

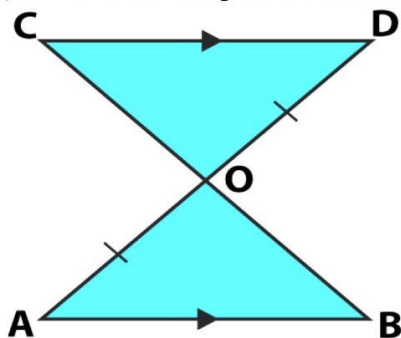
RS Aggarwal Solutions for Class 9 Maths Chapter 9 –
Congruence of Triangles and Inequalities in a Triangle

EXERCISE 9A

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1. In the given figure, $AB \parallel CD$ and O is the midpoint of AD . Show that

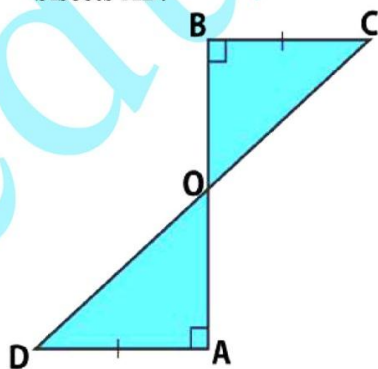
- (i) $\triangle AOB \cong \triangle DOC$
(ii) O is the midpoint of BC .



Solution:

- (i) From the figure $\triangle AOB$ and $\triangle DOC$
We know that $AB \parallel CD$ and $\angle BAO$ and $\angle CDO$ are alternate angles
So we get
 $\angle BAO = \angle CDO$
From the figure we also know that O is the midpoint of the line AD
We can write it as $AO = DO$
According to the figure we know that $\angle AOB$ and $\angle DOC$ are vertically opposite angles.
So we get $\angle AOB = \angle DOC$
Therefore, by ASA congruence criterion we get
 $\triangle AOB \cong \triangle DOC$
- (ii) We know that $\triangle AOB \cong \triangle DOC$
So we can write it as
 $BO = CO$ (c. p. c. t)
Therefore, it is proved that O is the midpoint of BC .

2. In the given figure, AD and BC are equal perpendiculars to a line segment AB . Show that CD bisects AB .



Solution:

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Based on the $\triangle AOD$ and $\triangle BOC$

From the figure we know that $\angle AOD$ and $\angle BOC$ are vertically opposite angles.

So we get

$$\angle AOD = \angle BOC$$

We also know that

$$\angle DAO = \angle CBO = 90^\circ$$

It is given that $AD = BC$

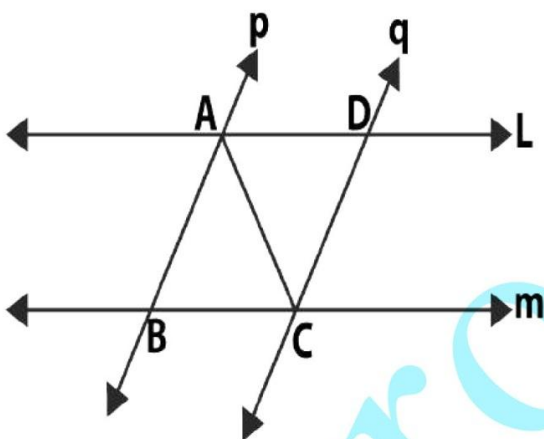
Therefore, by AAS congruence criterion we get

$$\triangle AOD \cong \triangle BOC$$

So we get $AO = BO$ (c. p. c. t)

Therefore, it is proved that CD bisects AB .

3. In the given figure, two parallel lines l and m are intersected by two parallel lines p and q . Show that $\triangle ABC \cong \triangle CDA$.



Solution:

Based on the $\triangle ABC$ and $\triangle CDA$

We know that $p \parallel q$ and $\angle BAC$ and $\angle DCA$ are alternate interior angles

So we get

$$\angle BAC = \angle DCA$$

We know that A and C are the common points for all the lines

So it can be written as

$$AC = CA$$

We know that $l \parallel m$ and $\angle BCA$ and $\angle DAC$ are alternate interior angles

So we get

$$\angle BCA = \angle DAC$$

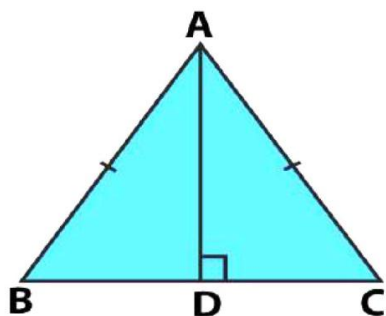
Therefore, by ASA congruence rule it is proved that

$$\triangle ABC \cong \triangle CDA$$

4. AD is an altitude of an isosceles $\triangle ABC$ in which $AB = AC$. Show that

(i) AD bisects BC ,

(ii) AD bisects $\angle A$.

**RS Aggarwal Solutions for Class 9 Maths Chapter 9 –
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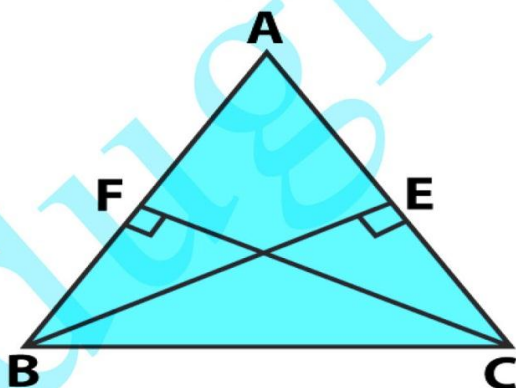
- (i) Based on the $\triangle BAD$ and $\triangle CAD$
We know that AD is the altitude so the angle is 90°
So we get
 $\angle ADB = \angle ADC = 90^\circ$
It is given that $AB = AC$ and we know that AD is common
Based on the RHS Congruence Criterion we get
 $\triangle BAD \cong \triangle CAD$
So we get $BD = CD$ (c. p. c. t)

Therefore, it is proved that AD bisects BC.

- (ii) We also know that $\angle BAD = \angle CAD$ (c. p. c. t)
Therefore, it is proved that AD bisects $\angle A$.

5. In the given figure, BE and CF are two equal altitudes of $\triangle ABC$. Show that

- (i) $\triangle ABE \cong \triangle ACF$,
(ii) $AB = AC$.

**Solution:**

- (i) Based on the $\triangle ABE$ and $\triangle ACF$
We know that
 $\angle AEB = \angle AFC = 90^\circ$
It is given that $BE = CF$
From the figure we know that $\angle A$ is common for both $\angle BAE$ and $\angle CAF$

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So we get

$$\angle BAE = \angle CAF = \angle A$$

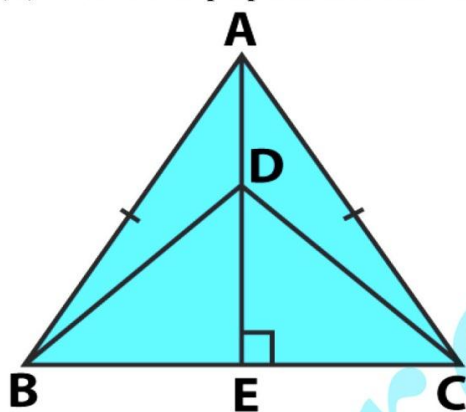
Therefore, by ASA congruence criterion we get

$$\triangle ABE \cong \triangle ACF$$

- (ii) As we know that $\triangle ABE \cong \triangle ACF$
It is proved that $AB = AC$.

6. $\triangle ABC$ and $\triangle DBC$ are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC . If AD is extended to intersect BC at E , show that

- (i) $\triangle ABD \cong \triangle ACD$
- (ii) $\triangle ABE \cong \triangle ACE$
- (iii) AE bisects $\angle A$ as well as $\angle D$
- (iv) AE is the perpendicular bisector of BC .



Solution:

- (i) Based on the $\triangle ABD$ and $\triangle ACD$
From $\triangle ABC$ we know that AB and AC are equal sides of isosceles triangle
So we get
 $AB = AC$
From $\triangle DBC$ we know that DB and DC are equal sides of isosceles triangle
So we get
 $DB = DC$
We also know that AD is common i.e. $AD = AD$
Therefore, by SSS congruence criterion we get
 $\triangle ABD \cong \triangle ACD$
- (ii) We know that $\triangle ABD \cong \triangle ACD$
We get $\angle BAD = \angle CAD$ (c. p. c. t)
It can be written as
 $\angle BAE = \angle CAE$ (1)

Considering $\triangle ABE$ and $\triangle ACE$

We know that AB and AC are the equal sides of isosceles $\triangle ABC$

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$$AB = AC$$

So by using equation (1) we get

$$\angle BAE = \angle CAE$$

We know that AE is common i.e. $AE = AE$

Therefore, by SAS congruence criterion we get

$$\triangle ABE \cong \triangle ACE$$

(iii) We know that $\triangle ABD \cong \triangle ACD$

We get $\angle BAD = \angle CAD$ (c. p. c. t)

It can be written as

$$\angle BAE = \angle CAE$$

Therefore, it is proved that AE bisects $\angle A$.

Considering $\triangle BDE$ and $\triangle CDE$

We know that BD and CD are equal sides of isosceles $\triangle ABC$

Since $\triangle ABE \cong \triangle ACE$

$$BE = CE \text{ (c. p. c. t)}$$

We know that DE is common i.e. $DE = DE$

Therefore, by SSS congruence criterion we get

$$\triangle BDE \cong \triangle CDE$$

We know that $\angle BDE = \angle CDE$ (c. p. c. t)

So DE bisects $\angle D$ which means that AE bisects $\angle D$

Hence it is proved that AE bisects $\angle A$ as well as $\angle D$.

(iv) We know that $\triangle BDE \cong \triangle CDE$

So we get

$$BE = CE \text{ and } \angle BED = \angle CED \text{ (c. p. c. t)}$$

From the figure we know that $\angle BED$ and $\angle CED$ form a linear pair of angles

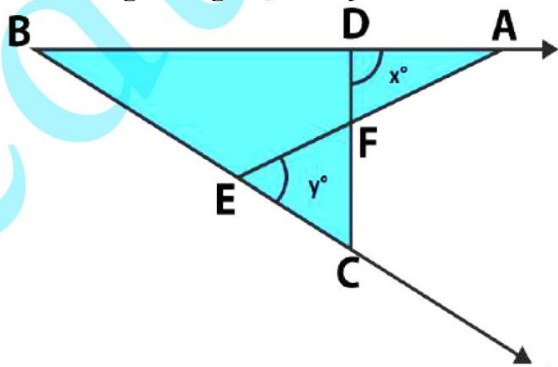
So we get

$$\angle BED = \angle CED = 90^\circ$$

We know that DE is the perpendicular bisector of BC

Therefore, it is proved that AE is the perpendicular bisector of BC.

7. In the given figure, if $x = y$ and $AB = CB$ then prove that $AE = CD$.



Solution:

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It is given that $x = y$ and $AB = CB$

By considering the $\triangle ABE$

We know that

Exterior $\angle AEB = \angle EBA + \angle BAE$

By substituting $\angle AEB$ as y we get

$y = \angle EBA + \angle BAE$

By considering the $\triangle BCD$

We know that

$x = \angle CBA + \angle BCD$

It is given that $x = y$

So we can write it as

$\angle CBA + \angle BCD = \angle EBA + \angle BAE$

On further calculation we can write it as

$\angle BCD = \angle BAE$

Based on both $\triangle BCD$ and $\triangle BAE$

We know that B is the common point

It is given that $AB = BC$

It is proved that $\angle BCD = \angle BAE$

Therefore, by ASA congruence criterion we get

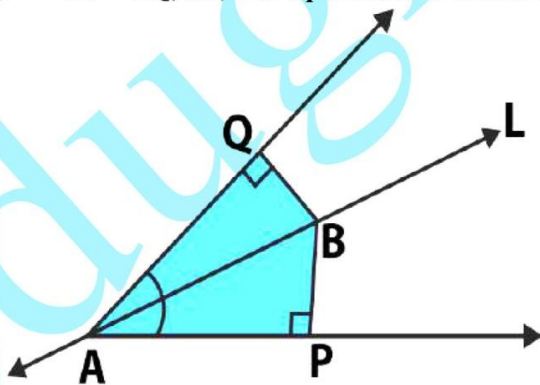
$\triangle BCD \cong \triangle BAE$

We know that the corresponding sides of congruent triangles are equal

Therefore, it is proved that $AE = CD$.

8. In the given figure, line l is the bisector of an angle $\angle A$ and B is any point on l . If BP and BQ are perpendiculars from B to the arms of $\angle A$, show that

- (i) $\triangle APB \cong \triangle AQB$
- (ii) $BP = BQ$, i.e., B is equidistant from the arms of $\angle A$.



Solution:

- (i) Considering $\triangle APB$ and $\triangle AQB$

We know that

$\angle APB = \angle AQB = 90^\circ$

From the figure we know that l is the bisector of $\angle A$

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So we get

$$\angle BAP = \angle BAQ$$

We know that AB is common i.e. $AB = AB$

Therefore, by AAS congruence criterion we get

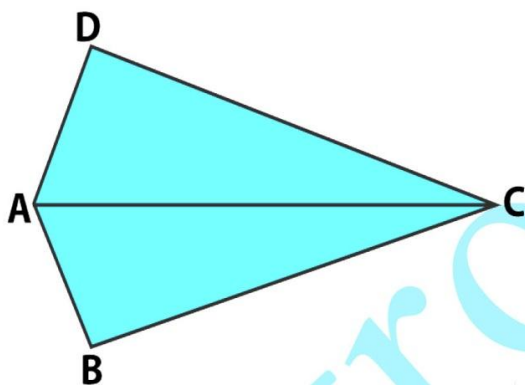
$$\triangle APB \cong \triangle AQB$$

(ii) We know that $\triangle APB \cong \triangle AQB$

So it is proved that $BP = BQ$ (c. p. c. t)

9. ABCD is a quadrilateral such that diagonal AC bisects the angles $\angle A$ and $\angle C$. Prove that $AB = AD$ and $CB = CD$.

Solution:



By considering $\triangle ABC$ and $\triangle ADC$

We know that AC bisects at $\angle A$

So we get

$$\angle BAC = \angle DAC$$

We know that AC is common i.e. $AC = AC$

From the figure we know that AC bisects at $\angle C$

$$\angle BCA = \angle DCA$$

By ASA congruence criterion we get

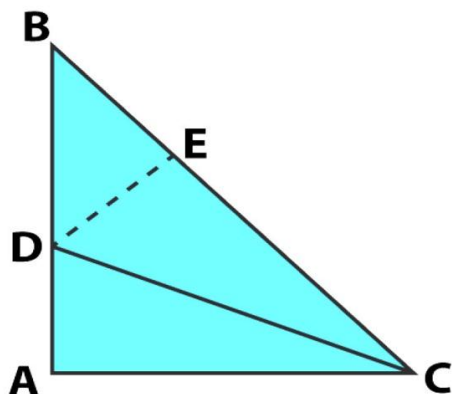
$$\triangle ABC \cong \triangle ADC$$

Therefore, it is proved that $AB = AD$ and $CB = CD$ (c. p. c. t)

10. $\triangle ABC$ is a right triangle right angled at A such that $AB = AC$ and bisector of $\angle C$ intersects the side AB at D. Prove that $AC + AD = BC$.

Solution:

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Construct a triangle ABC where DE is perpendicular to BC

Consider the $\triangle DAC$ and $\triangle DEC$

We know that

$$\angle BAC = \angle DAC = 90^\circ$$

From the figure we know that CD bisects $\angle C$

So we get

$$\angle DCA = \angle DCE$$

We know that CD is common i.e. $CD = CD$

By AAS congruence criterion

$$\triangle DAC \cong \triangle DEC$$

So we know that $DA = DE$ (1)

$AC = EC$ (c. p. c. t) (2)

It is given that $AB = AC$

We know that the angles opposite to equal sides are equal

$$\angle B = \angle C$$

We know that the sum of angles of $\triangle ABC$ is 180° .

$$\angle A + \angle B + \angle C = 180^\circ$$

By substituting the values

$$90^\circ + \angle B + \angle B = 180^\circ$$

On further calculation

$$2\angle B = 180^\circ - 90^\circ$$

$$2\angle B = 90^\circ$$

By division

$$\angle B = 45^\circ$$

Considering the $\triangle BED$

We know that $\angle BED = 90^\circ$

So we can write it as

$$\angle BDE + \angle B = 90^\circ$$

By substituting the values

$$\angle BDE + 45^\circ = 90^\circ$$

On further calculation

$$\angle BDE = 90^\circ - 45^\circ$$

By subtraction

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$$\angle BDE = 45^\circ$$

It can be written as

$$\angle BDE = \angle DBE = 45^\circ$$

We know that DE and BE are the equal sides of isosceles triangle

$$DE = BE \dots\dots\dots (3)$$

By comparing the equations (1) and (3)

We get

$$DA = DE = BE \dots\dots\dots (4)$$

We know that $BC = BE + EC$

By considering the equations (ii) and (iv)

We get

$$BC = DA + AC$$

We can also write it as

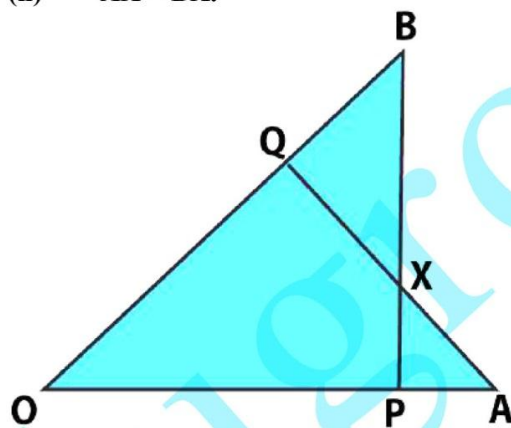
$$AC + AD = BC$$

Therefore, it is proved that $AC + AD = BC$.

11. In the given figure, $OA = OB$ and $OP = OQ$. Prove that

(i) $PX = QX$,

(ii) $AX = BX$.



Solution:

It is given that $OA = OB$ and $OP = OQ$

By considering the $\triangle OAQ$ and $\triangle OPB$

Therefore, by SAS congruence criterion

$$\triangle OAQ = \triangle OPB$$

We know that the corresponding parts of congruent triangles are equal

So we get

$$\angle OBP = \angle OAQ \dots\dots\dots (1)$$

Consider $\triangle BXQ$ and $\triangle PXA$

We can write it as

$$BQ = OB - OQ \text{ and } PA = OA - OP$$

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We know that $OP = OQ$ and is given that $OA = OB$
So we get $BQ = PA$ (2)

In $\triangle BXQ$ and $\triangle PXA$

We know that $\angle BXQ$ and $\angle PXA$ are vertically opposite angles

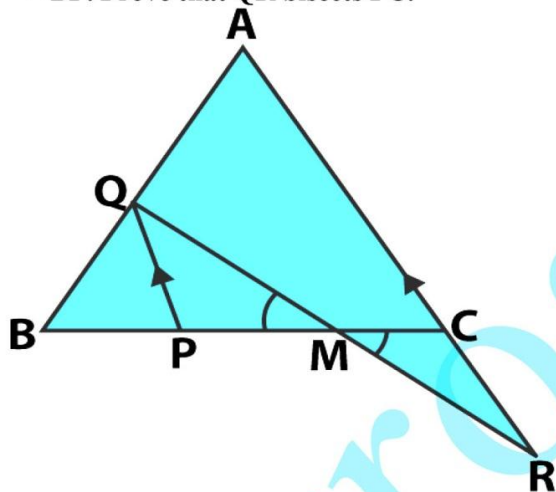
$\angle BXQ = \angle PXA$

From (1) and (2) and AAS congruence criterion we get

$\triangle BXQ \cong \triangle PXA$

So we get $PX = QX$ and $AX = BX$ (c. p. c. t)

12. In the given figure, ABC is an equilateral triangle; $PQ \parallel AC$ and AC is produced to R such that $CR = BP$. Prove that QR bisects PC .



Solution:

It is given that ABC is an equilateral triangle; $PQ \parallel AC$ and AC is produced to R such that $CR = BP$

Consider QR intersecting the line PC at point M

We know that $\triangle ABC$ is an equilateral triangle

So we get $\angle A = \angle ACB = 60^\circ$

From the figure we know that $PQ \parallel AC$ and $\angle BPQ$ and $\angle ACB$ are corresponding angles

So we get

$\angle BPQ = \angle ACB = 60^\circ$

Based on the $\triangle BPQ$ we know that

$\angle B = \angle ACB = 60^\circ$

It can be written as

$\angle BQP = 60^\circ$

According to the figure we know that $\triangle BPQ$ is an equilateral triangle

So we get

$PQ = BP = BQ$

It is given that $CR = BP$ so we get

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$$PQ = CR \dots\dots (1)$$

In the $\triangle PMQ$ and $\triangle CMR$ we know that $PQ \parallel AC$ and QR is the transversal

We know that $\angle PQM$ and $\angle CRM$ are alternate angles and $\angle PMQ$ and $\angle CMR$ are vertically opposite angles

$$\angle PQM = \angle CRM$$

$$\angle PMQ = \angle CMR$$

By considering equation (1) and AAS congruence criterion

$$\triangle PMQ \cong \triangle CMR$$

We know that the corresponding parts of congruent triangles are equal

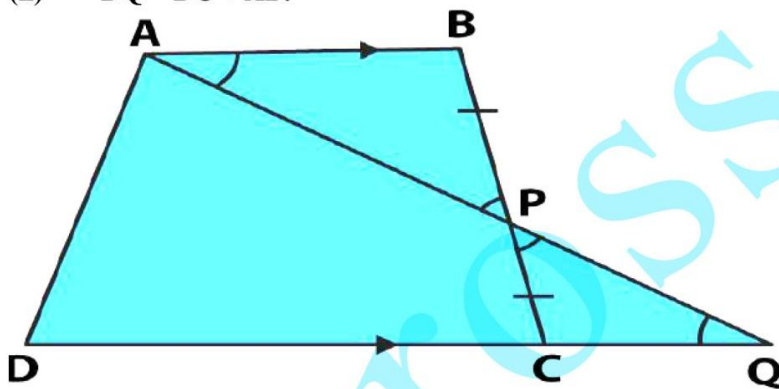
$$PM = MC$$

Therefore, it is proved that QR bisects PC .

13. In the given figure, ABCD is a quadrilateral in which $AB \parallel DC$ and P is the midpoint of BC . On producing, AP and DC meet at Q . Prove that

(i) $AB = CQ$,

(ii) $DQ = DC + AB$.



Solution:

It is given that

ABCD is a quadrilateral in which $AB \parallel DC$ and P is the midpoint of BC

Considering $\triangle ABP$ and $\triangle PCQ$

We know that $\angle PAB$ and $\angle PQC$ are alternate angles and $\angle APB$ and $\angle CPQ$ are vertically opposite angles

$$\angle PAB = \angle PQC$$

$$\angle APB = \angle CPQ$$

According to AAS congruence criterion

$$\triangle ABP \cong \triangle PCQ$$

$$AB = CQ \text{ (c. p. c. t.)} \dots\dots (i)$$

We know that

$$DQ = DC + CQ$$

By substituting CQ as AB we get

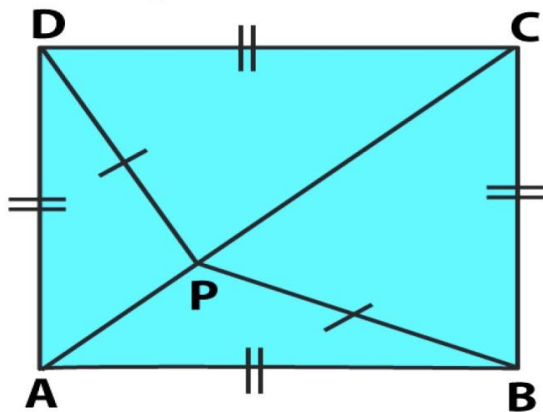
$$DQ = DC + AB \dots\dots (ii)$$

Therefore, it is proved that $AB = CQ$ and $DQ = DC + AB$.

14. In the given figure, ABCD is a square and P is a point inside it such that $PB = PD$. Prove that CPA

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is a straight line.



Solution:

It is given that ABCD is a square and P is a point inside it such that $PB = PD$

Considering $\triangle APD$ and $\triangle APB$

We know that all the sides are equal in a square

So we get $DA = AB$

AP is common i.e. $AP = AP$

According to SSS congruence criterion

$\triangle APD \cong \triangle APB$

We get $\angle APD = \angle APB$ (c. p. c. t)..... (1)

Considering $\triangle CPD$ and $\triangle CPB$

We know that all the sides are equal in a square

So we get $CD = CB$

CP is common i.e. $CP = CP$

According to SSS congruence criterion

$\triangle CPD \cong \triangle CPB$

We get $\angle CPD = \angle CPB$ (c. p. c. t)..... (2)

By adding both the equation (1) and (2)

$\angle APD + \angle CPD = \angle APB + \angle CPB$ (3)

From the figure we know that the angles surrounding the point P is 360°

So we get

$\angle APD + \angle CPD + \angle APB + \angle CPB = 360^\circ$

By grouping we get

$\angle APB + \angle CPB = 360^\circ - (\angle APD + \angle CPD)$ (4)

Now by substitution of (4) in (3)

$\angle APD + \angle CPD = 360^\circ - (\angle APD + \angle CPD)$

On further calculation

$2(\angle APD + \angle CPD) = 360^\circ$

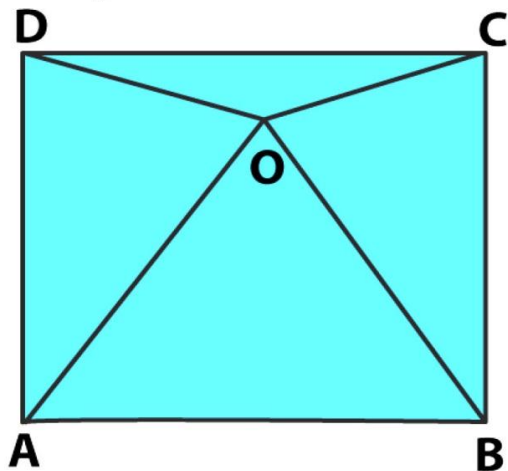
By division we get

$\angle APD + \angle CPD = 180^\circ$

Therefore, it is proved that CPA is a straight line.

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15. In the given figure, O is a point in the interior of square ABCD such that $\triangle OAB$ is an equilateral triangle. Show that $\triangle OCD$ is an isosceles triangle.



Solution:

We know that $\triangle OAB$ is an equilateral triangle

So it can be written as

$$\angle OAB = \angle OBA = \angle AOB = 60^\circ$$

From the figure we know that ABCD is a square

So we get

$$\angle A = \angle B = \angle C = \angle D = 90^\circ$$

In order to find the value of $\angle DAO$

We can write it as

$$\angle A = \angle DAO + \angle OAB$$

By substituting the values we get

$$90^\circ = \angle DAO + 60^\circ$$

On further calculation

$$\angle DAO = 90^\circ - 60^\circ$$

By subtraction

$$\angle DAO = 30^\circ$$

We also know that $\angle CBO = 30^\circ$

Considering the $\triangle OAD$ and $\triangle OBC$

We know that the sides of a square are equal

$$AD = BC$$

We know that the sides of an equilateral triangle are equal

$$OA = OB$$

By SAS congruence criterion

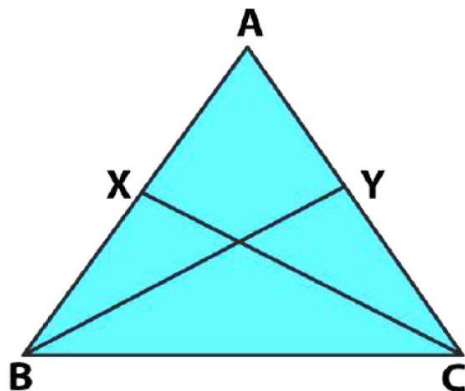
$$\triangle OAD \cong \triangle OBC$$

So we get $OD = OC$ (c. p. c. t)

Therefore, it is proved that $\triangle OCD$ is an isosceles triangle.

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16. In the adjoining figure, X and Y are respectively two points on equal sides AB and AC of $\triangle ABC$ such that $AX = AY$. Prove that $CX = BY$.



Solution:

It is given that $AX = AY$

Considering $\triangle AXC$ and $\triangle AYB$

$\angle A$ is common i.e. $\angle A = \angle A$

We know that the sides of equilateral triangle are equal so we get

$AC = AB$

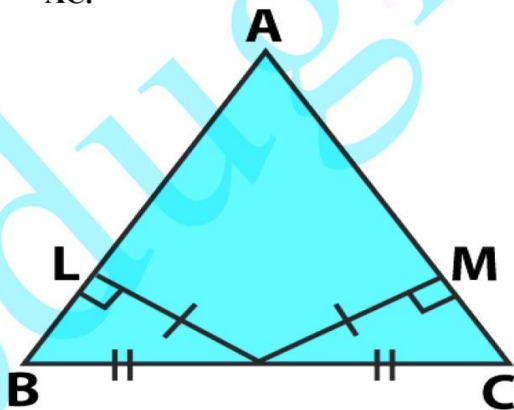
By SAS congruence criterion

$\triangle AXC \cong \triangle AYB$

$CX = YB$ (c. p. c. t)

Therefore, it is proved that $CX = BY$.

17. In $\triangle ABC$, D is the midpoint of BC. If $DL \perp AB$ and $DM \perp AC$ such that $DL = DM$, prove that $AB = AC$.



Solution:

It is given that D is the midpoint of BC

$DL \perp AB$ and $DM \perp AC$ such that $DL = DM$

Considering $\triangle BLD$ and $\triangle CMD$ as right angled triangle

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So we can write it as

$$\angle BLD = \angle CMD = 90^\circ$$

We know that $BD = CD$ and $DL = DM$

By RHS congruence criterion

$$\triangle BLD = \triangle CMD$$

$$\angle ABD = \angle ACD \text{ (c. p. c. t)}$$

Now, in $\triangle ABC$

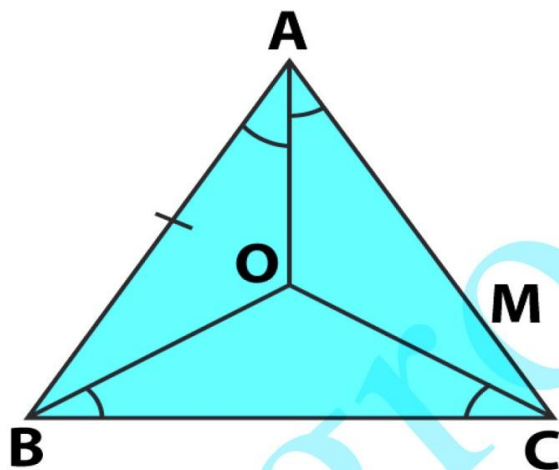
$$\angle ABD = \angle ACD$$

We know that the sides opposite to equal angles are equal so we get

$$AB = AC$$

Therefore, it is proved that $AB = AC$.

18. In $\triangle ABC$, $AB = AC$ and the bisectors $\angle B$ and $\angle C$ meet at a point O . Prove that $BO = CO$ and the ray AO is the bisector of $\angle A$.



Solution:

It is given that $AB = AC$ and the bisectors $\angle B$ and $\angle C$ meet at a point O

Consider $\triangle BOC$

So we get

$$\angle BOC = \frac{1}{2} \angle B \text{ and } \angle OCB = \frac{1}{2} \angle C$$

It is given that $AB = AC$ so we get $\angle B = \angle C$

So we get

$$\angle OBC = \angle OCB$$

We know that if the base angles are equal even the sides are equal

$$\text{So we get } OB = OC \text{ (1)}$$

$\angle B$ and $\angle C$ has the bisectors OB and OC so we get

$$\angle ABO = \frac{1}{2} \angle B \text{ and } \angle ACO = \frac{1}{2} \angle C$$

So we get

$$\angle ABO = \angle ACO \text{ (2)}$$

Considering $\triangle ABO$ and $\triangle ACO$ and equation (1) and (2)

It is given that $AB = AC$

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By SAS congruence criterion

$$\triangle ABO \cong \triangle ACO$$

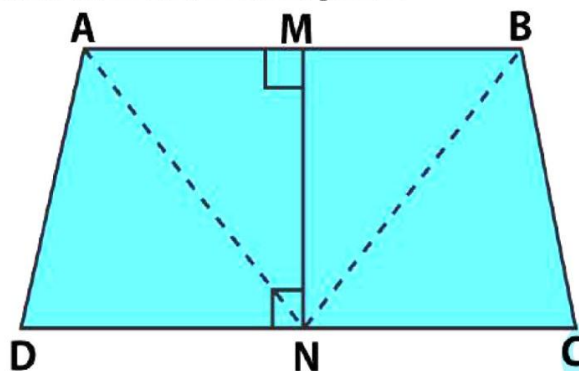
$$\angle BAO = \angle CAO \text{ (c. p. c. t)}$$

Therefore, it is proved that $BO = CO$ and the ray AO is the bisector of $\angle A$.

19. The line segments joining the midpoints M and N are parallel sides AB and DC respectively of a trapezium $ABCD$ is perpendicular to both the sides AB and DC . Prove that $AD = BC$.

Solution:

Construct AN and BN at the point N



Consider $\triangle ANM$ and $\triangle BNM$

We know that N is the midpoint of the line AB

So we get

$$AM = BM$$

From the figure we know that

$$\angle AMN = \angle BMN = 90^\circ$$

MN is common i.e. $MN = MN$

By SAS congruence criterion

$$\triangle ANM \cong \triangle BNM$$

$$AN = BN \text{ (c. p. c. t) } \dots\dots (1)$$

We know that

$$\angle ANM = \angle BNM \text{ (c. p. c. t)}$$

Subtracting LHS and RHS by 90°

$$90^\circ - \angle ANM = 90^\circ - \angle BNM$$

So we get

$$\angle AND = \angle BNC \dots\dots (2)$$

Now, consider $\triangle AND$ and $\triangle BNC$

$$AN = BN$$

$$\angle AND = \angle BNC$$

We know that N is the midpoint of the line DC

$$DN = CN$$

By SAS congruence criterion

$$\triangle AND \cong \triangle BNC$$

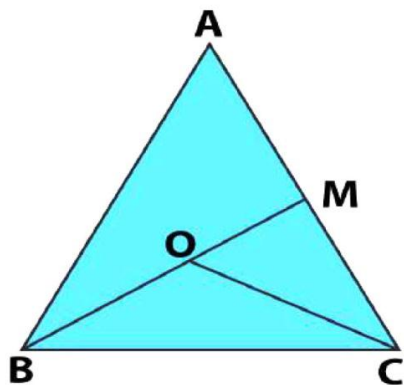
$$AD = BC \text{ (c. p. c. t)}$$

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Therefore, it is proved that $AD = BC$.

- 20. The bisectors $\angle B$ and $\angle C$ of an isosceles triangle with $AB = AC$ intersect each other at a point O . BO is produced to meet AC at a point M . Prove that $\angle MOC = \angle ABC$.**

Solution:



Consider $\triangle ABC$

It is given that $AB = AC$

So we get

$$\angle ABC = \angle ACB$$

Dividing both sides by 2 we get

$$\frac{1}{2} \angle ABC = \frac{1}{2} \angle ACB$$

So we get

$$\angle OBC = \angle OCB$$

By using the exterior angle property

We get

$$\angle MOC = \angle OBC + \angle OCB$$

We know that $\angle OBC = \angle OCB$

So we get

$$\angle MOC = 2 \angle OBC$$

We know that OB is the bisector of $\angle ABC$

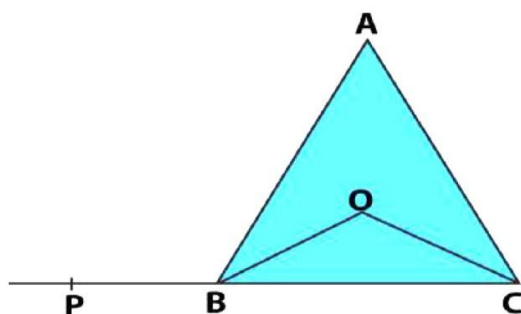
$$\angle MOC = \angle ABC$$

Therefore, it is proved that $\angle MOC = \angle ABC$.

- 21. The bisectors of $\angle B$ and $\angle C$ of an isosceles $\triangle ABC$ with $AB = AC$ intersect each other at a point O . Show that the exterior angle adjacent to $\angle ABC$ is equal to $\angle BOC$.**

Solution:

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Considering the $\triangle ABC$

It is given that $AB = AC$

So we get

$$\angle ABC = \angle ACB$$

Dividing by 2 both sides

$$\frac{1}{2} \angle ABC = \frac{1}{2} \angle ACB$$

So we get

$$\angle OBC = \angle OCB \dots\dots (1)$$

By using the angle sum property in $\triangle BOC$

$$\angle BOC + \angle OBC + \angle OCB = 180^\circ$$

Substituting equation (1)

$$\angle BOC + 2 \angle OBC = 180^\circ$$

So we get

$$\angle BOC + \angle ABC = 180^\circ$$

From the figure we know that $\angle ABC$ and $\angle ABP$ form a linear pair of angles so we get

$$\angle ABC + \angle ABP = 180^\circ$$

$$\angle ABC = 180^\circ - \angle ABP$$

By substituting the value in the above equation we get

$$\angle BOC + (180^\circ - \angle ABP) = 180^\circ$$

On further calculation

$$\angle BOC + 180^\circ - \angle ABP = 180^\circ$$

By subtraction

$$\angle BOC - \angle ABP = 180^\circ - 180^\circ$$

$$\angle BOC - \angle ABP = 0$$

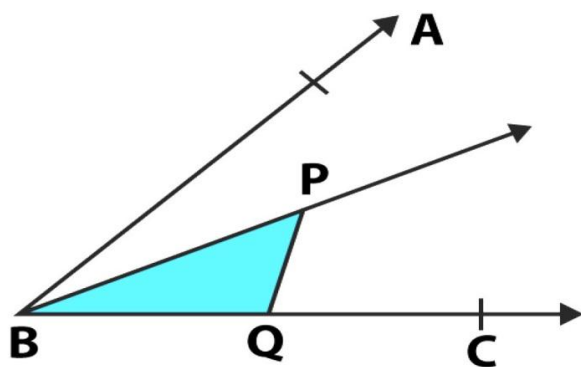
$$\angle BOC = \angle ABP$$

Therefore, it is proved that the exterior angle adjacent to $\angle ABC$ is equal to $\angle BOC$.

22. P is a point on the bisector of $\angle ABC$. If the line through P, parallel to BA meets BC at Q, prove that $\triangle BPQ$ is an isosceles triangle.

Solution:

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We know that $AB \parallel PQ$ and BP is a transversal

From the figure we know that $\angle ABP$ and $\angle BPQ$ are alternate angles

So we get

$$\angle ABP = \angle BPQ \dots (1)$$

We also know that BP is the bisector of $\angle ABC$

So we get

$$\angle ABP = \angle PBC \text{ and } \angle ABP = \angle BPQ \dots (2)$$

By considering equation (1) and (2) we get

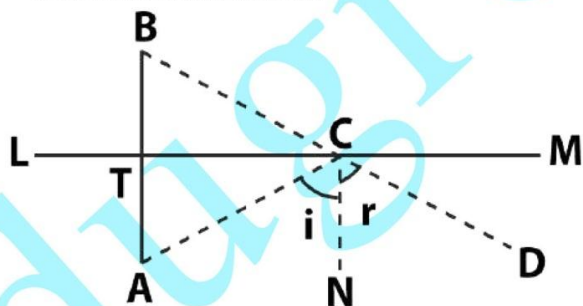
$$\angle BPQ = \angle PBQ$$

We know that the sides opposite to equal angles are equal

$$PQ = BQ$$

Therefore, it is proved that $\triangle BPQ$ is an isosceles triangle.

23. The image of an object placed at a point A before a plane mirror LM is seen at the point B by an observer at D , as shown in the figure. Prove that the image is as far behind the mirror as the object is in front of the mirror.



Solution:

According to the figure we need to prove that $AT = BT$

We know that

Angle of incidence = Angle of reflection

So we get

$$\angle ACN = \angle DCN \dots (1)$$

We know that $AB \parallel CN$ and AC is the transversal

From the figure we know that $\angle TAC$ and $\angle ACN$ are alternate angles

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$$\angle TAC = \angle CAN \dots\dots (2)$$

We know that $AB \parallel CN$ and BD is the transversal

From the figure we know that $\angle TBC$ and $\angle DCN$ are corresponding angles

$$\angle TBC = \angle DCN \dots\dots (3)$$

By considering the equation (1), (2) and (3)

We get

$$\angle TAC = \angle TBC \dots\dots (4)$$

Now in $\triangle ACT$ and $\triangle BCT$

$$\angle ATC = \angle BTC = 90^\circ$$

CT is common i.e. $CT = CT$

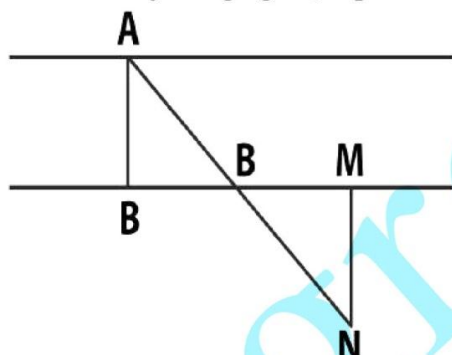
By AAS congruence criterion

$$\triangle ACT \cong \triangle BCT$$

$$AT = BT \text{ (c. p. c. t)}$$

Therefore, it is proved that the image is as far behind the mirror as the object is in front of the mirror.

24. In the adjoining figure, explain how one can find the breadth of the river without crossing it.



Solution:

Consider AB as the breadth of the river. Take a point M at a distance from B . Draw a perpendicular from the point M and name it as N so that it joins the point A as a straight line.

Now in $\triangle ABO$ and $\triangle NMO$

We know that

$$\angle OBA = \angle OMN = 90^\circ$$

We know that O is the midpoint of the line BM

So we get

$$OB = OM$$

From the figure we know that $\angle BAO$ and $\angle MON$ are vertically opposite angles

$$\angle BAO = \angle MON$$

By ASA congruence criterion

$$\triangle ABO \cong \triangle NMO$$

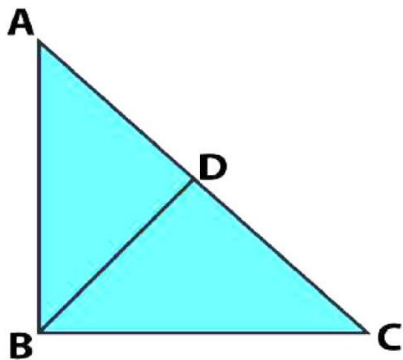
$$AB = NM \text{ (c. p. c. t)}$$

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Therefore, MN is the width of the river.

25. In a $\triangle ABC$, D is the midpoint of side AC such that $BD = \frac{1}{2} AC$. Show that $\angle ABC$ is a right angle.

Solution:



From the figure we know that D is the midpoint of the line AC

So we get

$$AD = CD = \frac{1}{2} AC$$

It is given that $BD = \frac{1}{2} AC$

So we can write it as

$$AD = BD = CD$$

Let us consider $AD = BD$

We know that the angles opposite to equal sides are equal

So we get

$$\angle BAD = \angle ABD \dots\dots(1)$$

Let us consider $CD = BD$

We know that the angles opposite to equal sides are equal

So we get

$$\angle BCD = \angle CBD \dots\dots (2)$$

By considering the angle sum property in $\triangle ABC$

We get

$$\angle ABC + \angle BAC + \angle BCA = 180^\circ$$

So we can write it as

$$\angle ABC + \angle BAD + \angle BCD = 180^\circ$$

By using equation (1) and (2) we get

$$\angle ABC + \angle ABD + \angle CBD = 180^\circ$$

So we get

$$\angle ABC + \angle ABC = 180^\circ$$

By addition

$$2 \angle ABC = 180^\circ$$

By division

$$\angle ABC = 90^\circ$$

Therefore, $\angle ABC$ is a right angle.

26. "If two sides and an angle of one triangle are equal to two sides and an angle of another triangle

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then the two triangles must be congruent.” Is the statement true? Why?

Solution:

The statement is false.

If two sides and the included angle of one triangle are equal to corresponding two sides and the included angle of another triangle then the two triangles must be congruent.

27. “If two angles and a side of one triangle are equal to two angles and a side of another triangle then the two triangles must be congruent.” Is the statement true? Why?

Solution:

The statement is false.

If two angles and the corresponding side of one triangle are equal to the two included angles and the corresponding side of another triangle then the two triangles must be congruent.

RS Aggarwal Solutions for Class 9 Maths Chapter 9 –
Congruence of Triangles and Inequalities in a Triangle**EXERCISE 9B****PAGE: 296**

1. Is it possible to construct a triangle with lengths of its sides as given below? Give reason for your answer.

- (i) 5cm, 4cm, 9cm
- (ii) 8cm, 7cm, 4cm
- (iii) 10cm, 5cm, 6cm
- (iv) 2.5cm, 5cm, 7cm
- (v) 3cm, 4cm, 8cm

Solution:

- (i) No. It is not possible to construct a triangle with lengths of its sides 5cm, 4cm and 9cm because the sum of two sides is not greater than the third side i.e. $5 + 4$ is not greater than 9.
- (ii) Yes. It is possible to construct a triangle with lengths of its sides 8cm, 7cm and 4cm because the sum of two sides of a triangle is greater than the third side.
- (iii) Yes. It is possible to construct a triangle with lengths of its sides 10cm, 5cm and 6cm because the sum of two sides of a triangle is greater than the third side.
- (iv) Yes. It is possible to construct a triangle with lengths of its sides 2.5cm, 5cm and 7cm because the sum of two sides of a triangle is greater than the third side.
- (v) No. It is not possible to construct a triangle with lengths of its sides 3cm, 4cm and 8cm because the sum of two sides is not greater than the third side.

2. In $\triangle ABC$, $\angle A = 50^\circ$ and $\angle B = 60^\circ$. Determine the longest and the shortest sides of the triangle.

Solution:

Consider $\triangle ABC$

Based on the sum property we can write it as

$$\angle A + \angle B + \angle C = 180^\circ$$

By substituting the values we get

$$50^\circ + 60^\circ + \angle C = 180^\circ$$

On further calculation

$$\angle C = 180^\circ - 50^\circ - 60^\circ$$

By subtraction

$$\angle C = 180^\circ - 110^\circ$$

$$\angle C = 70^\circ$$

So we have $\angle A < \angle B < \angle C$

We get

$$BC < AC < AB$$

Therefore, the longest side of the triangle is AB and the shortest side is BC.

3.

- (i) In $\triangle ABC$, $\angle A = 90^\circ$. What is the longest side?
- (ii) In $\triangle ABC$, $\angle A = \angle B = 45^\circ$. Which is its longest side?
- (iii) In $\triangle ABC$, $\angle A = 100^\circ$ and $\angle C = 50^\circ$. Which is its shortest side?

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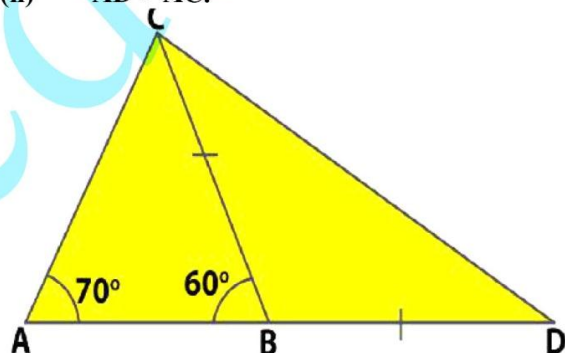
Solution:

- (i) It is given that $\angle A = 90^\circ$
 We know that in a right angled triangle the highest angle is 90° and the sum of all the angles is 180°
 So we get that $\angle A$ is the greatest angle in $\triangle ABC$
 Hence, BC is the longest side which is opposite to $\angle A$
- (ii) In $\triangle ABC$ it is given that $\angle A = \angle B = 45^\circ$
 Based on the sum property of the triangle
 $\angle A + \angle B + \angle C = 180^\circ$
 To find $\angle C$
 $\angle C = 180^\circ - \angle A - \angle B$
 By substituting the values in the above equation
 $\angle C = 180^\circ - 45^\circ - 45^\circ$
 By subtraction
 $\angle C = 180^\circ - 90^\circ$
 $\angle C = 90^\circ$
 So we get that $\angle C$ is the greatest angle in $\triangle ABC$
 Hence, AB is the longest side which is opposite to $\angle C$.
- (iii) In $\triangle ABC$ it is given that $\angle A = 100^\circ$ and $\angle C = 50^\circ$
 Based on the sum property of the triangle
 $\angle A + \angle B + \angle C = 180^\circ$
 To find $\angle B$
 $\angle B = 180^\circ - \angle A - \angle C$
 By substituting the values in the above equation
 $\angle B = 180^\circ - 100^\circ - 50^\circ$
 By subtraction
 $\angle B = 180^\circ - 150^\circ$
 $\angle B = 30^\circ$

 So we get $\angle B < \angle C < \angle A$
 i.e. $AC < AB < BC$
 Hence, AC is the shortest side in $\triangle ABC$.

4. In $\triangle ABC$, side AB is produced to D such that $BD = BC$. If $\angle A = 70^\circ$ and $\angle B = 60^\circ$, prove that

- (i) $AD > CD$
 (ii) $AD > AC$.



Solution:

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In $\triangle ABC$ it is given that $\angle A = 70^\circ$ and $\angle B = 60^\circ$

Based on the sum property of the triangle

$$\angle A + \angle B + \angle C = 180^\circ$$

To find $\angle C$

$$\angle C = 180^\circ - \angle A - \angle B$$

By substituting the values in the above equation

$$\angle C = 180^\circ - 70^\circ - 60^\circ$$

$$\angle C = 180^\circ - 130^\circ$$

By subtraction

$$\angle C = 50^\circ$$

Consider $\triangle BCD$

We know that $\angle CBD$ is the exterior angle of $\angle ABC$

So we get

$$\angle CBD = \angle DAC + \angle ACB$$

By substituting the values in the above equation

$$\angle CBD = 70^\circ + 50^\circ$$

By addition

$$\angle CBD = 120^\circ$$

It is given that $BC = BD$

So we can write it as

$$\angle BCD = \angle BDC$$

Based on the sum property of the triangle

$$\angle BCD + \angle BDC + \angle CBD = 180^\circ$$

So we get

$$\angle BCD + \angle BDC = 180^\circ - \angle CBD$$

By substituting values in the above equation

$$\angle BCD + \angle BDC = 180^\circ - 120^\circ$$

$$\angle BCD + \angle BDC = 60^\circ$$

It can be written as

$$2\angle BCD = 60^\circ$$

By division

$$\angle BCD = \angle BDC = 30^\circ$$

In $\triangle ACD$

It is given that $\angle A = 70^\circ$ and $\angle B = 60^\circ$

We can write it as

$$\angle ACD = \angle ACB + \angle BCD$$

By substituting the values we get

$$\angle ACD = 50^\circ + 30^\circ$$

By addition

$$\angle ACD = 80^\circ$$

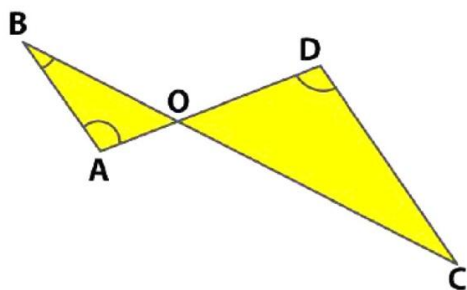
So we get to know that $\angle ACD$ is the greatest angle and the side opposite to it i.e. AD is the longest side.

Therefore, it is proved that $AD > CD$

We know that $\angle BDC$ is the smallest angle and the side opposite to it i.e. AC is the shortest side.

Therefore, it is proved that $AD > AC$.

5. In the given figure, $\angle B < \angle A$ and $\angle C < \angle D$. Show that $AD < BC$.

**RS Aggarwal Solutions for Class 9 Maths Chapter 9 –
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In the figure it is given that $\angle B < \angle A$ and $\angle C < \angle D$

Consider triangle AOB

Since $\angle B < \angle A$

We get

$AO < BO$ (1)

Consider triangle COD

Since $\angle C < \angle D$

$DO < CO$ (2)

By adding both the equations we get

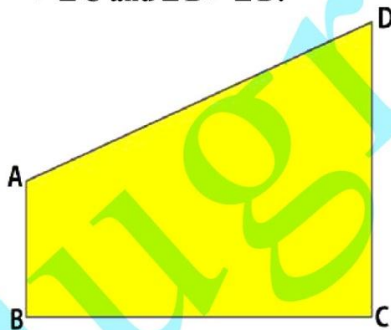
$AO + DO < BO + CO$

So we get

$AD < BC$

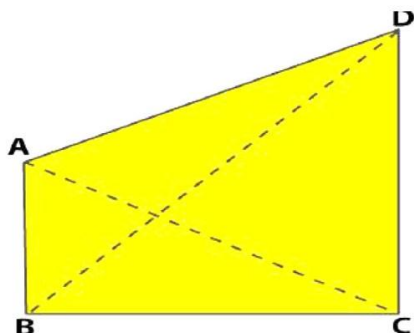
Therefore, it is proved that $AD < BC$.

6. AB and CD are respectively the smallest and largest sides of a quadrilateral ABCD. Show that $\angle A > \angle C$ and $\angle B > \angle D$.

**Solution:**

Construct two lines AC and BD in the given quadrilateral

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Consider $\triangle ABC$

We know that $BC > AB$

It can be written as

$$\angle BAC > \angle ACB \dots\dots (i)$$

Consider $\triangle ACD$

We know that $CD > AD$

It can be written as

$$\angle CAD > \angle ACD \dots\dots (ii)$$

By adding both the equations we get

$$\angle BAC + \angle CAD > \angle ACB + \angle ACD$$

So we get

$$\angle A > \angle C$$

Consider $\triangle ADB$

We know that $AD > AB$

It can be written as

$$\angle ABD > \angle ADB \dots\dots (iii)$$

Consider $\triangle BDC$

We know that $CD > BC$

It can be written as

$$\angle CBD > \angle BDC \dots\dots (iv)$$

By adding both the equations we get

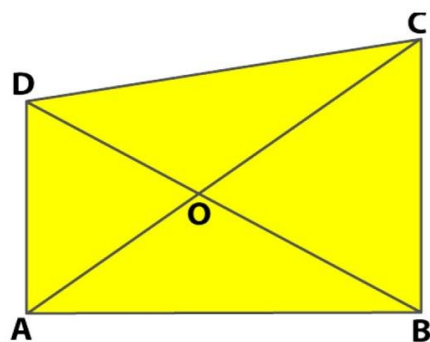
$$\angle ABD + \angle CBD > \angle ADB + \angle BDC$$

So we get

$$\angle B > \angle D$$

7. In a quadrilateral ABCD, show that $(AB + BC + CD + DA) > (AC + BD)$.

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Solution:

Consider $\triangle ABC$

We know that

$$AB + BC > AC \dots\dots (1)$$

Consider $\triangle ACD$

We know that

$$DA + CD > AC \dots\dots (2)$$

Consider $\triangle ADB$

We know that

$$DA + AB > BD \dots\dots (3)$$

Consider $\triangle BDC$

We know that

$$BC + CD > BD \dots\dots (4)$$

By adding all the equations

$$AB + BC + DA + CD + DA + AB + BC + CD > AC + AC + BD + BD$$

So we get

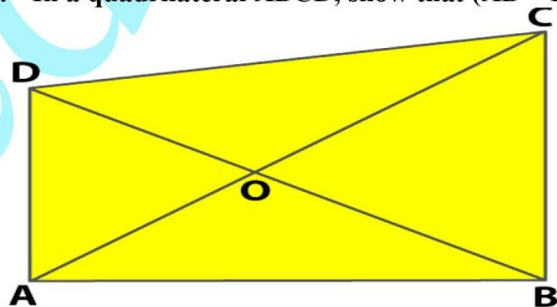
$$2(AB + BD + CD + DA) > 2(AC + BD)$$

Dividing by 2 both the sides

$$AB + BD + CD + DA > AC + BD$$

Therefore, it is proved that $AB + BD + CD + DA > AC + BD$.

8. In a quadrilateral ABCD, show that $(AB + BC + CD + DA) < 2(BD + AC)$.



Solution:

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Consider $\triangle AOB$

We know that

$$AO + BO > AB \dots (1)$$

Consider $\triangle BOC$

We know that

$$BO + CO > BC \dots (2)$$

Consider $\triangle COD$

We know that

$$CO + DO > CD \dots (3)$$

Consider $\triangle AOD$

We know that

$$DO + AO > DA \dots (4)$$

By adding all the equations

$$AO + BO + BO + CO + CO + DO + DO + AO > AB + BC + CD + DA$$

So we get

$$2(AO + CO) + 2(BO + DO) > AB + BC + CD + DA$$

On further calculation

$$2AC + 2BD > AB + BC + CD + DA$$

By taking 2 as common

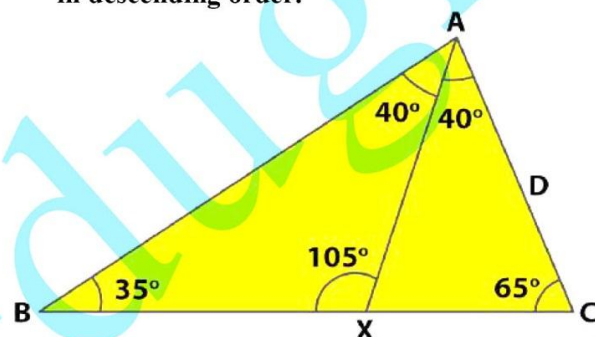
$$2(AC + BD) > AB + BC + CD + DA$$

So we get

$$AB + BC + CD + DA < 2(AC + BD)$$

Therefore, it is proved that $AB + BC + CD + DA < 2(AC + BD)$

9. In $\triangle ABC$, $\angle B = 35^\circ$, $\angle C = 65^\circ$ and the bisector of $\angle BAC$ meets BC in X . Arrange AX , BX and CX in descending order.



Solution:

Consider $\triangle ABC$

By sum property of a triangle

$$\angle A + \angle B + \angle C = 180^\circ$$

To find $\angle A$

$$\angle A = 180^\circ - \angle B - \angle C$$

By substituting the values

$$\angle A = 180^\circ - 35^\circ - 65^\circ$$

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By subtraction
 $\angle A = 180^\circ - 100^\circ$
 $\angle A = 80^\circ$

We know that
 $\angle BAX = \frac{1}{2} \angle A$
So we get
 $\angle BAX = \frac{1}{2} (80^\circ)$
By division
 $\angle BAX = 40^\circ$

Consider $\triangle ABX$
It is given that $\angle B = 35^\circ$ and $\angle BAX = 40^\circ$
By sum property of a triangle
 $\angle BAX + \angle BXA + \angle XBA = 180^\circ$
To find $\angle BXA$
 $\angle BXA = 180^\circ - \angle BAX - \angle XBA$
By substituting values
 $\angle BXA = 180^\circ - 35^\circ - 40^\circ$
By subtraction
 $\angle BXA = 180^\circ - 75^\circ$
 $\angle BXA = 105^\circ$

We know that $\angle B$ is the smallest angle and the side opposite to it i.e. AX is the smallest side.
So we get $AX < BX$ (1)

Consider $\triangle AXC$
 $\angle CAX = \frac{1}{2} \angle A$
So we get
 $\angle CAX = \frac{1}{2} (80^\circ)$
By division
 $\angle CAX = 40^\circ$

By sum property of a triangle
 $\angle AXC + \angle CAX + \angle CXA = 180^\circ$
To find $\angle AXC$
 $\angle AXC = 180^\circ - \angle CAX - \angle CXA$
By substituting values
 $\angle AXC = 180^\circ - 40^\circ - 65^\circ$
So we get
 $\angle AXC = 180^\circ - 105^\circ$
By subtraction
 $\angle AXC = 75^\circ$

So we know that $\angle CAX$ is the smallest angle and the side opposite to it i.e. CX is the smallest side.
We get
 $CX < AX$ (2)

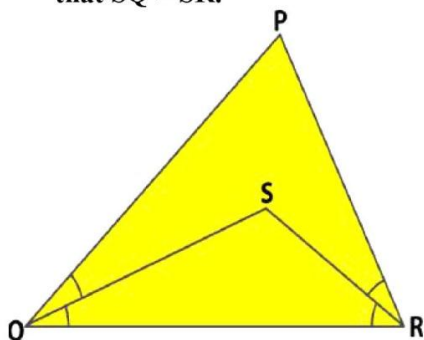
By considering equation (1) and (2)

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$$BX > AX > CX$$

Therefore, $BX > AX > CX$ is the descending order.

- 10. In the given figure, $PQ > PR$ and QS and RS are the bisectors of $\angle Q$ and $\angle R$ respectively. Show that $SQ > SR$.**



Solution:

Consider $\triangle PQR$

It is given that $PQ > PR$

So we get

$$\angle PRQ > \angle PQR$$

Dividing both sides by 2 we get

$$\frac{1}{2} \angle PRQ > \frac{1}{2} \angle PQR$$

From the figure we get

$$\angle SRQ > \angle SQR$$

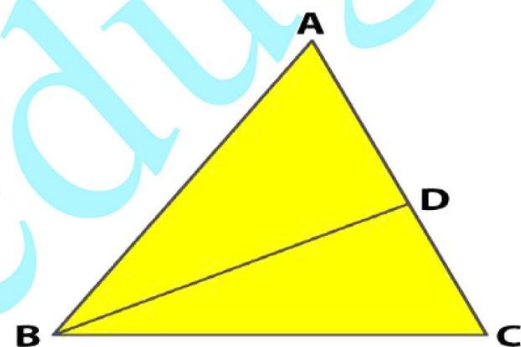
So we get

$$SQ > SR$$

Therefore, it is proved that $SQ > SR$.

- 11. D is any point on the side AC of $\triangle ABC$ with $AB = AC$. Show that $CD < BD$.**

Solution:



Consider $\triangle ABC$

It is given that $AB = AC$

So we get

$$\angle ABC = \angle ACB \dots\dots (1)$$

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From the figure we know that

$$\angle ABC = \angle ADB + \angle DBC$$

So we get

$$\angle ABC > \angle DBC$$

From equation (1)

$$\angle ACB > \angle DBC$$

$$\text{i.e. } \angle DCB > \angle DBC$$

It means that

$$BD > CD$$

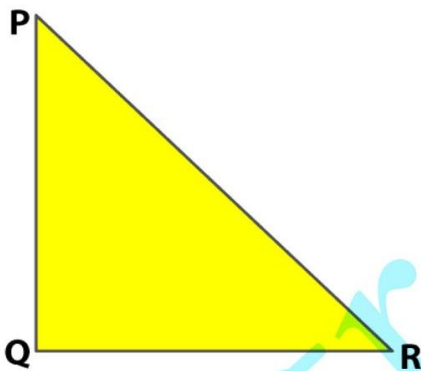
So we get

$$CD < BD$$

Therefore, it is proved that $CD < BD$.

12. Prove that in a triangle, other than an equilateral triangle, angle opposite the longest side is greater $\frac{2}{3}$ of a right angle.

Solution:



Consider $\triangle PQR$ where PR is the longest side

So we get $PR > PQ$

$$\text{i.e. } \angle Q > \angle R \dots\dots (1)$$

We also know that $PR > QR$

$$\text{i.e. } \angle Q > \angle P \dots\dots (2)$$

By adding both the equations

$$\angle Q + \angle Q > \angle R + \angle P$$

So we get

$$2\angle Q > \angle R + \angle P$$

By adding $\angle Q$ on both LHS and RHS

$$2\angle Q + \angle Q > \angle R + \angle P + \angle Q$$

We know that $\angle R + \angle P + \angle Q = 180^\circ$

So we get

$$3\angle Q > 180^\circ$$

By division

$$\angle Q > 60^\circ$$

So we get

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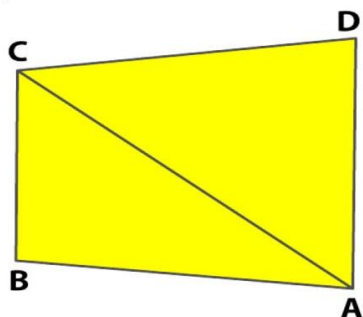
$$\angle Q > \frac{2}{3} (90^\circ)$$

i.e. $\angle Q > \frac{2}{3}$ of a right angle

Therefore, it is proved that in a triangle, other than an equilateral triangle, angle opposite the longest side is greater $\frac{2}{3}$ of a right angle.

13. In the given figure, prove that

- (i) $CD + DA + AB > BC$
(ii) $CD + DA + AB + BC > 2AC$.



Solution:

- (i) Consider $\triangle CDA$
We know that $CD + DA > AC$ (1)

Consider $\triangle ABC$
We know that $AC + AB > BC$ (2)

By adding both the equations we get

$$CD + DA + AC + AB > AC + BC$$

By subtracting AC on both the sides

$$CD + DA + AC + AB - AC > AC + BC - AC$$

So we get

$$CD + DA + AB > BC$$

Therefore, it is proved that $CD + DA + AB > BC$.

- (ii) Consider $\triangle CDA$
We know that $CD + DA > AC$ (1)

Consider $\triangle ABC$
We know that $AB + BC > AC$ (2)

By adding both the equations we get

$$CD + DA + AB + BC > AC + AC$$

$$\text{So we get } CD + DA + AB + BC > 2AC$$

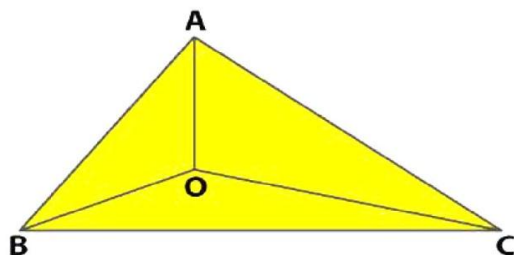
Therefore, it is proved that $CD + DA + AB + BC > 2AC$.

14. If O is a point within $\triangle ABC$, show that

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- (i) $AB + AC > OB + OC$
 (ii) $AB + BC + CA > OA + OB + OC$
 (iii) $OA + OB + OC > \frac{1}{2} (AB + BC + CA)$

Solution:



- (i) It is given that O is a point within $\triangle ABC$
 Consider $\triangle ABC$
 We know that $AB + AC > BC$ (1)

Consider $\triangle OBC$
 We know that $OB + OC > BC$ (2)

By subtracting both the equations we get
 $(AB + AC) - (OB + OC) > BC - BC$

So we get

$$(AB + AC) - (OB + OC) > 0$$

$$AB + AC > OB + OC$$

Therefore, it is proved that $AB + AC > OB + OC$.

- (ii) We know that $AB + AC > OB + OC$
 In the same way we can write
 $AB + BC > OA + OC$ and $AC + BC > OA + OB$
 By adding all the equations we get
 $AB + AC + AB + BC + AC + BC > OB + OC + OA + OC + OA + OB$
 So we get
 $2 (AB + BC + AC) > 2 (OA + OB + OC)$
 Dividing by 2 both sides
 $AB + BC + AC > OA + OB + OC$

- (iii) Consider $\triangle OAB$
 We know that $OA + OB > AB$ (1)

Consider $\triangle OBC$
 We know that $OB + OC > BC$ (2)

Consider $\triangle OCA$
 $OC + OA > CA$ (3)

By adding all the equations

$$OA + OB + OB + OC + OC + OA > AB + BC + CA$$

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So we get

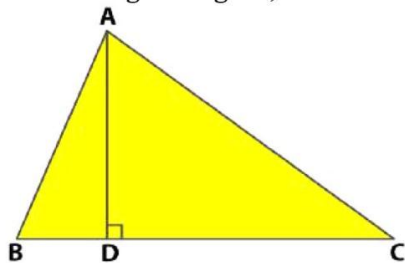
$$2(OA + OB + OC) > AB + BC + CA$$

Dividing by 2

$$OA + OB + OC > \frac{1}{2}(AB + BC + CA)$$

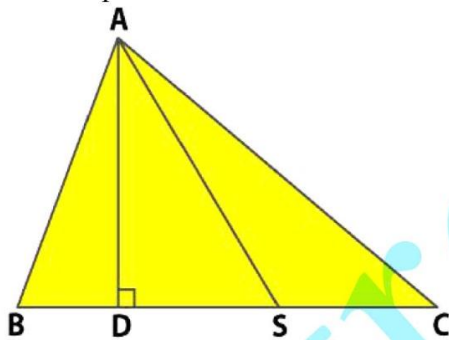
Therefore, it is proved that $OA + OB + OC > \frac{1}{2}(AB + BC + CA)$.

15. In the given figure, $AD \perp BC$ and $CD > BD$. Show that $AC > AB$.



Solution:

Consider point S on the line BC so that $BD = SD$ and join AS.



Consider $\triangle ADB$ and $\triangle ADS$

We know that $SD = BD$

Since AD is a perpendicular we know that

$$\angle ADB = \angle ADS = 90^\circ$$

AD is common i.e. $AD = AD$

By SAS congruence criterion

$$\triangle ADB \cong \triangle ADS$$

$$AB = AS \text{ (c. p. c. t)}$$

Consider $\triangle ABS$

We know that $AB = AS$

From the figure we know that $\angle ASB$ and $\angle ABS$ are angles opposite to the equal sides

$$\angle ASB = \angle ABS \dots (1)$$

Consider $\triangle ACS$

From the figure we know that $\angle ASB$ and $\angle ACS$ are angles opposite to the equal sides

$$\angle ASB = \angle ACS \dots (2)$$

Considering the equations (1) and (2)

$$\angle ABS > \angle ACS$$

It can be written as

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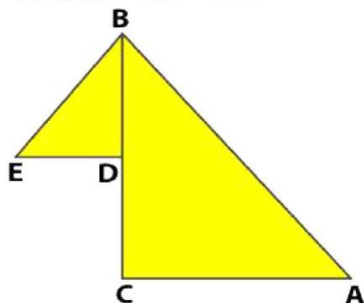
$$\angle ABC > \angle ACB$$

So we get

$$AC > AB$$

Therefore, it is proved that $AC > AB$.

16. In the given figure, D is a point on side BC of a $\triangle ABC$ and E is a point such that $CD = DE$. Prove that $AB + AC > BE$.



Solution:

Consider $\triangle ABC$

We know that

$$AB + AC > BC$$

It can be written as

$$AB + AC > BD + DC$$

We know that $CD = DE$

So we get

$$AB + AC > BD + DE \dots\dots (1)$$

Consider $\triangle BED$

We know that

$$BD + DE > BE \dots\dots (2)$$

Considering both the equations we get

$$AB + AC > BE.$$