**RS Aggarwal Solutions for Class 9 Maths Chapter 7 –
Lines and Angles**

EXERCISE 7(A) PAGE: 198

1. Define the following terms:
(i) Angle
(ii) Interior of an angle
(iii) Obtuse angle
(iv) Reflex angle
(v) Complementary angles
(vi) Supplementary angles
Solution:

(i) Angle – When two rays originate from the same end point, then an angle is formed.

(ii) Interior of an angle – The interior of ∠BAC is the set of all points in its plane which lie on the same side of AB as C and also on the same side of AC as B.

(iii) Obtuse angle – An angle whose measure is more than 90^o but less than 180^o is called an obtuse angle.

(iv) Reflex angle – An angle whose measure is more than 180^o but less than 360^o is called a reflex angle.

(v) Complementary angles – Two angles are said to be complementary, if the sum of their measure is 90^o.

(vi) Supplementary angles – Two angles are said to be supplementary if the sum of their measures is 180^o.

2. Find the complement of each of the following angles:
(i) 55^o
(ii) 16^o
(iii) 90^o
(iv) 2/3 of a right angle
Solution:

(i) We know that the complement of 55^o can be written as
$55^o = 90^o - 55^o = 35^o$

(ii) We know that the complement of 16^o can be written as
$16^o = 90^o - 16^o = 74^o$

(iii) We know that the complement of 90^o can be written as
$90^o = 90^o - 90^o = 0^o$

(iv) We know that 2/3 of a right angle can be written as $2/3 \times 90^o = 60^o$
$60^o = 90^o - 60^o = 30^o$

3. Find the supplement of each of the following angles:
(i) 42^o
(ii) 90^o
(iii) 124^o
(iv) 3/5 of a right angle
Solution:

(i) We know that the supplement of 42^o can be written as
$42^o = 180^o - 42^o = 138^o$

(ii) We know that the supplement of 90^o can be written as
$90^o = 180^o - 90^o = 90^o$

(iii) We know that the supplement of 124^o can be written as
$124^o = 180^o - 124^o = 56^o$

(v) We know that 3/5 of a right angle can be written as $3/5 \times 90^o = 54^o$
$54^o = 180^o - 54^o = 126^o$

4. Find the measure of an angle which is
(i) Equal to its complement
(ii) Equal to its supplement
Solution:

(i) Consider the required angle as x^o
We know that the complement can be written as $90^o - x^o$
To find that the measure of an angle is equal to its complement
We get
$x^o = 90^o - x^o$
We can also write it as
$x + x = 90$
So we get
$2x = 90$
By division we get
$x = 90/2$
$x^o = 45^o$

Therefore, the measure of an angle which is equal to its complement is 45^o

(ii) Consider the required angle as x^o
We know that the supplement can be written as $180^o - x^o$
To find that the measure of an angle is equal to its complement
We get
$x^o = 180^o - x^o$
We can also write it as
$x + x = 180$
So we get
$2x = 180$
By division we get
$x = 180/2$
$x^o = 90^o$

Therefore, the measure of an angle which is equal to its complement is 90^o

5. Find the measure of an angle which is 36^o more than its complement.
Solution:

Consider the required angle as x^o
We know that the complement can be written as $90^o - x^o$
$x^o = (90^o - x^o) + 36^o$
We can also write it as
$x + x = 90 + 36$
So we get
$2x = 126$
By division we get
$x = 126/2$
$x^o = 63^o$

Therefore, the measure of an angle which is 36^o more than its complement is 63^o.

6. Find the measure of an angle which is 30^o less than its supplement.
Solution:

Consider the required angle as x^o
We know that the supplement can be written as $180^o - x^o$
$x^o = (180^o - x^o) – 30^o$
We can also write it as
$x + x = 180 - 30$
So we get
$2x = 150$
By division we get
$x = 150/2$
$x^o = 75^o$

Therefore, the measure of an angle which is 30^o more than its supplement is 75^o.

7. Find the angle which is four times its complement.
Solution:

Consider the required angle as x^o
We know that the complement can be written as $90^o - x^o$
$x^o = 4(90^o - x^o)$
We can also write it as
$x = 360 – 4x$
So we get
$5x = 360$
By division we get
$x = 360/5$
$x^o = 72^o$

Therefore, the angle which is four times its complement is 72^o.

8. Find the angle which is five times its supplement.
Solution:

Consider the required angle as x^o
We know that the supplement can be written as $180^o - x^o$

$x^o = 5(180^o - x^o)$
We can also write it as
$x = 900 – 5x$
So we get
$6x = 900$
By division we get
$x = 900/6$
$x^o = 150^o$

Therefore, the angle which is five times its supplement is 150^o.

9. Find the angle whose supplement is four times its complement.
Solution:

Consider the required angle as x^o
We know that the complement can be written as $90^o - x^o$ and the supplement can be written as $180^o - x^o$
$180^o - x^o = 4(90^o - x^o)$
We can also write it as
$180^o - x^o = 360^o - 4x^o$
So we get
$4x^o - x^o = 360^o - 180^o$
$3x^o = 180^o$
By division we get
$x^o = 180/3$
$x^o = 60^o$

Therefore, the angle whose supplement is four times its complement is 60^o.

10. Find the angle whose complement is one third of its supplement.
Solution:

Consider the required angle as x^o
We know that the complement can be written as $90^o - x^o$ and the supplement can be written as $180^o - x^o$
$90^o - x^o = 1/3(180^o - x^o)$
We can also write it as
$90^o - x^o = 60^o – (1/3) x^o$
So we get
$x^o – (1/3)x^o = 90^o - 60^o$
$(2/3) x^o = 30^o$
By division we get
$x^o = ((30×3)/2)$
$x^o = 45^o$

Therefore, the angle whose complement is one third of its supplement is 45^o.

11. Two complementary angles are in the ratio 4:5. Find the angles.
Solution:

Consider the required angle as x^o and $90^o - x^o$
According to the question it can be written as

$x^\circ / 90^\circ - x^\circ = 4/5$
By cross multiplication we get
$5x = 4(90 - x)$
$5x = 360 - 4x$
On further calculation we get
$5x + 4x = 360$
$9x = 360$
By division
$x = 360/9$
So we get
$x = 40$

Therefore, the angles are 40° and $90^\circ - x^\circ = 90^\circ - 40^\circ = 50^\circ$

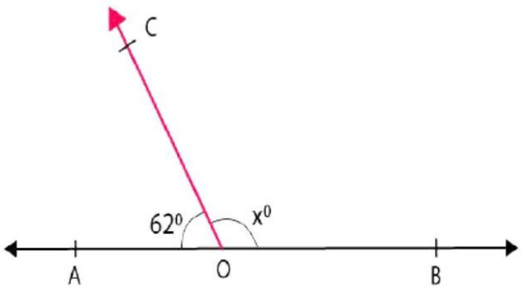
12. Find the value of x for which the angles $(2x - 5)^\circ$ and $(x - 10)^\circ$ are the complementary angles.
Solution:

It is given that $(2x - 5)^\circ$ and $(x - 10)^\circ$ are the complementary angles.
So we can write it as
$(2x - 5)^\circ + (x - 10)^\circ = 90^\circ$
$2x - 5^\circ + x - 10^\circ = 90^\circ$
On further calculation
$3x - 15^\circ = 90^\circ$
So we get
$3x = 105^\circ$
By division
$x = 105/3$
$x = 35^\circ$

Therefore, the value of x for which the angles $(2x - 5)^\circ$ and $(x - 10)^\circ$ are the complementary angles is 35°.

EXERCISE 7(B)

1. In the adjoining figure, AOB is a straight line. Find the value of x.

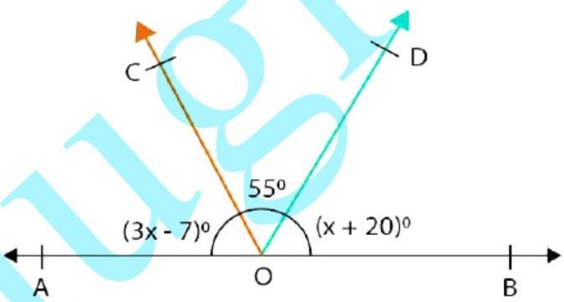


Solution:

From the figure we know that ∠AOC and ∠BOC are a linear pair of angles
So we get
∠AOC + ∠BOC = 180°
We know that
62° + x° = 180°
On further calculation
x° = 180° - 62°
By subtraction
x° = 118°

Therefore, the value of x is 118.

2. In the adjoining figure, AOB is a straight line. Find the value of x. Hence, find ∠AOC and ∠BOD.



Solution:

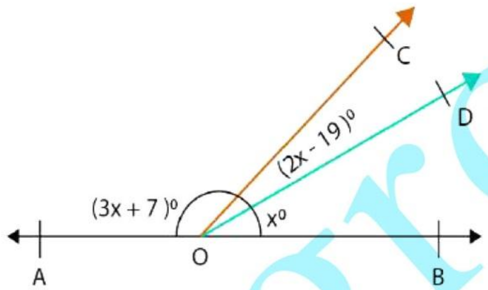
From the figure we know that ∠AOB is a straight line
So we get
∠AOB = 180°
It can also be written as
∠AOC + ∠COD + ∠BOD = 180°
By substituting the values
(3x – 7) ° + 55° + (x + 20) ° = 180°
3x – 7° + 55° + x + 20° = 180°

On further calculation
$4x + 68^o = 180^o$
$4x = 112^o$
By division
$x = 28^o$

By substituting the value of x we get
$\angle AOC = (3x - 7)^o$
$= 3(28^o) - 7^o$
On further calculation
$= 84^o - 7^o$
By subtraction
$= 77^o$

$\angle BOD = (x + 20)^o$
$= (28 + 20)^o$
By addition
$= 48^o$

3. In the adjoining figure, AOB is a straight line. Find the value of x. Hence, find ∠AOC, ∠COD and ∠BOD.



Solution:
From the figure we know that ∠BOD and ∠AOD are a linear pair of angles
So we get
$\angle BOD + \angle AOD = 180^o$
It can also be written as
$\angle BOD + \angle COD + \angle COA = 180^o$
By substituting the values
$x^o + (2x - 19)^o + (3x + 7)^o = 180^o$
$x + 2x - 19^o + 3x + 7^o = 180^o$
On further calculation
$6x - 12^o = 180^o$
$6x = 180^o + 12^o$
So we get
$6x = 192^o$
By division
$x = 32^o$

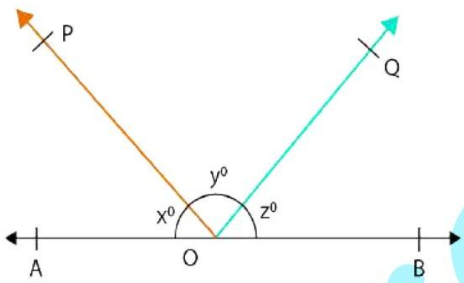
By substituting the value of x we get

∠AOC = $(3x + 7)^{o}$
= $3(32^{o}) + 7^{o}$
On further calculation
= $96^{o} + 7^{o}$
By addition
= 103^{o}

∠COD = $(2x - 19)^{o}$
= $(2(32^{o}) - 19)^{o}$
So we get
= $(64 – 19)^{o}$
By subtraction
= 45^{o}

∠BOD = $x^{o} = 32^{o}$

4. In the adjoining figure, x: y: z = 5: 4: 6. If XOY is a straight line, find the values of x, y and z.



Solution:

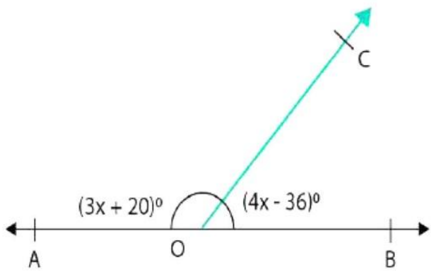
From the figure it is given that
x: y: z = 5: 4: 6
We can also write it as
x + y + z = 5 + 4 + 6 = 15
It is given that XOY is a straight line
So we know that
$x + y + z = 180^{o}$

As we know the sum of ratio is 15 then we can write that the measure of x as 5
The sum of all the angles in a straight line is 180^{o}
So we get the measure of x as
x = (5/15) × 180
On further calculation
x = 60
As we know the sum of ratio is 15 then we can write that the measure of y as 4
The sum of all the angles in a straight line is 180^{o}
So we get the measure of y as
y = (4/15) × 180
On further calculation
y = 48

In order to find the value of z
We know that
$x + y + z = 180^o$
Substituting the values of x and y we get
$60^o + 48^o + z = 180^o$
On further calculation
$z = 180^o - 60^o - 48^o$
By subtraction we get
$z = 72^o$

Therefore, the values of x, y and z are 60^o, 48^o and 72^o.

5. In the adjoining figure, what value of x will make AOB, a straight line?

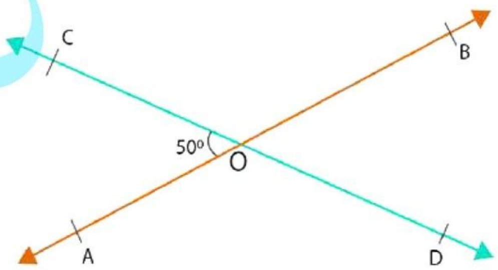


Solution:

We know that AOB will be a straight line only if the adjacent angles form a linear pair.
$\angle BOC + \angle AOC = 180^o$
By substituting the values we get
$(4x – 36)^o + (3x + 20)^o = 180^o$
$4x - 36^o + 3x + 20^o = 180^o$
On further calculation we get
$7x = 180^o - 20^o + 36^o$
$7x = 196^o$
By division we get
$x = 196/7$
$x = 28$

Therefore, the value of x is 28.

6. Two lines AB and CD intersect at O. If $\angle AOC = 50^o$, find $\angle AOD$, $\angle BOD$ and $\angle BOC$.



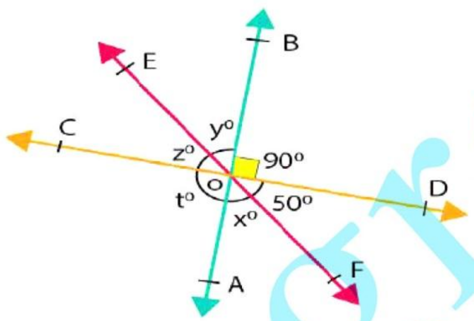
Solution:

From the figure we know that ∠AOC and ∠AOD form a linear pair.
It can also be written as
∠AOC + ∠AOD = 180°
By substituting the values
50° + ∠AOD = 180°
∠AOD = 180° - 50°
By subtraction
∠AOD = 130°

According to the figure we know that ∠AOD and ∠BOC are vertically opposite angles
So we get
∠AOD = ∠BOC = 130°

According to the figure we know that ∠AOC and ∠BOD are vertically opposite angles
So we get
∠AOC = ∠BOD = 50°

7. **In the adjoining figure, three coplanar lines AB, CD and EF intersect at a point O, forming angles as shown. Find the values of x, y, z and t.**



Solution:

From the figure we know that ∠COE and ∠DOF are vertically opposite angles
∠COE = ∠DOF = ∠z = 50°

From the figure we know that ∠BOD and ∠COA are vertically opposite angles
∠BOD = ∠COA = ∠t = 90°

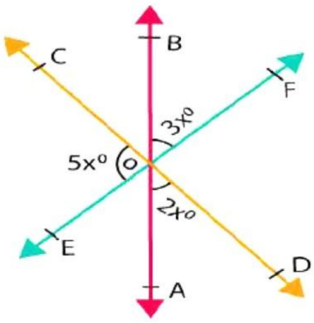
We also know that ∠COA and ∠AOD form a linear pair
∠COA + ∠AOD = 180°
It can also be written as
∠COA + ∠AOF + ∠FOD = 180°
By substituting values in the above equation we get
90° + x° + 50° = 180°
On further calculation we get
x° + 140° = 180°

$x^o = 180^o - 140^o$
By subtraction
$x^o = 40^o$

From the figure we know that ∠EOB and ∠AOF are vertically opposite angles
∠EOB = ∠AOF = x = y = 40

Therefore, the values of x, y, z and t are 40, 40, 50 and 90.

8. In the adjoining figure, three coplanar lines AB, CD and EF intersect at a point O. Find the value of x. Hence, find ∠AOD, ∠COE and ∠AOE.



Solution:

From the figure we know that ∠COE and ∠EOD form a linear pair
∠COE + ∠EOD = 180°
It can also be written as
∠COE + ∠EOA + ∠AOD = 180°
By substituting values in the above equation we get
5x + ∠EOA + 2x = 180°
From the figure we know that ∠EOA and ∠BOF are vertically opposite angles
∠EOA = ∠BOF
So we get
5x + ∠BOF + 2x = 180°
5x + 3x + 2x = 180°
On further calculation
10x = 180°
By division
x = 180/10 = 18

By substituting the value of x
∠AOD = 2x°
So we get
∠AOD = 2 (18) ° = 36°

∠EOA = ∠BOF = 3x°
So we get
∠EOA = ∠BOF = 3 (18) ° = 54°

∠COE = 5x°
So we get
∠COE = 5 (18) ° = 90°

9. Two adjacent angles on a straight line are in the ratio 5:4. Find the measure of each one of these angles.
Solution:

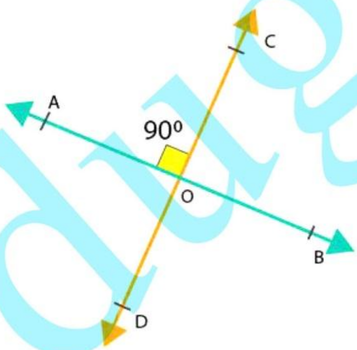
Consider the two adjacent angles as 5x and 4x.
We know that the two adjacent angles form a linear pair
So it can be written as
5x + 4x = 180°
On further calculation
9x = 180°
By division
x = 180/90
x = 20°

Substituting the value of x in two adjacent angles
5x = 5 (20) ° = 100°
4x = 4 (20) ° = 80°

Therefore, the measure of each one of these angles is 100° and 80°

10. If two straight lines intersect each other in such a way that one of the angles formed measures 90°, show that each of the remaining angles measures 90°.
Solution:

Consider two lines AB and CD intersecting at a point O with ∠AOC = 90°



From the figure we know that ∠AOC and ∠BOD are vertically opposite angles
∠AOC = ∠BOD = 90°

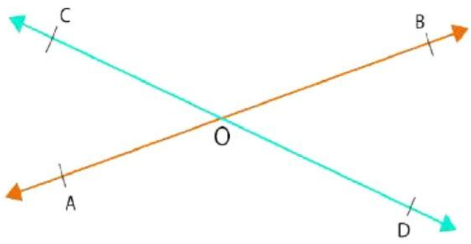
From the figure we also know that ∠AOC and ∠AOD form a linear pair
So it can be written as
∠AOC + ∠AOD = 180°
Substituting the values
90° + ∠AOD = 180°

On further calculation
∠AOD = 180º - 90º
By subtraction
∠AOD = 90º

From the figure we also know that ∠BOC = ∠AOD are vertically opposite angles
∠BOC = ∠AOD = 90º

Therefore, it is proved that each of the remaining angle is 90º

11. Two lines AB and CD intersect at a point O such that ∠BOC + ∠AOD = 280º, as shown in the figure. Find all the four angles.



Solution:

From the figure we know that ∠AOD and ∠BOC are vertically opposite angles
∠AOD = ∠BOC

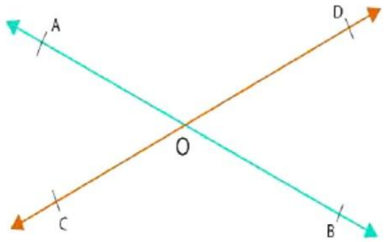
It is given that
∠BOC + ∠AOD = 280º
We know that ∠AOD = ∠BOC
So it can be written as
∠AOD + ∠AOD = 280º
On further calculation
2 ∠AOD = 280º
By division
∠AOD = 280/2
∠AOD = ∠BOC = 140º

From the figure we know that ∠AOC and ∠AOD form a linear pair
So it can be written as
∠AOC + ∠AOD = 180º
Substituting the values
∠AOC + 140º = 180º
On further calculation
∠AOC = 180º - 140º
By subtraction
∠AOC = 40º

From the figure we know that ∠AOC and ∠BOD are vertically opposite angles
∠AOC = ∠BOD = 40º

Therefore, ∠AOC = 40°, ∠BOC = 140°, ∠AOD = 140° and ∠BOD = 40°

12. Two lines AB and CD intersect each other at a point O such that ∠AOC: ∠AOD = 5:7. Find all the angles.



Solution:

Consider ∠AOC as 5x and ∠AOD as 7x
From the figure we know that ∠AOC and ∠AOD for linear pair of angles
So it can be written as
∠AOC + ∠AOD = 180°
By substituting the values
5x + 7x = 180°
On further calculation
12x = 180°
By division
x = 180/12
x = 15°

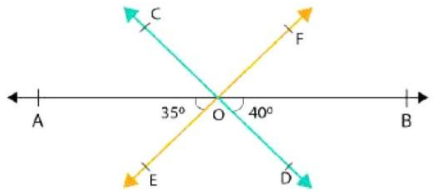
By substituting the value of x
∠AOC = 5x
So we get
∠AOC = 5 (15°) = 75°

∠AOD = 7x
So we get
∠AOD = 7 (15°) = 105°

From the figure we know that ∠AOC and ∠BOD are vertical angles
So we get
∠AOC = ∠BOD = 75°

From the figure we know that ∠AOD and ∠BOC are vertical angles
So we get
∠AOD = ∠BOC = 105°

13. In the given figure, three lines AB, CD and EF intersect at a point O such that ∠AOE = 35° and ∠BOD = 40°. Find the measure of ∠AOC, ∠BOF, ∠COF and ∠DOE.



Solution:

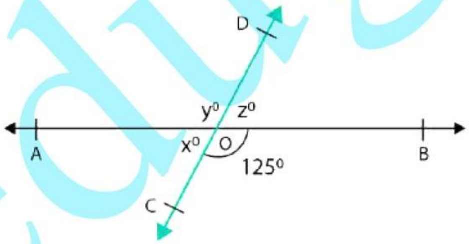
It is given that ∠BOD = 40°
From the figure we know that ∠BOD and ∠AOC are vertically opposite angles
∠AOC = ∠BOD = 40°

It is given that ∠AOE = 35°
From the figure we know that ∠BOF and ∠AOE are vertically opposite angles
∠AOE = ∠BOF = 35°

From the figure we know that AOB is a straight line
So it can be written as
∠AOB = 180°
We can write it as
∠AOE + ∠EOD + ∠BOD = 180°
By substituting the values
35° + ∠EOD + 40° = 180°
On further calculation
∠EOD = 180° - 35° - 40°
By subtraction
∠EOD = 105°

From the figure we know that ∠COF and ∠EOD are vertically opposite angles
∠COF = ∠EOD = 105°

14. In the given figure, the two lines AB and CD intersect at a point O such that ∠BOC = 125°. Find the values of x, y and z.



Solution:

From the figure we know that ∠AOC and ∠BOC form a linear pair of angles
So it can be written as
∠AOC + ∠BOC = 180°
By substituting the values we get
x + 125° = 180°

On further calculation
x = 180° - 125°
By subtraction
x = 55°

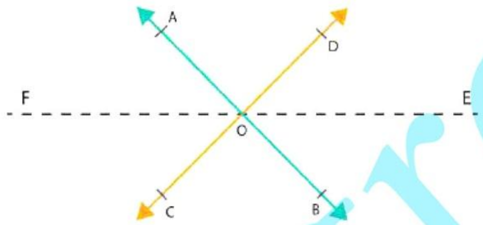
From the figure we know that ∠AOD and ∠BOC are vertically opposite angles
So we get
y = 125°

From the figure we know that ∠BOD and ∠AOC are vertically opposite angles
So we get
z = 55°

Therefore, the values of x, y and z are 55°, 125° and 55°

15. If two straight lines intersect each other than prove that the ray opposite to the bisector of one of the angles so formed bisects the vertically opposite angle.
Solution:

Given: Let us consider AB and CD as the two lines intersecting at a point O where OE is the ray bisecting ∠BOD and OF is the ray bisecting ∠AOC.



To Prove: ∠AOF = ∠COF

Proof: EF is a straight line passing through the point O where $\overrightarrow{OE} and \overrightarrow{OF}$ are two opposite rays.

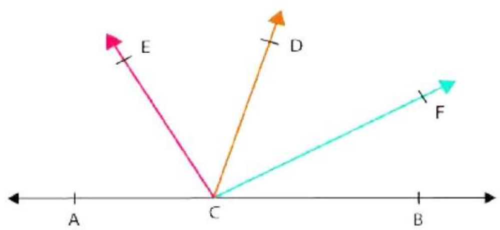
From the figure we know that ∠AOF and ∠BOE, ∠COF and ∠DOE are vertically opposite angles
So it can be written as
∠AOF = ∠BOE and ∠COF = ∠DOE
It is given that ∠BOE = ∠DOE
So we can write it as ∠AOF = ∠COF

Therefore, it is proved that ∠AOF = ∠COF.

16. Prove that the bisectors of two adjacent supplementary angles include a right angle.
Solution:

Given: $\overrightarrow{CE}$ is the bisector of ∠ACD and $\overrightarrow{CF}$ is the bisector of ∠BCD

To Prove: ∠ECF = 90°

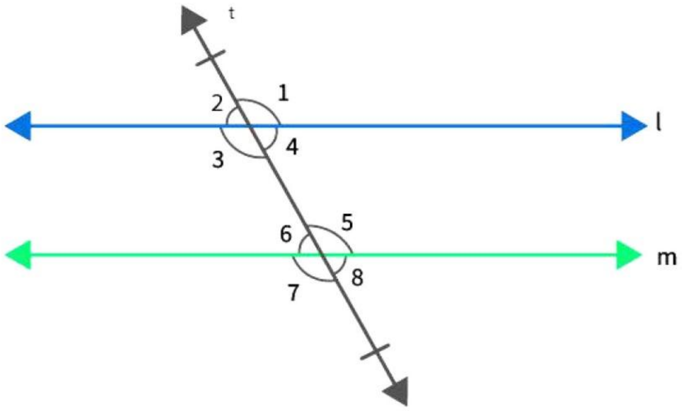


Proof:
From the figure we know that
∠ACD and ∠BCD form a linear pair of angles
So we can write it as
$\angle ACD + \angle BCD = 180^o$
We can also write it as
$\angle ACE + \angle ECD + \angle DCF + \angle FCB = 180^o$
From the figure we also know that
∠ACE = ∠ECD and ∠DCF = ∠FCB
So it can be written as
$\angle ECD + \angle ECD + \angle DCF + \angle DCF = 180^o$
On further calculation we get
$2\ \angle ECD + 2\ \angle DCF = 180^o$
Taking out 2 as common we get
$2\ (\angle ECD + \angle DCF) = 180^o$
By division we get
$(\angle ECD + \angle DCF) = 180/2$
$\angle ECD + \angle DCF = 90^o$

Therefore, it is proved that $\angle ECF = 90^o$

EXERCISE 7(C) PAGE: 223

1. **In the given figure, l || m and a transversal t cuts them. If ∠1 = 120°, find the measure of each of the remaining marked angles.**



Solution:

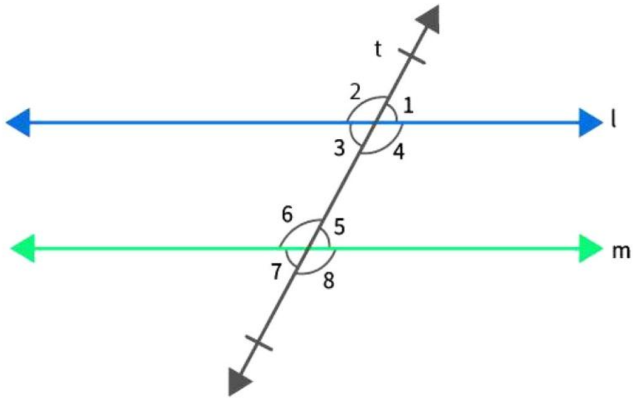
It is given that ∠1 = 120°
From the figure we know that ∠1 and ∠2 form a linear pair of angles
So it can be written as
∠1 + ∠2 = 180°
By substituting the values
120° + ∠2 = 180°
On further calculation
∠2 = 180° - 120°
By subtraction
∠2 = 60°

From the figure we know that ∠1 and ∠3 are vertically opposite angles
So we get
∠1 = ∠3 = 120°

From the figure we know that ∠2 and ∠4 are vertically opposite angles
So we get
∠2 = ∠4 = 60°

It is given that, l || m and t is a transversal
So the corresponding angles according to the figure is written as
∠1 = ∠5 = 120°
∠2 = ∠6 = 60°
∠3 = ∠7 = 120°
∠4 = ∠8 = 60°

2. **In the figure, l || m and a transversal t cuts them. If ∠7 = 80°, find the measure of each of the remaining marked angles.**



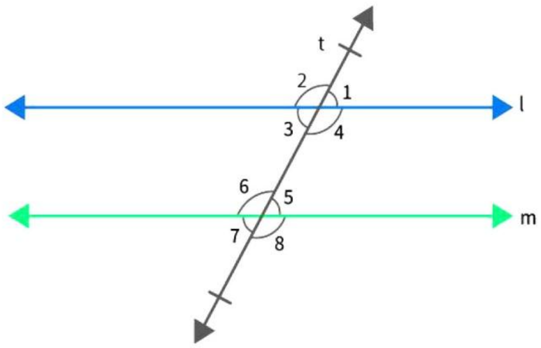
Solution:

It is given that ∠7 = 80°
From the figure we know that ∠7 and ∠8 form a linear pair of angles
So it can be written as
∠7 + ∠8 = 180°
By substituting the values
80° + ∠8 = 180°
On further calculation
∠8 = 180° - 80°
By subtraction
∠8 = 100°

From the figure we know that ∠7 and ∠5 are vertically opposite angles
So we get
∠7 = ∠5 = 80°
From the figure we know that ∠6 and ∠8 are vertically opposite angles
So we get
∠6 = ∠8 = 100°

It is given that, l || m and t is a transversal
So the corresponding angles according to the figure is written as
∠1 = ∠5 = 80°
∠2 = ∠6 = 100°
∠3 = ∠7 = 80°
∠4 = ∠8 = 100°

3. In the figure, l || m and a transversal t cuts them. If ∠1: ∠2 = 2: 3, find the measure of each of the marked angles.



Solution:

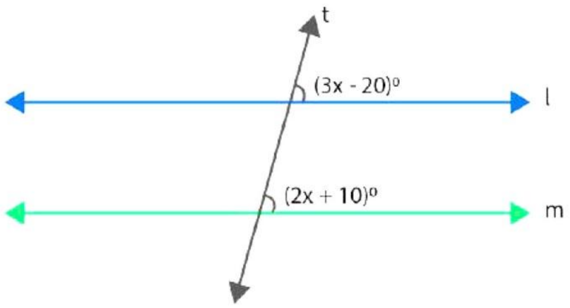
It is given that ∠1: ∠2 = 2: 3
From the figure we know that ∠1 and ∠2 form a linear pair of angles
So it can be written as
∠1 + ∠2 = 180°
By substituting the values
2x + 3x = 180°
On further calculation
5x = 180°
By division
x = 180°/5
x = 36°

By substituting the value of x we get
∠1 = 2x = 2 (36°) = 72°
∠2 = 3x = 3 (36°) = 108°

From the figure we know that ∠1 and ∠3 are vertically opposite angles
So we get
∠1 = ∠3 = 72°
From the figure we know that ∠2 and ∠4 are vertically opposite angles
So we get
∠2 = ∠4 = 108°

It is given that, l || m and t is a transversal
So the corresponding angles according to the figure is written as
∠1 = ∠5 = 72°
∠2 = ∠6 = 108°
∠3 = ∠7 = 72°
∠4 = ∠8 = 108°

4. For what value of x will the lines l and m be parallel to each other?

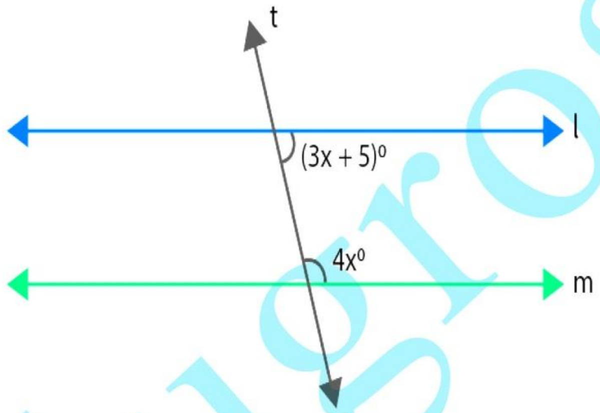


Solution:

If the lines l and m are parallel it can be written as
3x – 20 = 2x + 10
We know that the two lines are parallel if the corresponding angles are equal
3x – 2x = 10 + 20
On further calculation
x = 30

Therefore, the value of x is 30.

5. For what value of x will the lines l and m be parallel to each other?

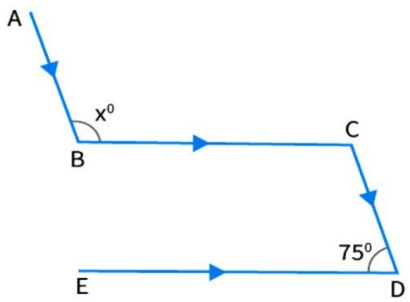


Solution:

We know that both the angles are consecutive interior angles
So it can be written as
3x + 5 + 4x = 180
On further calculation we get
7x = 180 – 5
By subtraction
7x = 175
By division we get
x = 175/ 7
x = 25

Therefore, the value of x is 25.

6. In the figure, AB || CD and BC || ED. Find the value of x.



Solution:

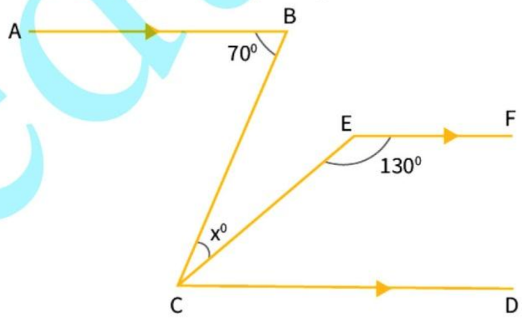
From the given figure we know that
AB and CD are parallel line and BC is a transversal
We know that ∠BCD and ∠ABC are alternate angles
So we can write it as
∠BCD + ∠ABC = x°

We also know that BC || ED and CD is a transversal
From the figure we know that ∠BCD and ∠EDC form a linear pair of angles
So it can be written as
∠BCD + ∠EDC = 180°
By substituting the values we get
∠BCD + 75° = 180°
On further calculation we get
∠BCD = 180° - 75°
By subtraction
∠BCD = 105°

From the figure we know that ∠BCD and ∠ABC are vertically opposite angles
So we get
∠BCD = ∠ABC = x = 105°
∠ABC = x = 105°

Therefore, the value of x is 105°.

7. In the figure, AB || CD || EF. Find the value of x.



Solution:

It is given that AB || CD and BC is a transversal
From the figure we know that ∠BCD and ∠ABC are alternate interior angles
So we get
∠ABC = ∠BCD

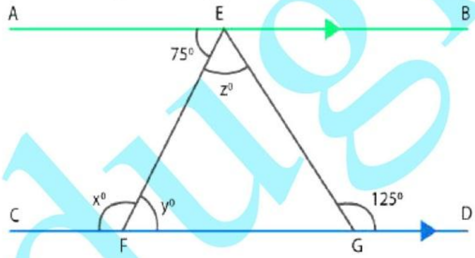
In order to find the value of x we can write it as
x^o + ∠ECD = 70^o ……. (1)

It is given that CD || EF and CE is a transversal
From the figure we know that ∠ECD and ∠CEF are consecutive interior angles
So we get
∠ECD + ∠CEF = 180^o
By substituting the values
∠ECD + 130^o = 180^o
On further calculation we get
∠ECD = 180^o - 130^o
By subtraction
∠ECD = 50^o

Now by substituting ∠ECD in equation (1) we get
x^o + ∠ECD = 70^o
$x^o + 50^o = 70^o$
On further calculation we get
$x^o = 70^o - 50^o$
By subtraction
$x^o = 20^o$

Therefore, the value of x is 20^o.

8. In the given figure, AB || CD. Find the values of x, y and z.



Solution:

It is given that AB || CD and EF is a transversal
From the figure we know that ∠AEF and ∠EFG are alternate angles
So we get
∠AEF = ∠EFG = 75^o
∠EFG = y = 75^o

From the figure we know that ∠EFC and ∠EFG form a linear pair of angles
So we get
∠EFC + ∠EFG = 180^o
It can also be written as

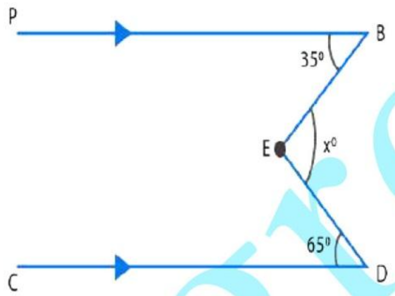
$x + y = 180^\circ$
By substituting the value of y we get
$x + 75^\circ = 180^\circ$
On further calculation we get
$x = 180^\circ - 75^\circ$
By subtraction
$x = 105^\circ$

From the figure based on the exterior angle property it can be written as
$\angle EGD = \angle EFG + \angle FEG$
By substituting the values in the above equation we get
$125^\circ = y + z$
$125^\circ = 75^\circ + z$
On further calculation we get
$z = 125^\circ - 75^\circ$
By subtraction
$z = 50^\circ$

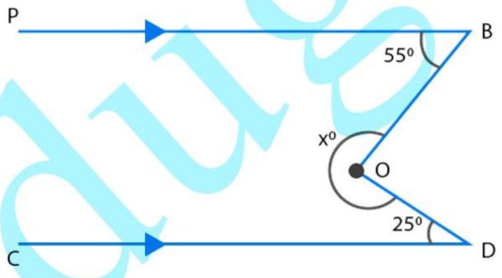
Therefore, the values of x, y and z are 105°, 75° and 50°.

9. In each of the figures given below, AB || CD. Find the value of x in each case.

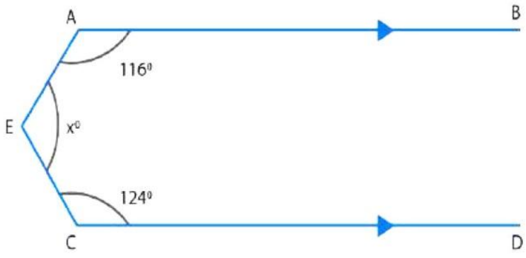
(i)



(ii)

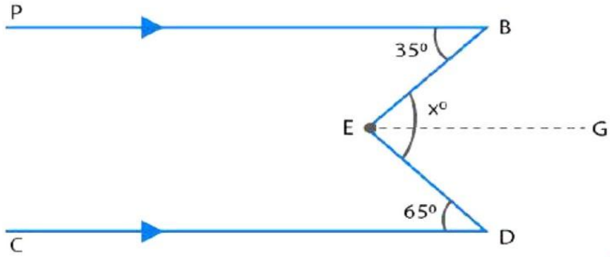


(iii)



Solution:

(i) Draw a line at point E which is parallel to CD and name it as EG



It is given that EG || CD and ED is a transversal

From the figure we know that ∠GED and ∠EDC are alternate interior angles
So we get
∠GED = ∠EDC = 65°

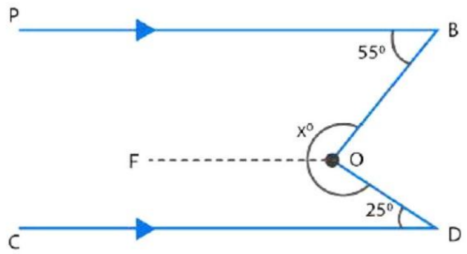
EG || CD and AB || CD
So we get EG || AB and EB is a transversal

From the figure we know that ∠BEG and ∠ABE are alternate interior angles
So we get
∠BEG = ∠ABE = 35°
∠DEB = x°

From the figure we can write ∠DEB as
∠DEB = ∠BEG + ∠GED
By substituting the values
x° = 35° + 65°
By addition
x° = 100°

Therefore, the value of x is 100.

(ii) Draw a line OF which is parallel to CD



So we get
OF || CD and OD is a transversal

From the figure we know that ∠CDO and ∠FOD are consecutive angles
So we get
$\angle CDO + \angle FOD = 180^\circ$
By substituting the values
$25^\circ + \angle FOD = 180^\circ$
On further calculation
$\angle FOD = 180^\circ - 25^\circ$
By subtraction
$\angle FOD = 155^\circ$

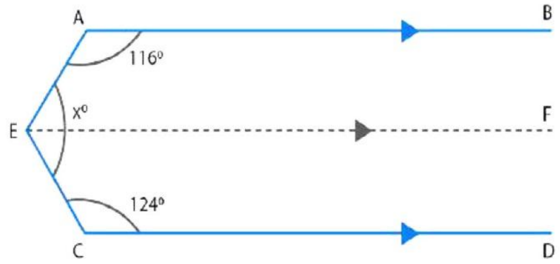
We also know that OF || CD and AB || CD
So we get OF || AB and OB is a transversal

From the figure we know that ∠ABO and ∠FOB are consecutive angles
So we get
$\angle ABO + \angle FOB = 180^\circ$
By substituting the values
$55^\circ + \angle FOB = 180^\circ$
On further calculation
$\angle FOB = 180^\circ - 55^\circ$
By subtraction
$\angle FOB = 125^\circ$

In order to find the value of x
$x^\circ = \angle FOB + \angle FOD$
By substituting the values
$x^\circ = 125^\circ + 155^\circ$
By addition
$x^\circ = 280^\circ$

Therefore, the value of x is 280.

(iii) Draw a line EF through point O which is parallel to CD



So we get EF || CD and EC is a transversal

From the figure we know that ∠FEC and ∠ECD are consecutive interior angles
So we get
∠FEC + ∠ECD = 180°
By substituting the values
∠FEC + 124° = 180°
On further calculation
∠FEC = 180° - 124°
By subtraction
∠FEC = 56°

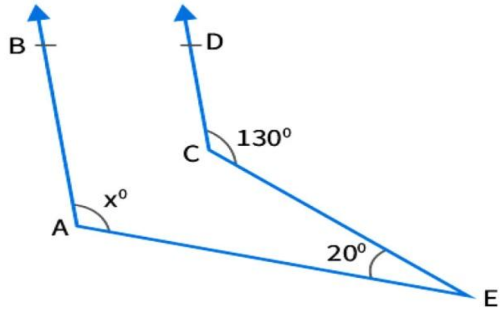
We know that EF || CD and AB || CD
So we get EF || AB and AE is a transversal

From the figure we know that ∠BAE and ∠FEA are consecutive interior angles
So we get
∠BAE + ∠FEA = 180°
By substituting the values
116° + ∠FEA = 180°
On further calculation
∠FEA = 180° - 116°
By subtraction
∠FEA = 64°

In order to find the value of x
x° = ∠FEA + ∠FEC
By substituting the values
x° = 64° + 56°
By addition
x° = 120°

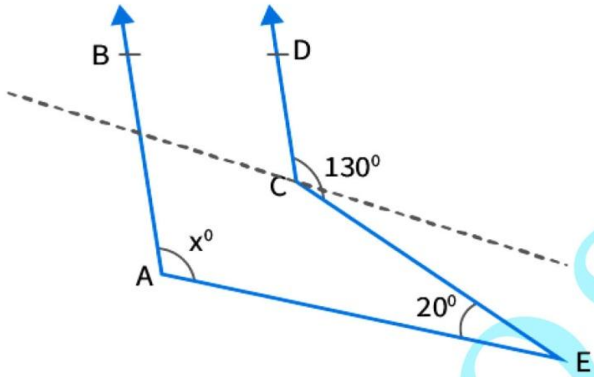
Therefore, the value of x is 120°

10. In the given figure, AB || CD. Find the value of x.



Solution:

Draw a line through point C and name it as FG where FG || AE



We know that CG || BE and CE is a transversal

From the figure we know that ∠GCE and ∠CEA are alternate angles
So we get
∠GCE = ∠CEA = 20°

It can also be written as
∠DCG = ∠DCE - ∠GCE
By substituting the values we get
∠DCG = 130° - 20°
By subtraction we get
∠DCG = 110°

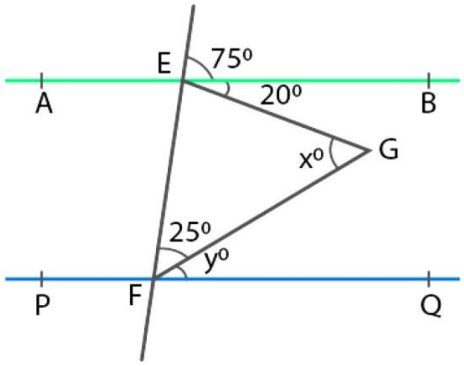
We also know that AB || CD and FG is a transversal

From the figure we know that ∠BFC and ∠DCG are corresponding angles
So we get
∠BFC = ∠DCG = 110°

We know that FG || AE and AF is a transversal
From the figure we know that ∠BFG and ∠FAE are corresponding angles
So we get
∠BFG = ∠FAE = 110°
∠FAE = x = 110°

Therefore, the value of x is 110.

11. In the given figure, AB || PQ. Find the values of x and y.



Solution:

It is given that AB || PQ and EF is a transversal
From the figure we know that ∠CEB and ∠EFQ are corresponding angles
So we get
∠CEB = ∠EFQ = 75°

It can be written as
∠EFQ = 75°
Where
∠EFG + ∠GFQ = 75°
By substituting the values
25° + y° = 75°
On further calculation
y° = 75° - 25°
By subtraction
y° = 50°

From the figure we know that ∠BEF and ∠EFQ are consecutive interior angles
So we get
∠BEF + ∠EFQ = 180°
By substituting the values
∠BEF + 75° = 180°
On further calculation
∠BEF = 180° - 75°
By subtraction
∠BEF = 105°

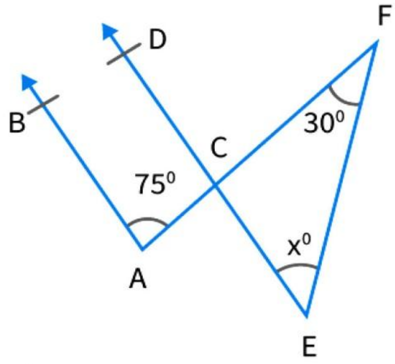
We know that ∠BEF can be written as
∠BEF = ∠FEG + ∠GEB
105° = ∠FEG + 20°
On further calculation
∠FEG = 105° - 20°
By subtraction

∠FEG = 85°

According to the △ EFG
We can write
x° + 25° + ∠FEG = 180°
By substituting the values
x° + 25° + 85° = 180°
On further calculation
x° = 180° - 25° - 85°
By subtraction
x° = 180° – 110°
x° = 70°

Therefore, the value of x is 70.

12. In the given figure, AB || CD. Find the value of x.



Solution:

It is given that AB || CD and AC is a transversal.
From the figure we know that ∠BAC and ∠ACD are consecutive interior angles
So we get
∠BAC + ∠ACD = 180°
By substituting the values
75° + ∠ACD = 180°
On further calculation
∠ACD = 180° - 75°
By subtraction
∠ACD = 105°

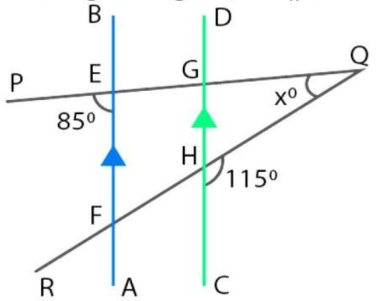
From the figure we know that ∠ECF and ∠ACD are vertically opposite angles
So we get
∠ECF = ∠ACD = 105°

According to the △ CEF
We can write
∠ECF + ∠CEF + ∠EFC = 180°
By substituting the values
105° + x° + 30° = 180°
On further calculation

$x^o = 180^o - 105^o - 30^o$
By subtraction
$x^o = 180^o – 135^o$
$x^o = 45^o$

Therefore, the value of x is 45.

13. In the given figure, AB || CD. Find the value of x.



Solution:

It is given that AB || CD and PQ is a transversal
From the figure we know that ∠PEF and ∠EGH are corresponding angles
So we get
∠PEF = ∠EGH = 85^o

From the figure we also know that ∠EGH and ∠QGH form a linear pair of angles
So we get
∠EGH + ∠QGH = 180^o
By substituting the values we get
85^o + ∠QGH = 180^o
On further calculation
∠QGH = $180^o - 85^o$
By subtraction
∠QGH = 95^o

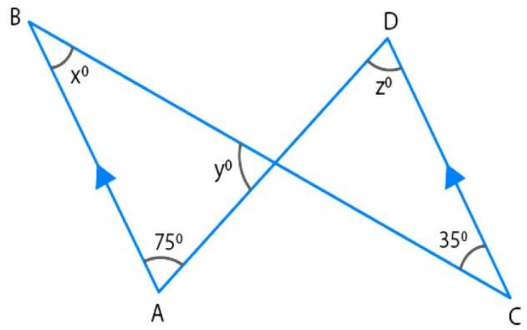
We can also find the ∠GHQ
∠GHQ + ∠CHQ = 180^o
By substituting the values
∠GHQ + $115^o = 180^o$
On further calculation
∠GHQ = $180^o - 115^o$
By subtraction
∠GHQ = 65^o

According to the △ GHQ
We can write
∠GQH + ∠GHQ + ∠QGH = 180^o
By substituting the values
$x^o + 65^o + 95^o = 180^o$
On further calculation

$x^o = 180^o - 65^o - 95^o$
By subtraction
$x^o = 180^o – 160^o$
$x^o = 20^o$

Therefore, the value of x is 20.

14. In the given figure, AB || CD. Find the value of x, y and z.



Solution:

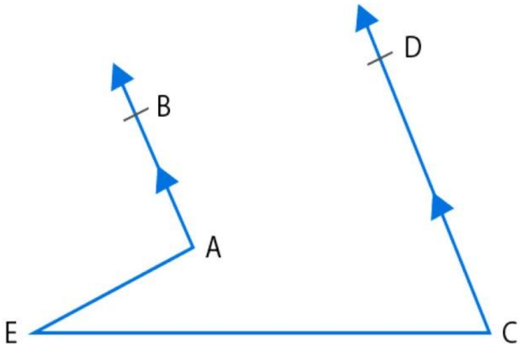
It is given that AB || CD and BC is a transversal
From the figure we know that ∠ABC = ∠BCD
So we get
x = 35

It is also given that AB || CD and AD is a transversal
From the figure we know that ∠BAD = ∠ADC
So we get
z = 75

According to the △ ABO
We can write
$\angle ABO + \angle BAO + \angle BOA = 180^o$
By substituting the values
$x^o + 75^o + y^o = 180^o$
$35^o + 75^o + y^o = 180^o$
On further calculation
$y^o = 180^o - 35^o - 75^o$
By subtraction
$y^o = 180^o – 110^o$
$y^o = 70^o$

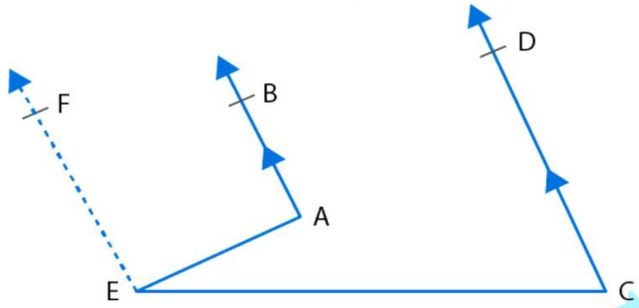
Therefore, the value of x, y and z is 35, 70 and 75.

15. In the given figure, AB || CD. Prove that ∠BAE - ∠ECD = ∠AEC.



Solution:

Construction a line EF which is parallel to AB and CD through the point E



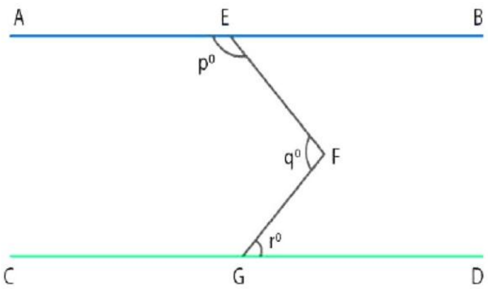
We know that EF || AB and AE is a transversal
From the figure we know that ∠BAE and ∠AEF are supplementary
So it can be written as
$\angle BAE + \angle AEF = 180^\circ$….. (1)

We also know that EF || CD and CE is a transversal
From the figure we know that ∠DCE and ∠CEF are supplementary
So it can be written as
$\angle DCE + \angle CEF = 180^\circ$

According to the diagram the above equation can be written as
$\angle DCE + (\angle AEC + \angle AEF) = 180^\circ$
From equation (1) we know that ∠AEF can be written as $180^\circ - \angle BAE$
So we get
$\angle DCE + \angle AEC + 180^\circ - \angle BAE = 180^\circ$
So we get
$\angle BAE - \angle DCE = \angle AEC$

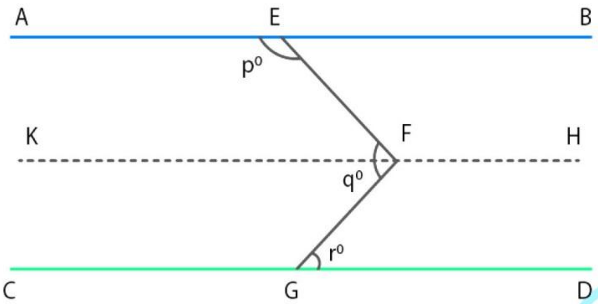
Therefore, it is proved that $\angle BAE - \angle DCE = \angle AEC$

16. In the given figure, AB || CD. Prove that p + q – r = 180.



Solution:

Draw a line KH passing through the point F which is parallel to both AB and CD
We know that KF || CD and FG is a transversal



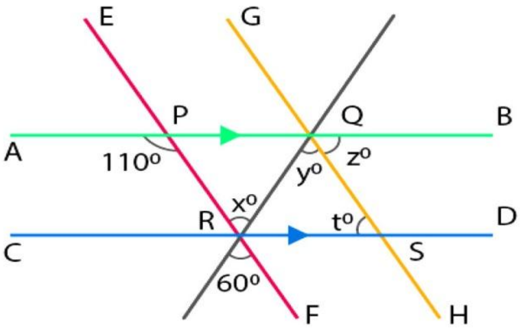
From the figure we know that ∠KFG and ∠FGD are alternate angles
So we get
∠KFG = ∠FGD = r° ……. (1)

We also know that AE || KF and EF is a transversal
From the figure we know that ∠AEF and ∠KFE are alternate angles
So we get
∠AEF + ∠KFE = 180°
By substituting the values we get
p° + ∠KFE = 180°
So we get
∠KFE = 180° - p° ……. (2)

By adding both the equations (1) and (2) we get
∠KFG + ∠KFE = 180° - p° + r°
From the figure ∠KFG + ∠KFE can be written as ∠EFG
∠EFG = 180° - p° + r°
We know that ∠EFG = q°
q° = 180° - p° + r°
It can be written as
p + q – r = 180°

Therefore, it is proved that p + q – r = 180°

17. In the given figure, AB || CD and EF || GH. Find the values of x, y, z and t.



Solution:

From the figure we know that ∠PRQ = x° = 60° as the vertically opposite angles are equal
We know that EF || GH and RQ is a transversal

From the figure we also know that ∠PRQ and ∠RQS are alternate angles
So we get
∠PRQ = ∠RQS
∠x = ∠y = 60°

We know that AB || CD and PR is a transversal
From the figure we know that ∠PRD and ∠APR are alternate angles
So we get
∠PRD = ∠APR
It can be written as
∠PRQ + ∠QRD = ∠APR
By substituting the values we get
x + ∠QRD = 110°
60° + ∠QRD = 110°
On further calculation
∠QRD = 110° - 60°
By subtraction
∠QRD = 50°

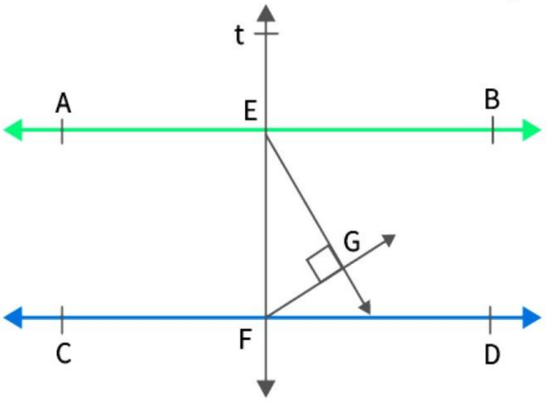
According to the △ QRS
We can write
∠QRD + ∠QSR + ∠RQS = 180°
By substituting the values
∠QRD + t° + y° = 180°
50° + t° + 60° = 180°
On further calculation
t° = 180° - 50° - 60°
By subtraction
t° = 180° – 110°
t° = 70°

We know that AB || CD and GH is a transversal
From the figure we know that z° and t° are alternate angles

So we get
$z^\circ = t^\circ = 70^\circ$

Therefore, the values of x, y, z and t are 60º, 60º, 70º and 70º.

18. In the given figure, AB || CD and a transversal t cuts them at E and F respectively. If EG and FG are the bisectors of ∠BEF and ∠EFD respectively, prove that ∠EGF = 90º.



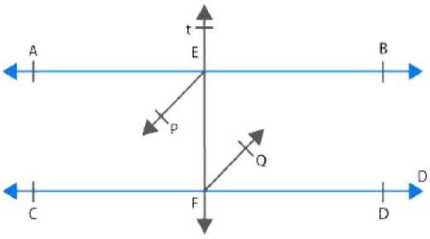
Solution:

We know that AB || CD and t is a transversal cutting at points E and F
From the figure we know that ∠BEF and ∠DFE are interior angles
So we get
∠BEF + ∠DFE = 180º
Dividing the entire equation by 2 we get
(1/2)∠BEF + (1/2) ∠DFE = 90º
According to the figure the above equation can further be written as
∠GEF + ∠GFE = 90º ……. (1)

According to the △ GEF
We can write
∠GEF + ∠GFE + ∠EGF = 180º
Based on equation (1) we get
90º + ∠EGF = 180º
On further calculation
∠EGF = 180º - 90º
By subtraction
∠EGF = 90º

Therefore, it is proved that ∠EGF = 90º

19. In the given figure, AB || CD and a transversal t cuts them at E and F respectively. If EP and FQ are the bisectors of ∠AEF and ∠EFD respectively, prove that EP || FQ.

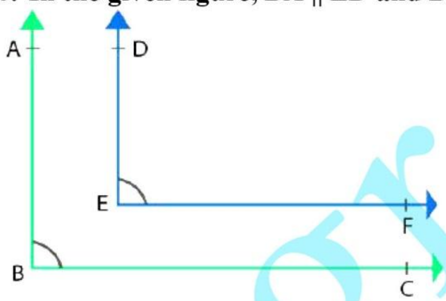


Solution:

We know that AB || CD and t is a transversal
From the figure we know that ∠AEF and ∠EFD are alternate angles
So we get
∠AEF = ∠EFD
Dividing both the sides by 2 we get
(1/2) ∠AEF = (1/2) ∠EFD
So we get
∠PEF = ∠EFQ
The alternate interior angles are formed only when the transversal EF cuts both FQ and EP.

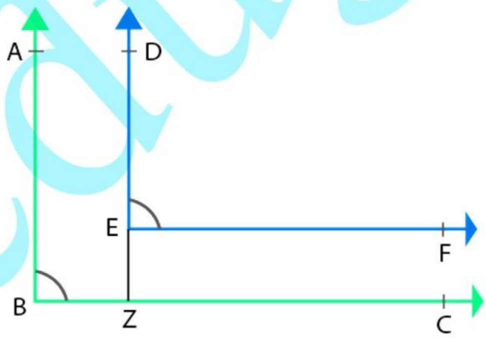
Therefore, it is proved that EP || FQ.

20. In the given figure, BA || ED and BC || EF. Show that ∠ABC = ∠DEF.



Solution:

Extend the line DE to meet the line BE at the point Z.



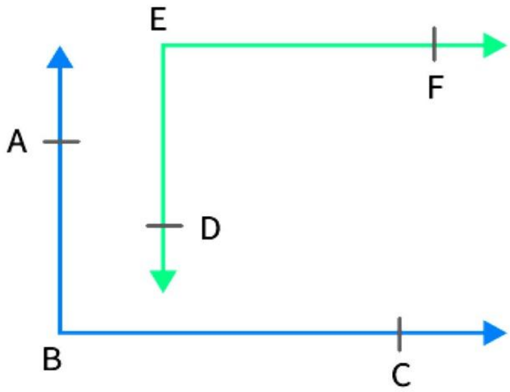
We know that AB || DZ and BC is a transversal
From the figure we know that ∠ABC and ∠DZC are corresponding angles
So we get
∠ABC = ∠DZC ….. (1)

We also know that EF || BC and DZ is a transversal
From the figure we know that ∠DZC and ∠DEF are corresponding angles
So we get
∠DZC = ∠DEF …… (2)

Considering both the equation (1) and (2) we get
∠ABC = ∠DEF

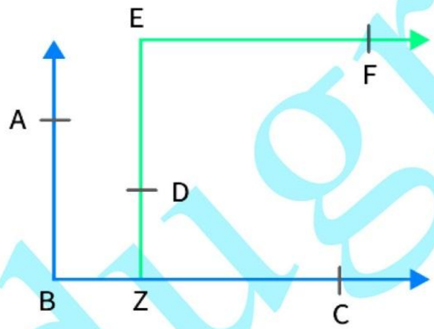
Therefore, it is proved that ∠ABC = ∠DEF

21. In the given figure, BA || ED and BC || EF. Show that ∠ABC + ∠DEF = 180º.



Solution:

Extend the line ED to meet the line BC at the point Z



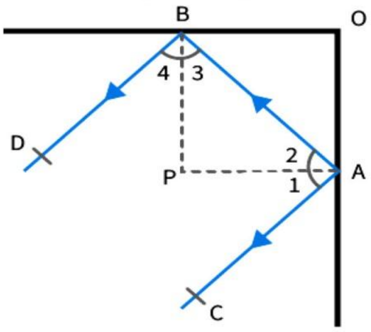
We know that AB || EZ and BC is a transversal
From the figure we know that ∠ABZ and ∠EZB are interior angles
So we get
∠ABZ + ∠EZB = 180º
∠ABZ can also be written as ∠ABC
∠ABC + ∠EZB = 180º …… (1)

We know that EF || BC and EZ is a transversal
From the figure we know that ∠BZE and ∠ZEF are alternate angles
So we get
∠BZE = ∠ZEF
∠ZEF can also be written as ∠DEF
∠BZE = ∠DEF ….. (2)

By substituting equation (1) in (2) we get
∠ABC + ∠DEF = 180°

Therefore, it is proved that ∠ABC + ∠DEF = 180°

22. In the given figure, m and n are two plane mirrors perpendicular to each other. Show that the incident ray CA is parallel to the reflected ray BD.



Solution:

Construct a line m and n from A and B intersect at P
So we get
OB ⊥ m and OC ⊥ n
So m ⊥ n

We can also write it as
OB ⊥ OC

Since APB is a right angle triangle
We know that ∠APB = 90°

So we can write it as
∠APB = ∠PAB + ∠PBA
By substituting the values
90° = ∠2 + ∠3

We know that angle of incidence is equal to the angle of reflection
So we get
∠1 = ∠2 and ∠4 = ∠3
It can be written as
∠1 + ∠4 = ∠2 + ∠3 = 90°
We can write it as
∠1 + ∠2 + ∠3 + ∠4 = 180°
We know that ∠1 + ∠2 = ∠CAB and ∠3 + ∠4 = ∠ABD
∠CAB + ∠ABD = 180°

According to the diagram ∠CAB and ∠ABD are consecutive interior angles when the transversal AB cuts BD and CA.

$\angle 2 = \angle 3 = 90^\circ$

We know that $\angle 2$ and $\angle 3$ are corresponding angles when the transversal n cuts p and q.
So we get p || q.

Therefore, it is shown that the two lines which are perpendicular to two parallel lines are parallel to each other.