$$6x^2 + 2x + 9x + 3 = 0$$

$$2x(3x + 1) + 3(3x + 1) = 0$$

$$(2x + 3)(3x + 1) = 0$$

either
$$(2x + 3) = 0$$
 or $(3x + 1) = 0$

$$x = -1/3$$
 or $-3/2$

Question 14:

$$6x^2 + x - 12 = 0$$

Solution:

$$6x^2 + x - 12 = 0$$

$$6x^2 + 9x - 8x - 12 = 0$$

$$3x(2x + 3) - 4(2x + 3) = 0$$

$$(2x+3)(3x-4)=0$$

Either,
$$2x + 3 = 0$$
, then $2x = -3 \Rightarrow x = \frac{-3}{2}$

or
$$3x - 4 = 0$$
, then $3x = 4 \Rightarrow x = \frac{4}{3}$

$$x = \frac{-3}{2}$$
 or $\frac{4}{3}$

Question 15:

$$3x^2 - 2x - 1 = 0$$



Solution:

$$3x^2 - 2x - 1 = 0$$
$$3x^2 - 3x + x - 1 = 0$$

$$3x(x-1)+1(x-1)=0$$

$$(x-1)(3x+1)=0$$

Either,
$$x - 1 = 0$$
, then $x = 1$

or
$$3x + 1 = 0$$
, then $x = \frac{-1}{3}$

$$x = 1$$
 or $\frac{-1}{3}$

Question 16:

$$4x^2 - 9x = 100$$

Solution:

$$4x^2 - 9x = 100$$
$$4x^2 - 9x - 100 = 0$$

$$4x^2 - 25x + 16x - 100 = 0$$

$$x(4x-25)+4(4x-25)=0$$

$$(4x-25)(x+4)=0$$

Either,
$$4x - 25 = 0$$
, then $x = \frac{25}{4}$

or
$$x + 4 = 0$$
, then $x = -4$

$$x = -4 \text{ or } \frac{25}{4}$$

Question 17:

$$15x^2 - 28 = x$$

$$15x^2 - 28 = x$$

$$15x^2 - x - 28 = 0$$

$$15x^2 - (21x - 20x) - 28 = 0$$



$$15x^2 - 21x + 20x -$$

$$28 = 0$$

$$3x(5x-7)+4(5x-7)=0$$

$$(3x+4)(5x-7) = 0$$

$$3x + 4 = 0$$
 or $5x - 7 = 0$

$$x = -4/3$$
 or $7/5$

Question 18:

$$4 - 11x = 3x^2$$

Solution:

$$4 - 11x = 3x^2$$

$$3x^2 + 11x - 4 = 0$$

$$3x^2 + 12x - x - 4 = 0$$

$$3x(x+4)-1(x+4) = 0$$

$$(x+4)(3x-1)=0$$

Either
$$x + 4 = 0$$
 or $3x - 1 = 0$

$$x = -4$$
 or $1/3$

Question 19:

$$48x^2 - 13x - 1 = 0$$

$$48x^2 - 13x - 1 = 0$$

$$48x^2 - (16x - 3x) - 1 = 0$$

$$48x^2 - 16x + 3x - 1 = 0$$

$$16x(3x-1)+1(3x-1)=0$$

$$(16x+1)(3x-1)=0$$

Either
$$16x + 1 = 0$$
 or $3x - 1 = 0$

$$x = -1/16$$
 or $1/3$



Question 20:

$$x^2 + 2\sqrt{2} x - 6 = 0$$

Solution:

$$x^2 + 2\sqrt{2}x - 6 = 0$$

$$x^2 + 3\sqrt{2}x - \sqrt{2}x - 6 = 0$$

$$x(x + 3\sqrt{2}) - \sqrt{2}(x + 3\sqrt{2}) = 0$$

$$(x + 3\sqrt{2})(x - \sqrt{2}) = 0$$

Either,
$$(x + 3\sqrt{2}) = 0$$
, then $x = -3\sqrt{2}$

or
$$x - \sqrt{2} = 0$$
, then $x = \sqrt{2}$

$$x = \sqrt{2}$$
 or $-3\sqrt{2}$

Question 21;

$$\sqrt{3} x^2 + 10x + 7\sqrt{3} = 0$$

Solution:

$$\sqrt{3} x^2 + 10x + 7\sqrt{3} = 0$$

$$\sqrt{3} x^2 + 10x + 7\sqrt{3} = 0$$

$$\sqrt{3} x^2 + 3x + 7x + 7\sqrt{3} = 0$$

$$\sqrt{3} \times (x + \sqrt{3}) + 7(x + \sqrt{3}) = 0$$

$$(x + \sqrt{3})(\sqrt{3} x + 7) = 0$$

either
$$\sqrt{3} x + 7 = 0$$
 or $x + \sqrt{3} = 0$

$$x = -\sqrt{3} \text{ or } -7/\sqrt{3}$$

Question 22:

$$\sqrt{3} x^2 + 11x + 6\sqrt{3} = 0$$

$$\sqrt{3} x^2 + 11x + 6\sqrt{3} = 0$$

$$\sqrt{3} x^2 + 9x + 2x + 6\sqrt{3} = 0$$



$$\sqrt{3} \times (x + 3\sqrt{3}) + 2(x + 3\sqrt{3}) = 0$$

$$(\sqrt{3}x + 2)(x + 3\sqrt{3}) = 0$$

either
$$(\sqrt{3}x + 2) = 0$$
 or $(x + 3\sqrt{3}) = 0$

$$x = -3\sqrt{3}$$
 or $-2\sqrt{3}/3$

Question 23:

$$3\sqrt{7} x^2 + 4x + \sqrt{7} = 0$$

Solution:

$$3\sqrt{7} x^2 + 4x - \sqrt{7} = 0$$

$$3\sqrt{7} x^2 + 4x - \sqrt{7} = 0$$

$$3\sqrt{7} x^2 - 3x + 7x - \sqrt{7} = 0$$

$$3x(\sqrt{7}x - 1) + \sqrt{7}(\sqrt{7}x - 1) = 0$$

$$(3x + \sqrt{7})(\sqrt{7}x - 1) = 0$$

either
$$(3x + \sqrt{7}) = 0$$
 or $(\sqrt{7}x - 1) = 0$

$$x = -\sqrt{7/3}$$
 or $1/\sqrt{7}$

Question 24:

$$\sqrt{7} x^2 - 6x - 13\sqrt{7} = 0$$

$$\sqrt{7} x^2 - 6x - 13\sqrt{7} = 0$$



$$\sqrt{7} x^2 + 7x - 13x - 13\sqrt{7} = 0$$

$$\sqrt{7} x (x + \sqrt{7}) - 13 (x + \sqrt{7}) = 0$$

$$(x + \sqrt{7})(\sqrt{7}x - 13) = 0$$

Either,
$$x + \sqrt{7} = 0$$
, then $x = -\sqrt{7}$

or
$$\sqrt{7} x - 13 = 0$$
, then $\sqrt{7} x = 13$

$$x = -\sqrt{7} \text{ or } \frac{13\sqrt{7}}{7}$$

Question 25:

$$4\sqrt{6} x^2 - 13x - 2\sqrt{6} = 0$$

Solution:

$$4\sqrt{6}x^2 - 13x - 2\sqrt{6} = 0$$

$$4\sqrt{6}x^2 - 16x + 3x - 2\sqrt{6} = 0$$

$$4\sqrt{2}x(\sqrt{3}x-2\sqrt{2})+\sqrt{3}(\sqrt{3}x-2\sqrt{2})=0$$

$$(\sqrt{3}x - 2\sqrt{2})(4\sqrt{2}x + \sqrt{3}) = 0$$

Either,
$$\sqrt{3} x - 2 \sqrt{2} = 0$$
, then $\sqrt{3} x = 2 \sqrt{2}$

$$x = \frac{2\sqrt{6}}{3}$$
 and

$$4\sqrt{2}x + \sqrt{3} = 0$$
, then $4\sqrt{2}x = -\sqrt{3}$

$$x = \frac{-\sqrt{6}}{8}$$

$$x = \frac{2\sqrt{2}}{\sqrt{3}}$$

$$x = \frac{2\sqrt{6}}{3}, \frac{-\sqrt{6}}{8}$$

Question 26.



$$3x^2 - 2\sqrt{6}x + 2 = 0$$

Solution:

$$3x^2 - 2\sqrt{6}x + 2 = 0$$

$$3x^2 - \sqrt{6}x - \sqrt{6}x + 2 = 0$$

$$\sqrt{3} x(\sqrt{3} x - \sqrt{2}) - \sqrt{2} (\sqrt{3} x - \sqrt{2}) = 0$$

$$(\sqrt{3} x - \sqrt{2})(\sqrt{3} x - \sqrt{2}) = 0$$

Either,
$$\sqrt{3} x - \sqrt{2} = 0$$
, then $x = \frac{\sqrt{2}}{\sqrt{3}}$

or
$$\sqrt{3} x - \sqrt{2} = 0$$
, then $x = \frac{\sqrt{2}}{\sqrt{3}}$

$$x = \frac{\sqrt{2}}{\sqrt{3}}$$
 or $\frac{\sqrt{2}}{\sqrt{3}}$

Question 27:

$$\sqrt{3} x^2 - 2\sqrt{2} x - 2\sqrt{3} = 0$$

Solution:

$$\sqrt{3} x^2 - 2\sqrt{2} x - 2\sqrt{3} = 0$$

$$\sqrt{3} x^2 - 3\sqrt{2} x + \sqrt{2} x - 2\sqrt{3} = 0$$

$$\sqrt{3}x(x-\sqrt{6}) + \sqrt{2}(x-\sqrt{6}) = 0$$

$$(\sqrt{3}x + \sqrt{2})(x - \sqrt{6}) = 0$$

either
$$(\sqrt{3}x + \sqrt{2}) = 0$$
 or $(x - \sqrt{6}) = 0$

$$x = \sqrt{6}$$
 or $-\sqrt{2}/\sqrt{3}$

Question 28.

$$x^2 - 3\sqrt{5} x + 10 = 0$$



$$x^2 - 3\sqrt{5} x + 10 = 0$$

$$x^2 - 2\sqrt{5}x - \sqrt{5}x + 10 = 0$$

$$x(x-2\sqrt{5})-\sqrt{5}(x-2\sqrt{5})=0$$

$$(x-2\sqrt{5})(x-\sqrt{5})=0$$

Either,
$$x - 2\sqrt{5} = 0$$
, then $x = 2\sqrt{5}$

or
$$x - \sqrt{5} = 0$$
, then $x = \sqrt{5}$

Hence,
$$x = 2\sqrt{5}$$
 or $\sqrt{5}$

Question 29:

$$x^2 - (\sqrt{3} + 1) x + \sqrt{3} = 0$$

Solution:

$$x^2 - (\sqrt{3} + 1) x + \sqrt{3} = 0$$

$$x^2 - (\sqrt{3} + 1) x + \sqrt{3} = 0$$

$$x^2 - \sqrt{3}x - x + \sqrt{3} = 0$$

$$x(x - \sqrt{3}) - (x - \sqrt{3}) = 0$$

$$(x-1)(x-\sqrt{3})=0$$

either
$$(x - 1) = 0$$
 or $(x - \sqrt{3}) = 0$

$$x = 1 \text{ or } \sqrt{3}$$

Question 30:

$$x^2 + 3\sqrt{3} x - 30 = 0$$



$$x^2 + 3\sqrt{3} x - 30 = 0$$

$$x^2 + 5\sqrt{3} x - 2\sqrt{3} - 30 = 0$$

$$x(x + 5\sqrt{3}) - 2\sqrt{3}(x + 5\sqrt{3}) = 0$$

$$(x - 2\sqrt{3})(x + 5\sqrt{3}) = 0$$

either
$$(x - 2\sqrt{3}) = 0$$
 or $(x + 5\sqrt{3}) = 0$

$$x = -5\sqrt{3} \text{ or } 2\sqrt{3}$$





Exercise 4B

Page No: 185

Solve each of the following equations by using the method of completing the square: Question 1:

$$x^2 - 6x + 3 = 0$$

Solution:

$$x^2 - 6x + 3 = 0$$

 $x^2 - 6x + 3 = 0$

$$x^2 - 6x = -3$$

$$x^2 - 2(x)3 + 3^2 = -3 + 3^2$$

(adding 3² on both sides)

$$(x-3)^2 = -3 + 9 = 6$$

Using algebraic identity: $a^2 - 2ab + b^2 = (a - b)^2$

$$x - 3 = \pm \sqrt{6}$$

$$x = 3 \pm \sqrt{6}$$

$$x = (3 + \sqrt{6}) \text{ or } (3 - \sqrt{6})$$

Question 2:

$$x^2 - 4x + 1 = 0$$

Solution:

$$x^2 - 4x + 1 = 0$$

$$x^2 - 4x = -1$$

$$x^2 - 2(x)(2) + 2^2 = -1 + 2^2$$

(adding 2² on both sides)

$$(x - 2)^2 = 3$$



Using algebraic identity: $a^2 - 2ab + b^2 = (a - b)^2$

$$x-2=\pm\sqrt{3} \implies x=2\pm\sqrt{3}$$

$$x = (2 + \sqrt{3}) \text{ or } (2 - \sqrt{3})$$

Question 3:

$$x^2 + 8x - 2 = 0$$

Solution:

$$x^2 + 8x - 2 = 0$$

$$x^2 + 8x = 2$$

$$x^2 - 2 \cdot x \cdot 4 + 42 = 2 + 42$$

(adding 2² on both sides)

$$(x + 4)^2 = 18$$

Using algebraic identity: $a^2 - 2ab + b^2 = (a - b)^2$

$$x + 4 = \pm 3\sqrt{2}$$

$$x = -4 \pm 3\sqrt{2}$$

$$x = (-4 + 3\sqrt{2}) \text{ or } (-4 - 3\sqrt{2})$$

Question 4:

$$4x^2 + 4\sqrt{3}x + 3 = 0$$



$$4x^2 + 4\sqrt{3}x + 3 = 0$$

$$(2x)^2 + 2 \times 2x \times \sqrt{3} = -3$$

Adding $(\sqrt{3})^2$ to both sides,

$$(2x)^2 + 2 \times 2x \times \sqrt{3} + (\sqrt{3})^2 = -3 + (\sqrt{3})^2$$

$$(2x + \sqrt{3})^2 = -3 + \sqrt{3} = 0$$

$$2x + \sqrt{3} = 0 \Rightarrow 2x = -\sqrt{3}$$

So,

$$x=\frac{-\sqrt{3}}{2},\,\frac{-\sqrt{3}}{2}$$

Question 5:

$$2x^2 + 5x - 3 = 0$$

Solution:

$$2x^2 + 5x - 3 = 0$$

$$4x^2 + 10x - 6 = 0$$

(multiplying both sides by 2)

$$4x^2 + 10x = 6$$

(adding (5/2)² on both sides)

$$(2x + 5/2)^2 = 6 + 25/4 = 49/4$$

Using algebraic identity: $a^2 - 2ab + b^2 = (a - b)^2$

Taking square root,

$$2x + \frac{5}{2} = \pm \frac{7}{2}$$

$$2x + 5/2 = 7/2$$
 or $2x + 5/2 = -7/2$

$$x = 1/2$$
 or -3



Question 6: $3x^2 - x - 2 = 0$ Solution:

$$3x^2 - x - 2 = 0$$

$$9x^2 - 3x - 6 = 0$$
 (multiplying both sides by 3)

$$9x^2 - 3x = 6$$

Adding (1/2)^2 on both the sides.

$$(3x)^2 - 2.3x.\frac{1}{2} + (\frac{1}{2})^2 = 6 + (\frac{1}{2})^2$$

$$\left(3x - \frac{1}{2}\right)^2 = 6 + \frac{1}{4} = \frac{25}{4} = \left(\frac{5}{2}\right)^2$$

$$3x - 1/2 = 5/2$$
 or $3x - 1/2 = -5/2$

$$x = 1$$
 or $x = -2/3$

Question 7:

$$8x^2 - 14x - 15 = 0$$

Solution:

$$8x^2 - 14x - 15 = 0$$

$$16x^2 - 28x - 30 = 0$$

(multiplying both sides by 2)

Adding (7/2)^2 on both the sides

$$(4x)^2 - 2.4x \cdot \frac{7}{2} + \left(\frac{7}{2}\right)^2 = 30 + \left(\frac{7}{2}\right)^2$$

$$\left(4x - \frac{7}{2}\right)^2 = 30 + \frac{49}{4} = \frac{169}{4} = \left(\frac{13}{2}\right)^2$$

$$4x - 7/2 = 13/2$$
 or $4x - 7/2 = -13/2$

$$x = 5/2$$
 or $x = -3/4$



Question 8:

 $7x^2 + 3x - 4 = 0$

Solution:

$$7x^2 + 3x - 4 = 0$$

 $49x^2 + 21x - 28 = 0$ (multiplying both sides by 7)

Adding (3/2)^2 on both the sides,

$$(7x)^2 + 2.7x.\frac{3}{2} + \left(\frac{3}{2}\right)^2 = 28 + \left(\frac{3}{2}\right)^2$$

$$\left(7x + \frac{3}{2}\right)^2 = 28 + \frac{9}{4} = \frac{121}{4} = \left(\frac{11}{2}\right)^2$$

$$7x + 3/2 = 11/2$$
 or $7x + 3/2 = -11/2$

$$x = -1$$
 or $x = 4/7$

Question 9:

$$3x^2 - 2x - 1 = 0$$

Solution:

$$3x^2 - 2x - 1 = 0$$

$$9x^2 - 6x = 3$$

(multiplying both sides by 3)

Adding (1)^2 on both the sides

$$(3x)^2 - 2.3x.1 + (1)^2 = 3 + (1)^2$$

$$(3x - 1)^2 = 2^2$$

$$3x - 1 = 2$$
 or $3x - 1 = -2$

$$x = -1$$
 or $x = -1/3$



Question 10: $5x^2 - 6x - 2 = 0$ Solution:

$$5x^2 - 6x - 2 = 0$$

$$25x^2 - 30x - 10 = 0$$
 (multiplying both sides by 5)

$$25x^2 - 30x = 10$$

Adding (3)² both the sides

$$(5x)^2 - 2.5x.3 + (3)^2 = 10 + (3)^2$$

$$(5x-2)^2 = 10 + 9 = 19$$

$$5x - 3 = \sqrt{19}$$
 or $5x - 3 = -\sqrt{19}$

$$x = (3 + \sqrt{19})/5$$
 or $x = (3 - \sqrt{19})/5$

Question 11:

$$2/x^2 - 5/x + 2 = 0$$

Solution:

$$2/x^2 - 5/x + 2 = 0$$

$$\frac{2 - 5x + 2x^2}{x^2} = 0$$

$$2x^2 - 5x + 2 = 0$$

$$4x^2 - 10x = -4$$
 (multiplying both sides by 2)

Adding (5/2)² both the sides

$$(2x)^2 - 2.2x.\frac{5}{2} + \left(\frac{5}{2}\right)^2 = -4 + \left(\frac{5}{2}\right)^2$$



$$(2x)^2 - 2.2x.\frac{5}{2} + \left(\frac{5}{2}\right)^2 = -4 + \left(\frac{5}{2}\right)^2$$

$$2x - 5/2 = 3/2$$
 or $2x - 5/2 = -3/2$

$$x = 2$$
 or $x = 1/2$

Question 12:

$$4x^2 + 4bx - (a^2 - b^2) = 0$$

Solution:

$$4x^2 + 4bx - (a^2 - b^2) = 0$$

$$x^2 + bx - \frac{a^2 - b^2}{4} = 0$$

(Dividing by 4)

Adding (b/2)^2 both sides,

$$(x)^2+2\times x+\frac{b}{2}+\left(\frac{b}{2}\right)^2$$

$$=\frac{a^2-b^2}{4}+\left(\frac{b}{2}\right)^2=\frac{a^2-b^2}{4}+\frac{b^2}{4}$$

$$\left(x+\frac{b}{2}\right)^2 = \frac{a^2-b^2+b^2}{4} = \frac{a^2}{4} = \left(\pm\frac{a}{2}\right)^2$$

$$x + b/2 = a/2$$
 or $x + b/2 = -a/2$

$$x = (a-b)/2$$
 or $x = -(a+b)/2$

Question 13:

$$x^2 - (\sqrt{2} + 1) x + \sqrt{2} = 0$$



$$x^2 - (\sqrt{2} + 1)x + \sqrt{2} = 0$$

$$x^2 - 2 \times \left(\frac{\sqrt{2} + 1}{2}\right) \times x = -\sqrt{2}$$

Adding,
$$\left(\frac{\sqrt{2}+1}{2}\right)^2$$
 to both sides

$$x^2 - 2\left(\frac{\sqrt{2}+1}{2}\right)x + \left(\frac{\sqrt{2}+1}{2}\right)^2.$$

$$=-\sqrt{2}+\left(\frac{\sqrt{2}+1}{2}\right)^2$$

$$\left(x - \frac{\sqrt{2} + 1}{2}\right)^2 = \frac{-\sqrt{2}}{1} + \frac{2 + 1 + 2\sqrt{2}}{4}$$

$$= \frac{-4\sqrt{2} + 2 + 1 + 2\sqrt{2}}{4} = \frac{2 + 1 - 2\sqrt{2}}{4}$$

$$=\left(\pm\frac{\sqrt{2}-1}{2}\right)^2$$

$$x = \frac{\sqrt{2} + 1}{2} \pm \frac{\sqrt{2} - 1}{2}$$



$$x = \frac{\sqrt{2} + 1}{2} \pm \frac{\sqrt{2} - 1}{2}$$

$$x = \frac{\sqrt{2} + 1 + \sqrt{2} - 1}{2} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

or
$$x = \frac{\sqrt{2} + 1 - \sqrt{2} + 1}{2} = \frac{2}{2} = 1$$

$$x = 1$$
 or $\sqrt{2}$

Question 14:

$$\sqrt{2} x^2 - 3x - 2\sqrt{2} = 0$$

Solution:

$$\sqrt{2} x^2 - 3x - 2\sqrt{2} = 0$$

Dividing each side by $\sqrt{2}$

$$x^2 - \frac{3}{\sqrt{2}}x - 2 = 0$$

$$(x)^2 - 2 \times x \times \frac{3}{2\sqrt{2}} = 2$$

Adding, $\left(\frac{3}{2\sqrt{2}}\right)^2$ to both sides,

$$(x)^2-2\times x\times \frac{3}{2\sqrt{2}}+\left(\frac{3}{2\sqrt{2}}\right)^2$$

$$=2+\left(\frac{3}{2\sqrt{2}}\right)^2$$



$$\left(x - \frac{3}{2\sqrt{2}}\right)^2 = 2 + \frac{9}{8} = \frac{25}{8} = \left(\pm \frac{5}{2\sqrt{2}}\right)^2$$

$$x - \frac{3}{2\sqrt{2}} = \left(\pm \frac{5}{2\sqrt{2}}\right)$$

$$x = \frac{3}{2\sqrt{2}} \pm \frac{5}{2\sqrt{2}}$$

$$x = \frac{3}{2\sqrt{2}} + \frac{5}{2\sqrt{2}} = \frac{8}{2\sqrt{2}} = \frac{4}{\sqrt{2}}$$

$$=\frac{4\sqrt{2}}{\sqrt{2}\times\sqrt{2}}=2\sqrt{2}$$

or
$$x = \frac{3}{2\sqrt{2}} - \frac{5}{2\sqrt{2}} = \frac{-2}{2\sqrt{2}} = \frac{-1}{\sqrt{2}} = \frac{\sqrt{2}}{4}$$

$$x = \frac{-1}{\sqrt{2}} \text{ or } 2\sqrt{2}$$

Question 15:

$$\sqrt{3} x^2 + 10x + 7\sqrt{3} = 0$$

Solution:

$$\sqrt{3} x^2 + 10x - 7\sqrt{3} = 0$$

Dividing each side by $\sqrt{3}$

$$x^2 + \frac{10}{\sqrt{3}}x + 7 = 0$$

$$(x)^2 + 2 \times x \times \frac{5}{\sqrt{3}} = -7$$

Adding,
$$\left(\frac{5}{\sqrt{3}}\right)^2$$
 to both sides



$$(x)^2 + 2 \times x \times \frac{5}{\sqrt{3}} + \left(\frac{5}{\sqrt{3}}\right)^2 = -7 + \left(\frac{5}{\sqrt{3}}\right)^2$$

$$\left(x + \frac{5}{\sqrt{3}}\right)^2 = -7 + \frac{25}{3}$$
$$= \frac{-21 + 25}{3} = \frac{4}{3} = \left(\pm \frac{2}{\sqrt{3}}\right)^2$$

$$x + \frac{5}{\sqrt{3}} = \pm \frac{2}{\sqrt{3}}$$

$$x = \frac{-5}{\sqrt{3}} \pm \frac{2}{\sqrt{3}}$$

$$x = -\sqrt{3} \text{ or } \frac{-7}{\sqrt{3}}$$

Question 16:

By using the method of completing the square, show that the equation $2x^2 + x + 4 = 0$ has no real roots:

Solution:

$$2x^2 + x + 4 = 0$$

$$4x^2 + 2x + 8 = 0$$

(multiplying both sides by 2)

$$4x^2 + 2x = -8$$

Adding (1/2)^2 both sides

$$(2x)^2 + 2.2x \cdot \frac{1}{2} + \left(\frac{1}{2}\right)^2 = -8 + \left(\frac{1}{2}\right)^2$$

$$\left(2x + \frac{1}{2}\right)^2 = -8 + \frac{1}{4} = -\frac{31}{4} < 0$$

But $(2x + 1/2)^2$ cannot be negative for any real value of x

The given equation has no real roots.



Exercise 4C

Page No: 191

Find the discriminant of each of the following equations:

Question 1:

(i)
$$2x^2 - 7x + 6 = 0$$

(ii)
$$3x^2 - 2x + 8 = 0$$

(iii)
$$2x^2 - 5\sqrt{2}x + 4 = 0$$

(iv)
$$\sqrt{3} x^2 + 2\sqrt{2} x - 2\sqrt{3} = 0$$

(v)
$$(x-1)(2x-1)=0$$

(vi)
$$1 - x = 2x^2$$

Solution:

(i)
$$2x^2 - 7x + 6 = 0$$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

Here,
$$a = 2$$
, $b = -7$ and $c = 6$

Discriminant formula: D = b^2 - 4ac

$$(-7)^2 - 4 \times 2 \times 6$$

= 1

(ii)
$$3x^2 - 2x + 8 = 0$$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

Here,
$$a = 3$$
, $b = -2$, $c = 8$

Discriminant formula: D = b^2 - 4ac

$$= (-2)^2 - 4.3.8$$

$$= 4 - 96$$

$$= -92$$



(iii)
$$2x^2 - 5\sqrt{2}x + 4 = 0$$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

Here, a = 2, $b = -5\sqrt{2}$, c = 4

Discriminant formula: D = b^2 - 4ac

$$= (-5\sqrt{2})^2 - 4.2.4$$

$$= 50 - 32$$

(iv)
$$\sqrt{3} x^2 + 2\sqrt{2} x - 2\sqrt{3} = 0$$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

Here, $a = \sqrt{3}$, $b = 2\sqrt{2}$, $c = -2\sqrt{3}$

Discriminant formula: $D = b^2 - 4ac$

$$= (2\sqrt{2})^2 - 4(\sqrt{3})(-2\sqrt{3})$$

$$(v)(x-1)(2x-1)=0$$

$$2x^2 - 3x + 1 = 0$$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

Here, a = 2, b = -3, c = -1

Discriminant formula: D = b^2 - 4ac

$$= (-3)^2 - 4x2x1$$

= 1



(vi)
$$1 - x = 2x^2$$

$$1 - x = 2x^2$$

$$2x^2 + x - 1 = 0$$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

Here,
$$a = 2$$
, $b = 1$, $c = -1$

Discriminant formula: D = b^2 - 4ac

$$= (1)^2 - 4x2x-1$$

= 9

Find the roots of each of the following equations, if they exist, by applying the quadratic formula:

Question 2:

$$x^2 - 4x - 1 = 0$$

Solution:

$$x^2 - 4x - 1 = 0$$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

Here,
$$a = 1$$
, $b = -4$, $c = -1$

Find Discriminant:

$$D = b^2 - 4ac$$

$$= (-4)^2 - 4x1x-1$$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$



$$x = \frac{-(-4)) \pm \sqrt{20}}{2 \times 1}$$
$$= \frac{4 \pm 2\sqrt{5}}{2}$$
$$= 2 \pm \sqrt{5}$$

Therefore, $x = 2 + \sqrt{5}$ and $x = 2 - \sqrt{5}$

Question 3:

$$x^2 - 6x + 4 = 0$$

Solution:

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

$$a = 1, b = -6, c = 4$$

Find Discriminant:

$$D = b^2 - 4ac$$

$$= (-6)^2 - 4.1.4$$

Roots of equation are real.

Find the Roots:

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{-(-6) \pm \sqrt{20}}{2 \times 1} = \frac{6 \pm 2\sqrt{5}}{2}$$

$$=3\pm\sqrt{5}$$



$$x = 3 + \sqrt{5}$$
 and $x = 3 - \sqrt{5}$

Question 4:

$$2x^2 + x - 4 = 0$$

Solution:

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

$$a = 2, b = 1, c = -4$$

Find Discriminant:

$$D = b^2 - 4ac$$

$$= (1)^2 - 4.2.4$$

$$= 1 + 32$$

$$= 33 > 0$$

Roots of equation are real.

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$=\frac{-1\pm\sqrt{33}}{4}$$

Root are:

$$x = \frac{(-1 + \sqrt{33})}{4}$$

and

$$x=\frac{(-1-\sqrt{33})}{4}$$



Question 5:

$$25x^2 + 30x + 7 = 0$$

Solution:

$$25x^2 + 30x + 7 = 0$$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

$$a = 25, b = 30, c = 7$$

Find Discriminant:

$$D = b^2 - 4ac$$

$$= (30)^2 - 4.25.7$$

$$= 900 - 700$$

$$= 200 > 0$$

Roots of equation are real.

Find the Roots:

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$x = \frac{-30 \pm \sqrt{200}}{2 \times 25}$$
$$-30 \pm 10\sqrt{2}$$
$$50$$
$$-3 \pm \sqrt{2}$$

Roots of the equations are:

$$x = \frac{-3 + \sqrt{2}}{5}$$



And

$$x = \frac{-3 - \sqrt{2}}{5}$$

Question 6:

$$16x^2 = 24x + 1$$

Solution:

$$16x^2 - 24x - 1 = 0$$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

$$a = 16$$
, $b = -24$, $c = -1$

Find Discriminant:

$$D = b^2 - 4ac$$

$$= (-24)^2 - 4.16.-1$$

$$= 576 + 64$$

$$= 640 > 0$$

Roots of equation are real.

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$3 \pm \sqrt{10}$$

Roots are:

$$x = \frac{3 + \sqrt{10}}{4}$$
 or $x = \frac{3 - \sqrt{10}}{4}$



Question /:

$$15x^2 - 28 = x$$

Solution:

$$15x^2 - x - 28 = 0$$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

$$a = 15, b = -1, c = -28$$

Find Discriminant:

$$D = b^2 - 4ac$$

$$= (-1)^2 - 4.15.(-28)$$

$$= 1 + 1680$$

$$= 1681 > 0$$

Roots of equation are real.

$$x=\frac{-b\pm\sqrt{D}}{2a}$$

$$= \frac{-(-1) \pm \sqrt{1681}}{2 \times 15} = \frac{1 \pm 41}{30}$$

$$X = 7/5$$
 and $x = -4/3$

Question 8:

$$2x^2 - 2\sqrt{2}x + 1 = 0$$

Solution:

$$2x^2 - 2\sqrt{2}x + 1 = 0$$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

$$a = 2, b = -2\sqrt{2}, c = 1$$



Find Discriminant:

$$D = b^2 - 4ac$$

$$=(-2\sqrt{2})^2-4.2.1$$

$$= 8 - 8$$

$$= 0$$

Equation has equal root.

Find roots:

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{2\sqrt{2} \pm 0}{4} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

$$x = 1/\sqrt{2}$$
 and $x = 1/\sqrt{2}$

Question 9:

$$\sqrt{2} x^2 + 7x + 5\sqrt{2} = 0$$

Solution:

$$\sqrt{2} x^2 + 7x + 5\sqrt{2} = 0$$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

$$a = \sqrt{2}$$
, $b = 7$, $c = 5\sqrt{2}$

Find Discriminant:

$$D = b^2 - 4ac$$

$$= (7)^2 - 4. \sqrt{2}. 5\sqrt{2}$$

$$= 49 - 40$$



$$= 9 > 0$$

Roots of equation are real.

Find roots:

$$x=\frac{-b\pm\sqrt{D}}{2a}$$

$$= \frac{-7 \pm \sqrt{9}}{2 \times \sqrt{2}} = \frac{-7 \pm 3}{2\sqrt{2}}$$

Roots are:

$$x = -\sqrt{2}$$
 and $x = -5/\sqrt{2}$

Question 10:

$$\sqrt{3} x^2 + 10x - 8\sqrt{3} = 0$$

Solution:

$$\sqrt{3} x^2 + 10x - 8\sqrt{3} = 0$$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

$$a = \sqrt{3}, b = 10, c = -8\sqrt{3}$$

Find Discriminant:

$$D = b^2 - 4ac$$

$$= (10)^2 - 4. \sqrt{3} \cdot - 8\sqrt{3}$$

$$= 100 + 96$$

$$= 196 > 0$$

Roots of equation are real.

Find roots:



$$x=\frac{-b\pm\sqrt{D}}{2a}$$

$$=\frac{-10\pm\sqrt{196}}{2\times\sqrt{3}}=\frac{-10\pm14}{2\sqrt{3}}$$

Roots are:

$$x = 2\sqrt{3}/3$$
 and $x = -4\sqrt{3}$

Question 11:

$$\sqrt{3} x^2 - 2\sqrt{2} x - 2\sqrt{3} = 0$$

Solution:

$$\sqrt{3} x^2 - 2\sqrt{2} x - 2\sqrt{3} = 0$$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

$$a = \sqrt{3}$$
, $b = -2\sqrt{2}$, $c = -2\sqrt{3}$

Find Discriminant:

$$D = b^2 - 4ac$$

$$= (-2\sqrt{2})^2 - 4 \cdot \sqrt{3} \cdot -2\sqrt{3}$$

$$= 8 + 24$$

$$= 32 > 0$$

Roots of equation are real.

Find roots:

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$=\frac{2\sqrt{2}\pm4\sqrt{2}}{2\sqrt{3}}$$



$$x = \sqrt{6}$$
 and $x = -\sqrt{2}/\sqrt{3}$

Question 12:

$$2x^2 + 6\sqrt{3} x - 60 = 0$$

Solution:

$$2x^2 + 6\sqrt{3} x - 60 = 0$$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

$$a = 2$$
, $b = 6\sqrt{3}$, $c = -60$

Find Discriminant:

$$D = b^2 - 4ac$$

$$= (6\sqrt{3})^2 - 4.2. - 60$$

$$= 108 + 480$$

$$= 588 > 0$$

Roots of equation are real.

Find roots:

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{-6\sqrt{3} \pm \sqrt{196 \times 3}}{2 \times 2}$$

$$=\frac{-3\sqrt{3}\pm7\sqrt{3}}{2}$$

Roots are:

$$x = 2\sqrt{3}$$
 and $x = -5\sqrt{3}$



Question 13:

$$4\sqrt{3} x^2 + 5x - 2\sqrt{3} = 0$$

Solution:

$$4\sqrt{3} x^2 + 5x - 2\sqrt{3} = 0$$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

$$a = 4\sqrt{3}$$
, $b = 5$, $c = -2\sqrt{3}$

Find Discriminant:

$$D = b^2 - 4ac$$

$$= (5)^2 - 4.4\sqrt{3} \cdot - 2\sqrt{3}$$

$$= 25 + 96$$

$$= 121 > 0$$

Roots of equation are real.

Find roots:

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{-5 \pm \sqrt{121}}{2 \times 4\sqrt{3}} = \frac{-5 \pm 11}{8\sqrt{3}}$$

Roots are:

$$x = \sqrt{3}/4$$
 and $x = -2/\sqrt{3}$

Question 14:

$$3x^2 - 2\sqrt{6}x + 2 = 0$$

Solution:

$$3x^2 - 2\sqrt{6}x + 2 = 0$$



Compare given

equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

$$a = 3, b = -2\sqrt{6}, c = 2$$

Find Discriminant:

$$D = b^2 - 4ac$$

$$= (-2\sqrt{6})^2 - 4.3.2$$

$$= 24 - 24$$

$$= 0$$

Roots of equation are equal.

Find roots:

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$=\frac{-(-2\sqrt{6})\pm\sqrt{0}}{2\times3}$$

$$=\frac{2\sqrt{6}}{6}$$

Roots are:

$$x = \sqrt{2}/\sqrt{3}$$
 and $x = x = \sqrt{2}/\sqrt{3}$

Question 15:

$$2\sqrt{3} x^2 - 5x + \sqrt{3} = 0$$

Solution:

$$2\sqrt{3} x^2 - 5x + \sqrt{3} = 0$$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

$$a = 2\sqrt{3}, b = -5, c = \sqrt{3}$$



Find Discriminant:

$$D = b^2 - 4ac$$

$$= (-5)^2 - 4.2\sqrt{3}.\sqrt{3}$$

Roots of equation are real.

Find roots:

$$x=\frac{-b\pm\sqrt{D}}{2a}$$

$$= \frac{-(-5) \pm \sqrt{1}}{2 \times 2\sqrt{3}} = \frac{5 \pm 1}{4\sqrt{3}}$$

Roots are:

$$x = \sqrt{3/2}$$
 and $x = 1/\sqrt{3}$



Exercise 4D

Page No: 199

Question 1:

Find the nature of the roots of the following quadratic equations:

(i)
$$2x^2 - 8x + 5 = 0$$

(ii)
$$3x^2 - 2\sqrt{6}x + 2 = 0$$

(iii)
$$5x^2 - 4x + 1 = 0$$

(iv)
$$5x(x-2)+6=0$$

(v)
$$12x^2 - 4\sqrt{15}x + 5 = 0$$

(vi)
$$x^2 - x + 2 = 0$$

Solution:

$$(i)2x^2 - 8x + 5 = 0$$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

$$a = 2, b = -8, c = 5$$

Using Discriminant Formula:

$$D = b^2 - 4ac$$

$$= (-8)^2 - 4.2.5$$

$$= 64 - 40$$

$$= 24 > 0$$

Hence the roots of equation are real and unequal.

(ii)
$$3x^2 - 2\sqrt{6}x + 2 = 0$$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

$$a = 3$$
, $b = -2\sqrt{6}$, $c = 2$



Using Discriminant Formula:

$$D = b^2 - 4ac$$

= $(-2\sqrt{6})^2 - 4.3.2$
= $24 - 24$

$$-24$$

Roots of equation are real and equal.

(iii)
$$5x^2 - 4x + 1 = 0$$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

$$a = 5$$
, $b = -4$, $c = 1$

Discriminant:

$$D = b^2 - 4ac$$

$$= (-4)^2 - 4.5.1$$

$$= 16 - 20$$

$$= -4 < 0$$

Equation has no real roots.

(iv)
$$5x(x-2)+6=0$$

$$5x^2 - 10x + 6 = 0$$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

$$a = 5$$
, $b = -10$, $c = 6$

Discriminant:

$$D = b^2 - 4ac$$

$$=(-10)2-4.5.6$$

$$= 100 - 120$$



$$= -20 < 0$$

Equation has no real roots.

(v)
$$12x^2 - 4\sqrt{15}x + 5 = 0$$

$$12x^2 - 4\sqrt{15} x + 5 = 0$$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

$$a = 12$$
, $b = -4\sqrt{15}$, $c = 5$

Discriminant:

$$D = b^2 - 4ac$$

= $(-4\sqrt{15})^2 - 4.12.5$

$$= 0$$

Equation has real and equal roots.

(vi)
$$x^2 - x + 2 = 0$$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

$$a = 1$$
, $b = -1$, $c = 2$

Discriminant:

$$D = b^2 - 4ac$$

$$= (-1)^2 - 4.1.2$$

$$= 1 - 8$$

$$= -7 < 0$$

Equation has no real roots.



Question 2:

If a and b are distinct real numbers, show that the quadratic equation $2(a^2 + b^2) x^2 + 2(a + b) x + 1 = 0$ has no real roots:

Solution:

$$2(a^2 + b^2) x^2 + 2 (a + b) x + 1 = 0$$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

$$a = 2 (a^2 + b^2), b = 2(a + b), c = 1$$

Discriminant:

$$D = b^2 - 4ac$$

$$=[2(a + b)]^2 - 4.2(a^2 + b^2).1$$

$$= 4a^2 + 4b^2 + 8ab - 8a^2 - 8b^2$$

$$= -4a^2 - 4b^2 + 8ab$$

$$= -4(a^2 + b^2 - 2ab)$$

$$= -4(a - b)^2$$

< 0

Hence the equation has no real roots.

Question 3:

Show that the roots of the equation $x^2 + px - q^2 = 0$ are real for all real values of p and q

Solution:

$$x^2 + px - q^2 = 0$$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

$$a = 1$$
, $b = p$, $c = -q^2$

Using discriminant formula:

$$D = b^2 - 4ac$$



=
$$(p)^2 - 4 \times 1 \times (-q^2)$$

= $p^2 + 4 q^2$
> 0

Hence roots are real for all real values of p and q.

Question 4:

For what values of k are the roots of the quadratic equation $3x^2 + 2kx + 27 = 0$ real and equal?

Solution:

$$3x^2 + 2kx + 27 = 0$$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

$$a = 3$$
, $b = 2k$, $c = 27$

Find discriminant:

$$D = b^{2} - 4ac$$

$$= (2k)^{2} - 4 \times 3 \times 27$$

$$= (2k)^{2} - 324$$
>0

Roots are real and equal

Find the value of k:

$$(2k)^2 - 324 = 0$$

 $(2k)^2 - (18)^2 = 0$
 $(k)^2 - (9)^2 = 0$
 $(k+9)(k-9) = 0$

Either
$$k + 9 = 0$$
 or $k - 9 = 0$

$$k = -9 \text{ or } k = 9$$



Hence, the value of k is k = 9 or -9

Question 5:

For what values of k are the roots of the quadratic equation kx $(x - 2\sqrt{5}) + 10 = 0$ real and equal?

Solution:

$$kx (x - 2\sqrt{5}) x + 10 = 0$$

$$kx^2 - 2\sqrt{5} kx + 10 = 0$$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

$$a = k, b = -2\sqrt{5} k, c = 10$$

Find discriminant:

$$D = b^2 - 4ac$$

$$= (-2 \text{ k})^2 - 4 \text{ x k x } 10 = 20 \text{k}^2 - 40 \text{k}$$

Since roots are real and equal (given), put D = 0

$$20k^2 - 40k = 0$$

$$k^2 - 2k = 0$$

$$k(k-2) = 0$$

Either,
$$k = 0$$
 or $k - 2 = 0$

Hence k = 0 or k = 2

Question 6:

For what values of p are the roots of the equation $4x^2 + px + 3 = 0$ real-and equal? Solution:

$$4x^2 + px + 3 = 0$$



Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

$$a = 4, b = p, c = 3$$

Find discriminant:

$$D = b^{2} - 4ac$$

$$= p^{2} - 4 \times 4 \times 3$$

$$= p^{2} - 48$$
Since roots are real and equal (given)
Put D = 0

Put D = 0

$$p^2 - 48 = 0$$

 $p^2 = 48 = (\pm 4\sqrt{3})^2$
 $p = \pm 4\sqrt{3}$
Hence p= 4 $\sqrt{3}$ or p = -4 $\sqrt{3}$

Question 7:

Find the nonzero value of k for which the roots of the quadratic equation $9x^2 - 3kx + k - 0$ are real and equal.

Solution:

$$9x^2 - 3kx + k = 0$$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

Here a = 9, b = -3k, c = k

Find discriminant:

D =
$$b^2 - 4ac$$

= $(-3k)^2 - 4 \times 9 \times k$
= $9k^2 - 36k$
Since roots are real and equal (given)
Put D = 0
 $9k^2 - 36k = 0$
 $9k (k - 4) = 0$



Either, k = 0 or k - 4 = 0

As, value of k is non-zero:

So,
$$k = 4$$

Question 8:

- (i) Find the values of k for which the quadratic equation $(3k + 1) x^2 + 2 (k + 1) x + 1 = 0$ has real and equal roots
- (ii) Find the value of k for which the equation $x^2 + k(2x + k 1) + 2 = 0$ has real and equal roots

Solution:

(i)
$$(3k + 1) x^2 + 2(k + 1) x + 1 = 0$$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

$$a = (3k + 1), b = 2(k + 1), c = 1$$

Find discriminant:

D =
$$b^2 - 4$$
 ac
= $(2(k + 1))^2 - 4(3k + 1) \times 1$
= $4k^2 + 4 + 8k - 12k - 4$
= $4k(k - 1)$

Since roots are real and equal (given)

Put
$$D = 0$$

4k $(k - 1) = 0$

Either,
$$k = 0$$
 or $k - 1 = 0$

$$k = 0, k = 1$$



(ii)
$$x^2 + k(2x + k - 1) + 2 = 0$$

Simplify above equation:

$$x^2 + 2kx + (k^2 - k + 2) = 0$$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

Here,
$$a = 1$$
, $b = 2k$, $c = (k^2 - k + 2)$

Find Discriminant:

$$D = b^2 - 4ac$$

$$= (2k)^2 - 4 \times 1 \times (k^2 - k + 2)$$

$$=4k^2-4k^2+4k-8$$

$$= 4k - 8$$

Since roots are real and equal (given)

Put
$$D = 0$$

$$4k - 8 = 0$$

$$k = 2$$

Hence, the value of k is 2.

Question 9:

Find the values of p for which the quadratic equation $(2p + 1) x^2 - (7p + 2) x + (7p - 3) = 0$ has real and equal roots.

Solution:

$$(2p + 1) x^2 - (7p + 2) x + (7p - 3) = 0$$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

$$a = (2p + 1)$$
, $b = -(7p + 2)$ and $c = (7p - 3)$

Discriminant:

$$D = b^2 - 4ac$$



$$= (-(7p + 2))^2 - 4.(2p + 1).(7p - 3)$$

$$= (49p^2 + 28p + 4) - 4(14p^2 + p - 3)$$

$$=49p^2 + 28p + 4 - 56p^2 - 4p + 12$$

$$= -7p2 + 24p + 16$$

Since roots are real and equal (given)

Put D = 0

$$7p^2 - 24p - 16 = 0$$

$$7p^2 - 28p + 4p - 16 = 0$$

$$7p(p-4) + 4(p-4) = 0$$

$$(7p + 4)(p - 4) = 0$$

Either
$$(7p + 4) = 0$$
 or $(p - 4) = 0$

$$p = -4/7$$
 or $p = 4$

Question 10:

Find the values of p for which the quadratic equation $(p + 1) x^2 - 6(p + 1) x + 3(p + 9) = 0$, $p \ne -1$ has equal roots: Hence, find the roots of the equation.

Solution:

The given quadratic equation is

$$(p+1) x^2 - 6(p+1) x + 3(p+9) = 0, p \ne -1$$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

$$a = (p + 1), b = -6(p + 1)$$
and $c = 3(p + 9)$



Discriminant:

$$D = b^2 - 4ac$$

$$= (-6(p+1))^2 - 4.(p+1).3(p+9)$$

$$= 36(p+1)(p+1) - 12(p+1)(p+9)$$

$$= 12(p+1)(3p+3-p-9)$$

$$= 12(p+1)(2p-6)$$

Since roots are real and equal (given)

Put
$$D = 0$$

$$12(p+1)(2p-6)=0$$

either
$$(p + 1) = 0$$
 or $(2p - 6) = 0$

$$p = -1 \text{ or } p = 3$$

Question 11:

If -5 is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, find the value of k.

Solution:

Given: -5 is a root of the quadratic equation $2x^2 + px - 15 = 0$

Substitute the value of x = -5

$$2(-5)^2 + p(-5) - 15 = 0$$

$$50 - 5p - 15 = 0$$

$$35 - 5p = 0$$

$$p = 7$$

Again,

In quadratic equation $p(x^2 + x) + k = 0$

$$7(x^2 + x) + k = 0$$
 (put value of p = 7)



$$7x^2 + 7x + k = 0$$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

$$a = 7, b = 7, c = k$$

Find Discriminant:

$$D = b^{2} - 4ac$$

$$= (7)^{2} - 4 \times 7 \times k$$

$$= 49 - 28k$$

Since roots are real and equal, put D = 0

$$49 - 28k = 0$$

 $28k = 49$

$$k = 7 / 4$$

The value of k is 7/4

Question 12:

If 3 is a root of the quadratic equation $x^2 - x + k - 0$, find the value of p so that the roots of the equation $x^2 + k(2x + k + 2) + p = 0$ are equal.

Solution:

Given: 3 is a root of equation $x^2 - x + k = 0$

Substitute the value of x = 3

$$(3)^2 - (3) + k = 0$$

$$9 - 3 + k = 0$$

$$k = -6$$

Now,
$$x^2 + k(2x + k + 2) + p = 0$$

$$x^2 + (-6)(2x - 6 + 2) + p = 0$$

$$x^2 - 12x + 36 - 12 + p = 0$$

$$x^2 - 12x + (24 + p) = 0$$



Compare given equation with the general form of quadratic equation, which is $ax^2 + bx$

$$+ c = 0$$

$$a = 1, b = -12, c = 24 + p$$

Find Discriminant:

$$D = b^2 - 4ac$$

$$= (-12)^2 - 4 \times 1 \times (24 + p)$$

$$= 144 - 96 - 4p = 48 - 4p$$

Since roots are real and equal, put D = 0

$$48 - 4p = 0$$

$$4p = 48$$

$$p = 12$$

The value of p is 12.

Question 13:

If -4 is a root of the equation $x^2 + 2x + 4p = 0$, find the value of k for which the quadratic equation $x^2 + px(1 + 3k) + 7(3 + 2k) = 0$ has equal roots.

Solution:

Given: -4 is a root of the equation $x^2 + 2x + 4p = 0$

Substitute the value of x = -4

$$(-4)^2 + 2(-4) + 4p = 0$$

$$16 - 8 + 4p = 0$$

$$8 + 4p = 0$$

$$4p = -8$$

or
$$p = -2$$

In the quadratic equation $x^2 + px (1 + 3k) + 7(3 + 2k) = 0$

$$x^2 - 2x (1 + 3k) + 7(3 + 2k) = 0$$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$



$$a = 1$$
, $b = -2(1 + 3k)$, $c = 7(3 + 2k)$

Find Discriminant:

$$D = b^2 - 4ac$$

$$= (-2(1+3k))^2 - 4 \times 1 \times 7(3+2k)$$

$$=4(1+9k^2+6k)-28(3+2k)$$

$$=36k^2-32k-80$$

Since roots are real and equal, put D = 0

$$36k^2 - 32k - 80 = 0$$

$$9k^2 - 8k - 20 = 0$$

$$9k^2 - 18k + 10k - 20 = 0$$

$$9k(k-2) + 10(k-2) = 0$$

$$(k-2)(9k+10)=0$$

Either,
$$k - 2 = 0$$
 or $9k + 10 = 0$

$$k = 2 \text{ or } k = -10/9$$

Question 14:

If the quadratic equation $(1 + m^2) x^2 + 2mcx + c^2 - a^2 = 0$ has equal roots, prove that $c^2 = a^2 (1 + m^2)$.

Solution:

$$(1 + m^2) x^2 + 2mcx + c^2 - a^2 = 0$$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

$$a = (1 + m^2)$$
, $b = 2mc$ and $c = c^2 - a^2$

Since roots are equal, so D = 0

$$(2mc)^2 - 4.(1 + m^2)(c^2 - a^2) = 0$$

$$4 \text{ m}^2\text{c}^2 - 4\text{c}^2 + 4\text{a}^2 - 4 \text{ m}^2\text{c}^2 + 4 \text{ m}^2\text{a}^2 = 0$$



$$a^2 + m^2 a^2 = c^2$$

or
$$c^2 = a^2 (1 + m^2)$$

Hence Proved

Question 15:

If the roots of the equation $(c^2 - ab) x^2 - 2(a^2 - bc)x + (b^2 - ac) = 0$ are real and equal, show that either a = 0 or $(a^3 + b^3 + c^3) = 3abc$

Solution:

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

$$a = (c^2 - ab) b = -2(a^2 - bc) c = (b^2 - ac)$$

Since roots are equal, so D = 0

$$(-2(a^2-bc))^2-4(c^2-ab)(b^2-ac)=0$$

$$4(a^4 - 2a^2bc + b^2c^2) - 4(b^2c^2 - ac^3 - ab3 + a^2bc) = 0$$

$$a^4 - 3a^2bc + ac^3 + ab^3 = 0$$

$$a (a^3 - 3abc + c^3 + b^3) = 0$$

either a = 0 or
$$(a^3 - 3abc + c^3 + b^3) = 0$$

$$a = 0 \text{ or } a^3 + c^3 + b^3 = 3abc$$

Hence Proved.

Question 16:

Find the values of p for which the quadratic equation $2x^2 + px + 8 = 0$ has real roots. Solution:



$2x^2 + px + 8 = 0$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx$

$$+ c = 0$$

$$a = 2, b = p, c = 8$$

Find D:

$$D = b^2 - 4ac$$

$$= p^2 - 4 \times 2 \times 8$$

$$= p^2 - 64$$

Since roots are real, so $D \ge 0$

$$p^2 - 64 \ge 0$$

$$p^2 \ge 64$$

$$\geq (\pm 8)^2$$

Either $p \ge 8$ or $p \le -8$

Question 17:

Find the value of a for which the equation $(\alpha - 12) x^2 + 2(\alpha - 12) x + 2 = 0$ has equal roots.

Solution:

$$(\alpha - 12) x^2 + 2(\alpha - 12) x + 2 = 0$$

Roots of given equation are equal (given)

So,
$$D = 0$$

$$4(\alpha - 12)(\alpha - 14) = 0$$

$$\alpha - 14 = 0 \{(\alpha - 12) \neq 0\}$$

$$\alpha = 14$$

Hence the value of α is 14



Question 18:

Find the value of k for which the roots of $9x^2 + 8kx + 16 = 0$ are real and equal. Solution:

$$9x^2 + 8kx + 16 = 0$$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

$$a = 9$$
, $b = 8k$, $c = 16$

Find D:

$$D = b^2 - 4ac$$

$$= (8k)^2 - 4 \times 9 \times 16$$
$$= 64k^2 - 576$$

Roots of given equation are equal (given)

So,
$$D = 0$$

$$64k^2 - 576 = 0$$

$$64k^2 = 576$$

$$k^2 = 9$$

$$k = \pm 3$$

Answer: k = 3, k = -3

Question 19:

Find the values of k for which the given quadratic equation has real and distinct roots.

(i)
$$kx^2 + 6x + 1 = 0$$

(ii)
$$x^2 - kx + 9 = 0$$

(iii)
$$9x^2 + 3kx + 4 = 0$$

(iv)
$$5x^2 - kx + 1 = 0$$

Solution:

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

(i)
$$a = k, b = 6, c = 1$$



For real and distinct roots, then D > 0

$$6^2 - 4k > 0$$

$$36 - 4k > 0$$

k < 9

(ii)

$$a = 1, b = -k, c = 9$$

For real and distinct roots, then D > 0

$$(-k)^2 - 36 > 0$$

$$k > 6$$
 or $k < -6$

$$a = 9$$
, $b = 3k$, $c = 4$

For real and distinct roots, then D > 0

$$(3k)^2 - 144 > 0$$

$$9k^2 > 144$$

$$k^2 > 16$$

$$k > 4$$
 or $k < -4$

$$a = 5, b = -k, c = 1$$

For real and distinct roots, then D > 0



$$k^2 - 20 > 0$$

$$k > 2\sqrt{5}$$
 or $k < -2\sqrt{5}$

Question 20:

If a and b are real and a \neq b then show that the roots of the equation (a - b) x^2 + 5(a + b) x - 2 (a - b) = 0 are real and unequal.

Solution:

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

$$a = (a - b), b = 5(a + b), c = -2(a - b)$$

Find Discriminant:

$$D = b^2 - 4ac$$

$$= (5(a + b))^2 - 4(a - b)(-2(a - b))$$

$$= 25(a + b)^2 + 8(a - b)^2$$

$$= 17(a + b)^2 + \{8(a + b)^2 + 8(a - b)^2\}$$

$$= 17(a + b)^2 + 16(a^2 + b^2)$$

Which is always greater than zero.

Equation has real and unequal roots.