

RS Aggarwal Solutions for Class 10 Maths Chapter 12 Trigonometric Ratios of Complementary Angles

Exercise 12

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Question 1.

Without using trigonometric tables, evaluate:

(i) $\frac{\sin 16^\circ}{\cos 74^\circ}$

(ii) $\frac{\sec 11^\circ}{\operatorname{cosec} 79^\circ}$

(iii) $\frac{\tan 27^\circ}{\cot 63^\circ}$

(iv) $\frac{\cos 35^\circ}{\sin 55^\circ}$

(v) $\frac{\operatorname{cosec} 42^\circ}{\sec 48^\circ}$

(vi) $\frac{\cot 38^\circ}{\tan 52^\circ}$

Solution:

(i) $\frac{\sin 16^\circ}{\cos 74^\circ}$

$$\frac{\sin 16^\circ}{\cos 74^\circ} = \frac{\sin 16^\circ}{\cos(90-16)^\circ} = \frac{\sin 16^\circ}{\sin 16^\circ} = 1$$

$\cos(90-\theta) = \sin \theta$ (lies in 1st quadrant, angle are positive)

(ii) $\frac{\sec 11^\circ}{\operatorname{cosec} 79^\circ}$

$$\frac{\sec 11^\circ}{\operatorname{cosec} 79^\circ} = \frac{1/\cos 11^\circ}{1/\sin 79^\circ}$$

$$= \frac{\sin 79^\circ}{\cos 11^\circ} = \frac{\sin 79^\circ}{\cos(90-79)^\circ} = \frac{\sin 79^\circ}{\sin 79^\circ}$$

$$= 1$$

(iii) $\frac{\tan 27^\circ}{\cot 63^\circ}$

$$\frac{\tan 27^\circ}{\cot 63^\circ} = \frac{\tan 27^\circ}{\cot(90-27)^\circ} = \frac{\tan 27^\circ}{\tan 27^\circ} = 1$$

(iv) $\frac{\cos 35^\circ}{\sin 55^\circ}$

$$\frac{\cos 35^\circ}{\sin 55^\circ} = \frac{\cos 35^\circ}{\sin(90-35)^\circ}$$

$$= \cos 35^\circ / \cos 35^\circ = 1$$

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$$(v) \frac{\operatorname{cosec} 42^{\circ}}{\sec 48^{\circ}}$$

$$\frac{\operatorname{cosec} 42^{\circ}}{\sec 48^{\circ}} = \frac{1/\sin 42^{\circ}}{1/\cos 48^{\circ}}$$

$$= \frac{\cos 48^{\circ}}{\sin(90-48)^{\circ}}$$

$$= \frac{\cos 48^{\circ}}{\cos 48^{\circ}} = 1$$

$$(vi) \frac{\cot 38^{\circ}}{\tan 52^{\circ}}$$

$$\frac{\cot 38^{\circ}}{\tan 52^{\circ}} = \frac{\cot 38^{\circ}}{\tan(90-38)^{\circ}}$$

$$= \frac{\cot 38^{\circ}}{\cot 38^{\circ}} = 1$$

Question 2:

Without using trigonometric tables, prove that:

$$(i) \cos 81^{\circ} - \sin 9^{\circ} = 0$$

$$(ii) \tan 71^{\circ} - \cot 19^{\circ} = 0$$

$$(iii) \operatorname{cosec} 80^{\circ} - \sec 10^{\circ} = 0$$

$$(iv) \operatorname{cosec}^2 72^{\circ} - \tan^2 18^{\circ} = 1$$

$$(v) \cos^2 75^{\circ} + \cos^2 15^{\circ} = 1$$

$$(vi) \tan^2 66^{\circ} - \cot^2 24^{\circ} = 0$$

$$(vii) \sin^2 48^{\circ} + \sin^2 42^{\circ} = 1$$

$$(viii) \cos^2 57^{\circ} - \sin^2 33^{\circ} = 0$$

$$(ix) (\sin 65^{\circ} + \cos 25^{\circ})(\sin 65^{\circ} - \cos 25^{\circ}) = 0$$

Solution:

$$(i) \text{LHS} = \cos 81^{\circ} - \sin 9^{\circ}$$

$$= \cos(90^{\circ} - 9^{\circ}) - \sin 9^{\circ}$$

$$= \sin 9^{\circ} - \sin 9^{\circ}$$

$$= 0$$

$$= \text{RHS}$$

$$(ii) \text{LHS} = \tan 71^{\circ} - \cot 19^{\circ}$$

$$= \tan(90^{\circ} - 19^{\circ}) - \cot 19^{\circ}$$

$$= \cot 19^{\circ} - \cot 19^{\circ}$$

$$= 0$$

$$= \text{RHS}$$

$$(iii) \text{LHS} = \operatorname{cosec} 80^{\circ} - \sec 10^{\circ}$$

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$$= \operatorname{cosec}(90^\circ - 10^\circ) - \sec(10^\circ)$$

$$= \sec 10^\circ - \sec 10^\circ$$

$$= 0$$

$$= \text{RHS}$$

$$\text{(iv) } \operatorname{cosec}^2 72^\circ - \tan^2 18^\circ = 1$$

$$\text{LHS} = \operatorname{cosec}^2 72^\circ - \tan^2 18^\circ$$

$$= \operatorname{cosec}^2 72^\circ - \tan^2 (90 - 72)^\circ$$

$$= \operatorname{cosec}^2 72^\circ - \cot^2 72^\circ$$

$$= 1$$

$$\text{(v) } \cos^2 75^\circ + \cos^2 15^\circ = 1$$

$$\text{LHS} = \cos^2 75^\circ + \cos^2 15^\circ$$

$$= \cos^2 75^\circ + \cos^2 (90 - 75)^\circ$$

$$= \cos^2 75^\circ + \sin^2 75^\circ$$

$$= 1$$

$$= \text{RHS}$$

$$\text{(vi) } \tan^2 66^\circ - \cot^2 24^\circ = 0$$

$$\text{LHS} = \tan^2 66^\circ - \cot^2 24^\circ$$

$$= \tan^2 66^\circ - \cot^2 (90 - 66)^\circ$$

$$= \tan^2 66^\circ - \tan^2 66^\circ$$

$$= 0$$

$$= \text{RHS}$$

$$\text{(vii) } \sin^2 48^\circ + \sin^2 42^\circ = 1$$

$$\text{LHS} = \sin^2 48^\circ + \sin^2 42^\circ$$

$$= \sin^2 48^\circ + \sin^2 (90 - 48)^\circ$$

$$= \sin^2 48^\circ + \cos^2 48^\circ$$

$$= 1$$

$$= \text{RHS}$$

$$\text{(viii) } \cos^2 57^\circ - \sin^2 33^\circ = 0$$

$$\text{LHS} = \cos^2 57^\circ - \sin^2 33^\circ$$

$$= \cos^2 57^\circ - \sin^2 (90 - 57)^\circ$$

$$= \cos^2 57^\circ - \cos^2 57^\circ$$

$$= 0$$

$$= \text{RHS}$$

$$\text{(ix) } (\sin 65^\circ + \cos 25^\circ)(\sin 65^\circ - \cos 25^\circ) = 0$$

$$\text{LHS} = (\sin 65^\circ + \cos 25^\circ)(\sin 65^\circ - \cos 25^\circ)$$

$$= \sin^2 65^\circ - \cos^2 25^\circ$$

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$$\begin{aligned}
 &= \sin^2 65^\circ - \cos^2 (90 - 65)^\circ \\
 &= \sin^2 65^\circ - \sin^2 65^\circ \\
 &= 0 \\
 &= \text{RHS}
 \end{aligned}$$

Question 3.

Without using trigonometric tables, prove that:

- (i) $\sin 53^\circ \cos 37^\circ + \cos 53^\circ \sin 37^\circ = 1$
- (ii) $\cos 54^\circ \cos 36^\circ - \sin 54^\circ \sin 36^\circ = 0$
- (iii) $\sec 70^\circ \sin 20^\circ + \cos 20^\circ \operatorname{cosec} 70^\circ = 2$
- (iv) $\sin 35^\circ \sin 55^\circ - \cos 35^\circ \cos 55^\circ = 0$
- (v) $(\sin 72^\circ + \cos 18^\circ)(\sin 72^\circ - \cos 18^\circ) = 0$
- (vi) $\tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ = 1$

Solution:

$$\begin{aligned}
 \text{(i) LHS} &= \sin 53^\circ \cos 37^\circ + \cos 53^\circ \sin 37^\circ \\
 &= \sin 53^\circ \cos (90^\circ - 53^\circ) + \cos 53^\circ \sin (90^\circ - 53^\circ) \\
 &= \sin 53^\circ \times \sin 53^\circ + \cos 53^\circ \times \cos 53^\circ \\
 &= \sin^2 53^\circ + \cos^2 53^\circ \\
 &= 1 \\
 &= \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii) LHS} &= \cos 54^\circ \cos 36^\circ - \sin 54^\circ \sin 36^\circ \\
 &= \cos 54^\circ \cos 36^\circ - \sin (90^\circ - 36^\circ) \sin (90^\circ - 54^\circ) \\
 &= \cos 54^\circ \cos 36^\circ - \cos 36^\circ \cos 54^\circ \\
 &= 0 \\
 &= \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii) LHS} &= \sec 70^\circ \sin 20^\circ + \cos 20^\circ \operatorname{cosec} 70^\circ \\
 &= \sec (90^\circ - 20^\circ) \sin 20^\circ + \cos 20^\circ \operatorname{cosec} (90^\circ - 20^\circ) \\
 &= \operatorname{cosec} 20^\circ \sin 20^\circ + \cos 20^\circ \sec 20^\circ \\
 &= 1 + 1 \\
 &= 2 \\
 &= \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv) LHS} &= \sin 35^\circ \sin 55^\circ - \cos 35^\circ \cos 55^\circ \\
 &= \sin (90^\circ - 55^\circ) \sin (90^\circ - 35^\circ) - \cos 35^\circ \cos 55^\circ \\
 &= \cos 55^\circ \cos 35^\circ - \cos 35^\circ \cos 55^\circ \\
 &= 0 \\
 &= \text{RHS}
 \end{aligned}$$

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$$\begin{aligned}
 \text{(v) LHS} &= (\sin 72^\circ + \cos 18^\circ)(\sin 72^\circ - \cos 18^\circ) \\
 &= (\sin^2 72^\circ - \cos^2 18^\circ) \\
 &= (\sin^2 72^\circ - \cos^2 (90^\circ - 72^\circ)) \\
 &= \sin^2 72^\circ - \sin^2 72^\circ \\
 &= 0 \\
 &= \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 \text{(vi) LHS} &= \tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ \\
 &= \tan 48^\circ \tan 23^\circ \tan (90^\circ - 48^\circ) \tan (90^\circ - 23^\circ) \\
 &= \tan 48^\circ \tan 23^\circ \cot 48^\circ \cot 23^\circ \\
 &= 1 \times 1 \\
 &= 1 \\
 &= \text{RHS}
 \end{aligned}$$

Question 4.

Prove that:

$$\text{(i) } \frac{\sin 70^\circ}{\cos 20^\circ} + \frac{\operatorname{cosec} 20^\circ}{\sec 70^\circ} - 2 \cos 70^\circ \operatorname{cosec} 20^\circ = 0$$

$$\text{(ii) } \frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \operatorname{cosec} 31^\circ = 2$$

$$\text{(iii) } \frac{2 \sin 68^\circ}{\cos 22^\circ} - \frac{2 \cot 15^\circ}{5 \tan 75^\circ} - \frac{3 \tan 45^\circ \tan 20^\circ \tan 40^\circ \tan 50^\circ \tan 70^\circ}{5} = 1$$

$$\text{(iv) } \frac{\sin 18^\circ}{\cos 72^\circ} + \sqrt{3} (\tan 10^\circ \tan 30^\circ \tan 40^\circ \tan 50^\circ \tan 80^\circ) = 2$$

$$\text{(v) } \frac{7 \cos 55^\circ}{3 \sin 35^\circ} - \frac{4(\cos 70^\circ \operatorname{cosec} 20^\circ)}{3(\tan 5^\circ \tan 25^\circ \tan 45^\circ \tan 65^\circ \tan 85^\circ)} = 1$$

Solution:

(i)

$$\frac{\sin 70^\circ}{\cos 20^\circ} + \frac{\operatorname{cosec} 20^\circ}{\sec 70^\circ} - 2 \cos 70^\circ \operatorname{cosec} 20^\circ = 0$$

LHS =

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$$= \frac{\sin 70^\circ}{\cos 20^\circ} + \frac{\operatorname{cosec} 20^\circ}{\sec 70^\circ} - 2 \cos 70^\circ \operatorname{cosec} 20^\circ$$

$$= \frac{\sin 70^\circ}{\cos(90-70)^\circ} + \frac{\operatorname{cosec}(90-70)^\circ}{\sec 70^\circ} - 2 \cos 70^\circ \operatorname{cosec}(90-70)^\circ$$

$$= \frac{\sin 70^\circ}{\sin 70^\circ} + \frac{\sec 70^\circ}{\sec 70^\circ} - 2 \cos 70^\circ \sec 70^\circ$$

$$= 1 + 1 - 2$$

$$= 0$$

$$= \text{RHS}$$

Hence proved.

(ii)

$$\text{LHS} =$$

$$= \frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \operatorname{cosec} 31^\circ$$

$$= \frac{\cos 80^\circ}{\sin(90-80)^\circ} + \cos 59^\circ \operatorname{cosec}(90-59)^\circ$$

$$= \frac{\cos 80^\circ}{\cos 80^\circ} + \cos 59^\circ \sec 59^\circ$$

$$= 1 + 1$$

$$= 2$$

$$= \text{RHS}$$

Hence proved.

(iii)

$$\text{LHS} =$$

$$= \frac{2 \sin 68^\circ}{\cos 22^\circ} - \frac{2 \cot 15^\circ}{5 \tan 75^\circ} - \frac{3 \tan 45^\circ \tan 20^\circ \tan 40^\circ \tan 50^\circ \tan 70^\circ}{5}$$

$$=$$

$$= \frac{2 \sin 68^\circ}{\cos(90-68)^\circ} - \frac{2 \cot 15^\circ}{5 \tan(90-15)^\circ} - \frac{3 \tan 45^\circ \tan 20^\circ \tan 40^\circ \tan(90-50)^\circ \tan(90-20)^\circ}{5}$$

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$$= \frac{2 \sin 68^\circ}{\sin 68^\circ} - \frac{2 \cot 15^\circ}{5 \cot 15^\circ} - \frac{3 \tan 45^\circ \tan 20^\circ \tan 40^\circ \cot 40^\circ \cot 20^\circ}{5}$$

$$= 2 - (2/5) - 3/5$$

$$= (10 - 2 - 3)/5$$

$$= 1$$

$$= \text{RHS}$$

Hence proved.

(iv)

LHS =

$$\frac{\sin 18^\circ}{\cos 72^\circ} + \sqrt{3}(\tan 10^\circ \tan 30^\circ \tan 40^\circ \tan 50^\circ \tan 80^\circ)$$

$$= \frac{\sin(90^\circ - 72^\circ)}{\cos 72^\circ} + \sqrt{3} \left(\tan 10^\circ \times \frac{1}{\sqrt{3}} \tan 40^\circ \tan(90^\circ - 40^\circ) \tan(90^\circ - 10^\circ) \right)$$

$$= \frac{\cos 72^\circ}{\cos 72^\circ} + \tan 10^\circ \tan 40^\circ \cot 40^\circ \cot 10^\circ$$

$$= 1 + (\tan 10^\circ \cot 10^\circ)(\tan 40^\circ \cot 40^\circ)$$

$$= 1 + 1$$

$$= 2$$

$$= \text{RHS}$$

Hence proved.

(v)

LHS =

$$\frac{7 \cos 55^\circ}{3 \sin 35^\circ} - \frac{4(\cos 70^\circ \operatorname{cosec} 20^\circ)}{3(\tan 5^\circ \tan 25^\circ \tan 45^\circ \tan 65^\circ \tan 85^\circ)}$$

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$$= \frac{7 \cos 55^\circ}{3 \sin(90 - 55)^\circ} - \frac{4(\cos 70^\circ \operatorname{cosec}(90 - 70)^\circ)}{3(\tan 5^\circ \tan 25^\circ \tan 45^\circ \tan(90 - 25)^\circ \tan(90 - 5^\circ))}$$

$$= \frac{7 \cos 55^\circ}{3 \cos 55^\circ} - \frac{4(\cos 70^\circ \sec 70^\circ)}{3(\tan 5^\circ \tan 25^\circ \times 1 \times \cot 25^\circ \cot 5^\circ)}$$

$$= \frac{7 \cos 55^\circ}{3 \cos 55^\circ} - \frac{4(\cos 70^\circ \sec 70^\circ)}{3(\tan 5^\circ \cot 5^\circ \times 1 \times \tan 25^\circ \cot 25^\circ)}$$

$$(7/3) - (4/3)$$

$$= 3/3$$

$$= 1$$

$$= \text{RHS}$$

Hence proved.

Question 5.

Prove that:

(i)

$$\sin \theta \cos(90^\circ - \theta) + \sin(90^\circ - \theta) \cos \theta = 1$$

(ii)

$$\frac{\sin \theta}{\cos(90 - \theta)} + \frac{\cos \theta}{\sin(90 - \theta)} = 2$$

(iii)

$$\frac{\sin \theta \cos(90^\circ - \theta) \cos \theta}{\sin(90^\circ - \theta)} + \frac{\cos \theta \sin(90^\circ - \theta) \sin \theta}{\cos(90^\circ - \theta)} = 1$$

$$(iv) \frac{\cos(90^\circ - \theta) \sec(90^\circ - \theta) \tan \theta}{\operatorname{cosec}(90^\circ - \theta) \sin(90^\circ - \theta) \cot(90^\circ - \theta)} + \frac{\tan(90^\circ - \theta)}{\cot \theta} = 2$$

$$(v) \frac{\cos(90^\circ - \theta)}{1 + \sin(90^\circ - \theta)} + \frac{1 + \sin(90^\circ - \theta)}{\cos(90^\circ - \theta)} = 2 \operatorname{cosec} \theta$$

$$(vi) \frac{\sec(90^\circ - \theta) \operatorname{cosec} \theta - \tan(90^\circ - \theta) \cot \theta + \cos^2 25^\circ + \cos^2 65^\circ}{3 \tan 27^\circ \tan 63^\circ} = \frac{2}{3}$$

(vii)

$$\cot \theta \tan(90^\circ - \theta) - \sec(90^\circ - \theta) \operatorname{cosec} \theta + \sqrt{3} \tan 12^\circ \tan 60^\circ \tan 78^\circ = 2$$

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Solution:

(i)

$$\text{LHS} = \sin \theta \cos (90^\circ - \theta) + \sin (90^\circ - \theta) \cos \theta$$

$$= \sin \theta \sin \theta + \cos \theta \cos \theta$$

$$= \sin^2 \theta + \cos^2 \theta$$

$$= 1$$

$$= \text{R.H.S.}$$

Hence proved.

(ii)

$$\text{LHS} =$$

$$= \frac{\sin \theta}{\cos (90^\circ - \theta)} + \frac{\cos \theta}{\sin (90^\circ - \theta)}$$

$$= \frac{\sin \theta}{\sin \theta} + \frac{\cos \theta}{\cos \theta}$$

$$= 1 + 1$$

$$= 2$$

$$= \text{R.H.S.}$$

Hence proved.

(iii)

$$\text{LHS} =$$

$$= \frac{\sin \theta \cos (90^\circ - \theta) \cos \theta}{\sin (90^\circ - \theta)} + \frac{\cos \theta \sin (90^\circ - \theta) \sin \theta}{\cos (90^\circ - \theta)}$$

$$= \frac{\sin \theta \sin \theta \cos \theta}{\cos \theta} + \frac{\cos \theta \cos \theta \sin \theta}{\sin \theta}$$

$$= \frac{\sin^3 \theta \cos \theta + \cos^3 \theta \sin \theta}{\cos \theta \sin \theta}$$

$$= \frac{\cos \theta \sin \theta (\cos^2 \theta + \sin^2 \theta)}{\cos \theta \sin \theta}$$

$$= \frac{\cos \theta \sin \theta (1)}{\cos \theta \sin \theta}$$

$$= 1$$

$$= \text{RHS}$$

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(iv)

LHS:

$$= \frac{\cos(90^\circ - \theta) \sec(90^\circ - \theta) \tan \theta}{\operatorname{cosec}(90^\circ - \theta) \sin(90^\circ - \theta) \cot(90^\circ - \theta)} + \frac{\tan(90^\circ - \theta)}{\cot \theta}$$

$$= \frac{(\sin \theta \operatorname{cosec} \theta) \tan \theta}{(\sec \theta \cos \theta) \tan \theta} + \frac{\cot \theta}{\cot \theta}$$

$$= \frac{1 \times \tan \theta}{1 \times \tan \theta} + 1$$

$$= 1 + 1$$

$$= 2$$

$$= \text{RHS}$$

(v)

LHS:

$$= \frac{\cos(90^\circ - \theta)}{1 + \sin(90^\circ - \theta)} + \frac{1 + \sin(90^\circ - \theta)}{\cos(90^\circ - \theta)}$$

$$= \frac{\sin \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta} = \frac{\sin^2 \theta + (1 + \cos \theta)^2}{\sin \theta (1 + \cos \theta)}$$

$$= \frac{(\sin^2 \theta + \cos^2 \theta) + 1 + 2 \cos \theta}{\sin \theta (1 + \cos \theta)}$$

$$= \frac{1 + 1 + 2 \cos \theta}{\sin \theta (1 + \cos \theta)} = \frac{2(1 + \cos \theta)}{\sin \theta (1 + \cos \theta)}$$

$$= 2/\sin \theta$$

$$= 2 \operatorname{cosec} \theta$$

$$= \text{R.H.S.}$$

Hence, proved.

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(vi)

LHS=

$$\begin{aligned}
 &= \frac{\sec(90 - \theta) \operatorname{cosec} \theta - \tan(90 - \theta) \cot \theta + \cos^2 25^\circ + \cos^2 65^\circ}{3 \tan 27^\circ \tan 63^\circ} \\
 &= \frac{\operatorname{cosec} \theta \operatorname{cosec} \theta - \cot \theta \cot \theta + \cos^2 25^\circ + \cos^2 (90 - 25)^\circ}{3 \tan 27^\circ \tan (90 - 27)^\circ} \\
 &= \frac{(\operatorname{cosec}^2 \theta - \cot^2 \theta) + (\cos^2 25^\circ + \sin^2 25^\circ)}{3 \tan 27^\circ \cot 27^\circ}
 \end{aligned}$$

$$\begin{aligned}
 &= (1+1)/(3 \times 1) \\
 &= 2/3
 \end{aligned}$$

=RHS

Hence proved.

(vii)

$$\begin{aligned}
 \text{LHS} &= \cot \theta \tan (90^\circ - \theta) - \sec (90^\circ - \theta) \operatorname{cosec} \theta + \sqrt{3} \tan 12^\circ \tan 60^\circ \tan 78^\circ \\
 &= \cot \theta \cot \theta - \operatorname{cosec} \theta \operatorname{cosec} \theta + \sqrt{3} \tan 60^\circ \tan 12^\circ \tan 78^\circ \\
 &= \cot^2 \theta - \operatorname{cosec}^2 \theta + \sqrt{3} \tan 60^\circ \tan 12^\circ \tan (90 - 12)^\circ \\
 &= -(\operatorname{cosec}^2 \theta - \cot^2 \theta) + \sqrt{3} \tan 60^\circ \tan 12^\circ \cot 12^\circ \\
 &= -1 + \sqrt{3}(\sqrt{3} \times 1) \\
 &= -1 + 3 \\
 &= 2 \\
 &= \text{R.H.S.}
 \end{aligned}$$

Hence proved.

Question 6:**Prove that:**

(i) $\tan 5^\circ \tan 25^\circ \tan 30^\circ \tan 65^\circ \tan 85^\circ = 1/\sqrt{3}$

(ii) $\cot 12^\circ \cot 38^\circ \cot 52^\circ \cot 60^\circ \cot 78^\circ = 1/\sqrt{3}$

(iii) $\cos 15^\circ \cos 35^\circ \operatorname{cosec} 55^\circ \cos 60^\circ \operatorname{cosec} 75^\circ = 1/2$

(iv) $\cos 1^\circ \cos 2^\circ \cos 3^\circ \dots \cos 180^\circ = 0$

(v) $\left(\frac{\sin 49^\circ}{\cos 41^\circ}\right)^2 + \left(\frac{\cos 41^\circ}{\sin 49^\circ}\right)^2 = 2$

Solution:

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$$\begin{aligned}\text{L.H.S.} &= \tan 5^\circ \tan 25^\circ \tan 30^\circ \tan 65^\circ \tan 85^\circ \\&= \tan 5^\circ \tan 25^\circ \times \frac{1}{\sqrt{3}} \times \tan (90^\circ - 25^\circ) \tan (90^\circ - 5^\circ) \\&= \tan 5^\circ \tan 25^\circ \times \frac{1}{\sqrt{3}} \times \cot 25^\circ \cot 5^\circ \\&= (\tan 5^\circ \cot 5^\circ)(\tan 25^\circ \cot 25^\circ) \times \frac{1}{\sqrt{3}} \\&= 1 \times \frac{1}{\sqrt{3}} \\&= \frac{1}{\sqrt{3}} \\&= \text{R.H.S.}\end{aligned}$$

(ii)

$$\begin{aligned}\text{L.H.S.} &= \cot 12^\circ \cot 38^\circ \cot 52^\circ \cot 60^\circ \cot 78^\circ \\&= \cot 12^\circ \cot 38^\circ \cot (90^\circ - 38^\circ) \times \frac{1}{\sqrt{3}} \times \cot (90^\circ - 12^\circ) \\&= \cot 12^\circ \cot 38^\circ \tan 38^\circ \times \frac{1}{\sqrt{3}} \times \tan 12^\circ \\&= (\cot 12^\circ \tan 12^\circ)(\cot 38^\circ \tan 38^\circ) \times \frac{1}{\sqrt{3}} \\&= 1 \times \frac{1}{\sqrt{3}} \\&= \frac{1}{\sqrt{3}} \\&= \text{R.H.S.}\end{aligned}$$

(iii)

$$\begin{aligned}\text{L.H.S.} &= \cos 15^\circ \cos 35^\circ \operatorname{cosec} 55^\circ \cos 60^\circ \operatorname{cosec} 75^\circ \\&= \cos 15^\circ \cos 35^\circ \operatorname{cosec} (90^\circ - 35^\circ) \times \frac{1}{2} \operatorname{cosec} (90^\circ - 15^\circ) \\&= \cos 15^\circ \cos 35^\circ \sec 35^\circ \times \frac{1}{2} \sec 15^\circ \\&= \cos 15^\circ \cos 35^\circ \times \frac{1}{\cos 35^\circ} \times \frac{1}{2} \times \frac{1}{\cos 15^\circ} \\&= \frac{1}{2} \\&= \text{R.H.S.}\end{aligned}$$

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(iv)

$$\begin{aligned}
 \text{L.H.S.} &= \cos 1^\circ \cos 2^\circ \cos 3^\circ \dots \cos 180^\circ \\
 &= \cos 1^\circ \cos 2^\circ \cos 3^\circ \dots \cos 90^\circ \dots \cos 180^\circ \\
 &= \cos 1^\circ \cos 2^\circ \cos 3^\circ \dots \times 0 \times \dots \cos 180^\circ \\
 &= 0 \\
 &= \text{R.H.S.}
 \end{aligned}$$

(v)

$$\begin{aligned}
 \text{L.H.S.} &= \left(\frac{\sin 49^\circ}{\cos 41^\circ} \right)^2 + \left(\frac{\cos 41^\circ}{\sin 49^\circ} \right)^2 \\
 &= \left(\frac{\sin(90^\circ - 41^\circ)}{\cos 41^\circ} \right)^2 + \left(\frac{\cos(90^\circ - 49^\circ)}{\sin 49^\circ} \right)^2 \\
 &= \left(\frac{\cos 41^\circ}{\cos 41^\circ} \right)^2 + \left(\frac{\sin 49^\circ}{\sin 49^\circ} \right)^2 \\
 &= 1^2 + 1^2 \\
 &= 1 + 1 \\
 &= 2 \\
 &= \text{R.H.S.}
 \end{aligned}$$

Question 7:**Prove that**

(i) $\sin(70^\circ + \theta) - \cos(20^\circ - \theta) = 0$

(ii) $\tan(55^\circ - \theta) - \cot(35^\circ + \theta) = 0$

(iii) $\operatorname{cosec}(67^\circ + \theta) - \sec(23^\circ - \theta) = 0$

(iv) $\operatorname{cosec}(65^\circ + \theta) \sec(25^\circ - \theta) - \tan(55^\circ - \theta) + \cot(35^\circ + \theta) = 0$

(v) $\sin(50^\circ + \theta) - \cos(40^\circ - \theta) + \tan 1^\circ \tan 10^\circ \tan 80^\circ \tan 89^\circ = 1.$

Solution:

L.H.S.

$$\begin{aligned}
 &= \sin(70^\circ + \theta) - \cos(20^\circ - \theta) \\
 &= \sin(70^\circ + \theta) - \cos[90^\circ - (70^\circ + \theta)] \\
 &= \sin(70^\circ + \theta) - \sin(70^\circ + \theta) \\
 &= 0 \\
 &= \text{R.H.S.}
 \end{aligned}$$

Hence Proved.

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(ii)

L.H.S.

$$\begin{aligned}
 &= \tan (55^\circ - \theta) - \cot (35^\circ + \theta) \\
 &= \tan (90^\circ - (35^\circ + \theta)) - \cot (35^\circ + \theta) \\
 &= \cot (35^\circ + \theta) - \cot (35^\circ + \theta) \\
 &= 0
 \end{aligned}$$

=RHS

Hence Proved.

(iii)

L.H.S.

$$\begin{aligned}
 &= \operatorname{cosec} (67^\circ + \theta) - \sec (23^\circ - \theta) \\
 &= \operatorname{cosec} (67^\circ + \theta) - \sec (90^\circ - (23^\circ + \theta)) \\
 &= \operatorname{cosec} (67^\circ + \theta) - \operatorname{cosec} (67^\circ + \theta) \\
 &= 0
 \end{aligned}$$

=RHS

Hence Proved.

(iv)

L.H.S.

$$\begin{aligned}
 &= \operatorname{cosec} (65^\circ + \theta) - \sec (25^\circ - \theta) - \tan (55^\circ - \theta) + \cot (35^\circ + \theta) \\
 &= \operatorname{cosec} (65^\circ + \theta) - \sec (90^\circ - (65^\circ + \theta)) - \tan (90^\circ - (35^\circ + \theta)) + \cot (35^\circ + \theta) \\
 &= \operatorname{cosec} (65^\circ + \theta) - \operatorname{cosec} (65^\circ + \theta) - \cot (35^\circ + \theta) + \cot (35^\circ + \theta) \\
 &= 0
 \end{aligned}$$

= R.H.S.

(v)

L.H.S.

$$\begin{aligned}
 &= \sin (50^\circ + \theta) - \cos (40^\circ - \theta) + \tan 1^\circ \tan 10^\circ \tan 80^\circ \tan 89^\circ \\
 &= \sin ((90^\circ - (40^\circ - \theta))) - \cos (40^\circ - \theta) + (\tan 1^\circ \tan 89^\circ)(\tan 10^\circ \tan 80^\circ) \\
 &= \cos (40^\circ - \theta) - \cos (40^\circ - \theta) + \{\tan 1^\circ \tan (90^\circ - 1^\circ)\}\{\tan 10^\circ \tan (90^\circ - 10^\circ)\} \\
 &= 0 + \{(\tan 1^\circ \cot 1^\circ)\}\{\tan 10^\circ \cot 10^\circ\}
 \end{aligned}$$

$$= 0 + 1 = 1$$

= R.H.S.

Question 8.

Express each of the following in terms of trigonometric ratios of angles lying between 0° and 45° .

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- (i) $\sin 67^\circ + \cos 75^\circ$
(ii) $\cot 65^\circ + \tan 49^\circ$
(iii) $\sec 78^\circ + \operatorname{cosec} 56^\circ$
(iv) $\operatorname{cosec} 54^\circ + \sin 72^\circ$

Solution:

(i)

$$\begin{aligned}\sin 67^\circ + \cos 75^\circ \\&= \sin(90^\circ - 23^\circ) + \cos(90^\circ - 15^\circ) \\&= \cos 23^\circ + \sin 15^\circ\end{aligned}$$

(ii)

$$\begin{aligned}\cot 65^\circ + \tan 49^\circ \\&= \cot(90^\circ - 25^\circ) + \tan(90^\circ - 41^\circ) \\&= \tan 25^\circ + \cot 41^\circ\end{aligned}$$

(iii)

$$\begin{aligned}\sec 78^\circ + \operatorname{cosec} 56^\circ \\&= \sec(90^\circ - 12^\circ) + \operatorname{cosec}(90^\circ - 34^\circ) \\&= \operatorname{cosec} 12^\circ + \sec 34^\circ\end{aligned}$$

(iv)

$$\begin{aligned}\operatorname{cosec} 54^\circ + \sin 72^\circ \\&= \operatorname{cosec}(90^\circ - 36^\circ) + \sin(90^\circ - 18^\circ) \\&= \sec 36^\circ + \cos 18^\circ\end{aligned}$$

Question 9.

If A, B and C are the angles of a ΔABC , prove that $\tan\left(\frac{C+A}{2}\right) = \cot\frac{B}{2}$.

Solution:

Given function is : $\tan\left(\frac{C+A}{2}\right) = \cot\frac{B}{2}$

Sum of all the angles of a triangle = 180 degree

So, $A + B + C = 180^\circ$

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$$\text{Or } A + C = 180^\circ - B$$

$$\text{And, } (A + C)/2 = (180^\circ - B)/2 = 90^\circ - B/2$$

$$\text{Now, } \tan (A + C)/2 = \tan(90^\circ - B/2) = \cot B/2$$

Hence Proved.

Question 10.

If $\cos 2\theta = \sin 4\theta$, where 2θ and 4θ are acute angles, then find the value of θ .

Solution:

$$\cos 2\theta = \sin 4\theta \quad \dots(1)$$

We know that,

$$\sin(90^\circ - \theta) = \cos \theta$$

So, equation (1) can be written as

$$\sin(90^\circ - 2\theta) = \sin 4\theta$$

On comparing both sides

$$90^\circ - 2\theta = 4\theta$$

$$90^\circ = 4\theta + 2\theta$$

$$6\theta = 90^\circ$$

$$\text{or } \theta = 15^\circ$$

The value of θ is 15°

Question 11: If $\sec 2A = \operatorname{cosec}(A - 42^\circ)$, where $2A$ is an acute angle, then find the value of A .

Solution: Given: $\sec 2A = \operatorname{cosec}(A - 42^\circ)$

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$$\operatorname{cosec}(90^\circ - 2A) = \operatorname{cosec}(A - 42^\circ)$$

$$90^\circ - 2A = A - 42^\circ$$

$$3A = 132$$

$$A = 44^\circ$$

The value of angle A is 44 degrees.

Question 12:

If $\sin 3A = \cos(A - 26^\circ)$, where $3A$ is an acute angle, find the value of A.

Solution:

$$\sin 3A = \cos(A - 26^\circ) \text{ (given)}$$

$$\text{or } \cos(90^\circ - 3A) = \cos(A - 26^\circ)$$

On comparing

$$90^\circ - 3A = A - 26^\circ$$

$$A + 3A = 90^\circ + 26^\circ$$

$$4A = 116^\circ = 29^\circ$$

The value of A is 29° .

Question 13:

If $\tan 2A = \cot(A - 12^\circ)$, where $2A$ is an acute angle, find the value of A.

Solution:

$$\tan 2A = \cot(A - 12^\circ)$$

$$\text{or } \cot(90^\circ - 2A) = \cot(A - 12^\circ)$$

On comparing

$$90^\circ - 2A = A - 12^\circ$$

$$A + 2A = 90^\circ + 12^\circ$$

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$$3A = 102^\circ$$

$$A = 34^\circ$$

The value of A is 34°

Question 14:

If $\sec 4A = \operatorname{cosec}(A - 15^\circ)$, where $4A$ is an acute angle, find the value of A.

Solution: $\sec 4A = \operatorname{cosec}(A - 15^\circ)$

or $\operatorname{cosec}(90^\circ - 4A) = \operatorname{cosec}(A - 15^\circ)$

On comparing

$$90^\circ - 4A = A - 15^\circ$$

$$A + 4A = 90^\circ + 15^\circ$$

$$5A = 105^\circ$$

$$A = 21^\circ$$

The value of A is 21° .

Question 15:

Prove that:

$$\frac{2}{3} \operatorname{cosec}^2 58^\circ - \frac{2}{3} \cot 58^\circ \tan 32^\circ - \frac{5}{3} \tan 13^\circ \tan 37^\circ \tan 45^\circ \tan 53^\circ \tan 77^\circ = -1$$

Solution:

$$= \frac{2}{3} \operatorname{cosec}^2 58^\circ - \frac{2}{3} \cot 58^\circ \tan 32^\circ - \frac{5}{3} \tan 13^\circ \tan 37^\circ \tan 45^\circ \tan 53^\circ \tan 77^\circ$$

$$= \frac{2}{3} \operatorname{cosec}^2 58^\circ - \frac{2}{3} \cot 58^\circ \tan (90-58)^\circ - \frac{5}{3} \tan 45^\circ (\tan 13^\circ \tan 77^\circ) (\tan 37^\circ \tan 53^\circ)$$

$$= \frac{2}{3} \operatorname{cosec}^2 58^\circ - \frac{2}{3} \cot 58^\circ \cot 58^\circ - \frac{5}{3} \times 1 \times (\tan 13^\circ \tan (90-13)^\circ) \times (\tan 37^\circ \tan (90-37)^\circ)$$

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$$= \frac{2}{3} \operatorname{cosec}^2 58^\circ - \frac{2}{3} \cot^2 58^\circ - \frac{5}{3} \times (\tan 13^\circ \cot 13^\circ) (\tan 37^\circ \cot 37^\circ)$$

$$= \frac{2}{3} [\operatorname{cosec}^2 58^\circ - \cot^2 58^\circ] - \frac{5}{3}$$

$$= \left(\frac{2}{3}\right) - \left(\frac{5}{3}\right)$$

$$= -1$$

$$= \text{R.H.S.}$$