

RS Aggarwal Solutions for Class 10 Maths Chapter 11 T-Ratios of Some Particular Angles**Exercise 11****Page No: 552**

Evaluate each of the following:
(Question 1 to Question 9)

Question 1: $\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$.

Solution:

We know that,

$$\sin 60^\circ = \sqrt{3}/2 = \cos 30^\circ$$

$$\text{and } \sin 30^\circ = 1/2 = \cos 60^\circ$$

Now,

$$\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ = (\sqrt{3}/2)(\sqrt{3}/2) + (1/2)(1/2)$$

$$= (3/4) + (1/4)$$

$$= 4/4$$

$$= 1$$

Question 2:

$\cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ$.

Solution:

We know that,

$$\cos 60^\circ = 1/2 = \sin 30^\circ$$

$$\text{and } \cos 30^\circ = \sqrt{3}/2 = \sin 60^\circ$$

Now,

$$\cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ = (1/2) \times (\sqrt{3}/2) - (\sqrt{3}/2) \times (1/2)$$

$$= (\sqrt{3}/4) - (\sqrt{3}/4)$$

$$= 0$$

Question 3:

$\cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$.

Solution:

We know that,

RS Aggarwal Solutions for Class 10 Maths Chapter 11 T-Ratios of Some Particular Angles

$$\cos 45^\circ = 1/\sqrt{2} = \sin 45^\circ$$

$$\cos 30^\circ = \sqrt{3}/2 \text{ and}$$

$$\sin 30^\circ = 1/2$$

Now,

$$\cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ = (1/\sqrt{2}) \times (\sqrt{3}/2) + (1/\sqrt{2})(1/2)$$

$$= (\sqrt{3} / 2\sqrt{2}) + (1 / 2\sqrt{2})$$

$$= (\sqrt{3} + 1) / (2\sqrt{2})$$

Question 4:

$$\frac{\sin 30^\circ}{\cos 45^\circ} + \frac{\cot 45^\circ}{\sec 60^\circ} - \frac{\sin 60^\circ}{\tan 45^\circ} + \frac{\cos 30^\circ}{\sin 90^\circ}$$

Solution:

We know that,

$$\sin 30^\circ = 1/2, \sin 60^\circ = \sqrt{3}/2, \sin 90^\circ = 1$$

$$\cos 30^\circ = \sqrt{3}/2, \cos 45^\circ = 1/\sqrt{2}, \cos 60^\circ = 1/2$$

$$\sec 60^\circ = 2, \tan 45^\circ = 1 \text{ and } \cot 45^\circ = 1$$

Now,

$$\begin{aligned} & \frac{\sin 30^\circ}{\cos 45^\circ} + \frac{\cot 45^\circ}{\sec 60^\circ} - \frac{\sin 60^\circ}{\tan 45^\circ} + \frac{\cos 30^\circ}{\sin 90^\circ} \\ &= \frac{\frac{1}{2}}{\frac{1}{\sqrt{2}}} + \frac{1}{2} - \frac{\frac{\sqrt{3}}{2}}{1} + \frac{\frac{\sqrt{3}}{2}}{1} \\ &= \frac{\sqrt{2}}{2} + \frac{1}{2} - \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} = \frac{\sqrt{2} + 1 - \sqrt{3} + \sqrt{3}}{2} \\ &= \left(\frac{\sqrt{2} + 1}{2} \right) \end{aligned}$$

Question 5: $(5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ) / (\sin^2 30^\circ + \cos^2 30^\circ)$

Solution:

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We know that,

$$\cos 30^\circ = \sqrt{3}/2 \Rightarrow \cos^2 30^\circ = 3/4$$

$$\cos 60^\circ = 1/2 \Rightarrow \cos^2 60^\circ = 1/4$$

$$\sec 30^\circ = (2/\sqrt{3}) \Rightarrow \sec^2 30^\circ = 4/3$$

$$\tan 45^\circ = 1 \Rightarrow \tan^2 45^\circ = 1$$

$$\sin 30^\circ = 1/2 \Rightarrow \sin^2 30^\circ = 1/4$$

Now,

$$(5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ) / (\sin^2 30^\circ + \cos^2 30^\circ)$$

$$= \frac{5(1/2)^2 + 4(2/\sqrt{3})^2 - (1)^2}{(1/2)^2 + (\sqrt{3}/2)^2}$$

$$= \frac{5 \times \frac{1}{4} + 4 \times \frac{4}{3} - 1}{\frac{1}{4} + \frac{3}{4}}$$

$$= \frac{\frac{5}{4} + \frac{16}{3} - 1}{\frac{1+3}{4}}$$

$$= \frac{15 + 64 - 12}{4}$$

$$= 67/12$$

Question 6:

$$2\cos^2 60^\circ + 3\sin^2 45^\circ - 3\sin^2 30^\circ + 2\cos^2 90^\circ.$$

Solution:

We know that,

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$$\sin 45^\circ = 1/\sqrt{2}, \cos 60^\circ = 1/2$$

$$\sin 30^\circ = 1/2 \text{ and } \cos 90^\circ = 0$$

Now,

$$2\cos^2 60^\circ + 3\sin^2 45^\circ - 3\sin^2 30^\circ + 2\cos^2 90^\circ$$

$$\begin{aligned} &= 2 \times \left(\frac{1}{2}\right)^2 + 3 \times \left(\frac{1}{\sqrt{2}}\right)^2 - 3 \times \left(\frac{1}{2}\right)^2 + 2(0)^2 \\ &= \frac{1}{2} + \frac{3}{2} - \frac{3}{4} \Rightarrow \frac{2+6-3}{4} = \frac{5}{4} \end{aligned}$$

Question 7:

$$\cot^2 30^\circ - 2\cos^2 30^\circ - \frac{3}{4} \sec^2 45^\circ + \frac{1}{4} \operatorname{cosec}^2 30^\circ.$$

Solution:

We know that,

$$\cot 30^\circ = \sqrt{3}, \cos 30^\circ = \sqrt{3}/2$$

$$\sec 45^\circ = \sqrt{2}, \operatorname{cosec} 30^\circ = 2$$

Now,

$$\cot^2 30^\circ - 2\cos^2 30^\circ - \frac{3}{4} \sec^2 45^\circ + \frac{1}{4} \operatorname{cosec}^2 30^\circ$$

$$\begin{aligned} &= (\sqrt{3})^2 - 2 \times \left(\frac{\sqrt{3}}{2}\right)^2 - \frac{3}{4} \times \left(\frac{\sqrt{2}}{1}\right)^2 + \frac{1}{4} \times (2)^2 \\ &= 3 - 2 \times \frac{3}{4} - \frac{3}{4} \times 2 + \frac{1}{4} \times 4 \\ &= 3 - \frac{3}{2} - \frac{3}{2} + 1 \end{aligned}$$

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$$= 1$$

Question 8:

$$(\sin^2 30^\circ + 4\cot^2 45^\circ - \sec^2 60^\circ)(\operatorname{cosec}^2 45^\circ \sec^2 30^\circ).$$

Solution:

$$\sin 30^\circ = 1/2 \Rightarrow \sin^2 30^\circ = 1/4$$

$$\cos 45^\circ = 1/\sqrt{2} = \sin 45^\circ$$

$$\cot 45^\circ = 1 \Rightarrow \cot^2 45^\circ = 1$$

$$\cos 60^\circ = 1/2 \Rightarrow \sec 60^\circ = 2 \Rightarrow \sec^2 60^\circ = 4$$

$$\cos 30^\circ = \sqrt{3}/2 \Rightarrow \sec 30^\circ = 2/\sqrt{3} \Rightarrow \sec^2 30^\circ = 4/3$$

$$\operatorname{cosec} 45^\circ = 1/\sin 45^\circ = \sqrt{2} \Rightarrow \operatorname{cosec}^2 45^\circ = 2$$

$$(\sin^2 30^\circ + 4\cot^2 45^\circ - \sec^2 60^\circ)(\operatorname{cosec}^2 45^\circ \sec^2 30^\circ)$$

$$= \left[\left(\frac{1}{2} \right)^2 + 4 \times (1)^2 - (2)^2 \right] \left[(\sqrt{2})^2 \times \left(\frac{2}{\sqrt{3}} \right)^2 \right]$$

$$= 1/4 \times 8/3$$

$$= 2/3$$

Question 9:

$$4/\cot^2 30^\circ + 1/\sin^2 30^\circ - 2\cos^2 45^\circ - \sin^2 0^\circ.$$

Solution:

$$4/\cot^2 30^\circ + 1/\sin^2 30^\circ - 2\cos^2 45^\circ - \sin^2 0^\circ$$

$$= \frac{4}{(\sqrt{3})^2} + \frac{1}{\left(\frac{1}{2}\right)^2} - 2 \times \left(\frac{1}{\sqrt{2}}\right)^2 - 0$$

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$$= 4/3 + 4/1 - 1 - 0$$

$$= 26/6$$

$$= 13/3$$

Question 10:

Show that:

$$(i) \frac{1 - \sin 60^\circ}{\cos 60^\circ} = \frac{\tan 60^\circ - 1}{\tan 60^\circ + 1}$$

$$(ii) \frac{\cos 30^\circ + \sin 60^\circ}{1 + \sin 30^\circ + \cos 60^\circ} = \cos 30^\circ$$

Solution:

(i)

$$\frac{1 - \sin 60^\circ}{\cos 60^\circ}$$

$$= \frac{1 - \frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{2 - \sqrt{3}}{1}$$

$$= 2 - \sqrt{3}$$

RHS:

$$\frac{\tan 60^\circ - 1}{\tan 60^\circ + 1} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

$$= \frac{\sqrt{3} - 1}{\sqrt{3} + 1} \times \frac{\sqrt{3} - 1}{\sqrt{3} - 1}$$

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$$= (4 - 2\sqrt{3})/2$$

$$= 2 - \sqrt{3}$$

(ii)

LHS:

$$\frac{\cos 30^\circ + \sin 60^\circ}{1 + \sin 30^\circ + \cos 60^\circ}$$

=

$$\frac{\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}}{1 + \frac{1}{2} + \frac{1}{2}} = \frac{\sqrt{3}}{2}$$

RHS:

$$\cos 30^\circ = \frac{\sqrt{3}}{2}$$

LHS = RHS

Question 11:

Verify each of the following:

- (i) $\sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ = \sin 30^\circ$
- (ii) $\cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ = \cos 30^\circ$
- (iii) $2 \sin 30^\circ \cos 30^\circ = \sin 60^\circ$
- (iv) $2 \sin 45^\circ \cos 45^\circ = \sin 90^\circ$

Solution:

(i) L.H.S. = $\sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$

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$$= (\sqrt{3}/2) \times (\sqrt{3}/2) - (1/2)(1/2)$$

$$= (3/4) - (1/4)$$

$$= 2/4$$

$$1/2$$

R.H.S.:

$$\sin 30^\circ = 1/2$$

$$\text{LHS} = \text{RHS}$$

(ii)

$$\text{L.H.S.} = \cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ$$

$$= (1/2) \times (\sqrt{3}/2) + (\sqrt{3}/2)(1/2)$$

$$= (\sqrt{3}/4) + (\sqrt{3}/4)$$

$$= \sqrt{3}/2$$

R.H.S.

$$\cos 30^\circ = \sqrt{3}/2$$

$$\text{L.H.S.} = \text{R.H.S.}$$

(iii)

$$\text{L.H.S.} = 2 \sin 30^\circ \cos 30^\circ$$

$$= 2 \times (1/2) \times (\sqrt{3}/2)$$

$$= \sqrt{3}/2$$

R.H.S.

$$\sin 60^\circ = \sqrt{3}/2$$

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$$\text{L.H.S.} = \text{R.H.S.}$$

$$\text{(iv) L.H.S.} = 2 \sin 45^\circ \cos 45^\circ$$

$$= 2 \times (1/\sqrt{2}) \times (1/\sqrt{2})$$

$$= (2 \times 1/2)$$

$$= 1$$

$$\text{R.H.S.} = \sin 90^\circ = 1$$

$$\text{L.H.S.} = \text{R.H.S.}$$

Question 12:

If $A = 45^\circ$, verify that:

$$\text{(i) } \sin 2A = 2 \sin A \cos A \quad \text{(ii) } \cos 2A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

Solution:

$$A = 45^\circ \text{ then } 2A = 90^\circ$$

$$\text{(i) } \sin 2A = \sin 90^\circ$$

RHS:

$$2 \sin 45^\circ \cos 45^\circ = 2 \times (1/\sqrt{2}) \times (1/\sqrt{2})$$

$$= 1$$

LHS:

$$\sin 90^\circ = 1$$

$$\text{L.H.S.} = \text{R.H.S.}$$

$$\text{(ii) } \cos 2A = \cos 90^\circ = 0$$

$$2 \cos^2 A - 1 = 2 \cos^2 45^\circ - 1$$

$$= 2 \left(\frac{1}{\sqrt{2}} \right)^2 - 1 = 1 - 1 = 0$$

$$1 - 2 \sin^2 A = 1 - 2 \sin^2 45^\circ = 1 - 2 \times \left(\frac{1}{\sqrt{2}} \right)^2 = 1 - 1 = 0$$

$$\therefore \cos 2A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

RS Aggarwal Solutions for Class 10 Maths Chapter 11 T-Ratios of Some Particular Angles**Question 13.****If $A = 30^\circ$, verify that:**

$$(i) \sin 2A = \frac{2 \tan A}{1 + \tan^2 A}$$

$$(ii) \cos 2A = \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

$$(iii) \tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Solution:

$$A = 30 \Rightarrow 2A = 60$$

(i)

$$\sin 2A = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\begin{aligned} \text{Also } \frac{2 \tan A}{1 + \tan^2 A} &= \frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ} = \frac{2 \times \frac{1}{\sqrt{3}}}{1 + \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1 + \frac{1}{3}} \\ &= \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}} = \frac{2}{\sqrt{3}} \times \frac{3}{4} = \frac{\sqrt{3}}{2} \end{aligned}$$

$$\text{Hence, } \sin 2A = \frac{2 \tan A}{1 + \tan^2 A}$$

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(ii)

$$\cos 2A = \cos 60^\circ = \frac{1}{2}$$

$$\begin{aligned} \text{Also, } \frac{1 - \tan^2 A}{1 + \tan^2 A} &= \frac{1 - \tan^2 30^\circ}{1 + \tan^2 30^\circ} = \frac{1 - \left(\frac{1}{\sqrt{3}}\right)^2}{1 + \left(\frac{1}{\sqrt{3}}\right)^2} \\ &= \frac{\left(1 - \frac{1}{3}\right)}{\left(1 + \frac{1}{3}\right)} = \frac{\left(\frac{2}{3}\right)}{\left(\frac{4}{3}\right)} = \left(\frac{2}{3} \times \frac{3}{4}\right) = \frac{1}{2} \end{aligned}$$

$$\text{Hence, } \cos 2A = \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

(iii)

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$A = 30 \text{ degrees}$$

Show that:

$$\tan 60^\circ = \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ}$$

RHS:

$$\frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ} = \frac{2 \times \frac{1}{\sqrt{3}}}{1 - \frac{1}{3}} = \frac{2/\sqrt{3}}{2/3}$$

$$= 3/\sqrt{3}$$

$$= \sqrt{3}$$

$$= \tan 60^\circ$$

$$= \text{LHS}$$

RS Aggarwal Solutions for Class 10 Maths Chapter 11 T-Ratios of Some Particular Angles**Question 14:**If $A = 60^\circ$ and $B = 30^\circ$, verify that:

(i) $\sin (A + B) = \sin A \cos B + \cos A \sin B$

(ii) $\cos (A + B) = \cos A \cos B - \sin A \sin B$

Solution:

(i) $\sin (A + B) = \sin A \cos B + \cos A \sin B$

If $A = 60^\circ$ and $B = 30^\circ$, then

To verify: $\sin 90^\circ = \sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$

RHS: $\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$

$$= (\sqrt{3}/2) \times (\sqrt{3}/2) + (1/2)(1/2)$$

$$= (3/4) + (1/4)$$

$$= 4/4$$

$$= 1$$

$$= \sin 90^\circ$$

$$= \text{LHS}$$

(ii) $\cos (A + B) = \cos A \cos B - \sin A \sin B$

If $A = 60^\circ$ and $B = 30^\circ$

Verify: $\cos (90^\circ) = \cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ$

R.H.S. = $\cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ$

$$= (1/2) \times (\sqrt{3}/2) - (\sqrt{3}/2)(1/2)$$

$$= (\sqrt{3}/4) - (\sqrt{3}/4)$$

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$$= 0$$

$$= \cos 90^\circ$$

$$= \text{L.H.S.}$$

Question 15:

If $A = 60^\circ$ and $B = 30^\circ$, verify that:

(i) $\sin(A - B) = \sin A \cos B - \cos A \sin B$

(ii) $\cos(A - B) = \cos A \cos B + \sin A \sin B$

(iii) $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

Solution:

(i) $\sin(A - B) = \sin A \cos B - \cos A \sin B$

If $A = 60^\circ$ and $B = 30^\circ$, then

LHS :

$$= \sin(60^\circ - 30^\circ)$$

$$= \sin 30^\circ$$

$$= \frac{1}{2}$$

$$\text{R.H.S.} = \sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$$

$$= \left(\frac{\sqrt{3}}{2}\right) \times \left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right) \left(\frac{1}{2}\right)$$

$$= \left(\frac{3}{4}\right) - \left(\frac{1}{4}\right)$$

$$= \frac{2}{4}$$

$$= \frac{1}{2}$$

$$\text{L.H.S.} = \text{R.H.S.}$$

(ii) $\cos(A - B) = \cos A \cos B + \sin A \sin B$

If $A = 60^\circ$ and $B = 30^\circ$, then

$$\text{Verify: } \cos(30^\circ) = \cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ$$

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$$\text{R.H.S.} = \cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ$$

$$= (1/2) \times (\sqrt{3}/2) + (\sqrt{3}/2)(1/2)$$

$$= (\sqrt{3}/4) + (\sqrt{3}/4)$$

$$= \sqrt{3}/2$$

$$= \cos 30^\circ$$

$$= \text{L.H.S.}$$

(iii) If $A = 60^\circ$ and $B = 30^\circ$, then

$$A - B = 30^\circ$$

$$\tan (A - B) = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

Now,

$$\frac{\tan A - \tan B}{1 + \tan A \tan B} = \frac{\tan 60^\circ - \tan 30^\circ}{1 + \tan 60^\circ \tan 30^\circ}$$

$$= \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3} \times \frac{1}{\sqrt{3}}}$$

$$= \frac{3 - 1}{\sqrt{3} \times 2}$$

$$= 1/\sqrt{3}$$

$$\tan (A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Verified.

RS Aggarwal Solutions for Class 10 Maths Chapter 11 T-Ratios of Some Particular Angles**Question 16:**

If A and B are acute angles such that $\tan A = 1/3$, $\tan B = 1/2$ and

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B},$$

Show that $A + B = 45^\circ$.

Solution:

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A + B) = \frac{\left(\frac{1}{3} + \frac{1}{2}\right)}{1 - \frac{1}{3} \times \frac{1}{2}} \left[\because \tan A = \frac{1}{3}, \tan B = \frac{1}{2} \right]$$

$$= (5/6) / (5/6)$$

$$= 1$$

This implies, $\tan(A + B) = 1$

$$= \tan 45^\circ$$

Or $A + B = 45^\circ$. Proved

Question 17:

Using the formula, $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$, find the value of $\tan 60^\circ$, it being given that $\tan 30^\circ = 1/\sqrt{3}$.

Solution:

Put $A = 30^\circ \Rightarrow 2A = 60^\circ$

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$$\tan 60^\circ = \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ}$$

$$= \frac{2 \times \frac{1}{\sqrt{3}}}{1 - \left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= \frac{2 \times \frac{1}{\sqrt{3}}}{1 - \frac{1}{3}}$$

$$= \sqrt{3}$$

The value of $\tan 60^\circ$ is $\sqrt{3}$.

Question 18:

Using the formula, $\cos A = \sqrt{\frac{1 + \cos 2A}{2}}$, find the value of $\cos 30^\circ$, it being given that $\cos 60^\circ = 1/2$.

Solution:

Put $A = 30^\circ$ then $2A = 60^\circ$

$$\cos A = \sqrt{\frac{1 + \cos 2A}{2}}$$

$$\begin{aligned} \cos 30^\circ &= \sqrt{\frac{1 + \cos 60^\circ}{2}} = \sqrt{\frac{1 + \frac{1}{2}}{2}} = \sqrt{\frac{\frac{3}{2}}{2}} \\ &= \frac{\sqrt{3}}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{\sqrt{3}}{2} \end{aligned}$$

The value of $\cos 30^\circ$ is $\sqrt{3}/2$.

Question 19:

Using the formula, $\sin A = \sqrt{\frac{1 - \cos 2A}{2}}$, find the value of $\sin 30^\circ$, it being given that $\cos 60^\circ = 1/2$.

RS Aggarwal Solutions for Class 10 Maths Chapter 11 T-Ratios of Some Particular Angles**Solution:**Put $A = 30^\circ$ then $2A = 60^\circ$

$$\sin A = \sqrt{\frac{1 - \cos 2A}{2}}$$

Squaring both side, we get

$$\sin^2 A = \frac{1 - \cos 2A}{2}$$

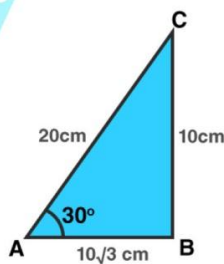
And,

$$\sin^2 30^\circ = \frac{1 - \cos 60^\circ}{2} = \frac{1 - \frac{1}{2}}{2} = \frac{1}{4}$$

$$\sin 30^\circ = \frac{1}{2}$$

Question 20:

In the adjoining figure, $\triangle ABC$ is a right-angled triangle in which $\angle B = 90^\circ$, $\angle A = 30^\circ$ and $AC = 20$ cm. Find (i) BC , (ii) AB .

Solution:Draw a right angled $\triangle ABC$ using given instructions:

$$\text{Here } \sin 30 = BC/AC$$

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$$\frac{1}{2} = \frac{BC}{20}$$

$$\text{Or } BC = 10 \text{ cm}$$

By Pythagoras theorem:

$$(AB)^2 = (AC)^2 - (BC)^2$$

$$= (20)^2 - (10)^2$$

$$= 300$$

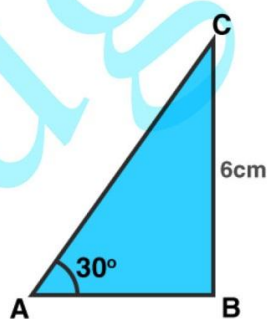
$$AB = 10\sqrt{3} \text{ cm}$$

Question 21:

In the adjoining figure, ΔABC is a right-angled at B and $\angle A = 30^\circ$. If $BC = 6 \text{ cm}$, Find (i) AB, (ii) AC.

Solution:

Draw a right angled ΔABC using given instructions:



$$\text{Here } \sin 30 = \frac{BC}{AC}$$

$$\frac{1}{2} = \frac{6}{AC}$$

$$\text{Or } AC = 12 \text{ cm}$$

RS Aggarwal Solutions for Class 10 Maths Chapter 11 T-Ratios of Some Particular Angles

By Pythagoras theorem:

$$(AB)^2 = (AC)^2 - (BC)^2$$

$$= (12)^2 - (6)^2$$

$$= 108$$

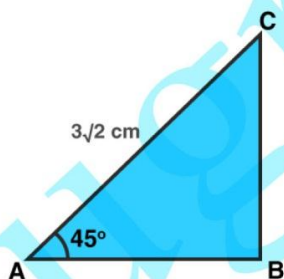
$$AB = 6\sqrt{3} \text{ cm}$$

Question 22:

In the adjoining figure, $\triangle ABC$ is a right-angled at B and $\angle A = 45^\circ$. If $AC = 3\sqrt{2}$ cm, Find (i) BC, (ii) AB.

Solution:

From right angled $\triangle ABC$,



(i)

$$\frac{BC}{AC} = \sin 45^\circ$$

$$\frac{BC}{3\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\text{Or } BC = 3$$

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(ii) By Pythagoras theorem

$$(AB)^2 = (AC)^2 - (BC)^2$$

$$= (3\sqrt{2})^2 - (3)^2$$

$$= 18 - 9$$

$$= 9$$

$$AB = 3 \text{ cm}$$

Question 23:

If $\sin(A + B) = 1$ and $\cos(A - B) = 1$, $0^\circ \leq (A + B) \leq 90^\circ$ and $A > B$, then find A and B.

Solution:

$$\sin(A + B) = 1 \text{ or } \sin(A + B) = \sin 90^\circ \text{ [As } \sin 90^\circ = 1]$$

$$A + B = 90^\circ \dots(1)$$

$$\text{Again, } \cos(A - B) = 1$$

$$= \cos 0^\circ$$

$$A - B = 0 \dots(2)$$

Adding (1) and (2), we get

$$2A = 90^\circ \text{ or } A = 45^\circ$$

Putting $A = 45^\circ$ in (1) we get

$$45^\circ + B = 90^\circ \text{ or } B = 45^\circ$$

Hence, $A = 45^\circ$ and $B = 45^\circ$.

Question 24:

If $\sin(A - B) = 1/2$ and $\cos(A + B) = 1/2$, $0^\circ < (A + B) < 90^\circ$ and $A > B$, then find A and B.

Solution:

$$\sin(A - B) = 1/2$$

RS Aggarwal Solutions for Class 10 Maths Chapter 11 T-Ratios of Some Particular Angles

$$\text{or } \sin(A - B) = \sin 30^\circ$$

$$A - B = 30^\circ \dots(1)$$

$$\text{Again, } \cos(A+B) = 1/2$$

$$= \cos 60^\circ$$

$$A + B = 60^\circ \dots(2)$$

Solving (1) and (2), we get

$$2A = 90^\circ \text{ or } A = 45^\circ$$

Putting $A = 45^\circ$ in (1), we get

$$45^\circ - B = 30^\circ \text{ or } B = 45^\circ - 30^\circ = 15^\circ$$

Therefore, $A = 45^\circ$, $B = 15^\circ$.

Question 25:

If $\tan(A - B) = 1/\sqrt{3}$ and $\tan(A + B) = \sqrt{3}$, $0^\circ < (A + B) < 90^\circ$ and $A > B$, then find A and B.

Solution:

$$\tan(A - B) = 1/\sqrt{3}$$

$$\text{or } \tan(A - B) = \tan 30^\circ$$

$$A - B = 30^\circ \dots(1)$$

$$\text{Again, } \tan(A+B) = \sqrt{3}$$

$$= \tan 60^\circ$$

$$A + B = 60^\circ \dots(2)$$

Solving (1) and (2), we get

$$2A = 90^\circ \text{ or } A = 45^\circ$$

Putting $A = 45^\circ$ in (1), we get

$$45^\circ - B = 30^\circ \text{ or } B = 45^\circ - 30^\circ = 15^\circ$$

Therefore, $A = 45^\circ$, $B = 15^\circ$

Question 26:

If $3x = \operatorname{cosec} \theta$ and $3/x = \cot \theta$, find the value of $3(x^2 - 1/x^2)$.

Solution:

RS Aggarwal Solutions for Class 10 Maths Chapter 11 T-Ratios of Some Particular Angles

Given: $3x = \operatorname{cosec} \theta$ and $3/x = \cot \theta$

We know that: $\operatorname{cosec}^2 \theta - \cot^2 \theta = 1$

Substituting the values, we get

$$(3x)^2 - (3/x)^2 = 1$$

$$9\left(x^2 - \frac{1}{x^2}\right) = 1$$

$$\left(x^2 - \frac{1}{x^2}\right) = \frac{1}{9}$$

Question 27:

If $\sin(A+B) = \sin A \cos B + \cos A \sin B$ and $\cos(A-B) = \cos A \cos B + \sin A \sin B$,

find the values of (i) $\sin 75^\circ$ and (ii) $\cos 15^\circ$.

Solution:

Given: $\sin(A+B) = \sin A \cos B + \cos A \sin B$ and

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

(i) To find: $\sin 75^\circ$

Put $A = 30^\circ$ and $B = 45^\circ$, then

$$\sin 75^\circ = \sin 30^\circ \cos 45^\circ + \cos 30^\circ \sin 45^\circ$$

$$= \left(\frac{1}{2}\right) \times \left(\frac{1}{\sqrt{2}}\right) + \left(\frac{\sqrt{3}}{2}\right) \times \left(\frac{1}{\sqrt{2}}\right)$$

$$= \left(\frac{1}{2\sqrt{2}}\right) + \left(\frac{\sqrt{3}}{2\sqrt{2}}\right)$$

$$= \frac{(1+\sqrt{3})}{2\sqrt{2}}$$

(ii) Find $\cos 75^\circ$

Put $A = 45^\circ$ and $B = 30^\circ$, then

$$\cos 15^\circ = \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$$

$$= \left(\frac{1}{\sqrt{2}}\right) \times \left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right) \times \left(\frac{1}{2}\right)$$

$$= \left(\frac{\sqrt{3}}{2\sqrt{2}}\right) + \left(\frac{1}{2\sqrt{2}}\right)$$

$$= \frac{(1+\sqrt{3})}{2\sqrt{2}}$$