EXERCISE 8 PAGE: 252

1. In △ ABC, if ∠B = 76° and ∠C = 48°, find ∠A.
Solution:

We know that the sum of all the angles in a triangle is 180°.
So we can write it as
$\angle A + \angle B + \angle C = 180^\circ$
By substituting the values in the above equation we get
$\angle A + 76^\circ + 48^\circ = 180^\circ$
On further calculation
$\angle A = 180^\circ - 76^\circ - 48^\circ$
By subtraction we get
$\angle A = 180^\circ - 124^\circ$
$\angle A = 56^\circ$

Therefore, the value of ∠A is 56°.

2. The angles of a triangle are in the ratio 2: 3: 4. Find the angles.
Solution:

Let us consider the measure of the angles in a triangle as $2x^\circ$, $3x^\circ$ and $4x^\circ$
We know that the sum of all the angles in a triangle is 180°.
So we can write it as
$2x + 3x + 4x = 180^\circ$
By addition
$9x = 180^\circ$
By division
$x = 180/9$
$x = 20$

By substituting the values of x
$2x^\circ = 2\,(20) = 40^\circ$
$3x^\circ = 3\,(20) = 60^\circ$
$4x^\circ = 4\,(20) = 80^\circ$

Therefore, the angles are 40°, 60° and 80°

3. In △ ABC, if 3 ∠A = 4 ∠B = 6 ∠C, calculate ∠A, ∠B and ∠C.
Solution:

Consider $3\angle A = 4\angle B = 6\angle C = x$
So we can write it as
$3\angle A = x$
$\angle A = x/3$

$4\angle B = x$
$\angle B = x/4$

$6\angle C = x$

∠C = x/6

We know that the sum of all the angles in a triangle is 180°.
So we can write it as
∠A + ∠B + ∠C = 180°
By substituting the values in the above equation we get
(x/3) + (x/4) + (x/6) = 180°
LCM of 3, 4 and 6 is 12
So we get
(4x + 3x + 2x)/12 = 180°
By addition
9x/12 = 180°
By cross multiplication
9x = 180 × 12
9x = 2160
By division
x = 2160/9
x = 240

By substituting the values of x
∠A = x/3 = 240/3 = 80°
∠B = x/4 = 240/4 = 60°
∠C = x/6 = 240/6 = 40°

Therefore, the value of ∠A, ∠B and ∠C is 80°, 60° and 40°.

4. In △ ABC, if ∠A + ∠B = 108° and ∠B + ∠C = 130°, find ∠A, ∠B and ∠C.
Solution:

It is given that ∠A + ∠B = 108° …… (1)
We know that the sum of all the angles in a triangle is 180°.
So we can write it as
∠A + ∠B + ∠C = 180°
By substituting ∠A + ∠B = 108° in the above equation
108° + ∠C = 180°
On further calculation
∠C = 180° - 108°
By subtraction
∠C = 72°

It is given that ∠B + ∠C = 130°
By substituting the value of ∠C
∠B + 72° = 130°
On further calculation
∠B = 130° - 72°
By subtraction
∠B = 58°

By substituting ∠B = 58° in equation (1)
So we get

$\angle A + \angle B = 108^\circ$
$\angle A + 58^\circ = 108^\circ$
On further calculation
$\angle A = 108^\circ - 58^\circ$
By subtraction
$\angle A = 50^\circ$

Therefore, $\angle A = 50^\circ$, $\angle B = 58^\circ$ and $\angle C = 72^\circ$

5. In $\triangle$ ABC, if $\angle A + \angle B = 125^\circ$ and $\angle A + \angle C = 113^\circ$, find $\angle A$, $\angle B$ and $\angle C$.
Solution:

It is given that $\angle A + \angle B = 125^\circ$ …. (1)
We know that the sum of all the angles in a triangle is 180°.
So we can write it as
$\angle A + \angle B + \angle C = 180^\circ$
By substituting $\angle A + \angle B = 125^\circ$ in the above equation
$125^\circ + \angle C = 180^\circ$
On further calculation
$\angle C = 180^\circ - 125^\circ$
By subtraction
$\angle C = 55^\circ$

It is given that $\angle A + \angle C = 113^\circ$
By substituting the value of $\angle C$
$\angle A + 55^\circ = 113^\circ$
On further calculation
$\angle A = 113^\circ - 55^\circ$
By subtraction
$\angle A = 58^\circ$

By substituting $\angle A = 58^\circ$ in equation (1)
So we get
$\angle A + \angle B = 125^\circ$
$58^\circ + \angle B = 125^\circ$
On further calculation
$\angle B = 125^\circ - 58^\circ$
By subtraction
$\angle B = 67^\circ$

Therefore, $\angle A = 58^\circ$, $\angle B = 67^\circ$ and $\angle C = 55^\circ$

6. In $\triangle$ PQR, if $\angle P - \angle Q = 42^\circ$ and $\angle Q - \angle R = 21^\circ$, find $\angle P$, $\angle Q$ and $\angle R$.
Solution:

It is given that $\angle P - \angle Q = 42^\circ$
It can be written as
$\angle P = 42^\circ + \angle Q$
We know that the sum of all the angles in a triangle is 180°.
So we can write it as

$\angle P + \angle Q + \angle R = 180^o$
By substituting $\angle P = 42^o + \angle Q$ in the above equation
$42^o + \angle Q + \angle Q + \angle R = 180^o$
On further calculation
$42^o + 2\angle Q + \angle R = 180^o$
$2\angle Q + \angle R = 180^o - 42^o$
By subtraction we get
$2\angle Q + \angle R = 138^o$ …. (i)

It is given that $\angle Q - \angle R = 21^o$
It can be written as
$\angle R = \angle Q - 21^o$
By substituting the value of $\angle R$ in equation (i)
$2\angle Q + \angle Q - 21^o = 138^o$
On further calculation
$3\angle Q - 21^o = 138^o$
$3\angle Q = 138^o + 21^o$
By addition
$3\angle Q = 159^o$
By division
$\angle Q = 159/3$
$\angle Q = 53^o$

By substituting $\angle Q = 53^o$ in $\angle P = 42^o + \angle Q$
So we get
$\angle P = 42^o + 53^o$
By addition
$\angle P = 95^o$

By substituting $\angle Q$ in $\angle Q - \angle R = 21^o$
$53^o - \angle R = 21^o$
On further calculation
$\angle R = 53^o - 21^o$
By subtraction
$\angle R = 32^o$

Therefore, $\angle P = 95^o$, $\angle Q = 53^o$ and $\angle R = 32^o$

7. The sum of two angles of a triangle is 116º and their difference is 24º. Find the measure of each angle of the triangle.

Solution:

Let us consider the sum of two angles as $\angle A + \angle B = 116^o$ and the difference can be written as $\angle A - \angle B = 24^o$
We know that the sum of all the angles in a triangle is 180^o.
So we can write it as
$\angle A + \angle B + \angle C = 180^o$
By substituting $\angle A + \angle B = 116^o$ in the above equation
$116^o + \angle C = 180^o$
On further calculation
$\angle C = 180^o - 116^o$

By subtraction
$\angle C = 64^\circ$

It is given that $\angle A - \angle B = 24^\circ$
It can be written as $\angle A = 24^\circ + \angle B$
Now by substituting $\angle A = 24^\circ + \angle B$ in $\angle A + \angle B = 116^\circ$
$\angle A + \angle B = 116^\circ$
$24^\circ + \angle B + \angle B = 116^\circ$
On further calculation
$24^\circ + 2\angle B = 116^\circ$
By subtraction
$2\angle B = 116^\circ - 24^\circ$
$2\angle B = 92^\circ$
By division
$\angle B = 92/2$
$\angle B = 46^\circ$

By substituting $\angle B = 46^\circ$ in $\angle A = 24^\circ + \angle B$
We get
$\angle A = 24^\circ + 46^\circ$
By addition
$\angle A = 70^\circ$

Therefore, $\angle A = 70^\circ$, $\angle B = 46^\circ$ and $\angle C = 64^\circ$

8. Two angles of a triangle are equal and the third angle is greater than each one of them by 18°. Find the angles.

Solution:

Consider $\angle A$ and $\angle B$ in a triangle is x°
We know that the sum of all the angles in a triangle is 180°.
So we can write it as
$\angle A + \angle B + \angle C = 180^\circ$
By substituting the values
$x^\circ + x^\circ + \angle C = 180^\circ$
By addition
$2x^\circ + \angle C = 180^\circ$ ….. (1)

According to the question we get
$\angle C = x^\circ + 18^\circ$ ……. (2)

By substituting (2) in (1) we get
$2x^\circ + x^\circ + 18^\circ = 180^\circ$
On further calculation
$3x^\circ + 18^\circ = 180^\circ$
By subtraction
$3x^\circ = 180^\circ - 18^\circ$
$3x^\circ = 162^\circ$
By division
$x^\circ = 162/3$

$x^\circ = 54^\circ$

By substituting the values of x
$\angle A = \angle B = 54^\circ$
$\angle C = 54^\circ + 18^\circ = 72^\circ$

Therefore, $\angle A = 54^\circ$, $\angle B = 54^\circ$ and $\angle C = 72^\circ$

9. Of the three angles of a triangle, one is twice the smallest and another one is thrice the smallest. Find the angles.
Solution:

Consider $\angle C$ is the smallest angle among $\angle ABC$
According to the question
We can write it as
$\angle A = 2\angle C$ and $\angle B = 3\angle C$
We know that the sum of all the angles in a triangle is 180°.
So we can write it as
$\angle A + \angle B + \angle C = 180^\circ$
By substituting the values
$2\angle C + 3\angle C + \angle C = 180^\circ$
By addition
$6\angle C = 180^\circ$
By division
$\angle C = 180/6$
$\angle C = 30^\circ$

Now by substituting the value of $\angle C$ we get
$\angle A = 2\angle C = 2(30^\circ) = 60^\circ$
$\angle B = 3\angle C = 3(30^\circ) = 90^\circ$

Therefore, $\angle A = 60^\circ$, $\angle B = 90^\circ$ and $\angle C = 30^\circ$.

10. In a right-angled triangle, one of the acute angles measures 53°. Find the measure of each angle of the triangle.
Solution:

Consider ABC as a right-angled triangle with $\angle C = 90^\circ$
We know that the sum of all the angles in a triangle is 180°.
So we can write it as
$\angle A + \angle B + \angle C = 180^\circ$
We can write it as
$\angle A + \angle B = 180^\circ - \angle C$
By substituting the values
$\angle A + \angle B = 180^\circ - 90^\circ$
By subtraction
$\angle A + \angle B = 90^\circ$

Let us consider $\angle A = 53^\circ$
By substituting the value of $\angle A$ in $\angle A + \angle B = 90^\circ$

We get
$53^o + \angle B = 90^o$
On further calculation
$\angle B = 90^o - 53^o$
By subtraction
$\angle B = 37^o$

Therefore, $\angle A = 53^o$, $\angle B = 37^o$ and $\angle C = 90^o$.

11. If one angle of a triangle is equal to the sum of the other two, show that the triangle is right angled.
Solution:

Consider ABC as a triangle
According to the question it can be written as
$\angle A = \angle B + \angle C$ ….. (1)

We know that the sum of all the angles in a triangle is 180^o.
So we can write it as
$\angle A + \angle B + \angle C = 180^o$
By substituting $\angle A$ in the above equation
$\angle B + \angle C + \angle B + \angle C = 180^o$
$2 (\angle B + \angle C) = 180^o$
By division
$\angle B + \angle C = 180/2$
$\angle B + \angle C = 90^o$

According to equation (1) we can write it as
$\angle A = 90^o$

Therefore, it is proved that the triangle is right angled.

12. If each angle of a triangle is less than the sum of the other two, show that the triangle is acute angled.
Solution:

Consider ABC as a triangle
According to the question it can be written as
$\angle A < \angle B + \angle C$
Add $\angle A$ to both the sides of the equation
So we get
$\angle A + \angle A < \angle A + \angle B + \angle C$
We know that the sum of all the angles in a triangle is 180^o.
So we can write it as
$\angle A + \angle B + \angle C = 180^o$
So we get
$2 \angle A < 180^o$
By division we get
$\angle A < 180/2$
$\angle A < 90^o$

In the same way we can also write
∠B < ∠A + ∠C
Add ∠B to both the sides of the equation
So we get
∠B + ∠B < ∠A + ∠B + ∠C
We know that the sum of all the angles in a triangle is 180°.
So we can write it as
∠A + ∠B + ∠C = 180°
So we get
2 ∠B < 180°
By division we get
∠B < 180/2
∠B < 90°

So we know that
∠C < ∠A + ∠B
Add ∠C to both the sides of the equation
So we get
∠C + ∠C < ∠A + ∠B + ∠C
We know that the sum of all the angles in a triangle is 180°.
So we can write it as
∠A + ∠B + ∠C = 180°
So we get
2 ∠C < 180°
By division we get
∠C < 180/2
∠C < 90°

Therefore, it is proved that the triangle ABC is acute angled.

13. If one angle of a triangle is greater than the sum of the other two, show that the triangle is obtuse angled.
Solution:

Consider ABC as a triangle
According to the question it can be written as
∠B > ∠A + ∠C …. (1)
We know that the sum of all the angles in a triangle is 180°.
So we can write it as
∠A + ∠B + ∠C = 180°
So we get
∠A + ∠C = 180° - ∠B
Substituting ∠A + ∠C in equation (1) we get
∠B > 180° - ∠B
Add ∠B to both the sides of the equation
So we get
∠B + ∠B > 180° - ∠B + ∠B
By addition we get
2 ∠B > 180°
By division we get

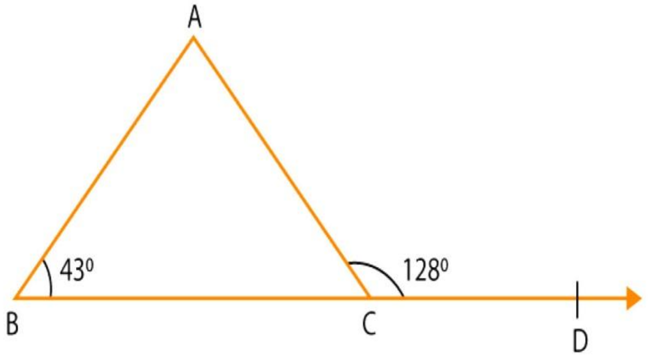
∠B > 180/2

∠B > 90°

So we know that ∠B > 90° which means that ∠B is an obtuse angle

Therefore, it is proved that the triangle ABC is obtuse angled.

14. In the given figure, side BC of △ ABC is produced to D. If ∠ACD = 128°, and ∠ABC = 43°, find ∠BAC and ∠ACB.



Solution:

From the figure we know that ∠ACB and ∠ACD form a linear pair of angles

So we get

∠ACB + ∠ACD = 180°

By substituting the values

∠ACB + 128° = 180°

On further calculation

∠ACB = 180° - 128°

By subtraction

∠ACB = 52°

We know that the sum of all the angles in a triangle is 180°.

So we can write it as

∠ABC + ∠ACB + ∠BAC = 180°

By substituting the values

43° + 52° + ∠BAC = 180°

On further calculation

∠BAC = 180° - 43° - 52°

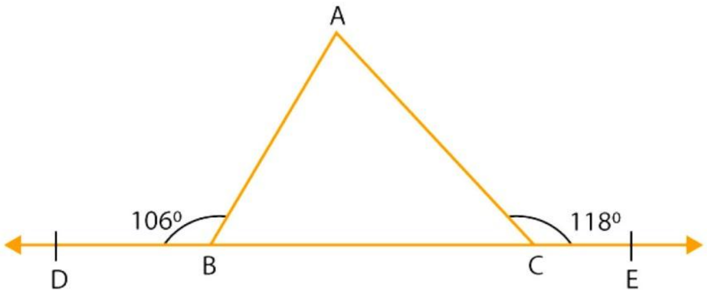
By subtraction

∠BAC = 180° - 95°

∠BAC = 85°

Therefore, ∠BAC = 85° and ∠ACB = 52°.

15. In the given figure, the side BC of △ ABC has been produced on the left-hand side from B to D and on the right-hand side from C to E. If ∠ABD = 106° and ∠ACE = 118°, find the measure of each angle of the triangle.



Solution:

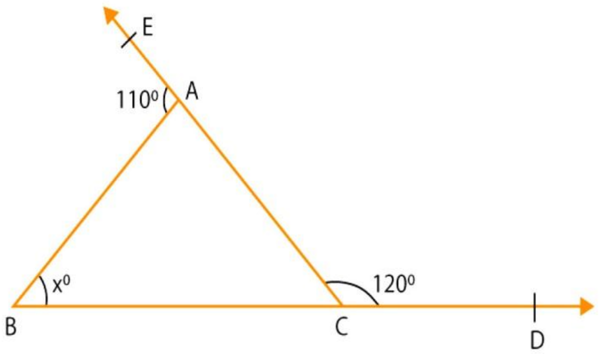
From the figure we know that ∠DBA and ∠ABC form a linear pair of angles
So we get
∠DBA + ∠ABC = 180°
By substituting the values
106° + ∠ABC = 180°
On further calculation
∠ABC = 180° - 106°
By subtraction
∠ABC = 74°

From the figure we know that ∠ACB and ∠ACE form a linear pair of angles
So we get
∠ACB + ∠ACE = 180°
By substituting the values
∠ACB + 118° = 180°
On further calculation
∠ACB = 180° - 118°
By subtraction
∠ACB = 62°

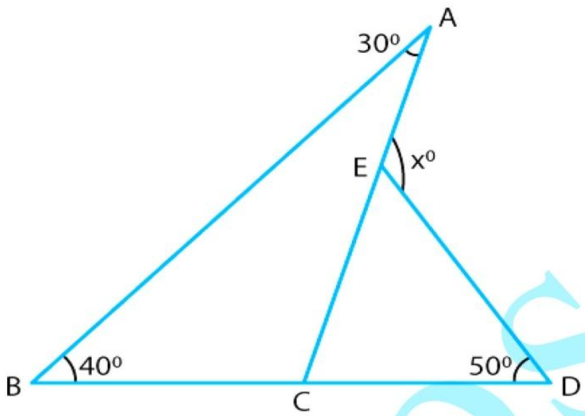
We know that the sum of all the angles in a triangle is 180°.
So we can write it as
∠ABC + ∠ACB + ∠BAC = 180°
By substituting the values
74° + 62° + ∠BAC = 180°
On further calculation
∠BAC = 180° - 74° - 62°
By subtraction
∠BAC = 180° - 136°
∠BAC = 44°

Therefore, the measure of each angle of the triangle is ∠A = 44°, ∠B = 74° and ∠C = 62°.

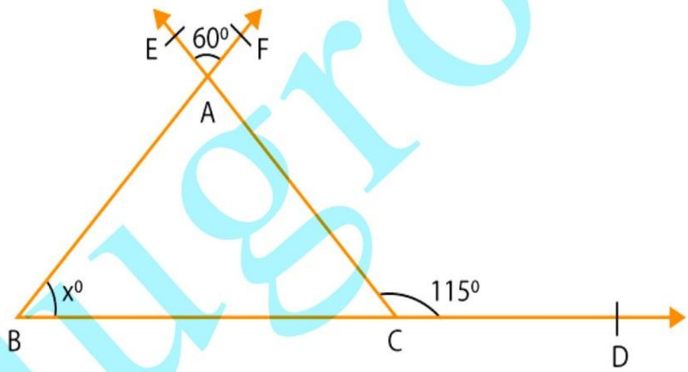
16. Calculate the value of x in each of the following figures.
(i)


E
A
110⁰
x⁰
120⁰
B
C
D

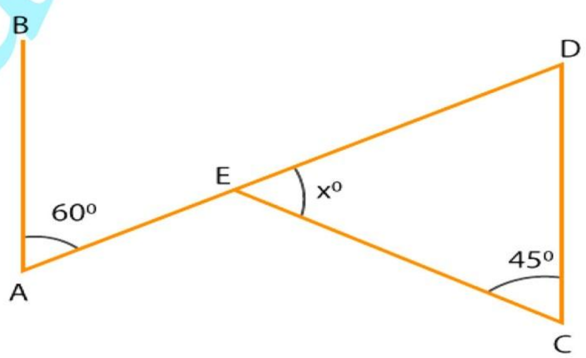
(ii)


A
30⁰
E
x⁰
40⁰
50⁰
B
C
D

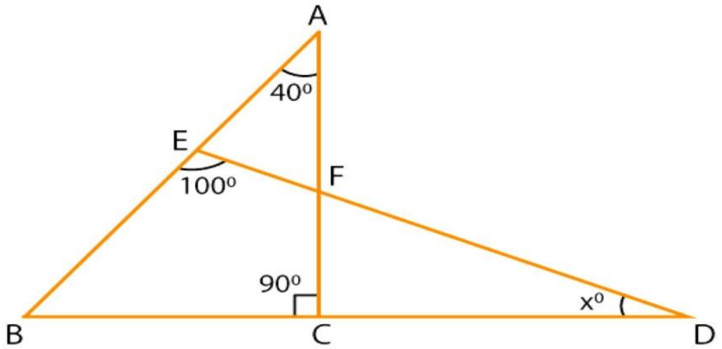
(iii)


E
60⁰
F
A
x⁰
115⁰
B
C
D

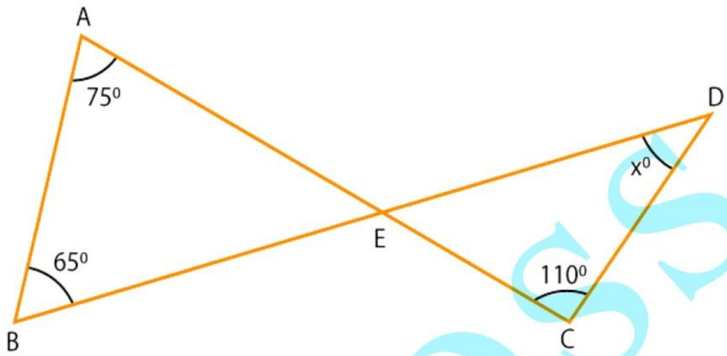
(iv)


B
D
E
x⁰
60⁰
45⁰
A
C

(v)



(vi)



Solution:

(i) From the figure we know that ∠EAB and ∠BAC form a linear pair of angles
So we get
∠EAB + ∠BAC = 180°
By substituting the values
110° + ∠BAC = 180°
On further calculation
∠BAC = 180° - 110°
By subtraction
∠BAC = 70°

From the figure we know that ∠BCA and ∠ACD form a linear pair of angles
So we get
∠BCA + ∠ACD = 180°
By substituting the values
∠BCA + 120° = 180°
On further calculation
∠BAC = 180° - 120°
By subtraction
∠BAC = 60°

We know that the sum of all the angles in a triangle is 180°.
∠ABC + ∠ACB + ∠BAC = 180°

By substituting the values
$x^o + 70^o + 60^o = 180^o$
On further calculation
$x^o = 180^o - 70^o - 60^o$
By subtraction
$x^o = 180^o - 130^o$
$x^o = 50^o$

(ii) We know that the sum of all the angles in triangle ABC is 180^o.
$\angle A + \angle B + \angle C = 180^o$
By substituting the values
$30^o + 40^o + \angle C = 180^o$
On further calculation
$\angle C = 180^o - 30^o - 40^o$
By subtraction
$\angle C = 180^o - 70^o$
$\angle C = 110^o$

From the figure we know that $\angle BCA$ and $\angle ACD$ form a linear pair of angles
So we get
$\angle BCA + \angle ACD = 180^o$
By substituting the values
$110^o + \angle ACD = 180^o$
On further calculation
$\angle ACD = 180^o - 110^o$
By subtraction
$\angle ACD = 70^o$

We know that the sum of all the angles in triangle ECD is 180^o.
$\angle ECD + \angle CDE + \angle CED = 180^o$
By substituting the values
$70^o + 50^o + \angle CED = 180^o$
On further calculation
$\angle CED = 180^o - 70^o - 50^o$
By subtraction
$\angle CED = 180^o - 120^o$
$\angle CED = 60^o$

From the figure we know that $\angle AED$ and $\angle CED$ form a linear pair of angles
So we get
$\angle AED + \angle CED = 180^o$
By substituting the values
$x^o + 60^o = 180^o$
On further calculation
$x^o = 180^o - 60^o$
By subtraction
$x^o = 120^o$

(iii) From the figure we know that $\angle EAF$ and $\angle BAC$ are vertically opposite angles
So we get

∠EAF = ∠BAC = 60^o

We know that in the triangle ABC exterior angle is equal to the sum of two opposite interior angles
So we can write it as
∠ACD = ∠BAC + ∠ABC
By substituting the values
$115^o = 60^o + x^o$
On further calculation
$x^o = 115^o - 60^o$
By subtraction
$x^o = 55^o$

(iv) We know that AB || CD and AD is a transversal
So we get ∠BAD = ∠ADC = 60^o
We know that the sum of all the angles in triangle ECD is 180^o.
∠E + ∠C + ∠D = 180^o
By substituting the values
$x^o + 45^o + 60^o = 180^o$
On further calculation
$x^o = 180^o - 45^o - 60^o$
By subtraction
$x^o = 180^o - 105^o$
$x^o = 75^o$

(v) We know that in the triangle AEF exterior angle is equal to the sum of two opposite interior angles
So we can write it as
∠BED = ∠EAF + ∠EFA
By substituting the values
$100^o = 40^o$ + ∠EFA
On further calculation
∠EFA = $100^o - 40^o$
By subtraction
∠EFA = 60^o

From the figure we know that ∠CFD and ∠EFA are vertically opposite angles
So we get
∠CFD = ∠EFA = 60^o

We know that in the triangle FCD exterior angle is equal to the sum of two opposite interior angles
So we can write it as
∠BCD = ∠CFD + ∠CDF
By substituting the values
$90^o = 60^o + x^o$
On further calculation
$x^o = 90^o - 60^o$
By subtraction
$x^o = 30^o$

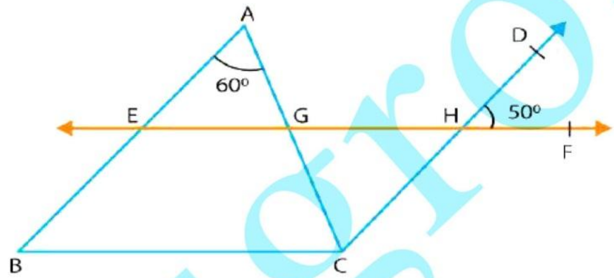
(vi) We know that the sum of all the angles in triangle ABE is 180^o.
∠A + ∠B + ∠E = 180^o

By substituting the values in the above equation
$75^o + 65^o + \angle E = 180^o$
On further calculation
$\angle E = 180^o - 75^o - 65^o$
By subtraction
$\angle E = 180^o - 140^o$
$\angle E = 40^o$

From the figure we know that ∠CED and ∠AEB are vertically opposite angles
So we get
$\angle CED = \angle AEB = 40^o$

We know that the sum of all the angles in triangle CED is 180^o.
$\angle C + \angle E + \angle D = 180^o$
By substituting the values
$110^o + 40^o + x^o = 180^o$
On further calculation
$x^o = 180^o - 110^o + 40^o$
By subtraction
$x^o = 180^o - 150^o$
$x^o = 30^o$

17. In the figure given alongside, AB || CD, EF || BC, ∠BAC = 60° and ∠DHF = 50°. Find ∠GCH and ∠AGH.



Solution:

We know that AB || CD and AC is a transversal
From the figure we know that ∠BAC and ∠ACD are alternate angles
So we get
$\angle BAC = \angle ACD = 60^o$
So we also get
$\angle BAC = \angle GCH = 60^o$

From the figure we also know that ∠DHF and ∠CHG are vertically opposite angles
So we get
$\angle DHF = \angle CHG = 50^o$

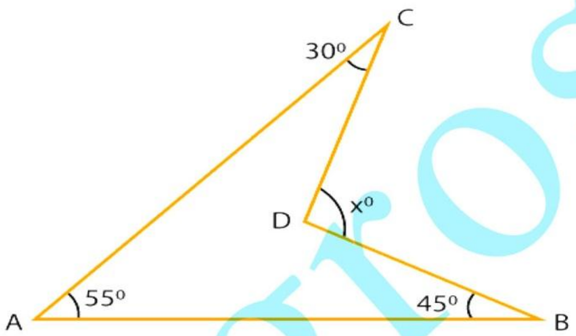
We know that the sum of all the angles in triangle GCH is 180^o.
So we can write it as
$\angle GCH + \angle CHG + \angle CGH = 180^o$
By substituting the values

60º + 50º + ∠CGH = 180º
On further calculation
∠CGH = 180º - 60º - 50º
By subtraction
∠CGH = 180º - 110º
∠CGH = 70º

From the figure we know that ∠CGH and ∠AGH form a linear pair of angles
So we get
∠CGH + ∠AGH = 180º
By substituting the values
70º + ∠AGH = 180º
On further calculation
∠AGH = 180º - 70º
By subtraction
∠AGH = 110º

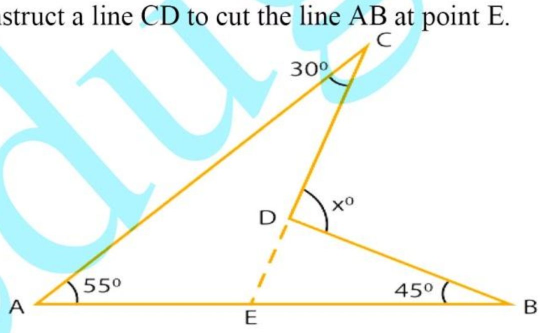
Therefore, ∠GCH = 60º and ∠AGH = 110º.

18. Calculate the value of x in the given figure.



Solution:

Construct a line CD to cut the line AB at point E.



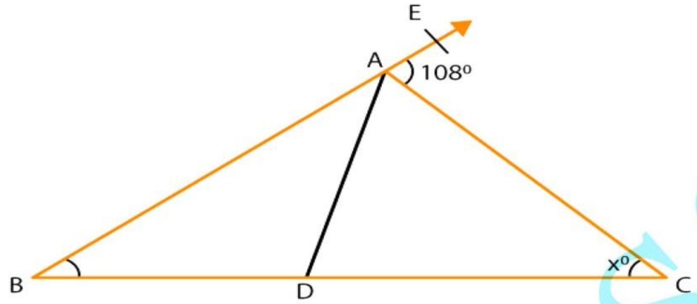
We know that in the triangle BDE exterior angle is equal to the sum of two opposite interior angles
So we can write it as
∠CDB = ∠CEB + ∠DBE
By substituting the values
xº = ∠CEB + 45º …. (1)

We know that in the triangle AEC exterior angle is equal to the sum of two opposite interior angles

So we can write it as
∠CEB = ∠CAB + ∠ACE
By substituting the values
∠CEB = $55^o + 30^o$
By addition
∠CEB = 85^o

By substituting ∠CEB in equation (1) we get
$x^o = 85^o + 45^o$
By addition
$x^o = 130^o$

19. In the given figure, AD divides ∠BAC in the ratio 1: 3 and AD = DB. Determine the value of x.



Solution:

It is given that AD divides ∠BAC in the ratio 1: 3
So let us consider ∠BAD and ∠DAC as y and 3y
According to the figure we know that BAE is a straight line

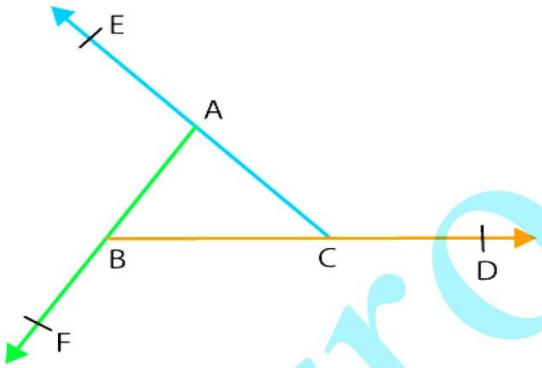
From the figure we know that ∠BAC and ∠CAE form a linear pair of angles
So we get
∠BAC + ∠CAE = 180^o
We know that
∠BAC = ∠BAD + ∠DAC
So it can be written as
∠BAD + ∠DAC + ∠CAE = 180^o
By substituting the values we get
$y + 3y + 108^o = 180^o$
On further calculation
$4y = 180^o - 108^o$
By subtraction
$4y = 72^o$
By division
y = 72/4
$y = 18^o$

We know that the sum of all the angles in triangle ABC is 180^o.
So we can write it as
∠ABC + ∠BCA + ∠BAC = 180^o
It is given that AD = DB so we can write it as ∠ABC = ∠BAD

From the figure we know that ∠BAC = y + 3y = 4y
By substituting the values
$y + x + 4y = 180^o$
On further calculation
$5y + x = 180^o$
By substituting the value of y
$5 (18^o) + x = 180^o$
By multiplication
$90^o + x = 180^o$
$x = 180^o - 90^o$
By subtraction we get
$x = 90^o$

Therefore, the value of x is 90.

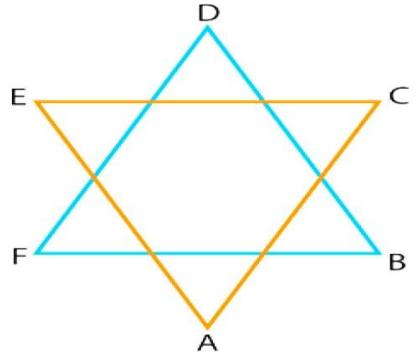
20. If the sides of a triangle are produced in order, prove that the sum of the exterior angles so formed is equal to four right angles.



Solution:

Given: △ ABC in which AB, BC and CA are produced to points D, E and F.
To prove: ∠DCA + ∠FAE + ∠FBD = 180°
Proof:
From the figure we know that
∠DCA = ∠A + ∠B …. (1)
∠FAE = ∠B + ∠C …. (2)
∠FBD = ∠A + ∠C …. (3)
By adding equation (1), (2) and (3) we get
∠DCA + ∠FAE + ∠FBD = ∠A + ∠B + ∠B + ∠C + ∠A + ∠C
So we get
∠DCA + ∠FAE + ∠FBD = 2 ∠A + 2 ∠B + 2 ∠C
Now by taking out 2 as common
∠DCA + ∠FAE + ∠FBD = 2 (∠A + ∠B + ∠C)
We know that the sum of all the angles in a triangle is 180°.
So we get
∠DCA + ∠FAE + ∠FBD = 2 (180°)
∠DCA + ∠FAE + ∠FBD = 360°
Therefore, it is proved.

21. In the adjoining figure, show that ∠A + ∠B +∠C + ∠D + ∠E + ∠F = 360º.



Solution:

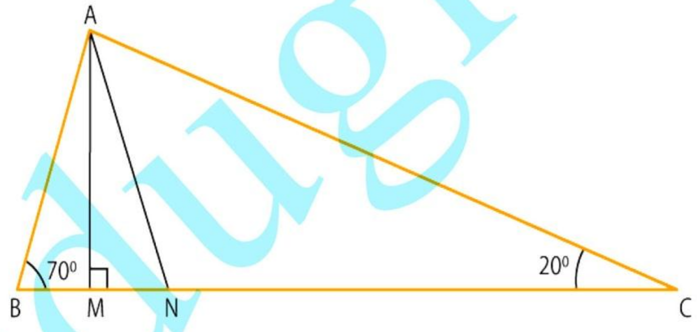
We know that the sum of all the angles in triangle ACE is 180º.
∠A + ∠C + ∠E = 180º ….. (1)

We know that the sum of all the angles in triangle BDF is 180º.
∠B + ∠D + ∠F = 180º ….. (2)

Now by adding both equations (1) and (2) we get
∠A + ∠C + ∠E + ∠B + ∠D + ∠F = 180º + 180º
On further calculation
∠A + ∠B + ∠C + ∠D + ∠E + ∠F = 360º

Therefore, it is proved that ∠A + ∠B + ∠C + ∠D + ∠E + ∠F = 360º.

22. In the given figure, AM ⊥ BC and AN is the bisector of ∠A. If ∠ABC = 70º and ∠ACB = 20º, find ∠MAN.



Solution:

We know that the sum of all the angles in triangle ABC is 180º.
∠A + ∠B + ∠C = 180º
By substituting the values
∠A + 70º + 20º = 180º
On further calculation
∠A = 180º - 70º - 20º
By subtraction
∠A = 180º - 90º

∠A = 90°

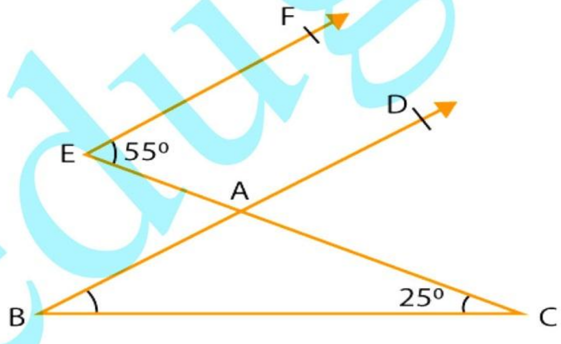
We know that the sum of all the angles in triangle ABM is 180°.
∠BAM + ∠ABM + ∠AMB = 180°
By substituting the values
∠BAM + 70° + 90° = 180°
On further calculation
∠BAM = 180° - 70° - 90°
By subtraction
∠BAM = 180° - 160°
∠BAM = 20°

It is given that AN is the bisector of ∠A
So it can be written as
∠BAN = (1/2) ∠A
By substituting the values
∠BAN = (1/2) (90°)
By division
∠BAN = 45°

From the figure we know that
∠MAN + ∠BAM = ∠BAN
By substituting the values we get
∠MAN + 20° = 45°
On further calculation
∠MAN = 45° - 20°
By subtraction
∠MAN = 25°

Therefore, ∠MAN = 25°.

23. In the given figure, BAD || EF, ∠AEF = 55° and ∠ACB = 25°, find ∠ABC.



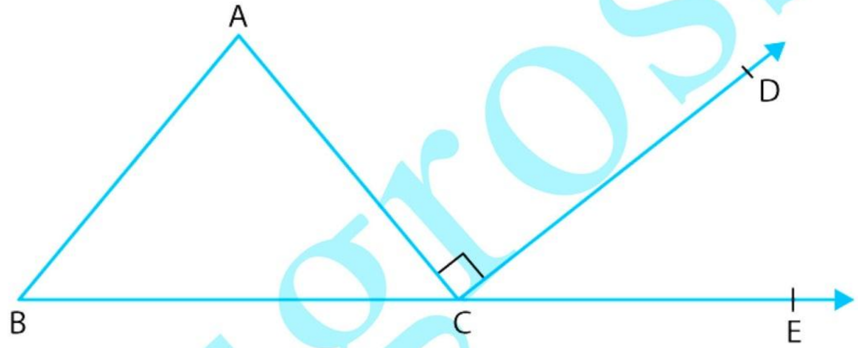
Solution:

We know that BAD || EF and EC is the transversal
From the figure we know that ∠AEF and ∠CAD are corresponding angles
So we get
∠AEF = ∠CAD = 55°

From the figure we know that ∠CAD and ∠CAB form a linear pair of angles
So we get
∠CAD + ∠CAB = 180°
By substituting the values
55° + ∠CAB = 180°
On further calculation
∠CAB = 180° - 55°
By subtraction
∠CAB = 125°

We know that the sum of all the angles in triangle ABC is 180°.
∠ABC + ∠CAB + ∠ACB = 180°
By substituting the values in the above equation we get
∠ABC + 125° + 25° = 180°
On further calculation
∠ABC = 180° - 125° - 25°
By subtraction
∠ABC = 180° - 150°
∠ABC = 30°

24. In a △ ABC, it is given that ∠A: ∠B: ∠C = 3: 2: 1 and CD ⊥ AC. Find ∠ECD.



Solution:

In a △ ABC, it is given that
∠A: ∠B: ∠C = 3: 2: 1
It can also be written as
∠A = 3x, ∠B = 2x and ∠C = x

We know that the sum of all the angles in triangle ABC is 180°.
∠A + ∠B + ∠C = 180°
By substituting the values we get
3x + 2x + x = 180°
By addition
6x = 180°
By division
x = 180/6
x = 30°

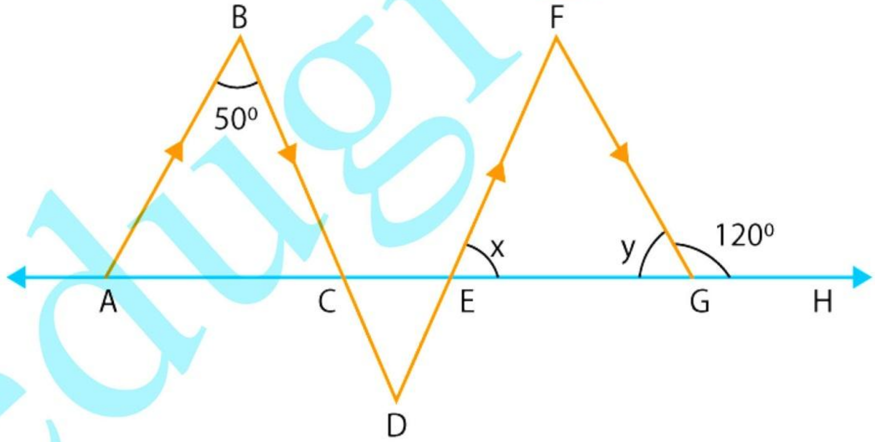
Now by substituting the value of x we get
$\angle A = 3x = 3\ (30^o) = 90^o$
$\angle B = 2x = 2\ (30^o) = 60^o$
$\angle C = x = 30^o$

We know that in the triangle ABC exterior angle is equal to the sum of two opposite interior angles
So we can write it as
$\angle ACE = \angle A + \angle B$
By substituting the values we get
$\angle ACE = 90^o + 60^o$
By addition
$\angle ACE = 150^o$

We know that $\angle ACE$ can be written as $\angle ACD + \angle ECD$
So we can write it as
$\angle ACE = \angle ACD + \angle ECD$
By substituting the values we get
$150^o = 90^o + \angle ECD$
It is given that $CD \perp AC$ so $\angle ACD = 90^o$
On further calculation
$\angle ECD = 150^o - 90^o$
By subtraction
$\angle ECD = 60^o$

Therefore, $\angle ECD = 60^o$.

25. In the given figure, AB || DE and BD || FG such that ∠ABC = 50º and ∠FGH = 120º. Find the values of x and y.



Solution:

From the figure we know that $\angle FGH$ and $\angle FGE$ form a linear pair of angles
So we get
$\angle FGH + \angle FGE = 180^o$
By substituting the values
$120^o + y = 180^o$
On further calculation

$y = 180^o - 120^o$
By subtraction
$y = 60^o$

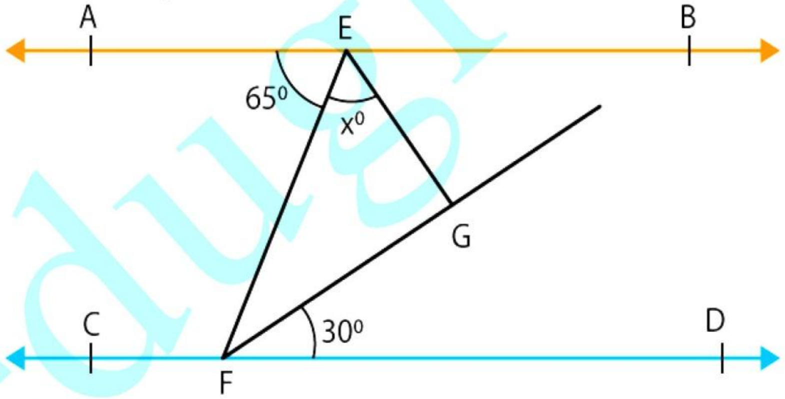
We know that AB || DF and BD is a transversal
From the figure we know that ∠ABC and ∠CDE are alternate angles
So we get
$\angle ABC = \angle CDE = 50^o$

We know that BD || FG and DF is the transversal
From the figure we know that ∠EFG and ∠CDE are alternate angles
So we get
$\angle EFG = \angle CDE = 50^o$

We know that the sum of all the angles in triangle EFG is 180^o.
$\angle FEG + \angle FGE + \angle EFG = 180^o$
By substituting the values we get
$x + y + 50^o = 180^o$
$x + 60^o + 50^o = 180^o$
On further calculation
$x = 180^o - 60^o - 50^o$
By subtraction
$x = 180^o - 110^o$
$x = 70^o$

Therefore, the values of $x = 70^o$ and $y = 60^o$.

26. In the given figure, AB || CD and EF is a transversal. If ∠AEF = 65º, ∠DFG = 30º, ∠EGF = 90º and ∠GEF = xº, find the value of x.



Solution:

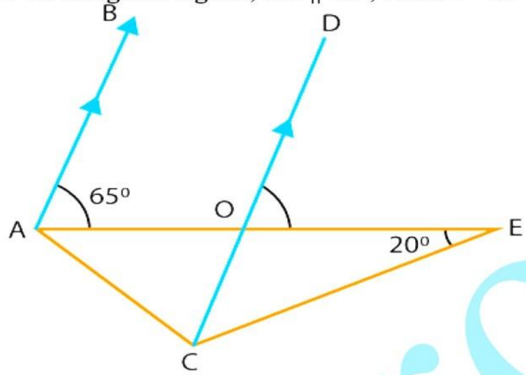
We know that AB || CD and EF is a transversal
From the figure we know that ∠AEF and ∠EFD are alternate angles
So we get
$\angle AEF = \angle EFG + \angle DFG$
By substituting the values
$65^o = \angle EFG + 30^o$
On further calculation

∠EFG = 65° - 30°
By subtraction
∠EFG = 35°

We know that the sum of all the angles in triangle GEF is 180°.
∠GEF + ∠EGF + ∠EFG = 180°
By substituting the values we get
x + 90° + 35° = 180°
On further calculation
x = 180° - 90° - 35°
By subtraction
x = 55°

Therefore, the value of x is 55°

27. In the given figure, AB || CD, ∠BAE = 65° and ∠OEC = 20°. Find ∠ECO.



Solution:

We know that AB || CD and AE is a transversal
From the figure we know that ∠BAE and ∠DOE are corresponding angles
So we get
∠BAE = ∠DOE = 65°

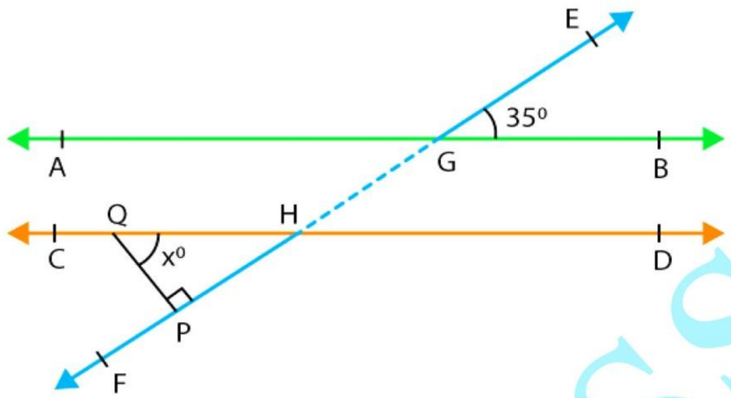
From the figure we know that ∠DOE and ∠COE form a linear pair of angles
So we get
∠DOE + ∠COE = 180°
By substituting the values
65° + ∠COE = 180°
On further calculation
∠COE = 180° - 65°
By subtraction
∠COE = 115°

We know that the sum of all the angles in triangle OCE is 180°.
∠OEC + ∠ECO + ∠COE = 180°
By substituting the values we get
20° + ∠ECO + 115° = 180°
On further calculation
∠ECO = 180° - 20° - 115°

By subtraction
∠ECO = 45°

Therefore, ∠ECO = 45°

28. In the given figure, AB || CD and EF is a transversal, cutting them at G and H respectively. If ∠EGB = 35° and QP ⊥ EF, find the measure of ∠PQH.



Solution:

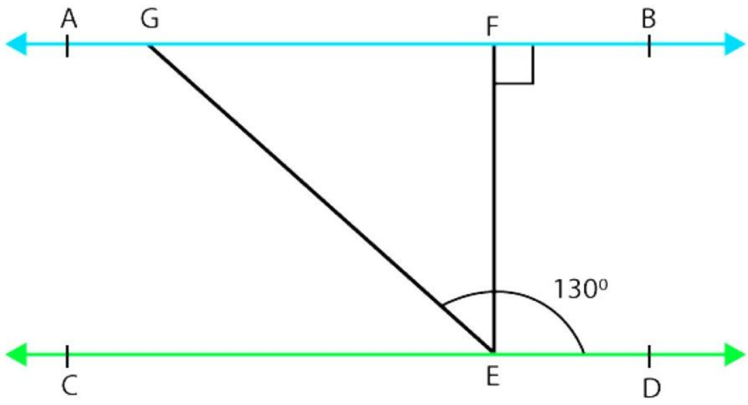
We know that AB || CD and EF is a transversal
From the figure we know that ∠EGB and ∠GHD are corresponding angles
So we get
∠EGB = ∠GHD = 35°

From the figure we know that ∠GHD and ∠QHP are vertically opposite angles
So we get
∠GHD = ∠QHP = 35°

We know that the sum of all the angles in triangle DQHP is 180°.
∠PQH + ∠QHP + ∠QPH = 180°
By substituting the values we get
∠PQH + 35° + 90° = 180°
On further calculation
∠PQH = 180° - 35° - 90°
By subtraction
∠PQH = 180° - 125°
∠PQH = 55°

Therefore, ∠PQH = 55°

29. In the given figure, AB || CD and EF ⊥ AB. If EG is the transversal such that ∠GED = 130°, find ∠EGF.



Solution:

We know that AB || CD and GE is the transversal
From the figure we know that ∠EGF and ∠GED are interior angles
So we get
$\angle EGF + \angle GED = 180^\circ$
By substituting the values
$\angle EGF + 130^\circ = 180^\circ$
On further calculation
$\angle EGF = 180^\circ - 130^\circ$
By subtraction
$\angle EGF = 50^\circ$

Therefore, $\angle EGF = 50^\circ$