RS Aggarwal Solutions for Class 9 Maths Chapter 3 – Factorisation of Polynomials

EXERCISE 3(A) PAGE: 99

Factorise

1. $9x^2 + 12xy$
2. $18x^2y - 24\,xyz$
3. $27a^3b^3 - 45a^4b^2$
4. $2a\,(x + y) - 3b(x + y)$
5. $2x\,(p^2 + q^2) + 4y(p^2 + q^2)$
6. x(a-5) + y(5-a)
7. 4(a+b) - $6(a + b)^2$
8. $8(3a - 2b)^2 - 10(3a - 2b)$
9. $x(x + y)^3 - 3x^2y(x + y)$
10. $x^3 + 2x^2 + 5x + 10$
11. $x^2 + xy - 2xz - 2yz$
12. $a^3b - a^2b$ + 5ab – 5b
13. $8 - 4a - 2a^3 + a^4$
14. $x^3 - 2x^2y + 3xy^2 - 6y^3$
15. $px - 5q + pq - 5x$
16. $x^2 + y - xy - x$
17. $(3a - 1)^2 - 6a + 2$
18. $(2x - 3)^2 - 8x + 12$
19. $a^3 + a - 3a^2 - 3$
20. 3ax – 6ay – 8by + 4bx
21. $abx^2 + a^2x + b^2x + ab$
22. $x^3 - x^2 + ax + x - a - 1$
23. 2x + 4y – 8xy – 1
24. $ab\left(x^2 + y^2\right) - xy\left(a^2 + b^2\right)$
25. $a^2 + ab(b + 1) + b^3$
26. $a^3 + ab(1 - 2a) - 2b^2$
27. $2a^2 + bc - 2ab - ac$
28. $(ax + by)^2 + (bx - ay)^2$
29. $a(a + b - c) - bc$
30. $a(a - 2b - c) + 2bc$
31. $a^2x^2 + \left(ax^2 + 1\right)x + a$
32. $ab\left(x^2 + 1\right) + x\left(a^2 + b^2\right)$
33. $x^2 - (a + b)x + ab$
34. $x^2 + \frac{1}{x^2} - 2 - 3x + \frac{3}{x}$

Solution:

1. Consider $9x^2 + 12xy$
   Now by taking 3x as common,
   We get,
   = 3x (3x + 4y)

2. $18x^2y - 24$ xyz
   Now by taking 6xy as common in the question
   We get,
   = 6xy (3x – 4z)

3. $27a^3b^3 - 45a^4b^2$
   Now by taking $9a^3b^2$ as common in the question
   We get,
   = $9a^3b^2$(3b – 5a)

4. 2a (x + y) − 3b(x + y)
   We can write 2a (x + y) − 3b(x + y) as
   = (x + y) (2a – 3b)

5. 2x $(p^2 + q^2) + 4y(p^2 + q^2)$
   We can write the given question as
   = (2x + 4y) $(p^2 + q^2)$
   By taking 2 as common in the first term
   = $2(p^2 + q^2)$(x + 2y)

6. x(a-5) + y(5-a)
   We can write the given question as
   = x (a – 5) + y (-1) (a – 5)
   So we get,
   = (x – y) (a – 5)

7. 4(a + b) - $6(a + b)^2$
   By taking (a + b) as common,
   = (a + b) [4 – 6(a + b)]
   So we get,
   = 2 (a + b) (2 – 3a – 3b)

8. $8(3a - 2b)^2 - 10(3a - 2b)$
   By taking (3a – 2b) in the given question,
   We get,
   = (3a – 2b) [8 (3a – 2b) – 10]
   By taking 2 as common in the second term,
   = (3a – 2b) 2[4 (3a – 2b) – 5]
   So we get
   = 2 (3a – 2b) (12a – 8b – 5)

9. $x(x + y)^3 - 3x^2y(x + y)$
   Taking x(x + y) as common in the given question,
   We get,
   = x (x + y) $[(x + y)^2 - 3xy]$
   According to the equation $(a + b)^2 = a^2 + b^2 + 2ab$
   = x (x + y) $(x^2 + y^2 + 2xy - 3xy)$
   So we get,
   = x (x + y) $(x^2 + y^2 - xy)$

10. $x^3 + 2x^2 + 5x + 10$
    On further simplification of the given question,
    We get,
    = $x^2(x + 2) + 5(x + 2)$

So we get,
= $(x^2 + 5)(x + 2)$

11. $x^2 + xy - 2xz - 2yz$
On further simplification of the given question,
We get,
= x (x + y) – 2z (x + y)
So we get,
= (x + y) (x – 2z)

12. $a^3b - a^2b$ + 5ab – 5b
By taking a^2b common in the first term and 5b as common in the second term
= $a^2b(a - 1) + 5b(a - 1)$
So we get,
= (a – 1) (a^2b + 5b)
By taking b as common
= (a – 1) b ($a^2 + 5$)
So we get,
= b (a – 1) ($a^2 + 5$)

13. $8 - 4a - 2a^3 + a^4$
By taking 4 as common in the first term and a^3as common in the second term
= 4 (2 – a) - a^3 (2 – a)
So we get,
= (2 – a) (4 - a^3)

14. $x^3 - 2x^2y + 3xy^2 - 6y^3$
By taking x^2 as common in the first term and $3y^2$ as common in the second term
= $x^2(x - 2y) + 3y^2(x - 2y)$
So we get,
= (x – 2y) ($x^2 + 3y^2$)

15. $px - 5q + pq - 5x$
By taking p as common in the first term and q as common in the second term
= p (x + q) – 5 (q + x)
So we get,
= (x + q) (p – 5)

16. $x^2 + y - xy - x$
By taking x as common in the first term and 1 as common in the second term
= x (x-y) – 1 (x – y)
So we get,
= (x-1) (x-y)

17. $(3a - 1)^2 - 6a + 2$
We can further write it as
= $(3a - 1)^2$ – 2 (3a-1)
By taking (3a – 1) as common
= (3a – 1) [(3a – 1) – 2]

So we get,
= (3a – 1) (3a – 3)
By taking 3 as common in the second term,
= 3 (3a – 1) (a – 1)

18. $(2x-3)^2 - 8x + 12$
We can further write it as
= $(2x-3)^2$ – 4 (2x – 3)
By taking (2x – 3) as common
= (2x – 3) (2x – 3 – 4)
So we get
= (2x – 3) (2x – 7)

19. $a^3 + a - 3a^2 - 3$
We can further write it as
= a $(a^2+1) - 3(a^2+1)$
So we get,
= (a – 3) (a^2+1)

20. 3ax – 6ay – 8by + 4bx
We can further write it as
= 3a (x – 2y) + 4b (x – 2y)
So we get,
= (x – 2y) (3a + 4b)

21. $abx^2 + a^2x + b^2x + ab$
We can further write it as
= ax (bx + a) + b (bx + a)
So we get,
= (ax + b) (bx + a)

22. $x^3 - x^2 + ax + x - a - 1$
We can further write it as
= $x^3 - x^2 + ax - a + x - 1$
By taking the common terms out
= $x^2(x-1) + a(x-1) + 1(x-1)$
So we get,
= (x – 1) $(x^2 + a + 1)$

23. 2x + 4y – 8xy – 1
We can further write it as
= 2x – 1 – 8xy + 4y
By taking the common terms out
= (2x – 1) – 4y (2x – 1)
So we get,
= (2x – 1) (1 – 4y)

24. $ab\ (x^2+y^2) - xy\ (a^2+b^2)$
By multiplying the terms
= $abx^2 + aby^2 - a^2xy - b^2xy$

We can further write it as
$= abx^2 - a^2xy + aby^2 - b^2xy$
By taking the common terms out
= ax (bx – ay) + by (ay – bx)
By taking the negative sign out
= ax (bx – ay) – by (bx – ay)
So we get,
= (ax – by) (bx – ay)

25. $a^2 + ab(b+1) + b^3$
By multiplying the terms
$= a^2 + ab^2 + ab + b^3$
We can further write it as
$= a^2 + ab + ab^2 + b^3$
By taking the common terms out
= a (a + b) + b^2(a + b)
So we get,
= (a +b^2) (a + b)

26. $a^3 + ab(1 - 2a) - 2b^2$
By multiplying the terms
$= a^3 + ab \ - 2a^2b - 2b^2$
By taking the common terms out
= a $(a^2 + b) - 2b(a^2 + b)$
So we get,
$= (a^2 + b)(a - 2b)$

27. $2a^2 + bc - 2ab - ac$
We can further write it as
$= 2a^2 - 2ab - ac + bc$
By taking the common terms out
= 2a (a – b) – c (a – b)
So we get,
= (a – b) (2a – c)

28. $(ax + by)^2 + (bx - ay)^2$
Using the formula $(a + b)^2$ *and* $(a - b)^2$
We get
$= a^2x^2 + b^2y^2 + 2abxy + b^2x^2 + a^2y^2 - 2abxy$
$= a^2x^2 + b^2y^2 + b^2x^2 + a^2y^2$
Rearranging the terms
$= a^2x^2 + b^2x^2 + b^2y^2 + a^2y^2$
By taking the common terms out
$= x^2(a^2 + b^2) + y^2(a^2 + b^2)$
So we get,
$= (a^2 + b^2)(x^2 + y^2)$

29. $a(a + b - c) - bc$
We can further write it as

= a^2 + ab – ac – bc
By taking the common terms out
= a (a + b) – c (a + b)
So we get,
= (a + b) (a – c)

30. $a(a - 2b - c) + 2bc$
By multiplying the terms
= a^2 – 2ab – ac + 2bc
By taking the common terms out
= a (a – 2b) – c (a – 2b)
So we get,
= (a – c) (a – 2b)

31. $a^2x^2 + (ax^2 + 1)x + a$
By multiplying the terms
= $a^2x^2 + ax^3 + x + a$
By taking the common terms out
= $ax^2(a + x) + 1(x + a)$
So we get,
= (a + x) $(ax^2 + 1)$

32. $ab(x^2 + 1) + x(a^2 + b^2)$
By multiplying the terms
= $abx^2 + ab + a^2x + b^2x$
= $abx^2 + a^2x + ab + b^2x$
By taking the common terms out
= ax (bx +a) + b (bx + a)
So we get,
= (bx + a) (ax + b)

33. $x^2 - (a + b)x + ab$
By multiplying the terms
= x^2 – ax – bx + ab
By taking the common terms out
= x (x – a) – b (x – a)
So we get,
= (x – a) (a – b)

34. $x^2 + \frac{1}{x^2} - 2 - 3x + \frac{3}{x}$
We can further write it as
= $(x - \frac{1}{x})^2$- 3 $(x - \frac{1}{x})$
By taking the common terms out
= $(x - \frac{1}{x})$ $(x - \frac{1}{x} - 3)$

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EXERCISE 3(B) PAGE: 105

Factorise

1. $9x^2 - 16y^2$
2. $(\frac{25}{4}x^2 - \frac{1}{9}y^2)$
3. $81 - 16x^2$
4. $5 - 20x^2$
5. $2x^4 - 32$
6. $3a^3b - 243ab^3$
7. $3x^3 - 48x$
8. $27a^2 - 48b^2$
9. $x - 64x^3$
10. $8ab^2 - 18a^3$
11. $150 - 6x^2$
12. $2 - 50x^2$
13. $20x^2 - 45$
14. $(3a + 5b)^2 - 4c^2$
15. $a^2 - b^2 - a - b$
16. $4a^2 - 9b^2 - 2a - 3b$
17. $a^2 - b^2 + 2bc - c^2$
18. $4a^2 - 4b^2 + 4a + 1$
19. $a^2 + 2ab + b^2 - 9c^2$
20. $108a^2 - 3(b - c)^2$
21. $(a + b)^3 - a - b$
22. $x^2 + y^2 - z^2 - 2xy$
23. $x^2 + 2xy + y^2 - a^2 + 2ab - b^2$
24. $25x^2 - 10x + 1 - 36y^2$
25. $a - b - a^2 + b^2$
26. $a^2 - b^2 - 4ac + 4c^2$
27. $9 - a^2 + 2ab - b^2$
28. $x^3 - 5x^2 - x + 5$
29. $1 + 2ab - (a^2 + b^2)$
30. $9a^2 + 6a + 1 - 36b^2$
31. $x^2 - y^2 + 6y - 9$
32. $4x^2 - 9y^2 - 2x - 3y$
33. $9a^2 + 3a - 8b - 64b^2$
34. $x^2 + \frac{1}{x^2} - 3$
35. $x^2 - 2 + \frac{1}{x^2} - y^2$
36. $x^4 + \frac{4}{x^4}$
37. $x^8 - 1$
38. $16x^4 - 1$
39. $81x^4 - y^4$
40. $x^4 - 625$

Solution:

1. $9x^2 - 16y^2$

We can further write it as
= $(3x)^2 - (4y)^2$
Based on the equation $a^2 - b^2 = (a+b)(a-b)$
We get,
= (3x + 4y) (3x – 4y)

2. $\left(\frac{25}{4}x^2 - \frac{1}{9}y^2\right)$
We can further write it as
= $(\frac{5}{2}x)^2 - \left(\frac{1}{3}y\right)^2$
Based on the equation $a^2 - b^2 = (a+b)(a-b)$
We get,
= $\left(\frac{5}{2}x + \frac{1}{3}y\right)\left(\frac{5}{2}x - \frac{1}{3}y\right)$

3. $81 - 16x^2$
We can further write it as
= $9^2 - (4x)^2$
Based on the equation $a^2 - b^2 = (a+b)(a-b)$
We get,
= (9 + 4x) (9 – 4x)

4. $5 - 20x^2$
By taking 5 as common
= 5 $(1 - 4x^2)$
We can further write it as
= 5 $[(1)^2 - (2x)^2]$
Based on the equation $a^2 - b^2 = (a+b)(a-b)$
We get,
= 5 [(1 + 2x) (1 – 2x)]
= 5 (1 + 2x) (1 – 2x)

5. $2x^4 - 32$
By taking 2 as common
= 2 $(x^4 - 16)$
We can further write it as
= 2 $[(x^2)^2 - 4^2]$
Based on the equation $a^2 - b^2 = (a+b)(a-b)$
We get,
= 2 $[(x^2 + 4)(x^2 - 4)]$
So we get,
= 2 $[(x^2 - 2^2)(x^2 + 4)]$
By using the formula
= 2 [(x + 2) (x – 2) $(x^2 + 4)$]
= 2 (x + 2) (x – 2) $(x^2 + 4)$

6. $3a^3b - 243ab^3$
By taking 3ab as common
= 3ab $(a^2 - 81b^2)$

Based on the equation $a^2 - b^2 = (a + b)(a - b)$
We get,
= 3ab ($a^2 - (9b)^2$)
We can write it as
= 3ab (a + 9b) (a – 9b)

7. $3x^3 - 48x$
By taking 3x as common
= 3x ($x^2 - 16$)
We can write it as
= 3x ($x^2 - 4^2$)
Based on the equation $a^2 - b^2 = (a + b)(a - b)$
We get,
= 3x (x + 4) (x – 4)

8. $27a^2 - 48b^2$
By taking 3 as common
= $3(9a^2 - 16b^2)$
We can write it as
= $3[(3a)^2 - (4b)^2]$
Based on the equation $a^2 - b^2 = (a + b)(a - b)$
We get,
= 3 (3a + 4b) (3a – 4b)

9. $x - 64x^3$
By taking x as common
= x (1 – $64x^2$)
We can write it as
= x [$1^2 - (8x)^2$]
Based on the equation $a^2 - b^2 = (a + b)(a - b)$
We get,
= x (1 + 8x) (1 – 8x)

10. $8ab^2 - 18a^3$
By taking 2a as common
= 2a ($4b^2 - 9a^2$)
We can write it as
= 2a [$(2b)^2 - (3a)^2$]
Based on the equation $a^2 - b^2 = (a + b)(a - b)$
We get,
= 2a (2b + 3a) (2b – 3a)

11. $150 - 6x^2$
By taking 6 as common
= 6 (25 - x^2)
We can write it as
= 6 ($5^2 - x^2$)
Based on the equation $a^2 - b^2 = (a + b)(a - b)$
We get,

= 6 (5 + x) (5 – x)

12. $2 - 50x^2$
By taking 2 as common
= 2 (1 - $25x^2$)
We can write it as
= 2 $[1^2 - (5x)^2]$
Based on the equation $a^2 - b^2 = (a + b)(a - b)$
We get,
= 2 (1 + 5x) (1 – 5x)

13. $20x^2 - 45$
By taking 5 as common
= 5 $(4x^2 - 9)$
We can write it as
= 5 $((2x)^2 - 3^2)$
Based on the equation $a^2 - b^2 = (a + b)(a - b)$
We get,
= 5 (2x + 3) (2x – 3)

14. $(3a + 5b)^2 - 4c^2$
We can write it as
= $(3a + 5b)^2 - (2c)^2$
Based on the equation $a^2 - b^2 = (a + b)(a - b)$
We get,
= (3a + 5b + 2c) (3a + 5b – 2c)

15. $a^2 - b^2 - a - b$
We can write it as
= $a^2 - b^2 - (a + b)$
Based on the equation $a^2 - b^2 = (a + b)(a - b)$
We get,
= (a + b) (a – b) – (a + b)
By taking (a + b) as common
= (a + b) (a – b – 1)

16. $4a^2 - 9b^2 - 2a - 3b$
We can write it as
= $(2a)^2 - (3b)^2 - (2a + 3b)$
Based on the equation $a^2 - b^2 = (a + b)(a - b)$
We get,
= (2a + 3b) (2a – 3b) – (2a + 3b)
By taking (2a + 3b) as common
= (2a + 3b) (2a – 3b – 1)

17. $a^2 - b^2 + 2bc - c^2$
We can write it as
= $a^2 - (b^2 - 2bc + c^2)$
Based on the equation $a^2 - b^2 = (a + b)(a - b)$

We get,
= $a^2 - (b-c)^2$
Applying the formula
= [a + (b – c)] [a – (b – c)]
= (a + b – c) (a – b + c)

18. $4a^2 - 4b^2 + 4a + 1$
We can write it as
= $(4a^2 + 4a + 1) - 4b^2$
Based on the equation $a^2 - b^2 = (a+b)(a-b)$
We get,
= $[(2a)^2 + 2 \times 2a \times 1 + 1^2] - (2b)^2$
On further calculation
= $(2a+1)^2 - (2b)^2$
Using the formula
= (2a + 1 + 2b) (2a + 1 – 2b)
So we get
= (2a + 2b + 1) (2a – 2b + 1)

19. $a^2 + 2ab + b^2 - 9c^2$
We can write the given question as
= $(a+b)^2 - (3c)^2$
Based on the equation $a^2 - b^2 = (a+b)(a-b)$
We get,
= (a + b + 3c) (a + b – 3c)

20. $108a^2 - 3(b-c)^2$
Taking 3 as common in the given question
= 3 $[36a^2 - (b-c)^2]$
We can write it as
= 3 $[(6a)^2 - (b-c)^2]$
Based on the equation $a^2 - b^2 = (a+b)(a-b)$
We get,
= 3 (6a + b – c) (6a – b + c)

21. $(a+b)^3 - a - b$
We can write it as
= $(a+b)^3 - (a+b)$
By taking (a + b) as common
= (a + b) $[(a+b)^2 - 1^2]$
Based on the equation $a^2 - b^2 = (a+b)(a-b)$
We get,
= (a + b) (a + b + 1) (a + b – 1)

22. $x^2 + y^2 - z^2 - 2xy$
We can write it as
= $(x^2 + y^2 - 2xy) - z^2$
So we get
= $(x-y)^2 - z^2$

Based on the equation $a^2 - b^2 = (a + b)(a - b)$
We get,
= (x – y + z) (x – y – z)

23. $x^2 + 2xy + y^2 - a^2 + 2ab - b^2$
We can further write it as
$= (x^2 + 2xy + y^2) - (a^2 - 2ab + b^2)$
So we get
$= (x + y)^2 - (a - b)^2$
Based on the equation $a^2 - b^2 = (a + b)(a - b)$
We get,
= [(x + y) + (a – b)] [(x + y) – (a – b)]
= (x + y + a – b) (x + y – a + b)

24. $25x^2 - 10x + 1 - 36y^2$
We can further write the given question as
$= (25x^2 - 10x + 1) - 36y^2$
So we get
$= [(5x)^2 - 2 \times 5x \times 1 + 1^2] - (6y)^2$
$= (5x - 1)^2 - (6y)^2$
Based on the equation $a^2 - b^2 = (a + b)(a - b)$
We get,
= (5x – 1 + 6y) (5x – 1 – 6y)

25. $a - b - a^2 + b^2$
We can write it as
$= (a - b) - (a^2 - b^2)$
Based on the equation $a^2 - b^2 = (a + b)(a - b)$
We get,
= (a – b) – (a + b) (a – b)
By taking (a – b) as common
= (a – b) (1 – a – b)

26. $a^2 - b^2 - 4ac + 4c^2$
We can write it as
$= a^2 - 4ac + 4c^2 - b^2$
So we get
$= a^2 - 2 \times a \times 2c + 2c^2 - b^2$
$= (a - 2c)^2 - b^2$
Based on the equation $a^2 - b^2 = (a + b)(a - b)$
We get,
= (a – 2c + b) (a – 2c – b)

27. $9 - a^2 + 2ab - b^2$
We can further write it as
$= 9 - (a^2 - 2ab + b^2)$
So we get
$= 3^2 - (a - b)^2$
Based on the equation $a^2 - b^2 = (a + b)(a - b)$

We get,
= (3 + a – b) (3 – a + b)

28. $x^3 - 5x^2 - x + 5$
By taking the common terms
= $x^2\ (x - 5) - 1(x - 5)$
So we get
= (x – 5) (x^2- 1)
Based on the equation $a^2 - b^2 = (a + b)(a - b)$
We get,
= (x – 5) (x + 1) (x – 1)

29. $1 + 2ab - (a^2 + b^2)$
We can further write it as
= $1 - (a^2 + b^2 - 2ab)$
So we get
= $1^2 - (a - b)^2$
Based on the equation $a^2 - b^2 = (a + b)(a - b)$
We get,
= [1 + (a – b)] [1 – (a – b)]
= (1 + a – b) (1 – a + b)

30. $9a^2 + 6a + 1 - 36b^2$
We can further write it as
= $(9a^2 + 6a + 1) - 36b^2$
So we get
= $[(3a)^2 + 2 \times 3a \times 1 + 1^2] - (6b)^2$
= $(3a + 1)^2$- $(6b)^2$
Based on the equation $a^2 - b^2 = (a + b)(a - b)$
We get,
= (3a + 1 + 6b) (3a + 1 – 6b)

31. $x^2 - y^2 + 6y - 9$
We can further write it as
= $x^2 - (y^2 - 6y + 9)$
So we get
= $x^2 - (y^2 - 2 \times y \times 3 + 3^2)$
= $x^2 - (y - 3)^2$
Based on the equation $a^2 - b^2 = (a + b)(a - b)$
We get,
= [x + (y – 3)] [x – (y – 3)]
= (x + y – 3) (x – y + 3)

32. $4x^2 - 9y^2 - 2x - 3y$
We can further write it as
= $(2x)^2$- $(3y)^2$- (2x + 3y)
Based on the equation $a^2 - b^2 = (a + b)(a - b)$
We get,
= (2x + 3y) (2x – 3y) – (2x + 3y)

Taking (2x + 3y) as common
= (2x + 3y) (2x – 3y – 1)

33. $9a^2 + 3a - 8b - 64b^2$
We can further write it as
= $9a^2 - 64b^2 + 3a - 8b$
So we get
= $(3a)^2 - (8b)^2 + (3a - 8b)$
Based on the equation $a^2 - b^2 = (a + b)(a - b)$
We get,
= (3a + 8b) (3a – 8b) +$(3a - 8b)$
Taking (3a – 8b) as common
= (3a – 8b) (3a + 8b + 1)

34. $x^2 + \frac{1}{x^2} - 3$
We can further write it as
= $x^2 + \frac{1}{x^2} - 2 - 1$
So we get,
= $\left(x^2 + \frac{1}{x^2} - 2\right) - 1$
It can also be written as
= $\left(x^2 + \frac{1}{x^2} - 2 \times x^2 \times \frac{1}{x^2}\right) - 1^2$
= $\left(x - \frac{1}{x}\right)^2 - 1^2$
Based on the equation $a^2 - b^2 = (a + b)(a - b)$
We get,
= $\left(x - \frac{1}{x} - 1\right)\left(x - \frac{1}{x} + 1\right)$

35. $x^2 - 2 + \frac{1}{x^2} - y^2$
We can further write it as
= $\left(x^2 - 2 \times x^2 \times \frac{1}{x^2} + \frac{1}{x^2}\right) - y^2$
So we get
= $\left(x - \frac{1}{x}\right)^2 - y^2$
Based on the equation $a^2 - b^2 = (a + b)(a - b)$
We get,
= $(x - \frac{1}{x} + y)\ (x - \frac{1}{x} - y)$

36. $x^4 + \frac{4}{x^4}$
We can further write it as
= $x^4 + \frac{4}{x^4}$ + 4 – 4
It can also be written as
= $(x^2)^2 + \left(\frac{2}{x^2}\right)^2 + 2 \times x^2 \times \left(\frac{2}{x^2}\right) - 2^2$
= $\left[(x^2)^2 + \left(\frac{2}{x^2}\right)^2 + 2 \times x^2 \times \left(\frac{2}{x^2}\right)\right] - 2^2$
On further simplification

$= (x^2 + \frac{2}{x^2})^2 - 2^2$
Based on the equation $a^2 - b^2 = (a+b)(a-b)$
We get,
$= (x^2 + \frac{2}{x^2} + 2)\ (x^2 + \frac{2}{x^2} - 2)$

37. $x^8 - 1$
We can further write it as
$= (x^4)^2 - 1^2$
Based on the equation $a^2 - b^2 = (a+b)(a-b)$
We get,
$= (x^4 + 1)(x^4 - 1)$
$(x^4 - 1)$ Can also be written using the formula
$= (x^4 + 1)((x^2)^2 - 1^2)$
So we get
$= (x^4 + 1)(x^2 + 1)(x^2 - 1)$
By expanding the terms using the formula
We get,
$= [(x^2)^2 + 1^2 + 2x^2 - 2x^2](x^2 + 1)(x + 1)(x - 1)$
On further simplification
$= [((x^2)^2 + 1^2 + 2x^2)\ - 2x^2](x^2 + 1)(x + 1)(x - 1)$
So we get
$= [(x^2 + 1) - (\sqrt{2}x)^2]\ (x^2 + 1)(x + 1)(x - 1)$
Using the formula we get,
$= (x^2 + 1 + \sqrt{2}x)\ (x^2 + 1 - \sqrt{2}x)\ (x^2 + 1)(x + 1)(x - 1)$

38. $16x^4 - 1$
We can further write it as
$= (4x^2)^2 - 1^2$
Based on the equation $a^2 - b^2 = (a+b)(a-b)$
We get,
$= (4x^2 + 1)(4x^2 - 1)$
On further simplification using the formula
$= (4x^2 + 1)(\ (2x)^2 - 1^2)$
So we get,
$= (4x^2 + 1)(2x + 1)(2x - 1)$

39. $81x^4 - y^4$
We can further write it as
$= (9x^2)^2 - (y^2)^2$
Based on the equation $a^2 - b^2 = (a+b)(a-b)$
We get,
$= (9x^2 + y^2)(9x^2 - y^2)$
On further simplification using the formula
$= (9x^2 + y^2)(\ (3x)^2 - y^2)$
So we get,
$= (9x^2 + y^2)(3x + y)(3x - y)$

40. $x^4 - 625$

We can further write it as
= $(x^2)^2 - 25^2$
Based on the equation $a^2 - b^2 = (a + b)(a - b)$
We get,
= $(x^2 + 25)(x^2 - 25)$
On further simplification using the formula
= $(x^2 + 25)(x^2 - 5^2)$
So we get,
= $(x^2 + 25)(x + 5)(x - 5)$

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Factorise:

1. $x^2 + 11x + 30$
2. $x^2 + 18x + 32$
3. $x^2 + 20x - 69$
4. $x^2 + 19x - 150$
5. $x^2 + 7x - 98$
6. $x^2 + 2\sqrt{3}x - 24$
7. $x^2 - 21x + 90$
8. $x^2 - 22x + 120$
9. $x^2 - 4x + 3$
10. $x^2 + 7\sqrt{6}x + 60$
11. $x^2 + 3\sqrt{3}x + 6$
12. $x^2 + 6\sqrt{6}x + 48$
13. $x^2 + 5\sqrt{5}x + 30$
14. $x^2 - 24x - 180$
15. $x^2 - 32x - 105$
16. $x^2 - 11x - 80$
17. $6 - x - x^2$
18. $x^2 - \sqrt{3}x - 6$
19. $40 + 3x - x^2$
20. $x^2 - 26x + 133$
21. $x^2 - 2\sqrt{3}x - 24$
22. $x^2 - 3\sqrt{5}x - 20$
23. $x^2 + \sqrt{2}x - 24$
24. $x^2 - 2\sqrt{2}x - 30$
25. $x^2 - x - 156$
26. $x^2 - 32x - 105$
27. $9x^2 + 18x + 8$
28. $6x^2 + 17x + 12$
29. $18x^2 + 3x - 10$
30. $2x^2 + 11x - 21$
31. $15x^2 + 2x - 8$
32. $2x^2 + 11x - 21$
33. $24x^2 - 41x + 12$
34. $3x^2 - 14x + 8$
35. $2x^2 + 3x - 90$
36. $\sqrt{5}x^2 + 2x - 3\sqrt{5}$
37. $2\sqrt{3}x^2 + x - 5\sqrt{3}$
38. $7x^2 + 2\sqrt{14}x + 2$
39. $6\sqrt{3}x^2 - 47x + 5\sqrt{3}$
40. $5\sqrt{5}x^2 + 20x + 3\sqrt{5}$
41. $\sqrt{3}x^2 + 10x + 8\sqrt{3}$
42. $\sqrt{2}x^2 + 3x + \sqrt{2}$
43. $2x^2 + 3\sqrt{3}x + 3$

44. $15x^2 - x - 28$
45. $6x^2 - 5x - 21$
46. $2x^2 - 7x - 15$
47. $5x^2 - 16x - 21$
48. $6x^2 - 11x - 35$
49. $9x^2 - 3x - 20$
50. $10x^2 - 9x - 7$
51. $x^2 - 2x + \frac{7}{16}$
52. $\frac{1}{3}x^2 - 2x - 9$
53. $x^2 + \frac{12}{35}x + \frac{1}{35}$
54. $21x^2 - 2x + \frac{1}{21}$
55. $\frac{3}{2}x^2 + 16x + 10$
56. $\frac{2}{3}x^2 - \frac{17}{3}x - 28$
57. $\frac{3}{5}x^2 - \frac{19}{5}x + 4$
58. $2x^2 - x + \frac{1}{8}$
59. $2(x+y)^2 - 9(x+y) - 5$
60. $9(2a-b)^2 - 4(2a-b) - 13$
61. $7(x-2y)^2 - 25(x-2y) + 12$
62. $10(3x + \frac{1}{x})^2 - \left(3x + \frac{1}{x}\right) - 3$
63. $6(2x - \frac{3}{x})^2 + 7\left(2x - \frac{3}{x}\right) - 20$
64. $(a+b)^2 + 101(a+2b) + 100$
65. $4x^4 + 7x^2 - 2$
66. **Evaluate** $\{(999)^2 - 1\}$.

Solution:

1. $x^2 + 11x + 30$
   We can further write it as
   $= x^2 + 6x + 5x + 30$
   By taking out the common terms
   = x (x + 6) + 5 (x + 6)
   So we get,
   = (x + 5) (x + 6)

2. $x^2 + 18x + 32$
   We can further write it as
   $= x^2 + 16x + 2x + 32$
   By taking out the common terms
   = x (x + 16) + 2 (x + 16)
   So we get,
   = (x + 2) (x + 16)

3. $x^2 + 20x - 69$
   We can further write it as
   $= x^2 + 23x - 3x - 69$

By taking out the common terms
= x (x + 23) – 3 (x + 23)
So we get,
= (x – 3) (x + 23)

4. $x^2 + 19x - 150$
We can further write it as
$= x^2 + 25x - 6x - 150$
By taking out the common terms
= x (x + 25) – 6 (x + 25)
So we get,
= (x – 6) (x + 25)

5. $x^2 + 7x - 98$
We can further write it as
$= x^2 + 14x - 7x - 98$
By taking out the common terms
= x (x + 14) – 7 (x + 14)
So we get,
= (x – 7) (x + 14)

6. $x^2 + 2\sqrt{3}x - 24$
We can further write it as
$= x^2 + 4\sqrt{3}x - 2\sqrt{3}x - 24$
By taking out the common terms
= x (x + 4$\sqrt{3}$) $-2\sqrt{3}$(x +4$\sqrt{3}$)
So we get,
= (x - 2$\sqrt{3}$) (x +4$\sqrt{3}$)

7. $x^2 - 21x + 90$
We can further write it as
$= x^2 - 6x - 15x + 90$
By taking out the common terms
= x (x – 6) – 15 (x – 6)
So we get,
= (x – 15) (x – 6)

8. $x^2 - 22x + 120$
We can further write it a
$= x^2 - 10x - 12x + 120$
By taking out the common terms
= x (x – 10) – 12 (x – 10)
So we get,
= (x – 12) (x – 10)

9. $x^2 - 4x + 3$
We can further write it as
$= x^2 - 3x - x + 3$
By taking out the common terms

= x (x – 3) – 1 (x – 3)
So we get,
= (x – 1) (x – 3)

10. $x^2 + 7\sqrt{6}x + 60$
We can further write it as
$= x^2 + 2\sqrt{6}x + 5\sqrt{6}x + 60$
By taking out the common terms
= x (x +2√6) + 5√6(x +2√6)
So we get,
= (x +5√6) (x +2√6)

11. $x^2 + 3\sqrt{3}x + 6$
We can further write it as
$= x^2 + 2\sqrt{3}x + \sqrt{3}x + 6$
By taking out the common terms
= x (x +2√3) + √3 (x +2√3)
So we get,
= (x +√3) (x +2√3)

12. $x^2 + 6\sqrt{6}x + 48$
We can further write it as
$= x^2 + 4\sqrt{6}x + 2\sqrt{6}x + 48$
By taking out the common terms
= x (x +4√6) + 2√6 (x +4√6)
So we get,
= (x + 2√6) (x +4√6)

13. $x^2 + 5\sqrt{5}x + 30$
We can further write it as
$= x^2 + 2\sqrt{5}x + 3\sqrt{5}x + 30$
By taking out the common terms
= x (x +2√5) + 3√5 (x +2√5)
So we get,
= (x +3√5) (x +2√5)

14. $x^2 - 24x - 180$
We can further write it as
$= x^2 - 30x + 6x - 180$
By taking out the common terms
= x (x – 30) + 6 (x – 30)
So we get,
= (x + 6) (x – 30)

15. $x^2 - 32x - 105$
We can further write it as
$= x^2 - 35x + 3x - 105$
By taking out the common terms

= x (x – 35) + 3 (x – 35)
So we get,
= (x + 3) (x – 35)

16. $x^2 - 11x - 80$
We can further write it as
$= x^2 - 16x + 5x - 80$
By taking out the common terms
= x (x – 16) + 5 (x – 16)
So we get,
= (x + 5) (x – 16)

17. $6 - x - x^2$
We can further write it as
$= 6 + 2x - 3x - x^2$
By taking out the common terms
= 2 (3 + x) – x (3 + x)
So we get,
= (2 – x) (3 + x)

18. $x^2 - \sqrt{3}x - 6$
We can further write it as
$= x^2 - 2\sqrt{3}x + \sqrt{3}x - 6$
By taking out the common terms
= x (x -2$\sqrt{3}$) + $\sqrt{3}$(x -2$\sqrt{3}$)
So we get,
= (x +$\sqrt{3}$) (x -2$\sqrt{3}$)

19. $40 + 3x - x^2$
We can further write it as
$= 40 + 8x - 5x - x^2$
By taking out the common terms
= 8 (5 + x) – x (5 + x)
So we get,
= (8 – x) (5 + x)

20. $x^2 - 26x + 133$
We can further write it as
$= x^2 - 19x - 7x + 133$
By taking out the common terms
= x (x – 19) – 7 (x – 19)
So we get,
= (x – 7) (x – 19)

21. $x^2 - 2\sqrt{3}x - 24$
We can further write it as
$= x^2 - 4\sqrt{3}x + 2\sqrt{3}x - 24$
By taking out the common terms
= x (x -4$\sqrt{3}$) + 2$\sqrt{3}$ (x -4$\sqrt{3}$)

So we get,
= $(x + 2\sqrt{3})(x - 4\sqrt{3})$

22. $x^2 - 3\sqrt{5}x - 20$
We can further write it as
$= x^2 - 4\sqrt{5}x + \sqrt{5}x - 20$
By taking out the common terms
= $x(x - 4\sqrt{5}) + \sqrt{5}(x - 4\sqrt{5})$
So we get,
= $(x + \sqrt{5})(x - 4\sqrt{5})$

23. $x^2 + \sqrt{2}x - 24$
We can further write it as
$= x^2 + 4\sqrt{2}x - 3\sqrt{2}x - 24$
By taking out the common terms
= $x(x + 4\sqrt{2}) - 3\sqrt{2}(x + 4\sqrt{2})$
So we get,
= $(x - 3\sqrt{2})(x + 4\sqrt{2})$

24. $x^2 - 2\sqrt{2}x - 30$
We can further write it as
$= x^2 - 5\sqrt{2}x + 3\sqrt{2}x - 30$
By taking out the common terms
= $x(x - 5\sqrt{2}) + 3\sqrt{2}(x - 5\sqrt{2})$
So we get,
= $(x + 3\sqrt{2})(x - 5\sqrt{2})$

25. $x^2 - x - 156$
We can further write it as
$= x^2 - 13x + 12x - 156$
By taking out the common terms
= $x(x - 13) + 12(x - 13)$
So we get,
= $(x + 12)(x - 13)$

26. $x^2 - 32x - 105$
We can further write it as
$= x^2 - 35x + 3x - 105$
By taking out the common terms
= $x(x - 35) + 3(x - 35)$
So we get,
= $(x + 3)(x - 35)$

27. $9x^2 + 18x + 8$
We can further write it as
$= 9x^2 + 12x + 6x + 8$
By taking out the common terms
= $3x(3x + 4) + 2(3x + 4)$

So we get,
= (3x + 2) (3x + 4)

28. $6x^2 + 17x + 12$
We can further write it as
$= 6x^2 + 9x + 8x + 12$
By taking out the common terms
= 3x (2x + 3) + 4 (2x + 3)
So we get,
= (3x + 4) (2x + 3)

29. $18x^2 + 3x - 10$
We can further write it as
$= 18x^2 - 12x + 15x - 10$
By taking out the common terms
= 6x (3x – 2) + 5 (3x – 2)
So we get,
= (6x + 5) (3x – 2)

30. $2x^2 + 11x - 21$
We can further write it as
$= 2x^2 + 14x - 3x - 21$
By taking out the common terms
= 2x (x + 7) – 3 (x + 7)
So we get,
= (2x – 3) (x + 7)

31. $15x^2 + 2x - 8$
We can further write it as
$= 15x^2 - 10x + 12x - 8$
By taking out the common terms
= 5x (3x – 2) + 4 (3x – 2)
So we get,
= (5x + 4) (3x – 2)

32. $2x^2 + 11x - 21$
We can further write it as
$= 2x^2 + 14x - 9x - 21$
By taking out the common terms
= 7x (3x + 2) – 3 (3x + 2)
So we get,
= (7x – 3) (3x + 2)

33. $24x^2 - 41x + 12$
We can further write it as
$= 24x^2 - 32x - 9x + 12$
By taking out the common terms
= 8x (3x – 4) – 3 (3x – 4)
So we get,
= (8x – 3) (3x – 4)

34. $3x^2 - 14x + 8$
We can further write it as
$= 3x^2 - 12x - 2x + 8$
By taking out the common terms
= 3x (x – 4) – 2 (x – 4)
So we get,
= (3x – 2) (x – 4)

35. $2x^2 + 3x - 90$
We can further write it as
$= 2x^2 - 12x + 15x - 90$
By taking out the common terms
= 2x (x – 6) + 15 (x – 6)
So we get,
= (2x + 15) (x – 6)

36. $\sqrt{5}x^2 + 2x - 3\sqrt{5}$
We can further write it as
$= \sqrt{5}x^2 - 3x + 5x - 3\sqrt{5}$
By taking out the common terms
$= x\ (\sqrt{5}x - 3) + \sqrt{5}\ (\sqrt{5}x - 3)$
So we get,
$= (x + \sqrt{5})\ (\sqrt{5}x - 3)$

37. $2\sqrt{3}x^2 + x - 5\sqrt{3}$
We can further write it as
$= 2\sqrt{3}x^2 + 6x - 5x - 5\sqrt{3}$
By taking out the common terms
$= 2\sqrt{3}x(x + \sqrt{3}) - 5(x + \sqrt{3})$
So we get,
$= (2\sqrt{3}x - 5)\ (x + \sqrt{3})$

38. $7x^2 + 2\sqrt{14}x + 2$
We can further write it as
$= 7x^2 + \sqrt{2}\sqrt{7}x + \sqrt{2}\sqrt{7}x + 2$
By taking out the common terms
$= \sqrt{7}x(\sqrt{7}x + \sqrt{2}) + \sqrt{2}(\sqrt{7}x + \sqrt{2})$
So we get,
$= (\sqrt{7}x + \sqrt{2})(\sqrt{7}x + \sqrt{2})$
It can also be written as
$= (\sqrt{7}x + \sqrt{2})^2$

39. $6\sqrt{3}x^2 - 47x + 5\sqrt{3}$
We can further write it as
$= 6\sqrt{3}x^2 - 45x - 2x + 5\sqrt{3}$
By taking out the common terms
$= 3\sqrt{3}x(2x - 5\sqrt{3}) - 1(2x - 5\sqrt{3})$

So we get,
$= (3\sqrt{3}x - 1)\,(2x - 5\sqrt{3})$

40. $5\sqrt{5}x^2 + 20x + 3\sqrt{5}$
We can further write it as
$= 5\sqrt{5}x^2 + 15x + 5x + 3\sqrt{5}$
By taking out the common terms
$= 5x(\sqrt{5}x + 3) + \sqrt{5}(\sqrt{5}x + 3)$
So we get,
$= (5x + \sqrt{5})(\sqrt{5}x + 3)$

41. $\sqrt{3}x^2 + 10x + 8\sqrt{3}$
We can further write it as
$= \sqrt{3}x^2 + 4x + 6x + 8\sqrt{3}$
By taking out the common terms
$= x\,(\sqrt{3}x + 4) + 2\sqrt{3}(\sqrt{3}x + 4)$
So we get,
$= (x + 2\sqrt{3})\,(\sqrt{3}x + 4)$

42. $\sqrt{2}x^2 + 3x + \sqrt{2}$
We can further write it as
$= \sqrt{2}x^2 + x + 2x + \sqrt{2}$
By taking out the common terms
$= x\,(\sqrt{2}x + 1) + \sqrt{2}(\sqrt{2}x + 1)$
So we get,
$= (x + \sqrt{2})\,(\sqrt{2}x + 1)$

43. $2x^2 + 3\sqrt{3}x + 3$
We can further write it as
$= 2x^2 + 2\sqrt{3}x + \sqrt{3}x + 3$
By taking out the common terms
$= 2x\,(x + \sqrt{3}) + \sqrt{3}\,(x + \sqrt{3})$
So we get,
$= (2x + \sqrt{3})\,(x + \sqrt{3})$

44. $15x^2 - x - 28$
We can further write it as
$= 15x^2 + 20x - 21x - 28$
By taking out the common terms
$= 5x\,(3x + 4) - 7\,(3x + 4)$
So we get,
$= (5x - 7)\,(3x + 4)$

45. $6x^2 - 5x - 21$
We can further write it as
$= 6x^2 + 9x - 14x - 21$
By taking out the common terms

= 3x (2x + 3) – 7 (2x + 3)
So we get,
= (3x – 7) (2x + 3)

46. $2x^2 - 7x - 15$
We can further write it as
$= 2x^2 - 10x + 3x - 15$
By taking out the common terms
= 2x (x – 5) + 3 (x – 5)
So we get,
= (2x + 3) (x – 5)

47. $5x^2 - 16x - 21$
We can further write it as
$= 5x^2 + 5x - 21x - 21$
By taking out the common terms
= 5x (x + 1) – 21 (x + 1)
So we get,
= (5x – 21) (x + 1)

48. $6x^2 - 11x - 35$
We can further write it as
$= 6x^2 - 21x + 10x - 35$
By taking out the common terms
= 3x (2x – 7) + 5 (2x – 7)
So we get,
= (3x + 5) (2x – 7)

49. $9x^2 - 3x - 20$
We can further write it as
$= 9x^2 - 15x + 12x - 20$
By taking out the common terms
= 3x (3x – 5) + 4 (3x – 5)
So we get,
= (3x + 4) (3x – 5)

50. $10x^2 - 9x - 7$
We can further write it as
$= 10x^2 + 5x - 14x - 7$
By taking out the common terms
= 5x (2x + 1) – 7 (2x + 1)
So we get,
= (5x – 7) (2x + 1)

51. $x^2 - 2x + \frac{7}{16}$
We can further write it as
$= \frac{1}{16}(16x^2 - 32x + 7)$
$= \frac{1}{16}(16x^2 - 4x - 28x + 7)$

By taking out the common terms
$= \frac{1}{16}[4x(4x-1)-7(4x-1)]$
So we get,
$= \frac{1}{16}(4x-7)(4x-1)$
$= (4x - 1)\left(\frac{x}{4} - \frac{7}{16}\right)$

52. $\frac{1}{3}x^2 - 2x - 9$
We can further write it as
$= \frac{1}{3}x^2 - 3x + x - 9$
By taking out the common terms
$= x\left(\frac{x}{3} - x\right) + (x - 9)$
$= \frac{x}{3}(x-9) + (x-9)$
Taking (x – 9) as common
$= (x - 9)\left(\frac{x}{3} + 1\right)$
So we get,
$= (x - 9)\left(\frac{x+3}{3}\right)$
$= \frac{1}{3}(x-9)(x+3)$

53. $x^2 + \frac{12}{35}x + \frac{1}{35}$
We can further write it as
$= x^2 + \frac{5x}{35} + \frac{x}{5} + \frac{1}{35}$
By taking out the common terms
$= 5x\left(\frac{x}{5} + \frac{1}{35}\right) + 1\left(\frac{x}{5} + \frac{1}{35}\right)$
So we get
$= (5x + 1)\left(\frac{x}{5} + \frac{1}{35}\right)$

54. $21x^2 - 2x + \frac{1}{21}$
We can further write it as
$= 21x^2 - x - x + \frac{1}{21}$
By taking out the common terms
$= 21x\left(x - \frac{1}{21}\right) - 1\left(x - \frac{1}{21}\right)$
So we get
$= (21x - 1)\left(x - \frac{1}{21}\right)$

55. $\frac{3}{2}x^2 + 16x + 10$
We can further write it as
$= \frac{3}{2}x^2 + x + 15x + 10$
By taking out the common terms
$= \frac{x}{2}(3x + 2) + 5(3x + 2)$
So we get

$= (\frac{x}{2} + 5)$ (3x + 2)

56. $\frac{2}{3}x^2 - \frac{17}{3}x - 28$
We can further write it as
$= \frac{2}{3}x^2 - \frac{7}{3}x - 8x - 28$
By taking out the common terms
$= \frac{x}{3}$ (2x + 7) – 4 (2x + 7)
So we get
$= (\frac{x}{3} - 4)$ (2x + 7)

57. $\frac{3}{5}x^2 - \frac{19}{5}x + 4$
We can further write it as
$= \frac{3}{5}x^2 - \frac{4}{5}x - 3x + 4$
By taking out the common terms
$= \frac{x}{5}\ (3x - 4) - 1(3x - 4)$
So we get
$= (\frac{x}{5} - 1)(3x - 4)$

58. $2x^2 - x + \frac{1}{8}$
We can further write it as
$= 2x^2 - \frac{1}{2}x - \frac{1}{2}x + \frac{1}{8}$
By taking out the common terms
$= \frac{x}{2}\ (4x - 1) - \frac{1}{8}(4x - 1)$
So we get
$= (\frac{x}{2} - \frac{1}{8})\ (4x - 1)$

59. $2(x + y)^2 - 9(x + y) - 5$
Let us consider (x + y) = z

So we get,
$= 2z^2 - 9z - 5$
We can further write it as
$= 2z^2 - 10z + z - 5$
By taking out the common terms
= 2z (z – 5) + 1 (z – 5)
= (2z + 1) (z – 5)

Let us replace z by x + y
So we get,
$2(x + y)^2 - 9(x + y) - 5$
By replacing,
= (2(x + y) + 1) ((x + y) – 5)
= (2x + 2y + 1) (x + y – 5)

60. $9(2a-b)^2-4(2a-b)-13$
Let us consider 2a – b = c

So we get,
= $9c^2-4c-13$
We can further write it as
= $9c^2-13c+9c-13$
By taking out the common terms
= c (9c – 13) + 1 (9c – 13)
= (c + 1) (9c – 13)

Let us replace c by 2a – b
So we get,
$9(2a-b)^2-4(2a-b)-13$
Then,
= ((2a – b) + 1) (9(2a – b) – 13)
= (2a – b + 1) (18a – 9b – 13)

61. $7(x-2y)^2-25(x-2y)+12$
Let us consider x – 2y = z

So we get,
= $7z^2-25z+12$
We can further write it as
= $7z^2-21z-4z+12$
By taking out the common terms
= 7z (z – 3) – 4 (z – 3)
= (7z – 4) (z – 3)

Let us replace z by x – 2y
So we get,
$7(x-2y)^2-25(x-2y)+12$
Then,
= (7(x – 2y) – 4) ((x – 2y) – 3)
= (7x – 14y – 4) (x – 2y – 3)

62. $10(3x+\frac{1}{x})^2-\left(3x+\frac{1}{x}\right)-3$
Consider,
$3x+\frac{1}{x}$ = a

So we get,
= $10a^2-a-3$
We can further write it as
= $10a^2-6a+5a-3$
By taking out the common terms
= 2a (5a – 3) + 1 (5a – 3)
= (2a + 1) (5a – 3)

Let us replace a by $3x + \frac{1}{x}$
So we get,
$(2(3x + \frac{1}{x}) + 1)\ (5(3x + \frac{1}{x}) - 3)$
On further multiplication,
$= (6x + \frac{2}{x} + 1)\ (15x + \frac{5}{x} - 3)$

63. $6(2x - \frac{3}{x})^2 + 7\left(2x - \frac{3}{x}\right) - 20$
Consider,
$2x - \frac{3}{x} = a$

So we get,
$= 6a^2 + 7a - 20$
We can further write it as
$= 6a^2 + 15a - 8a - 20$
By taking the common terms out
= 3a (2a + 5) – 4 (2a + 5)
= (3a – 4) (2a + 5)

Let us replace a by $2x - \frac{3}{x}$
So we get,
$(3(2x - \frac{3}{x}) - 4)\ (2(2x - \frac{3}{x}) + 5)$
$= (6x - \frac{9}{x} - 4)\ (4x - \frac{6}{x} + 5)$

64. $(a + b)^2 + 101(a + 2b) + 100$
Consider,
$a + 2b = x$

So we get,
$= (x)^2 + 101x + 100$
We can further write it as
$= (x)^2 + 100x + x + 100$
By taking out the common terms
= x (x + 100) + 1 (x + 100)
= (x +1) (x + 100)

Let us replace x by a + 2b
So we get,
((a + 2b) +1) ((a + 2b) + 100)
= (a + 2b + 1) (a + 2b + 100)

65. $4x^4 + 7x^2 - 2$
Consider $x^2 = y$
So we get,
$= 4y^2 + 7y - 2$
We can write it as
$= 4y^2 + 8y - y - 2$

By taking common terms
= 4y (y + 2) – 1 (y + 2)
So we get
= (4y – 1) (y + 2)

Let us replace y by x^2
So we get,
(4x^2 – 1) (x^2 + 2)
= (2x + 1) (2x – 1) (x^2 + 2)

66. $\{(999)^2 - 1\}$
The given question can be written as
= $\{(999)^2 - 1^2\}$
Using the formula $(a^2 - b^2) = (a + b)(a - b)$
= [(999 + 1) (999 – 1)]
On further calculation,
= 1000 × 998
= 998000

EXERCISE 3(D) PAGE: 119

1. Expand

(i) $(a + 2b + 5c)^2$

(ii) $(2a - b + c)^2$

(iii) $(a - 2b - 3c)^2$

Solution:

(i) $(a + 2b + 5c)^2$

According to the equation,

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

So we get,

$(a + 2b + 5c)^2$

Using the formula we can it write it as

$= a^2 + (2b)^2 + (5c)^2 + 2a(2b) + 2(2b)(5c) + 2(5c)a$

On further calculation we get,

$= a^2 + 4b^2 + 25c^2 + 4ab + 20bc + 10ac$

(ii) $(2a - b + c)^2$

According to the equation,

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

So we get,

$(2a - b + c)^2$

Using the formula we can it write it as

$= (2a)^2 + (-b)^2 + c^2 + 2(2a)(-b) + 2(-b)c + 2c(2a)$

On further calculation we get,

$= 4a^2 + b^2 + c^2 - 4ab - 2bc + 4ac$

(iii) $(a - 2b - 3c)^2$

According to the equation,

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

So we get,

$(a - 2b - 3c)^2$

Using the formula we can it write it as

$= a^2 + (-2b)^2 + (-3c)^2 + 2a(-2b) + 2(-2b)(-3c) + 2(-3c)a$

On further calculation we get,

$= a^2 + 4b^2 + 9c^2 - 4ab + 12bc - 6ac$

2. Expand

(i) $(2a - 5b - 7c)^2$

(ii) $(-3a + 4b - 5c)^2$

(iii) $\left(\frac{1}{2}a - \frac{1}{4}b + 2\right)^2$

Solution:

(i) $(2a - 5b - 7c)^2$

According to the equation,

$(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

So we get,
$(2a-5b-7c)^2$
Using the formula we can it write it as
$= (2a)^2+(-5b)^2+(-7c)^2+2(2a)(-5b)+2(-5b)(-7c)+2(-7c)(2a)$
On further calculation we get,
$= 4a^2+25b^2+49c^2-20ab+70bc-28ca$

(ii) $(-3a+4b-5c)^2$
According to the equation,
$(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

So we get,
$(-3a+4b-5c)^2$
Using the formula we can it write it as
$= (-3a)^2+(4b)^2+(-5c)^2+2(-3a)(4b)+2(4b)(-5c)+2(-5c)(-3a)$
On further calculation we get,
$= 9a^2+16b^2+25c^2-24ab-40bc+30ca$

(iii) $\left(\frac{1}{2}a-\frac{1}{4}b+2\right)^2$
According to the equation,
$(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

So we get,
$\left(\frac{1}{2}a-\frac{1}{4}b+2\right)^2$
Using the formula we can it write it as

$= \left(\frac{1}{2}a\right)^2+\left(-\frac{1}{4}b\right)^2+2^2+2\left(\frac{1}{2}a\right)\left(-\frac{1}{4}b\right)+2\left(-\frac{1}{4}b\right)2+2(2)\left(\frac{1}{2}a\right)$
On further calculation we get,
$= \frac{1}{4}a^2+\frac{1}{16}b^2+4-\frac{1}{4}ab-b+2a$

Factorise
3. $4x^2+9y^2+16z^2+12xy-24yz-16zx$
Solution:

According to the equation,
$(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

So we get,
$4x^2+9y^2+16z^2+12xy-24yz-16zx$
Using the formula we can it write it as
$= (2x)^2+(3y)^2+(-4z)^2+2(2x)(3y)+2(3y)(-4z)+2(2x)(-4z)$
On further calculation we get,
$= (2x+3y-4z)^2$
It can be written as
= (2x + 3y – 4z) (2x + 3y – 4z)

4. $9x^2 + 16y^2 + 4z^2 - 24xy + 16yz - 12xz$
Solution:

According to the equation,
$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

So we get,
$9x^2 + 16y^2 + 4z^2 - 24xy + 16yz - 12xz$
Using the formula we can it write it as
$= (-3x)^2 + (4y)^2 + (2z)^2 + 2(-3x)(4y) + 2(4y)(2z) + 2(2z)(-3x)$
On further calculation we get,
$= (-3x + 4y + 2z)^2$
It can be written as
= (-3x + 4y + 2z) (-3x + 4y + 2z)

5. $25x^2 + 4y^2 + 9z^2 - 20xy - 12yz + 30xz$
Solution:

According to the equation,
$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

So we get,
$25x^2 + 4y^2 + 9z^2 - 20xy - 12yz + 30xz$
Using the formula we can it write it as
$= (5x)^2 + (-2y)^2 + (3z)^2 + 2(5x)(-2y) + 2(-2y)(3z) + 2(3z)(5x)$
On further calculation we get,
$= (5x - 2y + 3z)^2$
It can be written as
= (5x – 2y + 3z) (5x – 2y + 3z)

6. $16x^2 + 4y^2 + 9z^2 - 16xy - 12yz + 24xz$
Solution:

According to the equation,
$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

So we get,
$16x^2 + 4y^2 + 9z^2 - 16xy - 12yz + 24xz$
Using the formula we can it write it as
$= (4x)^2 + (-2y)^2 + (3z)^2 + 2(-2y)(3z) + 2(-2y)(3z) + 2(3z)(4x)$
$= [4x + (-2y) + 3z]^2$
On further calculation we get,
$= (4x - 2y + 3z)^2$
It can be written as
= (4x – 2y + 3z) (4x – 2y + 3z)

7. Evaluate
(i) $(99)^2$

(ii) $(\mathbf{995})^{2}$
(iii) $(\mathbf{107})^{2}$
Solution:

(i) $(99)^2$
We can write $(99)^2$as
= $(100 - 1)^2$

According to the equation
$(a - b)^2 = a^2 - 2ab + b^2$

So we get,
$(100 - 1)^2$
Using the formula we can it write it as
= $(100)^2 - 2(100)(1) + (1)^2$
On further calculation we get,
= 10000 – 200 + 1
= 9801

(ii) $(995)^2$
We can write $(995)^2$as
= $(1000 - 5)^2$

According to the equation
$(a - b)^2 = a^2 - 2ab + b^2$

So we get,
$(1000 - 5)^2$
Using the formula we can it write it as
= $(1000)^2 - 2(1000)(5) + (5)^2$
On further calculation we get,
= 1000000 – 10000 + 25
= 990025

(iii) $(107)^2$
We can write $(107)^2$as
= $(100 + 7)^2$

According to the equation
$(a + b)^2 = a^2 + 2ab + b^2$

So we get,
$(100 + 7)^2$
Using the formula we can it write it as
= $(100)^2 + 2(100)(7) + (7)^2$
On further calculation we get,
= 10000 + 1400 + 49
= 11449

EXERCISE 3(E) PAGE: 123

1. Expand

(i) $(3x+2)^3$

(ii) $\left(3a+\frac{1}{4b}\right)^3$

(iii) $\left(1+\frac{2}{3}a\right)^3$

Solution:

(i) $(3x+2)^3$

According to the equation,

$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

We get,

$(3x+2)^3$

Using the formula we can it write it as

$= (3x)^3 + (2)^3 + 3(3x)(2)(3x+2)$

On further calculation we get,

$= 27x^3 + 8 + 18x(3x+2)$

$= 27x^3 + 8 + 54x^2 + 36x$

(ii) $\left(3a+\frac{1}{4b}\right)^3$

According to the equation,

$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

We get,

$\left(3a+\frac{1}{4b}\right)^3$

Using the formula we can it write it as

$= (3a)^3 + (\frac{1}{4b})^3 + 3(3a)(\frac{1}{4b})(3a+\frac{1}{4b})$

On further calculation we get,

$= 27a^3 + \frac{1}{64b}b^3 + \frac{9a}{4b}(3a+\frac{1}{4b})$

$= 27a^3 + \frac{1}{64b}b^3 + \frac{27a^2}{4b} + \frac{9a}{16b^2}$

(iii) $\left(1+\frac{2}{3}a\right)^3$

According to the equation,

$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

We get,

$\left(1+\frac{2}{3}a\right)^3$

Using the formula we can it write it as

$= 1^3 + (\frac{2}{3}a)^3 + 3(1)(\frac{2}{3}a)(1+\frac{2}{3}a)$

On further calculation we get,

$= 1 + \frac{8}{27}a + 2a(1+\frac{2}{3}a)$

$= 1 + \frac{8}{27}a + 2a + \frac{4}{3}a^2$

2. Expand

(i) $(5a - 3b)^3$

(ii) $\left(3x - \frac{5}{x}\right)^3$

(iii) $\left(\frac{4}{5}a - 2\right)^3$

Solution:

(i) $(5a - 3b)^3$

According to the equation,

$(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

We get,

$(5a - 3b)^3$

Using the formula

$= (5a)^3 - (3b)^3 - 3(5a)(3b)(5a - 3b)$

We can further write it as

$= 125a^3 - 27b^3 - 45ab(5a - 3b)$

$= 125a^3 - 27b^3 - 225a^2b + 135ab^2$

(ii) $\left(3x - \frac{5}{x}\right)^3$

According to the equation,

$(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

We get,

$\left(3x - \frac{5}{x}\right)^3$

It can be written as

$= (3x)^3 - \left(\frac{5}{x}\right)^3 - 3(3x)(\frac{5}{x})(3x - \frac{5}{x})$

On further calculation

We get,

$= 27x^3 - (\frac{125}{x})^3 - 45(3x - \frac{5}{x})$

$= 27x^3 - \left(\frac{125}{x}\right)^3 - 135x + \frac{225}{x}$

(iii) $\left(\frac{4}{5}a - 2\right)^3$

According to the equation,

$(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

We get,

$\left(\frac{4}{5}a - 2\right)^3$

It can be written as

$= (\frac{4}{5}a)^3 - 2^3 - 3(\frac{4}{5}a)(2)(\frac{4}{5}a - 2)$

On further calculation
$= \frac{64}{125}a^3 - 8 - \frac{24}{5}a(\frac{4}{5}a - 2)$
$= \frac{64}{125}a^3 - 8 - \frac{96}{25}a^2 + \frac{48}{5}a$

Factorise
3. $8a^3 + 27b^3 + 36a^2b + 54ab^2$
Solution:

According to the equation,
$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

We get,
$8a^3 + 27b^3 + 36a^2b + 54ab^2$
Using the formula
$= (2a)^3 + (3b)^3 + 3(2a)(3b)(2a+3b)$
We can write it as
$= (2a+3b)^3$
$= (2a+3b)(2a+3b)(2a+3b)$

4. $64a^3 - 27b^3 - 144a^2b + 108ab^2$
Solution:

According to the equation,
$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

We get,
$64a^3 - 27b^3 - 144a^2b + 108ab^2$
Based on the formula
$= (4a)^3 - (3b)^3 - 3(4a)(3b)(4a-3b)$
We can write it as
$= (4a-3b)^3$
$= (4a-3b)(4a-3b)(4a-3b)$

5. $1 + \frac{27}{125}a^3 + \frac{9a}{5} + \frac{27a^2}{25}$
Solution:

According to the equation,
$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

We get,
$1 + \frac{27}{125}a^3 + \frac{9a}{5} + \frac{27a^2}{25}$
Using the formula
$= (1)^3 + (\frac{3}{5}a)^3 + 3(1)(\frac{3}{5}a)(1 + \frac{3}{5}a)$
We can write it as
$= \left(1 + \frac{3}{5}a\right)^3$
$= (1 + \frac{3}{5}a)\ (1 + \frac{3}{5}a)\ (1 + \frac{3}{5}a)$

6. $125x^3 - 27y^3 - 225x^2y + 135xy^2$
Solution:

According to the equation,
$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

We get,
$125x^3 - 27y^3 - 225x^2y + 135xy^2$
Using the formula
$= (5x)^3 - (3y)^3 - 3(5x)(3y)(5x-3y)$
We can write it as
$= (5x-3y)^3$
$= (5x-3y)(5x-3y)(5x-3y)$

7. $a^3x^3 - 3a^2bx^2 + 3ab^2x - b^3$
Solution:

According to the equation,
$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

We get,
$a^3x^3 - 3a^2bx^2 + 3ab^2x - b^3$
Using the formula,
$= (ax)^3 - (b)^3 - 3(ax)b(ax-b)$
We can write it as,
$= (ax-b)^3$
$= (ax-b)\ (ax-b)(ax-b)$

8. $\frac{64}{125}a^3 - \frac{96}{25}a^2 + \frac{48}{5}a - 8$
Solution:

According to the equation,
$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

We get,
$\frac{64}{125}a^3 - \frac{96}{25}a^2 + \frac{48}{5}a - 8$
Based on the formula

$= (\frac{4}{5}a)^3 - 2^3 - 3(\frac{4}{5}a)(2)(\frac{4}{5}a - 2)$
We can write it as,
$= (\frac{4}{5}a - 2)^3$
$= (\frac{4}{5}a - 2)\ (\frac{4}{5}a - 2)(\frac{4}{5}a - 2)$

9. $a^3 - 12a(a-4) - 64$
Solution:

According to the equation,

$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

We get,
$a^3 - 12a(a-4) - 64$
Based on the formula
$= a^3 - 4^3 - 3a(4)(a-4)$
We can write it as
$= (a-4)^3$
$= (a-4)\ (a-4)(a-4)$

10. Evaluate
(i) $(\mathbf{103})^3$
(ii) $(\mathbf{99})^3$
Solution:

(i) $(103)^3$
According to the equation,
$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

We get,
$(103)^3$
$= (100+3)^3$
By using the formula,
$= 100^3 + 3^3 + 3(100)(3)(100+3)$
= 1000000 + 27 + 900 (103)
On further calculation,
= 1000000 + 27 + 92700
= 1092727

(ii) $(99)^3$
According to the equation,
$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

We get,
$(99)^3$
$= (100-1)^3$
By using the formula,
$= 100^3 - 1^3 - 3(100)(1)(100-1)$
= 1000000 – 1 – 300 (99)
On further calculation,
= 1000000 – 1 – 29700
= 970299

EXERCISE 3(F) PAGE: 129

Factorise

1. $x^3 + 27$
2. $27a^3 + 64b^3$
3. $125a^3 + \frac{1}{8}$
4. $216x^3 + \frac{1}{125}$
5. $16x^4 + 54x$
6. $7a^3 + 56b^3$
7. $x^5 + x^2$
8. $a^3 + 0.008$
9. $1 - 27a^3$
10. $64a^3 - 343$
11. $x^3 - 512$
12. $a^3 - 0.064$
13. $8x^3 - \frac{1}{27y^3}$
14. $\frac{x^3}{216} - 8y^3$
15. $x - 8xy^3$
16. $32x^4 - 500x$
17. $3a^7b - 81a^4b^4$
18. $x^4y^4 - xy$
19. $8x^2y^3 - x^5$
20. $1029 - 3x^3$
21. $x^6 - 729$
22. $x^9 - y^9$
23. $(a+b)^3 - (a-b)^3$
24. $8a^3 - b^3 - 4ax + 2bx$
25. $a^3 + 3a^2b + 3ab^2 + b^3 - 8$
26. $a^3 - \frac{1}{a^3} - 2a + \frac{2}{a}$
27. $2a^3 + 16b^3 - 5a - 10b$
28. $a^6 + b^6$
29. $a^{12} - b^{12}$
30. $x^6 - 7x^3 - 8$
31. $x^3 - 3x^2 + 3x + 7$
32. $(x+1)^3 + (x-1)^3$
33. $(2a+1)^3 + (a-1)^3$
34. $8(x+y)^3 - 27(x-y)^3$
35. $(x+2)^3 + (x-2)^3$
36. $(x+2)^3 - (x-2)^3$
37. Prove that $\frac{0.85\times0.85\times0.85+0.15\times0.15\times0.15}{0.85\times0.85-0.85\times0.15+0.15\times0.15} = 1$
38. Prove that $\frac{59\times59\times59-9\times9\times9}{59\times59+59\times9+9\times9} = 50$

Solution:

1. $x^3 + 27$

According to the equation,
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

So we get,
$x^3 + 27$
$= x^3 + 3^3$
Using the equation,
$= (x + 3)(x^2 - 3x + 3^2)$
$= (x + 3)(x^2 - 3x + 9)$

2. $27a^3 + 64b^3$
According to the equation,
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

So we get,
$27a^3 + 64b^3$
$= (3a)^3 + (4b)^3$
Using the equation,
$= (3a + 4b)((3a)^2 - (3a)(4b) + (4b)^2)$
$= (3a + 4b)(9a^2 - 12ab + 16b^2)$

3. $125a^3 + \frac{1}{8}$
According to the equation,
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

So we get,
$125a^3 + \frac{1}{8}$
$= (5a)^3 + (\frac{1}{2})^3$
On the basis of formula,
$= (5a + \frac{1}{2})((5a)^2 - (5a)(\frac{1}{2}) + (\frac{1}{2})^2)$
$= (5a + \frac{1}{2})(25a^2 - \frac{5}{2}a + \frac{1}{4})$

4. $216x^3 + \frac{1}{125}$
According to the equation,
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

So we get,
$216x^3 + \frac{1}{125}$
$= (6x)^3 + (\frac{1}{5})^3$
Based on the equation,
$= (6x + \frac{1}{5})((6x)^2 - (6x)(\frac{1}{5}) + (\frac{1}{5})^2)$
$= \left(6x + \frac{1}{5}\right)\left(36x^2 - \frac{6x}{5} + \frac{1}{25}\right)$

5. $16x^4 + 54x$
We can write the given question as,

$16x^4 + 54x = 2x(8x^3 + 27)$

According to the equation,
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
So we get,
$2x(8x^3 + 27)$
$= 2x\ ((2x)^3 + 3^3)$
Using the equation,
$= 2x\ ((2x) + 3)((2x)^2 - (2x)(3) + (3)^2)$
$= 2x(2x + 3)(4x^2 - 6x + 9)$

6. $7a^3 + 56b^3$
We can write the given question as,
$7a^3 + 56b^3 = 7(a^3 + 8b^3)$

According to the equation,
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

So we get,
$7(a^3 + 8b^3)$
$= 7[(a)^3 + (2b)^3]$
Using the formula
$= 7[(a + 2b)(a^2 - a(2b) + (2b)^2]$
$= 7(a + 2b)(\ a^2 - 2ab + 4b^2)$

7. $x^5 + x^2$
We can write the given question as,
$x^5 + x^2 = x^2(x^3 + 1)$

According to the equation,
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

So we get,
$x^2(x^3 + 1)$
$= x^2(x^3 + 1^3)$
Based on the equation,
$= x^2[(x + 1)(x^2 - x + 1^2)]$
$= x^2(x + 1)(x^2 - x + 1^2)$

8. $a^3 + 0.008$
According to the equation,
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

So we get,
$a^3 + 0.008$
$= a^3 + 0.2^3$
Using the formula
$= (a + 0.2)(a^2 - 0.2a + 0.2^2)$
$= (a + 0.2)(a^2 - 0.2a + 0.04)$

9. $1 - 27a^3$
According to the equation,
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

So we get,
$1 - 27a^3$
$= 1^3 - (3a)^3$
Using the formula,
$= (1 - 3a)(1^2 + 3a + (3a)^2)$
$= (1 - 3a)(1^2 + 3a + 9a^2)$
$= (1 - 3a)(1 + 3a + 9a^2)$

10. $64a^3 - 343$
According to the equation,
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

So we get,
$64a^3 - 343$
$= (4a)^3 - 7^3$
Based on the equation,
$= (4a - 7)((4a)^2 + (4a)(7) + 7^2)$
$= (4a - 7)(16a^2 + 28a + 49)$

11. $x^3 - 512$
According to the equation,
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

So we get,
$x^3 - 512$
$= x^3 - 8^3$
Using the equation,
$= (x - 8)(x^2 + 8x + 8^2)$
$= (x - 8)(x^2 + 8x + 64)$

12. $a^3 - 0.064$
According to the equation,
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

So we get,
$a^3 - 0.064$
$= a^3 - (0.4)^3$
Based on the formula,
$= (a - 0.4)(a^2 + 0.4a + (0.4)^2)$
$= (a - 0.4)(a^2 + 0.4a + 0.16)$

13. $8x^3 - \frac{1}{27y^3}$
According to the equation,
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

So we get,
$8x^3 - \frac{1}{27y^3}$
$= (2x)^3 - (\frac{1}{3y})^3$
Based on the formula,
$= (2x - \frac{1}{3y})((2x)^2 + (2x)(\frac{1}{3y}) + (\frac{1}{3y})^2)$
$= (2x - \frac{1}{3y})(4x^2 + \frac{2x}{3y} + \frac{1}{9y^2})$

14. $\frac{x^3}{216} - 8y^3$
According to the equation,
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

So we get,
$\frac{x^3}{216} - 8y^3$
$= \left(\frac{x}{6}\right)^3 - (2y)^3$
Based on the formula,
$= (\frac{x}{6} - 2y)(\left(\frac{x}{6}\right)^2 + (\frac{x}{6})(2y) + (2y)^2)$
$= (\frac{x}{6} - 2y)(\frac{x^2}{36} + \frac{xy}{3} + 4y^2)$

15. $x - 8xy^3$
We can write the given question as,
$x - 8xy^3 = x(1 - 8y^3)$

According to the equation,
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

So we get,
$x(1 - 8y^3)$
$= x(1^3 - (2y)^3)$
Using the equation,
$= x[(1 - 2y)(1^2 + 2y + (2y)^2)]$
$= x(1 - 2y)(1 + 2y + 4y^2)$

16. $32x^4 - 500x$
We can write the given question as,
$32x^4 - 500x = 4x(8x^3 - 125)$

According to the equation,
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

So we get,
$4x(8x^3 - 125)$
$= 4x((2x)^3 - 5^3)$
Using the equation,
$= 4x(2x - 5)((2x)^2 + (2x)(5) + (5)^2)$

$= 4x(2x - 5)(4x^2 + 10x + 25)$

17. $3a^7b - 81a^4b^4$
We can write the given question as,
$3a^7b - 81a^4b^4 = 3a^4b(a^3 - 27b^3)$

According to the equation,
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

So we get,
$3a^4b(a^3 - 27b^3)$
$= 3a^4b(a^3 - (3b)^3)$
Using the equation,
$= 3a^4b[(a - 3b)(a^2 + a(3b) + (3b)^2)]$
$= 3a^4b(a - 3b)(a^2 + 3ab + 9b^2)$

18. $x^4y^4 - xy$
We can write the given question as,
$x^4y^4 - xy = xy(x^3y^3 - 1)$

According to the equation,
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

So we get,
$xy(x^3y^3 - 1)$
$= xy((xy)^3 - 1^3)$
Using the equation,
$= xy[(xy - 1)((xy)^2 + (xy)(1) + (1)^2)]$
$= xy(xy - 1)(x^2y^2 + xy + 1)]$

19. $8x^2y^3 - x^5$
We can write the given question as,
$8x^2y^3 - x^5 = x^2(8y^3 - x^3)$

According to the equation,
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

So we get,
$x^2(8y^3 - x^3)$
$= x^2((2y)^3 - x^3)$
Using the equation
We get,
$= x^2[(2y - x)((2y)^2 + (2y)(x) + x^2)]$
$= x^2(2y - x)(4y^2 + 2xy + x^2)$

20. $1029 - 3x^3$
We can write the given question as,
$1029 - 3x^3 = 3(343 - x^3)$

According to the equation,
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

So we get,
$3(343 - x^3)$
$= 3[7^3 - x^3]$
Based on the formula,
$= 3[(7 - x)(7^2 + 7x + x^2)]$
$= 3(7 - x)(49 + 7x + x^2)$

21. $x^6 - 729$
According to the equation,
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

So we get,
$x^6 - 729$
$= (x^2)^3 - 9^3$
Using the equation,
$= (x^2 - 9)((x^2)^2 + (x^2)(9) + 9^2)$
$= (x^2 - 9)(x^4 + 9x^2 + 81)$
Based on the equation,
$= (x + 3)(x - 3)[(x^2 + 9)^2 - (3x)^2]$
On further simplification
$= (x + 3)(x - 3)(x^2 + 9 + 3x)(x^2 + 9 - 3x)$

22. $x^9 - y^9$
According to the equation,
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

So we get,
$x^9 - y^9$
$= (x^3)^3 - (y^3)^3$
Using the equation
We get,
$= (x^3 - y^3)((x^3)^2 + (x^3)(y^3) + (y^3)^2)$
$= (x - y)(x^2 + xy + y^2)(x^6 + x^3y^3 + y^6)$

23. $(a + b)^3 - (a - b)^3$
According to the equation,
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

So we get,
$(a + b)^3 - (a - b)^3$
$= (a + b - (a - b))((a + b)^2 + (a + b)(a - b) + (a - b)^2)$
On further simplification
$= (a + b - a + b)(a^2 + b^2 + 2ab + a^2 - b^2 + a^2 + b^2 - 2ab)$
$= 2b(3a^2 + b^2)$

24. $8a^3 - b^3 - 4ax + 2bx$

According to the equation,
$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

So we get,
$8a^3 - b^3 - 4ax + 2bx$
$= (2a)^3 - b^3 - 2x(2a-b)$
$= (2a-b)((2a)^2 + (2a)b + b^2) - 2x(2a-b)$
On further simplification
$= (2a-b)(4a^2 + 2ab + b^2) - 2x(2a-b)$
$= (2a-b)(4a^2 + 2ab + b^2 - 2x)$

25. $a^3 + 3a^2b + 3ab^2 + b^3 - 8$
We can write the given question as,
$a^3 + 3a^2b + 3ab^2 + b^3 - 8 = (a+b)^3 - 2^3$

According to the equation,
$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

So we get,
$(a+b)^3 - 2^3$
$= ((a+b) - 2)((a+b)^2 + (a+b)2 + 2^2)$
$= (a+b-2)((a+b)^2 + 2(a+b) + 4)$

26. $a^3 - \frac{1}{a^3} - 2a + \frac{2}{a}$
$= a^3 - \frac{1}{a^3} - 2(a - \frac{1}{a})$

According to the equation,
$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

So we get,
$a^3 - \frac{1}{a^3} - 2(a + \frac{1}{a})$
$= (a - \frac{1}{a})(a^2 + a(\frac{1}{a}) + \left(\frac{1}{a}\right)^2) - 2(a - \frac{1}{a})$
On further simplification
$= (a - \frac{1}{a})(a^2 + 1 + \frac{1}{a^2} - 2)$
$= (a - \frac{1}{a})(a^2 + \frac{1}{a^2} - 1)$

27. $2a^3 + 16b^3 - 5a - 10b$
$= 2(a^3 + 8b^3) - 5(a+2b)$

According to the equation,
$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
So we get,
$2(a^3 + 8b^3) - 5(a+2b)$
We can write it as,
$= 2(a^3 + (2b)^3) - 5(a+2b)$
$= 2(a+2b)(a^2 - a(2b) + (2b)^2) - 5(a+2b)$

$= 2(a + 2b)(a^2 - 2ab + 4b^2) - 5(a + 2b)$
By taking $(a + 2b)$as common,
We get,
$= (a + 2b)(2(a^2 - 2ab + 4b^2) - 5)$

28. $a^6 + b^6$
The given question can be written as
$= (a^2)^3 + (b^2)^3$
According to the equation
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
We can write the question as,
$(a^2)^3 + (b^2)^3$
$= (a^2 + b^2)((a^2)^2 - a^2b^2 + (b^2)^2)$
So we get,
$= (a^2 + b^2)(a^4 - a^2b^2 + b^4)$

29. $a^{12} - b^{12}$
The given question can be written as
$= (a^6)^2 - (b^6)^2$
According to the equation
$a^2 - b^2 = (a + b)(a - b)$
So we get,
$(a^6)^2 - (b^6)^2$
$= (a^6 + b^6)(a^6 - b^6)$
Now we can write it as,
$= [(a^2)^3 + (b^2)^3][(a^3)^2 - (b^3)^2]$
So according to the equation
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
$(a^2 - b^2) = (a + b)(a - b)$
So we get,
$= (a^2 + b^2)((a^2)^2 - a^2b^2 + (b^2)^2)\ (a^3 + b^3)(a^3 - b^3)$
According to the equation
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
We get,
$= (a^2 + b^2)(a^4 - a^2b^2 + b^4)\ (a + b)(a^2 - ab + b^2)\ (a - b)(a^2 + ab + b^2)$
$= (a + b)\ (a - b)\ (a^2 + b^2)\ (a^4 - a^2b^2 + b^4)\ (a^2 - ab + b^2)\ (a^2 + ab + b^2)$

30. $x^6 - 7x^3 - 8$
By substituting $x^3 = y$ in the given equation
We get,
$= y^2 - 7y - 8$
$= y^2 - 8y + y - 8$
Taking y as common in the first term and 1 as common in the second term
$= y(y - 8) + 1(y - 8)$
$= (y + 1)(y - 8)$
Now by replacing $y = x^3$
We get,
$= (x^3 + 1)(x^3 - 8)$

$= (x^3 + 1^3)(x^3 - 2^3)$
According to the equation
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
We get,
$= (x + 1)(x^2 - x + 1^2)\ (x - 2)(x^2 + 2x + 2^2)$
On further simplification
$= (x + 1)(x^2 - x + 1)\ (x - 2)(x^2 + 2x + 4)$
$= (x + 1)\ (x - 2)\ (x^2 - x + 1)\ (x^2 + 2x + 4)$

31. $x^3 - 3x^2 + 3x + 7$
We can write the given question as
$= x^3 - 3x^2 + 3x - 1 + 8$
By grouping the terms
$= (x^3 - 3x^2 + 3x - 1) + 8$
We know that $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
So we get,
$= (x - 1)^3 + 2^3$
According to the equation
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
We get,
$= ((x - 1) + 2)((x - 1)^2 - 2(x - 1) + 2^2)$
According to the equation
$(a - b)^2 = a^2 - 2ab + b^2$
$= (x - 1 + 2)(\ x^2 - 2x(1) + 1^2 - 2x + 2 + 4)$
On further simplification
$= (x + 1)(\ x^2 - 2x + 1 - 2x + 6)$
$= (x + 1)(\ x^2 - 4x + 7)$

32. $(x + 1)^3 + (x - 1)^3$
According to the equation
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
We get,
$= ((x + 1) + (x - 1))((x + 1)^2 - (x + 1)(x - 1) + (x - 1)^2)$
According to the equation
$(a + b)^2 = a^2 + 2ab + b^2$
$(a - b)^2 = a^2 - 2ab + b^2$
We get,
$= (x + 1 + x - 1)((\ x^2 + 2x + 1^2) - (x^2 - 1^2) + x^2 - 2x + 1^2)$
$= 2x(x^2 + 2x + 1 - x^2 + 1 + x^2 - 2x + 1)$
$= 2x(x^2 + 3)$

33. $(2a + 1)^3 + (a - 1)^3$
According to the equation
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
We get,
$= ((2a + 1) + (a - 1))((2a + 1)^2 - (2a + 1)(a - 1) + (a - 1)^2)$
According to the equation,
$(a + b)^2 = a^2 + 2ab + b^2$

$= (x^3 + 1^3)(x^3 - 2^3)$
According to the equation
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
We get,
$= (x + 1)(x^2 - x + 1^2)\ (x - 2)(x^2 + 2x + 2^2)$
On further simplification
$= (x + 1)(x^2 - x + 1)\ (x - 2)(x^2 + 2x + 4)$
$= (x + 1)\ (x - 2)\ (x^2 - x + 1)\ (x^2 + 2x + 4)$

31. $x^3 - 3x^2 + 3x + 7$
We can write the given question as
$= x^3 - 3x^2 + 3x - 1 + 8$
By grouping the terms
$= (x^3 - 3x^2 + 3x - 1) + 8$
We know that $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
So we get,
$= (x - 1)^3 + 2^3$
According to the equation
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
We get,
$= ((x - 1) + 2)((x - 1)^2 - 2(x - 1) + 2^2)$
According to the equation
$(a - b)^2 = a^2 - 2ab + b^2$
$= (x - 1 + 2)(\ x^2 - 2x(1) + 1^2 - 2x + 2 + 4)$
On further simplification
$= (x + 1)(\ x^2 - 2x + 1 - 2x + 6)$
$= (x + 1)(\ x^2 - 4x + 7)$

32. $(x + 1)^3 + (x - 1)^3$
According to the equation
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
We get,
$= ((x + 1) + (x - 1))((x + 1)^2 - (x + 1)(x - 1) + (x - 1)^2)$
According to the equation
$(a + b)^2 = a^2 + 2ab + b^2$
$(a - b)^2 = a^2 - 2ab + b^2$
We get,
$= (x + 1 + x - 1)((\ x^2 + 2x + 1^2) - (x^2 - 1^2) + x^2 - 2x + 1^2)$
$= 2x(x^2 + 2x + 1 - x^2 + 1 + x^2 - 2x + 1)$
$= 2x(x^2 + 3)$

33. $(2a + 1)^3 + (a - 1)^3$
According to the equation
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
We get,
$= ((2a + 1) + (a - 1))((2a + 1)^2 - (2a + 1)(a - 1) + (a - 1)^2)$
According to the equation,
$(a + b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$
So we get,
$= (2a+1+a-1)(\,(2a)^2 + 2(2a)(1) + 1^2 - 2a^2 + 2a - a + 1 + a^2 - 2a(1) + 1^2)$
$= 3a(4a^2 + 4a + 1 - 2a^2 + a + 1 + a^2 - 2a + 1)$
$= 3a(3a^2 + 3a + 3)$
By taking 3 as common
$= 9a(a^2 + a + 1)$

34. $8(x+y)^3 - 27(x-y)^3$
We can write the given question as
$= 2^3(x+y)^3 - 3^3(x-y)^3$
According to the equation
$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
We get,
$= (2(x+y) - 3(x-y))(\left(2(x+y)\right)^2 + 2(x+y)3(x-y) + (3(x-y))^2)$
According to the equation,
$(a+b)^2 = a^2 + 2ab + b^2$
$(a-b)^2 = a^2 - 2ab + b^2$
So we get,
$= (2x + 2y - 3x + 3y)((4(x^2 + 2xy + y^2) + 6(x^2 - y^2) + (9(x^2 - 2xy + y^2))$
$= (-x + 5y)(4x^2 + 8xy + 4y^2 + 6x^2 - 6y^2 + 9x^2 - 18xy + 9y^2)$
$= (-x + 5y)(19x^2 + 7y^2 - 10xy)$

35. $(x+2)^3 + (x-2)^3$
According to the equation
$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
We get,
$= ((x+2) + (x-2))((x+2)^2 - (x+2)(x-2) + (x-2)^2)$
According to the equation,
$(a+b)^2 = a^2 + 2ab + b^2$
$(a-b)^2 = a^2 - 2ab + b^2$
So we get,
$= (x + 2 + x - 2)(\,x^2 + 2(x)(2) + (2)^2 - (x^2 - 2^2) + x^2 - 2(x)(2) + 2^2)$
$= 2x(x^2 + 4x + 4 - x^2 + 4 + x^2 - 4x + 4)$
$= 2x(x^2 + 12)$

36. $(x+2)^3 - (x-2)^3$
According to the equation,
$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
So we get,
$= ((x+2) - (x-2))((x+2)^2 + (x+2)(x-2) + (x-2)^2)$
$= (x + 2 - x + 2)(\,(x+2)^2 + (x^2 - 2^2) + (x-2)^2)$
According to the equation,
$(a+b)^2 = a^2 + 2ab + b^2$
$(a-b)^2 = a^2 - 2ab + b^2$
So we get,
$= 4((x^2 + 2(x)(2) + 2^2) + x^2 - 2^2 + (x^2 - 2(x)(2) + 2^2))$
$= 4(x^2 + 4x + 4 + x^2 - 4 + x^2 - 4x + 4)$
$= 4(3x^2 + 4)$

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Find the product.

1. $(\boldsymbol{x+y+z})(\boldsymbol{x^2+y^2+z^2-xy+yz+zx})$

Solution:

$(x+y+z)(x^2+y^2+z^2-xy+yz+zx)$

The given question can be written as

$=(x+y+(-z))(x^2+y^2+(-z)^2-xy+y(-z)+(-z)x)$

According to the equation,

$a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

We get,

$=(x+y+(-z))(x^2+y^2+(-z)^2-xy+y(-z)+(-z)x)$

$=x^3+y^3-z^3+3xyz$

2. $(\boldsymbol{x-y-z})(\boldsymbol{x^2+y^2+z^2+xy-yz+xz})$

Solution:

$(x-y-z)(x^2+y^2+z^2+xy-yz+xz)$

According to the equation,

$a^3-b^3-c^3-3abc=(a-b-c)(a^2+b^2+c^2+ab-bc+ca)$

The given question can be written as

$=(x^3+xy^2+xz^2+x^2y-xyz+x^2z-x^2y-y^3-yz^2-xy^2+y^2z-xyz-x^2z-y^2z-z^3-xyz+yz^2-xz^2)$

So we get,

$=(x^3-y^3-z^3-xyz-xyz-xyz)$

$=x^3-y^3-z^3-3xyz$

3. $(\boldsymbol{x-2y+3})(\boldsymbol{x^2+4y^2+2xy+6y-3x+9})$

Solution:

$(x-2y+3)(x^2+4y^2+2xy+6y-3x+9)$

The given question can be written as

$=(x+(-2y)+3)(x^2+(-2y)^2+3^2-(x)(-2y)-(-2y)(3)-(3)(x))$

According to the equation,

$a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

We get,

$=x^3+(-2y)^3+3^3-3\times x\times(-2y)\times 3$

$=x^3-8y^3+27+18xy$

4. $(\boldsymbol{3x-5y+4})(\boldsymbol{9x^2+25y^2+15xy-20y+12x+16})$

Solution:

$(3x-5y+4)(9x^2+25y^2+15xy-20y+12x+16)$

The given question can be written as

$=(3x+(-5y)+4)((3x)^2+(-5y)^2-3x\times(-5y)-(-5y)\times 4-4\times 3x+4^2)$

According to the equation,
$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
We get,
$= (3x)^3 + (-5y)^3 + 4^3 - 3 \times 3x \times (-5y) \times 4$
$= 27x^3 - 125y^3 + 64 + 180xy$

Factorise:
5. $\mathbf{125a^3 + b^3 + 64c^3 - 60abc}$
Solution:

$125a^3 + b^3 + 64c^3 - 60abc$
It can be written as
$= (5a)^3 + b^3 + (4c)^3 - 3 \times 5a \times b \times 4c$
According to the equation,
$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
We get,
$= (5a + b + 4c)((5a)^2 + b^2 + (4c)^2 - (5a)b - b(4c) - (4c)(5a))$
$= (5a + b + 4c)(25a^2 + b^2 + 16c^2 - 5ab - 4bc - 20ca)$

6. $\mathbf{a^3 + 8b^3 + 64c^3 - 24abc}$
Solution:

$a^3 + 8b^3 + 64c^3 - 24abc$
It can be written as
$= a^3 + (2b)^3 + (4c)^3 - 3 \times a \times 2b \times 4c$
According to the equation,
$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
We get,
$= (a + 2b + 4c)(a^2 + (2b)^2 + (4c)^2 - a(2b) - (2b)(4c) - (4c)a)$
$= (a + 2b + 4c)(a^2 + 4b^2 + 16c^2 - 2ab - 8bc - 4ca)$

7. $\mathbf{1 + b^3 + 8c^3 - 6bc}$
Solution:

$1 + b^3 + 8c^3 - 6bc$
It can be written as
$= 1^3 + b^3 + (2c)^3 - 3 \times 1 \times b \times 2c$
According to the equation,
$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
We get,
$= (1 + b + 2c)(1^2 + b^2 + (2c)^2 - (1)b - b(2c) - (2c)(1))$
$= (1 + b + 2c)(1 + b^2 + 4c^2 - b - 2bc - 2c)$

8. $\mathbf{216 + 27b^3 + 8c^3 - 108bc}$
Solution:

$216 + 27b^3 + 8c^3 - 108bc$
It can be written as
$= 6^3 + (3b)^3 + (2c)^3 - 3 \times 6 \times 3b \times 2c$

According to the equation,
$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
We get,
$= (6 + 3b + 2c)(6^2 + (3b)^2 + (2c)^2 - (6)(3b) - (3b)(2c) - (2c)(6))$
$= (6 + 3b + 2c)(36 + 9b^2 + 4c^2 - 18b - 6bc - 12c)$

9. $27a^3 - b^3 + 8c^3 + 18abc$
Solution:

$27a^3 - b^3 + 8c^3 + 18abc$
It can be written as
$= (3a)^3 + (-b)^3 + (2c)^3 + 3 \times 3a \times (-b) \times 2c$
According to the equation,
$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
We get,
$= (3a + (-b) + 2c)((3a)^2 + (-b)^2 + (2c)^2 - (3a)(-b) - (-b)(2c) - (2c)(3a))$
$= (3a - b + 2c)(9a^2 + b^2 + 4c^2 + 3ab + 2bc - 6ca)$

10. $8a^3 + 125b^3 - 64c^3 + 120abc$
Solution:

$8a^3 + 125b^3 - 64c^3 + 120abc$
It can be written as
$= (2a)^3 + (5b)^3 + (-4c)^3 + 3 \times 2a \times 5b \times (-4c)$
According to the equation,
$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
We get,
$= (2a + 5b + (-4c))((2a)^2 + (5b)^2 + (-4c)^2 - (2a)(5b) - (5b)(-4c) - (-4c)(2a))$
$= (2a + 5b - 4c)(4a^2 + 25b^2 + 16c^2 - 10ab + 20bc + 8ca)$

11. $8 - 27b^3 - 343c^3 - 126bc$
Solution:

$8 - 27b^3 - 343c^3 - 126bc$
It can be written as
$= 2^3 + (-3b)^3 + (-7c)^3 + 3 \times 2 \times (-3b) \times (-7c)$
According to the equation,
$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
We get,
$= (2 + (-3b) + (-7c))(2^2 + (-3b)^2 + (-7c)^2 - (2)(-3b) - (-3b)(-7c) - (-7c)(2))$
$= (2 - 3b - 7c)(4 + 9b^2 + 49c^2 + 6b - 21bc + 14c)$

12. $125 - 8x^3 - 27y^3 - 90xy$
Solution:

$125 - 8x^3 - 27y^3 - 90xy$
It can be written as
$= (5)^3 + (-2x)^3 + (-3y)^3 - 3 \times 5 \times (-2x) \times (-3y)$
According to the equation,

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
We get,
$= (5 + (-2x) + (-3y))(5^2 + (-2x)^2 + (-3y)^2 - (5)(-2x) - (-2x)(-3y) - (-3y)(5))$
$= (5 - 2x - 3y)(25 + 4x^2 + 9y^2 + 10x - 6xy + 15y)$

13. $2\sqrt{2}a^3 + 16\sqrt{2}b^3 + c^3 - 12abc$
Solution:

$2\sqrt{2}a^3 + 16\sqrt{2}b^3 + c^3 - 12abc$
It can be written as
$= (\sqrt{2}a)^3 + (2\sqrt{2}b)^3 + c^3 - 3 \times \sqrt{2}a \times 2\sqrt{2}b \times c$
According to the equation,
$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
We get,
$= (\sqrt{2}a + 2\sqrt{2}b + c)((\sqrt{2}a)^2 + (2\sqrt{2}b)^2 + c^2 - (\sqrt{2}a)(2\sqrt{2}b) - (2\sqrt{2}b)c - c(\sqrt{2}a))$
$= (\sqrt{2}a + 2\sqrt{2}b + c)(2a^2 + 8b^2 + c^2 - 4ab - 2\sqrt{2}bc - \sqrt{2}ac)$

14. $27x^3 - y^3 - z^3 - 9xyz$
Solution:

$27x^3 - y^3 - z^3 - 9xyz$
It can be written as
$= (3x)^3 + (-y)^3 + (-z)^3 - 3 \times 3x \times (-y) \times (-z)$
According to the equation,
$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
We get,
$= (3x + (-y) + (-z))((3x)^2 + (-y)^2 + (-z)^2 - (3x)(-y) - (-y)(-z) - (-z)(3x))$
$= (3x - y - z)(9x^2 + y^2 + z^2 + 3xy - yz + 3xz)$

15. $2\sqrt{2}a^3 + 3\sqrt{3}b^3 + c^3 - 3\sqrt{6}abc$
Solution:

$2\sqrt{2}a^3 + 3\sqrt{3}b^3 + c^3 - 3\sqrt{6}abc$
It can be written as
$= (\sqrt{2}a)^3 + (\sqrt{3}b)^3 + c^3 - 3 \times \sqrt{2}a \times \sqrt{3}b \times c$
According to the equation,
$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
We get,
$= (\sqrt{2}a + \sqrt{3}b + c)((\sqrt{2}a)^2 + (\sqrt{3}b)^2 + c^2 - (\sqrt{2}a)(\sqrt{3}b) - (\sqrt{3}b)c - c(\sqrt{2}a))$
$= (\sqrt{2}a + \sqrt{3}b + c)(2a^2 + 3b^2 + c^2 - \sqrt{6}ab - \sqrt{3}bc - \sqrt{2}ac)$

16. $3\sqrt{3}a^3 - b^3 - 5\sqrt{5}c^3 - 3\sqrt{15}abc$
Solution:

$3\sqrt{3}a^3 - b^3 - 5\sqrt{5}c^3 - 3\sqrt{15}abc$
It can be written as
$= (\sqrt{3}a)^3 + (-b)^3 + (-\sqrt{5}c)^3 - 3 \times \sqrt{3}a \times (-b) \times (-\sqrt{5}c)$

According to the equation,
$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
We get,
$= (\sqrt{3}a + (-b) + (-\sqrt{5}c))((\sqrt{3}a)^2 + (-b)^2 + (-\sqrt{5}c)^2 - (\sqrt{3}a)(-b) - (-b)(-\sqrt{5}c) - (-\sqrt{5}c)(\sqrt{3}a))$
$= (\sqrt{3}a - b - \sqrt{5}c)(3a^2 + b^2 + 5c^2 + \sqrt{3}ab - \sqrt{5}bc + \sqrt{15}ac)$

17. $(\boldsymbol{a - b})^3 + (\boldsymbol{b - c})^3 + (\boldsymbol{c - a})^3$
Solution:

Let us consider $(a - b)^3 = x, (b - c)^3 = y$ and $(c - a)^3 = z$
It can be written as
$= x^3 + y^3 + z^3$
We know that
$(x + y + z) = (a - b + b - c + c - a) = 0$
So we get,
$x^3 + y^3 + z^3 = 3xyz$
Now replacing the values of x, y and z
= 3 (a – b) (b – c) (c – a)

18. $(\boldsymbol{a - 3b})^3 + (\boldsymbol{3b - c})^3 + (\boldsymbol{c - a})^3$
Solution:

Let us consider $(a - 3b)^3 = x, (3b - c)^3 = y$ and $(c - a)^3 = z$
It can be written as
$= x^3 + y^3 + z^3$
We know that
$(x + y + z) = (a - 3b + 3b - c + c - a) = 0$
So we get,
$x^3 + y^3 + z^3 = 3xyz$
Now replacing the values of x, y and z
= 3 (a – 3b) (3b – c) (c – a)

19. $(\boldsymbol{3a - 2b})^3 + (\boldsymbol{2b - 5c})^3 + (\boldsymbol{5c - 3a})^3$
Solution:

Let us consider $(3a - 2b)^3 = x, (2b - 5c)^3 = y$ and $(5c - 3a)^3 = z$
It can be written as
$= x^3 + y^3 + z^3$
We know that
$(x + y + z) = (3a - 2b + 2b - 5c + 5c - 3a) = 0$
So we get,
$x^3 + y^3 + z^3 = 3xyz$
Now replacing the values of x, y and z
= 3 (3a – 2b) (2b – 5c) (5c – 3a)

20. $(\boldsymbol{5a - 7b})^3 + (\boldsymbol{7b - 9c})^3 + (\boldsymbol{9c - 5a})^3$
Solution:

Let us consider $(5a - 7b)^3 = x, (7b - 9c)^3 = y$ and $(9c - 5a)^3 = z$

It can be written as
$= x^3 + y^3 + z^3$
We know that
$(x + y + z) = (5a - 7b + 7b - 9c + 9c - 5a) = 0$
So we get,
$x^3 + y^3 + z^3 = 3xyz$
Now replacing the values of x, y and z
= 3 (5a – 7b) (7b – 9c) (9c – 5a)

21. $\boldsymbol{a^3(b-c)^3 + b^3(c-a)^3 + c^3(a-b)^3}$
Solution:

The given question can be written as
$(a(b-c))^3 + (b(c-a))^3 + (c(a-b))^3$
Let us consider $(a(b-c))^3 = x, (b(c-a))^3 = y$ and $(c(a-b))^3 = z$
It can be written as
$= x^3 + y^3 + z^3$
We know that
$(x + y + z)$
$= a(b-c) + b(c-a) + c(a-b)$
$= ab - ac + bc - ab + ac - bc$
$= 0$
So we get,
$x^3 + y^3 + z^3 = 3xyz$
Now replacing the values of x, y and z
= 3 (a (b – c)) (b (c – a)) (c (a – b))
= 3abc (b – c) (c – a) (a – b)

22. Evaluate
(i) $\boldsymbol{(-12)^3 + 7^3 + 5^3}$
(ii) $\boldsymbol{(28)^3 + (-15)^3 + (-13)^3}$
Solution:

(i) It is given that
$(-12)^3 + 7^3 + 5^3$

If x + y + z = 0 then $x^3 + y^3 + z^3 = 3xyz$
According to that
-12 + 7 + 5 = 0
So we get
$(-12)^3 + 7^3 + 5^3 = 3(-12)(7)(5)$
= - 1260

(ii) It is given that
$(28)^3 + (-15)^3 + (-13)^3$

If x + y + z = 0 then $x^3 + y^3 + z^3 = 3xyz$
According to that
28 – 15 – 13 = 0

So we get
$(28)^3 + (-15)^3 + (-13)^3 = 3(28)(-15)(-13)$
$= 16380$

23. Prove that $(a+b+c)^3 - a^3 - b^3 - c^3 = 3(a+b)(b+c)(c+a)$.
Solution:

Given LHS
$= (a+b+c)^3 - a^3 - b^3 - c^3$
The given equation can be written as
$= ((a+b)+c)^3 - a^3 - b^3 - c^3$
According to the equation
$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$
So we get,
$= (a+b)^3 + c^3 + 3(a+b)(c)((a+b)+c) - a^3 - b^3 - c^3$
By again substituting the equation
$= a^3 + b^3 + 3ab(a+b) + c^3 + 3(a+b)(c)((a+b)+c) - a^3 - b^3 - c^3$
$= 3ab(a+b) + 3(a+b)(c)((a+b)+c)$
Take $3(a+b)$ as common in both the terms
$= 3(a+b)(ab + c(a+b) + c^2)$
Multiply c with the terms inside the bracket
$= 3(a+b)(ab + ac + bc + c^2)$
Taking a and c as common in the second term
$= 3(a+b)(a(b+c) + c(b+c))$
So we get,
$= 3(a+b)(b+c)(c+a)$
= RHS
Hence proved

24. If a, b, c are all non-zero and a + b + c = 0, prove that $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = 3$.
Solution:

Given LHS
$= \frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}$
By LCM we get
$= \frac{a^3+b^3+c^3}{abc}$
We know that,
If x + y + z = 0, we get $x^3 + y^3 + z^3 = 3xyz$
Based on that
$= \frac{3abc}{abc}$
Cancelling similar terms on numerator and denominator
= 3
= RHS
Hence proved.

25. If a + b + c = 9 and $a^2 + b^2 + c^2 = 35$, find the value of $(a^3 + b^3 + c^3 - 3abc)$.
Solution:

We know the equation,
$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
This can further be written as
$= (a + b + c)((a^2 + b^2 + c^2) - (ab - bc - ca))$ …….. (1)
We know the equation,
$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
By substituting the values given in the question
$(9)^2 = 35 + 2(ab + bc + ca)$
$81 = 35 + 2(ab + bc + ca)$
$2(ab + bc + ca) = 81 - 35$
$2(ab + bc + ca) = 46$
By dividing 46 by 2
$ab + bc + ca = \frac{46}{2}$
We get
$ab + bc + ca = 23$
Now by substituting the values in equation (1)
$a^3 + b^3 + c^3 - 3abc = (a + b + c)((a^2 + b^2 + c^2) - (ab - bc - ca))$
$= 9\ (35 - 23)$
$= 9\ (12)$
$= 108$