RS Aggarwal Solutions for Class Chapter 13 Trigonometric Identities

Exercise 13A

Prove each of the following identities:
Question 1:
(i) (1 – cos^2θ) cosec^2θ = 1
(ii) (1 + cot^2θ) sin^2θ = 1

Solution:
(i) (1 – cos^2θ) cosec^2θ = 1

L.H.S. = (1 – cos^2θ) cosec^2θ

= (sin^2θ) × cosec^2θ

(Using identity sin^2θ + cos^2 θ = 1)

= 1/ cosec^2θ × cosec^2θ

= 1

= R.H.S.
Hence Proved.

(ii) (1 + cot^2θ) sin^2θ = 1

L.H.S. = (1 + cot^2θ) × sin^2 θ
= (cosec^2 θ) × sin^2 θ
(Using identity 1 + cot^2 θ = cosec^2 θ)

= 1/ sin^2θ × sin^2 θ

= 1

= R.H.S.
Hence Proved.

Question 2:
(i) (sec^2θ – 1) cot^2θ = 1
(ii) (sec^2θ – 1) (cosec^2θ – 1) = 1
(iii) (1– cos^2θ) sec^2θ = tan^2θ

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Solution:

(i) (sec^2θ – 1) cot^2θ = 1

L.H.S. = (sec^2 θ – 1) × cot^2 θ

= (tan^2θ) x cot^2θ
(using identity 1 + tan^2 θ = sec^2 θ)

= 1/cot^2θ x cot^2θ

= 1

= R.H.S.

Hence Proved.

(ii) (sec^2θ – 1) (cosec^2θ – 1) = 1
L.H.S. = (sec^2 θ – 1)(cosec^2 θ – 1)

= (tan^2θ) × cot^2θ

(using identity 1 + cot^2 θ = cosec^2 θ and 1 + tan^2 θ = sec^2 θ)

= tan^2θ x 1/tan^2θ

= 1

= R.H.S.

Hence Proved.

(iii) (1– cos^2θ) sec^2θ = tan^2θ

L.H.S. = (1 – cos^2 θ) sec^2 θ

= (sin^2θ) × (1/cos^2θ)

(using identity sin^2 θ = 1- cos^2 θ)

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= tan^2 θ

= R.H.S.

Hence Proved.

Question 3: Prove

(i) $\sin^2\theta + \frac{1}{1+\tan^2\theta} = 1$

(ii) $\frac{1}{1+\tan^2\theta} + \frac{1}{1+\cot^2\theta} = 1$

Solution:

(i)
L.H.S.

$$\sin^2\theta + \frac{1}{1+\tan^2\theta}$$

= (sin^2θ) + (1/sec^2 θ)

(using 1 + tan^2 θ = sec^2 θ)

= (sin^2θ) + (cos^2θ)

We know, sin^2 θ + cos^2 θ = 1

= 1

= R.H.S.

Hence proved

(ii)
LHS

$$\frac{1}{1+\tan^2\theta} + \frac{1}{1+\cot^2\theta}$$

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= (1/sec^2θ) + (1/cosec^2 θ)

(Using 1 + tan^2 θ = sec^2 θ and 1 + tan^2 θ = sec^2 θ)

= (cos^2 θ) + (sin^2 θ)

we know, sin^2 θ + cos^2 θ = 1

= 1

= R.H.S.

Hence proved

Question 4: Prove
(i) (1 + cos θ) (1 – cos θ) (1 + cot^2θ) = 1
(ii) cosec θ (1 + cos θ) (cosec θ – cot θ) = 1

Solution:

(i)(1 + cos θ) (1 – cos θ) (1 + cot^2θ) = 1

LHS: (1 + cos θ) (1 – cos θ) (1 + cot^2θ)

= (1 – cos^2 θ) × cosec^2 θ

(Using sin^2 θ + cos^2 θ = 1)

= (sin^2 θ) × cosec^2 θ

= sin^2 θ x 1/sin^2 θ

= 1
= R.H.S.
Hence Proved

(ii) cosec θ (1 + cos θ) (cosec θ – cot θ) = 1
L.H.S.
cosec θ (1 + cos θ) (cosec θ – cot θ)

= (cosec θ + cosec θ cos θ)(cosec θ – cot θ)

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We know, cosec θ = 1/sin θ and cot θ = cosθ/sinθ

= (cosec θ + cot θ)(cosec θ – cotθ)

Apply formula: (a + b)(a - b) = a^2 - b^2

= cosec^2 θ - cot^2 θ

= 1
= R.H.S.
Hence proved.

Question 5: Prove

(i) $\cot^2\theta - \dfrac{1}{\sin^2\theta} = -1$

(ii) $\tan^2\theta - \dfrac{1}{\cos^2\theta} = -1$

(iii) $\cos^2\theta + \dfrac{1}{1+\cot^2\theta} = 1$

Solution:

(i)
L.H.S.
= cot^2 θ – 1/sin^2θ

= cos^2θ/sin^2θ - 1/sin^2θ

= (cos^2θ - 1)/sin^2θ

=-sin^2θ/sin^2θ

= -1

= R.H.S

(ii)
L.H.S.
= tan^2 θ – 1/cos^2θ

= sin^2θ/cos^2θ - 1/cos^2θ

= (sin^2θ - 1)/cos^2θ

=-cos^2θ/cos^2θ

= -1

= R.H.S

(iii)
L.H.S.
= cos^2 θ + 1/(1+cot^2θ

= cos^2 θ + 1/cose^2θ

= cos^2 θ + sin^2θ

= 1

= R.H.S

Question 6: Prove

$$\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}=2\sec^2\theta$$

Solution:

L.H.S.

$$=\frac{1}{(1+\sin\theta)}+\frac{1}{(1-\sin\theta)}$$

$$=\frac{(1-\sin\theta)+(1+\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$$

$$=\frac{2}{1-\sin^2\theta}$$

$$=\frac{2}{\cos^2\theta}$$

$$=2\sec^2\theta$$

= R.H.S.

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Question 7: Prove
(i) sec θ (1 − sin θ) (sec θ + tan θ) = 1
(ii) sin θ(1 + tan θ) + cos θ(1 + cot θ) = (sec θ + cosec θ)

Solution:

(i) L.H.S.
= sec θ (1 − sin θ) (sec θ + tan θ)

$$= \left(\frac{1}{\cos\theta}\right) \times (1 - \sin\theta) \times \left(\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\right)$$

$$= \left(\frac{1}{\cos\theta}\right) \times (1 - \sin\theta) \times \left(\frac{1+\sin\theta}{\cos\theta}\right)$$

$$= \frac{1-\sin^2\theta}{\cos^2\theta}$$

= cos^2θ/cos^2θ

= 1

= R.H.S.

(ii) L.H.S. = sin θ (1 + tan θ) + cos θ (1+ cot θ)

= sin θ (1 + sin θ/cos θ) + cos θ (1+ cos θ/sinθ)

= sin θ{(cosθ +sinθ)/cos θ} + cos θ{(sinθ+cosθ)/sinθ}

=(cos θ + sin θ) (sinθ/cos θ + cos θ/sinθ)

= (cos θ + sin θ)/cos θ sin θ

= cosec θ + sec θ

= R.H.S.

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Question 8:

(i) $1+\frac{\cot^2\theta}{1+\operatorname{cosec}\theta}=\operatorname{cosec}\theta$

(ii) $1+\frac{\tan^2\theta}{1+\sec\theta}=\sec\theta$

Solution:

$\text{L.H.S.} = 1+\frac{\cot^2\theta}{1+\operatorname{cosec}\theta}$

$= 1+\frac{\frac{\cos^2\theta}{\sin^2\theta}}{1+\frac{1}{\sin\theta}}$

$= 1+\frac{\cos^2\theta}{1+\sin\theta}\times\frac{\sin\theta}{\sin^2\theta}$

$= 1+\frac{\cos^2\theta}{(1+\sin\theta)\sin\theta}$

$= \frac{\sin\theta+\sin^2\theta+\cos^2\theta}{\sin\theta+\sin^2\theta}$

$= \frac{\sin\theta+1}{\sin\theta(1+\sin\theta)}$

= 1/sin θ
= cosec θ
= R.H.S.

(ii)

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$$\text{L.H.S.} = 1 + \frac{\tan^2\theta}{1+\sec\theta}$$

$$= 1 + \frac{\frac{\sin^2\theta}{\cos^2\theta}}{1+\frac{1}{\cos\theta}}$$

$$= 1 + \frac{\sin^2\theta}{1+\cos\theta} \times \frac{\cos\theta}{\cos^2\theta}$$

$$= 1 + \frac{\sin^2\theta}{(1+\cos\theta)\cos\theta}$$

$$= \frac{\cos\theta + \cos^2\theta + \sin^2\theta}{\cos\theta + \cos^2\theta}$$

$$= \frac{\cos\theta+1}{\cos\theta(1+\cos\theta)}$$

= 1/cos θ
= sec θ
= R.H.S.

Question 9: Prove

$$\frac{(1+\tan^2\theta)\cot\theta}{\operatorname{cosec}^2\theta} = \tan\theta$$

Solution:

$$\text{L.H.S.} = \frac{(1+\tan^2\theta)\cot\theta}{\operatorname{cosec}^2\theta}$$

$$= \frac{\left(1+\frac{\sin^2\theta}{\cos^2\theta}\right)\times\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin^2\theta}}$$

$$= \frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta} \times \frac{\cos\theta}{\sin\theta} \times \sin^2\theta$$

$$= 1 \times \frac{\sin\theta}{\cos\theta}$$

$$= \tan\theta$$

= R.H.S.

Question 10: Prove

$$\frac{\tan^2\theta}{1+\tan^2\theta}+\frac{\cot^2\theta}{1+\cot^2\theta}=1$$

Solution:

$$\text{L.H.S.} = \frac{\tan^2\theta}{1+\tan^2\theta}+\frac{\cot^2\theta}{1+\cot^2\theta}$$

$$= \frac{\frac{\sin^2\theta}{\cos^2\theta}}{1+\frac{\sin^2\theta}{\cos^2\theta}}+\frac{\frac{\cos^2\theta}{\sin^2\theta}}{1+\frac{\cos^2\theta}{\sin^2\theta}}$$

$$= \frac{\sin^2\theta}{1+\sin^2\theta}+\frac{\cos^2\theta}{1+\cos^2\theta}$$

$$= \frac{\sin^2\theta+\sin^2\theta\cos^2\theta+\cos^2\theta+\cos^2\theta\sin^2\theta}{(1+\sin^2\theta)(1+\cos^2\theta)}$$

$$= \frac{\sin^2\theta+\sin^2\theta\cos^2\theta+\cos^2\theta+\cos^2\theta\sin^2\theta}{\sin^2\theta+\sin^2\theta\cos^2\theta+\cos^2\theta+\cos^2\theta\sin^2\theta}$$

= 1

= R.H.S.

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Exercise 13B   Page No: 594

Question 1: If a cos θ + b sin θ = m and a sin θ – b cos θ = n, prove that (m^2 + n^2) = (a^2 + b^2).

Solution:

a cos θ + b sin θ = m

Squaring equation, we get

a^2 cos^2 θ + b^2 sin^2 θ + 2ab cos θ sin θ = m^2 ……(1)

Again Square equation, a sin θ – b cos θ = n

a^2 sin^2 θ + b^2 cos^2 θ – 2ab cos θ sin θ = n^2 ……(2)

Add (1) and (2)

a^2 cos^2 θ + b^2 sin^2 θ + 2ab cos θ sin θ + a^2 sin^2 θ + b^2 cos^2 θ – 2ab cos θ sin θ = m^2 +n^2

a^2 (cos^2 θ + sin^2θ) + b^2 (cos^2 θ + sin^2 θ) = m^2 + n^2

(Using cos^2 θ + sin^2θ = 1)

a^2 + b^2 = m^2 + n^2

Hence Proved.

Question 2: If x = a sec θ + b tan θ and y = a tan θ + b sec θ, prove that (x^2 – y^2) = (a^2 – b^2).

Solution:

a sec θ + b tan θ = x

a tan θ + b sec θ = y

Squaring above equations:

a^2 sec^2 θ + b^2 tan^2 θ + 2ab sec θ tan θ = x^2 …..(1)

a^2 tan^2 θ + b^2 sec^2 θ + 2ab sec θ tan θ = y^2 ….(2)

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Subtract equation (2) from (1):

a^2 (sec^2 θ – tan^2 θ) + b^2 (tan^2 θ – sec^2 θ) = x^2 – y^2

(using sec^2 θ = 1 + tan^2 θ)

or a^2 – b^2 = x^2 – y^2

Hence proved.

Question 3:

If $\left(\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta\right) = 1$ and $\left(\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta\right) = 1$,

prove that $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$

Solution:
x/a sin θ – y/b cos θ = 1
x/a cos θ + y/b sin θ = 1

Squaring both the equations, we have
x^2/a^2 sin^2 θ + y^2/b^2 cos^2 θ – 2 cos θ sin θ = 1 ....(1)

x^2/a^2 cos2 θ + y^2/b^2 sin^2 θ + 2 cos θ sin θ = 1 ......(2)

Add (1) and (2), we get

x^2/a^2(sin^2 θ + cos^2 θ) + y^2/b^2 (sin^2 θ + cos^2 θ) = 1+1

(Using cos^2 θ + sin^2θ = 1)

x^2/a^2 + y^2/b^2 = 2

Question 4: If (sec θ + tan θ) = m and (sec θ − tan θ) = n, show that mn = 1.

Solution:
(sec θ + tan θ) = m ...(1) and

(sec θ − tan θ) = n ....(2)

Multiply (1) and (2), we have

(sec θ + tan θ) (sec θ – tan θ) = mn

(sec^2 θ – tan^2 θ) = mn

(Because sec^2 θ – tan^2 θ=1)
1 = mn
Or mn = 1
Hence Proved

Question 5: If (cosec θ + cot θ) = m and (cosec θ − cot θ) = n, show that mn = 1.

Solution:
(cosec θ + cot θ) = m ...(1) and
(cosec θ − cot θ) = n ...(2)

Multiply (1) and (2)

(cosec^2 θ - cot^2 θ) = mn

(Because cosec^2 θ - cot^2 θ = 1)
1 = mn
Or mn = 1
Hence Proved

Question 6: If x = a cos^3 θ and y = b sin^3 θ, prove that

$$\left(\frac{x}{a}\right)^{2/3}+\left(\frac{y}{b}\right)^{2/3}=1$$

Solution:

x = a cos^3 θ

y = b sin^3 θ

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L.H.S.

$$\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3}$$

$$= \left(\frac{a\cos^3\theta}{a}\right)^{2/3} + \left(\frac{b\sin^3\theta}{b}\right)^{2/3}$$

$$= (\cos^3\theta)^{2/3} + (\sin^3\theta)^{2/3}$$

= cos^2 θ + sin^2 θ
= 1
= R.H.S.

Question 7: If (tan θ + sin θ) = m and (tan θ − sin θ) = n, prove that (m^2 − n^2)^2 = 16mn.

Solution:

(tan θ + sin θ) = m and (tan θ − sin θ) = n

To Prove: (m^2 − n^2)^2 = 16mn

L.H.S. = (m^2 − n^2)^2

= [(tan θ + sin θ)^2 - (tan θ − sin θ)^2]^2

= (4tan θ sin θ)^2

= 16 tan^2 θ sin^2 θ ...(1)

R.H.S. = 16mn

= 16(tan θ + sin θ)(tan θ − sin θ)

= 16(tan^2 θ − sin^2 θ)

= 16 [{sin^2 θ (1-cos^2 θ)/cos^2θ]

= 16 x sin^2 θ/cos^2θ x (1-cos^2 θ)

= 16 tan^2 θ sin^2 θ ...(2)
From (1) and (2)
L.H.S. = R.H.S.

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Question 8: If (cot θ + tan θ) = m and (sec θ – cos θ) = n, prove that (m^2n)^(2/3) – (mn^2)^(2/3) = 1.

Solution:

(cot θ + tan θ) = m and (sec θ – cos θ) = n

m = 1/tanθ + tan θ = (1+ tan^2 θ)/ tan θ = sec^2 θ / tan θ
= 1/sinθcosθ

or m = 1/sinθcosθ

Again, n = sec θ – cos θ
= 1/cosθ – cos θ
= (1 – cos^2 θ)/cosθ

= sin^2 θ/cos θ

or n = sin^2 θ/cos θ

To prove: (m^2n)^(2/3) – (mn^2)^(2/3) = 1

L.H.S.

(m^2n)^(2/3) – (mn^2)^(2/3)

Substituting the values of m and n, we have

$$= \left[\left(\frac{1}{\sin\theta\cos\theta}\right)^2 \times \frac{\sin^2\theta}{\cos\theta}\right]^{2/3} - \left[\left(\frac{1}{\sin\theta\cos\theta}\right) \times \left(\frac{\sin^2\theta}{\cos\theta}\right)^2\right]^{2/3}$$

$$= \left[\frac{\sin^2\theta}{\sin^2\theta\cos^3\theta}\right]^{2/3} - \left[\frac{\sin^4\theta}{\sin\theta\cos^3\theta}\right]^{2/3}$$

$$= \left[\frac{1}{\cos^3\theta}\right]^{2/3} - \left[\frac{\sin^3\theta}{\cos^3\theta}\right]^{2/3}$$

$$= \frac{1}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}$$

= (1 – sin^2 θ)cos^2 θ

(We know, 1 – sin^2 θ = cos^2 θ)

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= cos^2 θ/ co^2 θ

= 1

=R.H.S.

Hence proved.

Question 9: If (cosec θ – sin θ) = a^3 and (sec θ – cos θ) = b^3, prove that a^2b^2(a^2 + b^2) = 1.

Solution:
(cosec θ – sin θ) = a^3 and (sec θ – cos θ) = b^3

(cosec θ – sin θ) = a^3

(1/sinθ – sin θ) = a^3

cos^2θ/sinθ = a^3

And a^2 = (a^3)^(2/3) = (cos^2θ/sinθ)^(2/3) .....(1)

Again

(sec θ – cos θ) = b^3

(1/cosθ – cos θ) = b^3

= sin^2 θ/cosθ = b^3

And, b^2 = (b^3)^(2/3) = (sin^2 θ/cosθ)^(2/3)

To Prove:a^2b^2(a^2 + b^2) = 1

L.H.S.

a^2b^2(a^2 + b^2)

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$$= \left[\left(\frac{\cos^2\theta}{\sin\theta}\right)\right]^{2/3} \times \left[\left(\frac{\sin^2\theta}{\cos\theta}\right)\right]^{\frac{2}{3}} \times \left(\left[\left(\frac{\cos^2\theta}{\sin\theta}\right)\right]^{\frac{2}{3}} + \left[\left(\frac{\sin^2\theta}{\cos\theta}\right)\right]^{\frac{2}{3}}\right)$$

$$= \left(\frac{\cos^2\theta\sin^2\theta}{\cos\theta\sin\theta}\right)^{2/3} \times \left(\left[\left(\frac{\cos^2\theta}{\sin\theta}\right)\right]^{\frac{2}{3}} + \left[\left(\frac{\sin^2\theta}{\cos\theta}\right)\right]^{\frac{2}{3}}\right)$$

$$= [\cos^3\theta]^{2/3} + [\sin^3\theta]^{2/3}$$

= sin^2 θ + cos^2 θ
= 1
=R.H.S.

Hence proved.

Question 10: If (2 sin θ + 3 cos θ) = 2, show that (3 sin θ − 2 cos θ) = ± 3.

Solution:

(2 sin θ + 3 cos θ) = 2 ...(1)

(2 sin θ + 3 cos θ)^2 + (3 sin θ – 2 cos θ)^2
= 4sin^2 θ + 9 cos^2 θ + 12sin θ cos θ + 9 sin^2 θ + 4 cos^2 θ – 12 sin θ cos θ

= 13sin^2 θ + 13 cos^2 θ

= 13(sin^2 θ + cos^2 θ)

= 13

(Because (sin^2 θ + cos^2 θ) = 1)

=> (2 sin θ + 3 cos θ)^2 + (3 sin θ – 2 cos θ)^2 = 13

Using equation (1)

=> (2)^2 + (3 sin θ – 2 cos θ)^2 = 13
=> (3 sin θ – 2 cos θ)^2 = 9
or (3 sin θ – 2 cos θ) = ± 3

Hence Proved.

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Exercise 13C

Question 1: Write the value of (1-sin^2 θ)sec^2 θ.

Solution:
(1 – sin^2θ) sec^2 θ = (cos^2 θ) × 1/cos^2 θ

= 1

Question 2: Write the value of (1-cos^2θ)cosec^2θ.

Solution:
(1-cos^2θ)cosec^2θ

= sin^2θ x 1/sin^2θ

= 1

Question 3: Write the value of (1+tan^2θ)cos^2θ.

Solution:

(1+tan^2θ)cos^2θ = sec^2 θ x 1/sec^2 θ

= 1

Question 4: Write the value of (1+cot^2θ)sin^2θ.

Solution:

(1+cot^2θ)sin^2θ = cose^2θ x 1/cose^2θ

= 1

Question 5: Write the value of sin^2θ + 1/(1+tan^2θ)

Solution:

sin^2θ + 1/(1+tan^2θ)

= sin^2θ + 1/(sec^2θ)

= sin^2θ + cos^2θ

= 1

Question 6: Write the value of (cot^2θ - 1/sin^2θ)

Solution:
(cot^2θ - 1/sin^2θ) = (cos^2θ/sin^2θ - 1/sin^2θ)

= (cos^2θ - 1)/sin^2θ

= -sin^2θ/sin^2θ

= -1

Question 7: Write the value of sinθ cos(90°-θ)+cosθ sin(90°-θ).

Solution:
sinθ cos(90°-θ)+cosθ sin(90°-θ) = sinθ x sinθ + cosθ x cosθ

= sin^2θ + cos^2θ

= 1

Question 8: Write the value of cosec^2(90°-θ) – tan^2θ.

Solution:
cosec^2(90°-θ) – tan^2θ = sec^2θ - tan^2θ

= 1

Question 9: Write the value of sec^2θ(1+sinθ)(1-sinθ).

Solution:
sec^2θ(1+sinθ)(1-sinθ) = sec^2θ(1-sin^2 θ)

= sec^2θ x cos^2θ

= 1/cos^2θ x cos^2θ

= 1

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Question 10: Write the value of cosec^2θ(1+cosθ)(1-cosθ).

Solution:

cosec^2θ(1+cosθ)(1-cosθ) = cosec^2θ (1-cos^2 θ)

= cosec^2θ x sin^2θ

= cosec^2θ x 1/cosec^2θ

= 1

Question 11: Write the value of sin^2θ cos^2θ(1+tan^2θ)(1+cot^2θ).

Solution:

sin^2θ cos^2θ(1+tan^2θ)(1+cot^2θ) = sin^2θ x cos^2θ x sec^2θ x cosec^2θ

= sin^2θ x cos^2θ x 1/cos^2θ x 1/sin^2θ

=1

Question 12: Write the value of (1+tan^2θ)(1+sinθ)(1-sinθ).

Solution:

(1+tan^2θ)(1+sinθ)(1-sinθ) = sec^2θ(1-sin^2 θ)

= sec^2θ x cos^2θ

= 1/cos^2θ x cos^2θ

= 1

Question 13: Write the value of 3cot^2θ – 3cosec^2θ.

Solution:

3cot^2θ – 3cosec^2θ = 3(cot^2θ – cosec^2θ)

= 3 x -1

= -3

Question 14: Write the value of 4tan^2θ - 4/cos^2θ

Solution:

4tan^2θ - 4/cos^2θ = 4 x sin^2θ/cos^2θ - 4/cos^2θ

= (4(sin^2θ - 1))/cos^2θ

= 4 (-cos^2θ) / cos^2θ

= -4

Question 15: Write the value of (tan^2θ - sec^2θ) / (cot^2θ - cose^2θ)

Solution:

(tan^2θ - sec^2θ) / (cot^2θ - cose^2θ) = -1/-1

= 1

(Using 1 + cot^2θ = cose^2θ and 1 + tan^2θ = sec^2θ)