EXERCISE 2(A) PAGE: 74

1. Which of the following expressions are polynomials? In case of a polynomial, write its degree.

(i) $x^5 - 2x^3 + x + \sqrt{3}$

(ii) $y^3 + \sqrt{3}y$

(iii) $t^2 - \frac{2}{5}t + \sqrt{5}$

(iv) $x^{100} - 1$

(v) $\frac{1}{\sqrt{2}}x^2 - \sqrt{2}x + 2$

(vi) $x^{-2} + 2x^{-1} + 3$

(vii) 1

(viii) $\frac{-3}{5}$

(ix) $\frac{x^2}{2} - \frac{2}{x^2}$

(x) $\sqrt[3]{2}\,x^2 - 8$

(xi) $\frac{1}{2x^2}$

(xii) $\frac{1}{\sqrt{5}}x^{\frac{1}{2}} + 1$

(xiii) $\frac{3}{5}x^2 - \frac{7}{3}x + 9$

(xiv) $x^4 - x^{\frac{3}{2}} + x - 3$

(xv) $2x^3 + 3x^2 + \sqrt{x} - 1$

Solution:

(i) $x^5 - 2x^3 + x + \sqrt{3}$

It is a polynomial as the expression has positive integral powers of x with its highest power as 5.
Therefore the degree of the given expression is 5.

(ii) $y^3 + \sqrt{3}y$

It is a polynomial as the expression has positive integral powers of x with its highest power as 3.
Therefore the degree of the given expression is 3.

(iii) $t^2 - \frac{2}{5}t + \sqrt{5}$

It is a polynomial as the expression has positive integral powers of x with its highest power as 2.
Therefore the degree of the given expression is 2.

(iv) $x^{100} - 1$

It is a polynomial as the expression has positive integral powers of x with its highest power as 100.
Therefore the degree of the given expression is 100.

(v) $\frac{1}{\sqrt{2}}x^2 - \sqrt{2}x + 2$

Yes. It is a polynomial of degree 2.

(vi) $x^{-2} + 2x^{-1} + 3$
The given expression is not a polynomial.

(vii) 1
The given expression is a polynomial of degree 0.

(viii) $\frac{-3}{5}$
The given expression is a polynomial of degree 0.

(ix) $\frac{x^2}{2} - \frac{2}{x^2}$
We can write $\frac{x^2}{2} - \frac{2}{x^2}$ as $\frac{x^2}{2} - 2x^{-2}$
It is not a polynomial as the expression has negative integral power of x.

(x) $\sqrt[3]{2}\, x^2 - 8$
It is a polynomial as the expression has positive integral powers of x with its highest power as 2.
Therefore the degree of the given expression is 2.

(xi) $\frac{1}{2x^2}$
We can write $\frac{1}{2x^2}$ as $2x^{-2}$.
It is not a polynomial as the expression has negative integral power of x.

(xii) $\frac{1}{\sqrt{5}}\, x^{\frac{1}{2}} + 1$
It is not a polynomial as the expression has $x^{\frac{1}{2}}$ which is not a non-negative integer.

(xiii) $\frac{3}{5}\, x^2 - \frac{7}{3}\, x + 9$
It is a polynomial as the expression has positive integral powers of x with its highest power as 2.
Therefore the degree of the given expression is 2.

(xiv) $x^4 - x^{\frac{3}{2}} + x - 3$
It is not a polynomial as the expression has $x^{\frac{3}{2}}$ which is not a non-negative integer.

(xv) $2x^3 + 3x^2 + \sqrt{x} - 1$
We can write $2x^3 + 3x^2 + \sqrt{x} - 1$ as $2x^3 + 3x^2 + x^{\frac{1}{2}} - 1$.
It is not a polynomial as the expression has $x^{\frac{1}{2}}$ which is not a non-negative integer.

2. Identify constant, linear, quadratic, cubic and quartic polynomials from the following.
(i) $-7 + x$
(ii) $6y$
(iii) $-z^3$
(iv) $1 - y - y^3$
(v) $x - x^3 + x^4$

(vi) $1 + x + x^2$
(vii) $-6x^2$
(viii) -13
(ix) –p
Solution:

(i) – 7 + x
We know that the degree of – 7 + x is 1.
Therefore, it is a linear polynomial.

(ii) 6y
We know that the degree of 6y is 1.
Therefore, it is a linear polynomial.

(iii) $-z^3$
We know that the degree of $-z^3$ is 3.
Therefore, it is a cubic polynomial.

(iv) $1 - y - y^3$
We know that the degree of $1 - y - y^3$ is 3.
Therefore, it is a cubic polynomial.

(v) $x - x^3 + x^4$
We know that the degree of $x - x^3 + x^4$ is 4.
Therefore, it is a quartic polynomial.

(vi) $1 + x + x^2$
We know that the degree of $1 + x + x^2$ is 2.
Therefore, it is a quadratic polynomial.

(vii) $-6x^2$
We know that the degree of $-6x^2$ is 2.
Therefore, it is a quadratic polynomial.

(viii) -13
We know that -13 is a constant.
Therefore, it is a constant polynomial.

(ix) – p
We know that the degree of –p is 1.
Therefore, it is a linear polynomial.

3. Write
(i) **The coefficient of x^3 in $x + 3x^2 - 5x^3 + x^4$.**
(ii) **The coefficient of x in $\sqrt{3} - 2\sqrt{2}\,x + 6x^2$.**
(iii) **The coefficient of x^2 in $2x - 3 + x^3$.**
(iv) **The coefficient of x in $\frac{3}{8}x^2 - \frac{2}{7}x + \frac{1}{6}$.**
(v) **The constant term in $\frac{\pi}{2}x^2 + 7x - \frac{2}{5}\pi$.**

Solution:

(i) In $x + 3x^2 - 5x^3 + x^4$ the coefficient of x^3 is -5.

(ii) In $\sqrt{3} - 2\sqrt{2}\,x + 6x^2$ the coefficient of x is $-2\sqrt{2}$.

(iii) $2x - 3 + x^3$ can be written as $x^3 + 0x^2 + 2x - 3$.
In $2x - 3 + x^3$ the coefficient of x^2 is 0.

(iv) In $\frac{3}{8}x^2 - \frac{2}{7}x + \frac{1}{6}$ the coefficient of x is $-\frac{2}{7}$.

(v) In $\frac{\pi}{2}x^2 + 7x - \frac{2}{5}\pi$ the constant term is $-\frac{2}{5}\pi$.

4. Determine the degree of each of the following polynomials.

(i) $\frac{4x-5x^2+6x^3}{2x}$
(ii) $y^2(y - y^3)$
(iii) $(3x-2)(2x^3+3x^2)$
(iv) $-\frac{1}{2}x + 3$
(v) **-8**
(vi) $x^{-2}(x^4 + x^2)$

Solution:

(i) $\frac{4x-5x^2+6x^3}{2x}$

We can write it separately as

$= \frac{4x}{2x} - \frac{5x^2}{2x} + \frac{6x^3}{2x}$

On further simplification we get

$= 2 - \frac{5}{2}x + 3x^2$

The degree of the given expression is 2.

(ii) $y^2(y - y^3)$

By multiplying the terms
We get

$= y^3 - y^5$

The degree of the given expression is 5.

(iii) $(3x-2)(2x^3+3x^2)$

By multiplying the terms we get

$= 6x^4 + 9x^3 - 4x^3 - 6x^2$

On further simplification

$= 6x^4 + 5x^3 - 6x^2$

The degree of the given expression is 4.

(iv) $-\frac{1}{2}x + 3$
The degree of the given expression is 1.

(v) -8
The given expression is a constant polynomial of degree zero.

(vi) $x^{-2}(x^4 + x^2)$

By taking common terms out

$= x^{-2}.x^2\ (x^2 + 1)$

On further simplification

$= x^{-2+2}(x^2 + 1)$

So we get

$= x^0(x^2 + 1)$

$= x^2 + 1$

The degree of the given expression is 2.

5. (i) Give an example of a monomial of degree 5.
(ii) Give an example of a binomial of degree 8.
(iii) Give an example of a trinomial of degree 4.
(iv) Give an example of a monomial of degree 0.

Solution:

(i) Example of a monomial of degree 5 is $4x^5$.

(ii) Example of a binomial of degree 8 is $x - 4x^8$.

(iii) Example of a trinomial of degree 4 is $1 + 3x + x^4$.

(iv) Example of a monomial of degree 0 is 1.

6. Rewrite each of the following polynomials in standard form.

(i) $x - 2x^2 + 8 + 5x^3$

(ii) $\frac{2}{3} + 4y^2 - 3y + 2y^3$

(iii) $6x^3 + 2x - x^5 - 3x^2$

(iv) $2 + t - 3t^3 + t^4 - t^2$

Solution:

(i) $x - 2x^2 + 8 + 5x^3$ in standard form is written as $5x^3 - 2x^2 + x + 8$.

(ii) $\frac{2}{3} + 4y^2 - 3y + 2y^3$ in standard form is written as $2y^3 + 4y^2 - 3y + \frac{2}{3}$.

(iii) $6x^3 + 2x - x^5 - 3x^2$ in standard form is written as $- x^5 + 6x^3 - 3x^2 + 2x$.

(iv) $2 + t - 3t^3 + t^4 - t^2$ in standard form is written as $t^4 - 3t^3 - t^2 + t + 2$.

EXERCISE 2(B)

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1. If p(x) = 5 – 4x + $2x^2$, find
(i) p(0)
(ii) p(3)
(iii) p(-2)
Solution:

It is given that p(x) = 5 – 4x + $2x^2$
(i) p(0)

By substituting 0 in the place of x

= 5 – 4 × 0 + 2 × 0^2

So we get

= 5 – 0

=5

(ii) p(3)

By substituting 3 in the place of x

= 5 – 4 × 3 + 2 × 3^2

= 5 – 12 + 18

So we get

= - 7 + 18

= 11

(iii) p(-2)

By substituting -2 in the place of x

= 5 – 4 × (-2) + 2 × $(-2)^2$

So we get

= 5 + 8 + 8

= 21

2. If p(y) = 4 + 3y - y^2+ $5y^3$, find
(i) p(0)

(ii) p(2)
(iii) p(-1)
Solution:

We know that
p(y) = 4 + 3y - y^2+ $5y^3$

(i) p(0)

By substituting 0 in the place of y

$= 4 + 3 \times 0 - 0^2 + 5 \times 0^3$

So we get,

= 4 + 0

= 4

(ii) p(2)

By substituting 2 in the place of y

$= 4 + 3 \times 2 - 2^2 + 5 \times 2^3$

= 4 + 6 – 4 + 40

So we get

= 6 + 40

= 46

(iii) p(-1)

By substituting -1 in the place of y

$= 4 + 3 \times (-1) - (-1)^2 + 5 \times (-1)^3$

On further calculation

= 4 – 3 – 1 – 5

So we get,

= 4 – 9

= - 5

3. If f(t) = $4t^2$ – 3t + 6, find

(i) f(0)
(ii) f(4)
(iii) f(-5)
Solution:

Given, f(t) = $4t^2$ – 3t + 6

(i) f(0)

By substituting 0 in the place of t

= $4 \times 0^2 - 3 \times 0 + 6$

So we get,

= 6

(ii) f(4)

By substituting 4 in the place of t

= $4 \times 4^2 - 3 \times 4 + 6$

So we get,

= 64 – 12 + 6

= 58

(iii) f(-5)

By substituting -5 in the place of t

= $4 \times (-5)^2 - 3 \times (-5) + 6$

So we get,

= 100 + 15 + 6

= 121

4. If p(x) = x^3- $3x^2$+ 2x, find p (0), p (1), p (2). What do you conclude?
Solution:

It is given that,
p (x) = x^3- $3x^2$+ 2x

So we get,
p (0)

Now substituting 0 in the place of x,

$= 0^3 - 3\times 0^2 + 2\times 0$

= 0

p (1)

Now substituting 1 in the place of x,

$= 1^3 - 3\times 1^2 + 2\times 1$

So we get,

= 1 – 3 + 2

= 0

p (2)

Now substituting 2 in the place of x,

$= 2^3 - 3\times 2^2 + 2\times 2$

So we get,

= 8 – 12 + 4

= 0

Therefore, 0, 1 and 2 are the zeros of p(x) = $x^3 - 3x^2 + 2x$.

5. If p(x) = $x^3 + x^2 - 9x - 9$, find p (0), p (3), p (-3) and p (-1). What do you conclude about the zeros of p (x)? Is 0 a zero of p(x)?
Solution:

We know that,
p(x) = $x^3 + x^2 - 9x - 9$

So we get,
p(0)

Now substituting 0 in the place of x,

$= 0^3 + 0^2 - 9\times 0 - 9$

= -9

p(3)

Now substituting 3 in the place of x,

= 3^3+ 3^2- 9× 3 – 9

So we get,

= 27 + 9 – 27 – 9

= 0

p(-3)

Now substituting -3 in the place of x,

= $(-3)^3$+ $(-3)^2$- 9× (-3) – 9

So we get

= - 27 + 9 + 27 – 9

= 0

p (-1)

Now substituting -1 in the place of x,

= $(-1)^3$+ $(-1)^2$- 9× (-1) – 9

So we get

= -1 + 1 + 9 – 9

= 0

Therefore, 0, 3 and -3 are the zeros of p(x) = x^3+ x^2- 9x – 9
0 is not a zero of p(x) = x^3+ x^2- 9x – 9 as p(0) ≠ 0.

6. Verify that
(i) 4 is a zero of the polynomial, p(x) = x – 4.
(ii) -3 is a zero of the polynomial, q(x) = x + 3.
(iii) $\frac{2}{5}$ is a zero of the polynomial, f(x) = 2-5x.
(iv) $\frac{-1}{2}$ is a zero of the polynomial, g(y) = 2y+1.
Solution:

(i) Given that, p(x) = x – 4
So we get,
p(4) = 4 – 4 = 0

Therefore, 4 is a zero of the polynomial, p(x) = x – 4.

(ii) Given that, q(x) = x + 3
So we get,
q(-3) = -3 + 3 = 0

Therefore, -3 is a zero of the polynomial, q(x) = x + 3.

(iii) Given that, f(x) = 2-5x
So we get,
$f(\frac{2}{5})$ = 2-5 × $\frac{2}{5}$ = 2 – 2 = 0

Therefore, $\frac{2}{5}$ is a zero of the polynomial, f(x) = 2-5x.

(iv) Given that, g(y) = 2y+1
So we get,
$g(\frac{-1}{2}) = 2(\frac{-1}{2})$+1 = -1 +1 = 0

Therefore, $\frac{-1}{2}$ is a zero of the polynomial, g(y) = 2y+1.

7. Verify that
(i) 1 and 2 are the zeros of the polynomial, p(x) = x^2- 3x + 2.
(ii) 2 and -3 are the zeros of the polynomial, q(x) = x^2+x -6.
(iii) 0 and 3 are the zeros of the polynomial, r(x) = x^2- 3x.
Solution:

(i) It is given that, p(x) = x^2- 3x + 2
In order to verify that 1 and 2 are the zeros of the given polynomial
p(1)

Now substituting 1 in the place of x,

= 1^2- 3× 1 + 2

So we get

= 1 – 3 + 2

=0

p(2)

Now substituting 2 in the place of x,

= 2^2- 3× 2 + 2

So we get

= 4 – 6 + 2

0

Thus, 1 and 2 are the zeros of the polynomial, p(x) = x^2- 3x + 2.

(ii) It is given that, q(x) = x^2+x -6
In order to verify that 2 and -3 are the zeros of the given polynomial
q(2)

Now substituting 2 in the place of x,

= 2^2+ 2 -6

So we get

= 4 + 2 – 6

= 6 – 6

= 0

q(-3)

Now substituting -3 in the place of x,

= $(-3)^2$+ (-3) -6

So we get

= 9 - 3 – 6

= 9 – 9

= 0

Thus, 2 and -3 are the zeros of the polynomial, q(x) = x^2+x -6.

(iii) It is given that, r(x) = x^2- 3x
In order to verify that 0 and 3 are the zeros of the given polynomial
r(0)

Now substituting 0 in the place of x,

= 0^2- 3 × 0

So we get

= 0

r(3)
Now substituting 3 in the place of x,

= 3^2 - 3 × 3

So we get

= 9 – 9

= 0

Thus, 0 and 3 are the zeros of the polynomial, r(x) = x^2 - 3x.

8. Find the zero of the polynomial:
(i) p(x) = x-5
(ii) q(x) = x+4
(iii) r(x) = 2x+5
(iv) f(x) = 3x+1
(v) g(x) = 5 - 4x
(vi) h(x) = 6x – 2
(vii) p(x) = ax, a ≠0
(viii) q(x) = 4x
Solution:

(i) Given,
p(x) = x-5

In order to find the zero of given polynomial
Let us consider,
p(x) = 0

x – 5 = 0

x = 5

Therefore, 5 is the zero of p(x) = x-5.

(ii) Given,
q(x) = x+4
In order to find the zero of given polynomial
Let us consider,
q(x) = 0

x + 4 = 0

x = - 4

Therefore, - 4 is the zero of q(x) = x+4.

(iii) Given,
r(x) = 2x+5
In order to find the zero of given polynomial
Let us consider,

r(x) = 0

2x+5 = 0

So we get

2x = -5

x = $\frac{-5}{2}$

Therefore, $\frac{-5}{2}$ is the zero of r(x) = 2x+5.

(iv) Given,
f(x) = 3x+1

In order find the zero of given polynomial
Let us consider,
f(x) = 0

3x+1 = 0

So we get

3x = -1

x = $\frac{-1}{3}$

Therefore, $\frac{-1}{3}$ is the zero of f(x) = 3x+1.

(v) Given,
g(x) = 5 - 4x

In order find the zero of given polynomial
Let us consider,
g(x) = 0

5 - 4x = 0

So we get,

4x = 5

x = $\frac{5}{4}$

Therefore, $\frac{5}{4}$ is the zero of g(x) = 5 - 4x.

(vi) Given,

h(x) = 6x – 2
In order find the zero of given polynomial
Let us consider,
h(x) = 0

6x – 2 = 0

6x = 2

So we get,

x = $\frac{2}{6}$

By calculating we get,

x = $\frac{1}{3}$

Therefore, $\frac{1}{3}$ is the zero of h(x) = 6x – 2.

(vii) Given,
p(x) = ax, a ≠0

In order find the zero of given polynomial
Let us consider,
p(x) = 0

ax = 0

x = 0

Therefore, 0 is the zero of p(x) = ax.

(viii) Given,
q(x) = 4x

In order find the zero of given polynomial
Let us consider,
q(x) = 0

So we get

4x = 0

x = 0

Therefore, 0 is the zero of q(x) = 4x.

9. If 2 and 0 are the zeros of the polynomial f(x) = $2x^3$- $5x^2$+ ax + b then find the values of 'a' and 'b'.
Solution:

We know that,
f(x) = $2x^3$- $5x^2$+ ax + b

Let us consider 2 as the zero of the given polynomial
So we get,
f(2) = 0

$2x^3$- $5x^2$+ ax + b = 0

By substituting 2 in the place of x we get

$2(2)^3$- $5(2)^2$+2a+ b = 0

On further calculation

2(8) – 5(4) + 2a + b = 0

So we get

16 – 20 + 2a + b = 0

2a + b – 4 = 0 ……… (1)

Now let us consider 0 as the zero of the given polynomial
So we get,
f(0) = 0

$2x^3$- $5x^2$+ ax + b = 0

By substituting 0 in the place of x we get

$2(0)^3$- $5(0)^2$+ (0)a+ b = 0

On further calculation

2(0) – 5(0) + (0)a + b = 0

So we get

0+ b = 0

b = 0 ……… (2)

Applying equation (2) in (1)
2a + b – 4 = 0

2a + 0 – 4 = 0

2a – 4 = 0
2a = 4

By dividing 4 by 2

a = $\frac{4}{2}$ = 2

Therefore, the values of ‘a’ and ‘b’ are 2 and 0 respectively.

EXERCISE 2(C) PAGE: 84

1. By actual division, find the quotient and the remainder when $(x^4 + 1)$ is divided by (x-1). Verify that remainder = f(1).
Solution:

We can write $(x^4 + 1)$ as $(x^4 + 0x^3 + 0x^2 + 0x + 1)$

$$
\begin{array}{r|l}
 & x^3 + x^2 + x + 1 \\
\hline
x - 1 & x^4 + 0x^3 + 0x^2 + 0x + 1 \\
 & x^4 - x^3 \\
\hline
 & x^3 + 0x^2 \\
 & x^3 - x^2 \\
\hline
 & x^2 + 0x \\
 & x^2 - x \\
\hline
 & x + 1 \\
 & x - 1 \\
\hline
 & 2 \\
\hline
\end{array}
$$

The quotient obtained from the division method is $x^3 + x^2 + x + 1$ and the remainder is 2.
By verification:
$f(x) = x^4 + 1$

By substituting 1 in the place of x

$f(1) = 1^4 + 1$

$f(1) = 1 + 1$

So we get

$f(1) = 2$ which is the remainder.

2. Verify the division algorithm for the polynomials
$p(x) = 2x^4 - 6x^3 + 2x^2 - x + 2$ and $g(x) = x+2$
Solution:

$$
\begin{array}{r|l}
 & 2x^3 - 10x^2 + 22x - 45 \\
\hline
x + 2 & 2x^4 - 6x^3 + 2x^2 - x + 2 \\
 & 2x^4 + 4x^3 \\
\hline
 & -10x^3 + 2x^2 \\
 & -10x^3 - 20x^2 \\
\hline
 & 22x^2 - x \\
 & 22x^2 + 44x \\
\hline
 & -45x + 2 \\
 & -45x - 90 \\
\hline
 & 92 \\
\hline
\end{array}
$$

We know that,

(x+2) $(2x^3 - 10x^2 + 22x - 45) + 92$

So we get,
= $2x^4 - 10x^3 + 22x^2 - 45x + 4x^3 - 20x^2 + 44x - 90 + 92$
= $2x^4 - 6x^3 + 2x^2 - x + 2$

= p(x)

Therefore, the division algorithm is verified.

Using the remainder theorem, find the remainder, when p(x) is divided by g(x), where
3. p(x) = $x^3 - 6x^2 + 9x + 3$, g(x) = x-1
Solution:

Given,
p(x) = $x^3 - 6x^2 + 9x + 3$

To find the value of x,
Consider,
g(x) = 0

x – 1 = 0

So we get

x = 1

According to the remainder theorem,
p(x) divided by (x-1) obtains the remainder as g(1).

Calculating g(1)

= $1^3 - 6(1)^2 + 9 \times 1 + 3$

On further simplification

= 1 – 6 + 9 + 3

So we get

= -5 + 12

= 7

Therefore, the remainder for the given expression is 7.

4. p(x) = $2x^3 - 7x^2 + 9x - 13$, g(x) = x-3
Solution:

Given,

p(x) = $2x^3 - 7x^2 + 9x - 13$

To find the value of x,
Consider,
g(x) = 0

x - 3 = 0

So we get

x = 3
According to the remainder theorem,
p(x) divided by (x-3) obtains the remainder as g(3).

Calculating g(3)

= $2(3)^3 - 7(3)^2 + 9 \times 3 - 13$

On further calculation

= 2(27) – 7(9) + 27 – 13

So we get

= 54 – 63 + 27 – 13

= 5

Therefore, the remainder for the given expression is 5.

5. p(x) = $3x^4 - 6x^2 - 8x - 2$, g(x) = x-2
Solution:

Given,
p(x) = $3x^4 - 6x^2 - 8x - 2$

To find the value of x,
Consider,
g(x) = 0

x - 2 = 0

So we get

x = 2

According to the remainder theorem,
p(x) divided by (x-2) obtains the remainder as g(2).

Calculating g(2)

= $3x^4 - 6x^2 - 8x - 2$

By substituting 2 in the place of x

= $3(2)^4 - 6(2)^2 - 8 \times 2 - 2$

So we get

= 3(16) – 6(4) – 16 – 2

= 48 – 24 – 16 – 2

= 6

Therefore, the remainder for the given expression is 6.

6. p(x) = $2x^3 - 9x^2 + x + 15$, g(x) = 2x -3
Solution:
Given,
p(x) = $2x^3 - 9x^2 + x + 15$
To find the value of x,
Consider,
g(x) = 0

2x - 3 = 0

So we get

2x = 3

By dividing

x = $\frac{3}{2}$

According to the remainder theorem,
p(x) divided by (2x - 3) obtains the remainder as g($\frac{3}{2}$).

Calculating g($\frac{3}{2}$)

= $2x^3 - 9x^2 + x + 15$

Substituting $\frac{3}{2}$ in the place of x

= $2(\frac{3}{2})^3 - 9(\frac{3}{2})^2 + \frac{3}{2} + 15$

= $2(\frac{27}{8}) - 9(\frac{9}{4}) + \frac{3}{2} + 15$

On further calculation

$= \frac{27}{4} - \frac{81}{4} + \frac{3}{2} + 15$

$= \frac{27-81+6+60}{4}$

By dividing we get

$= \frac{12}{4}$

= 3

Therefore, the remainder for the given expression is 3.

7. p(x) = $x^3 - 2x^2 - 8x - 1$, g(x) = x+1
Solution:

Given,
p(x) = $x^3 - 2x^2 - 8x - 1$

To find the value of x,
Consider,
g(x) = 0

x+1 = 0

So we get

x = -1

According to the remainder theorem,
p(x) divided by (x+1) obtains the remainder as g(-1).
Calculating g(-1)

$= x^3 - 2x^2 - 8x - 1$

By substituting the value of x as 1

$= (-1)^3 - 2(-1)^2 - 8(-1) - 1$

On further calculation

= -1 – 2 + 8 – 1

So we get

= - 3 + 7

= 4

Therefore, the remainder for the given expression is 4.

8. p(x) = $2x^3 + x^2 - 15x - 12$, g(x) = x+2
Solution:

Given,
p(x) = $2x^3 + x^2 - 15x - 12$

To find the value of x,
Consider,
g(x) = 0

x+2 = 0

x = -2

According to the remainder theorem,
p(x) divided by (x+2) obtains the remainder as g(-2).

Calculating g(-2)

$= 2x^3 + x^2 - 15x - 12$

Substituting the value -2 in the place of x

$= 2(-2)^3 + (-2)^2 - 15(-2) - 12$

So we get

= -16 + 4 + 30 – 12

On further calculation

= - 12 + 18

= 6

Therefore, the remainder for the given expression is 6.

9. p(x) = $6x^3 + 13x^2 + 3$, g(x) = 3x+2
Solution:

Given,
p(x) = $6x^3 + 13x^2 + 3$
To find the value of x,
Consider,
g(x) = 0

3x + 2 = 0

3x = -2

x = $\frac{-2}{3}$

According to the remainder theorem,
p(x) divided by (3x + 2) obtains the remainder as g($\frac{-2}{3}$).

Calculating g($\frac{-2}{3}$)

= $6x^3 + 13x^2 + 3$

By substituting the value $\frac{-2}{3}$ in the place of x

= $6(\frac{-2}{3})^3 + 13(\frac{-2}{3})^2 + 3$

So we get

= 6 ($\frac{-8}{27}$) + 13 ($\frac{4}{9}$) + 3

On further calculation

= $\frac{48}{27}$ + $\frac{52}{9}$ + 3

= $\frac{-16}{9}$ + $\frac{52}{9}$ + 3

= $\frac{-16+52+3}{9}$

By dividing

= $\frac{63}{9}$

We get

= 7

Therefore, the remainder for the given expression is 7.

10. p(x) = $x^3 - 6x^2 + 2x - 4$, g(x) = 1 - $\frac{3}{2}$x
Solution:

Given,
p(x) = $x^3 - 6x^2 + 2x - 4$

To find the value of x,
Consider,
g(x) = 0
$1 - \frac{3}{2}x = 0$

$\frac{3}{2}x = 1$

So we get

$x = \frac{2}{3}$

According to the remainder theorem,
p(x) divided by $(1 - \frac{3}{2}x)$ obtains the remainder as $g(\frac{2}{3})$.

Calculating $g(\frac{2}{3})$

$= x^3 - 6x^2 + 2x - 4$

Substituting the value $\frac{2}{3}$ in the place of x

$= (\frac{2}{3})^3 - 6\left(\frac{2}{3}\right)^2 + 2(\frac{2}{3}) - 4$

On further calculation we get

$= \frac{8}{27} - 6\,(\frac{4}{9}) + \frac{4}{3} - 4$

$= \frac{8}{27} - \frac{8}{3} + \frac{4}{3} - 4$

So we get

$= \frac{8-72+36-108}{27}$

$= \frac{-136}{27}$

Therefore, the remainder for the given expression is $\frac{-136}{27}$.

11. p(x) = $2x^3 + 3x^2 - 11x - 3$, g(x) = $(x+\frac{1}{2})$
Solution:

Given,
p(x) = $2x^3 + 3x^2 - 11x - 3$

To find the value of x,

Consider,
g(x) = 0

$x + \frac{1}{2} = 0$

So we get

$x = -\frac{1}{2}$

According to the remainder theorem,
p(x) divided by $(x + \frac{1}{2})$ obtains the remainder as $g(-\frac{1}{2})$.

Calculating $g(-\frac{1}{2})$

$= 2x^3 + 3x^2 - 11x - 3$
By substituting the value $-\frac{1}{2}$ in the place of x
$= 2(-\frac{1}{2})^3 + 3(-\frac{1}{2})^2 - 11(-\frac{1}{2}) - 3$

On further calculation we get

$= 2\,(-\frac{1}{8}) + 3\,(\frac{1}{4}) + \frac{11}{2} - 3$

So we get

$= -\frac{1}{4} + \frac{3}{4} + \frac{11}{2} - 3$

$= \frac{-1+3+22-12}{4}$

By dividing

$= \frac{12}{4}$

= 3

Therefore, the remainder for the given expression is 3.

12. p(x) = $x^3 - ax^2 + 6x - a$, g(x) = x-a
Solution:

Given,
$p(x) = x^3 - ax^2 + 6x - a$

To find the value of x,
Consider,
g(x) = 0

x – a = 0

So we get

x = a

According to the remainder theorem,
p(x) divided by (x - a) obtains the remainder as g(a).

Calculating g(a)

$= x^3 - ax^2 + 6x - a$

By substituting a in the place of x

$= a^3 - a(a)^2 + 6a - a$

So we get

$= a^3 - a^3 + 5a$

= 5a

Therefore, the remainder for the given expression is $5a$.

13. The polynomials $(2x^3 + x^2 - ax + 2)$ and $(2x^3 - 3x^2 - 3x + a)$ when divided by (x-2) leave the same remainder. Find the value of a.
Solution:

Consider $p(x) = (2x^3 + x^2 - ax + 2)$ and $q(x) = (2x^3 - 3x^2 - 3x + a)$
When p(x) and q(x) are divided by (x-2) the remainder obtained is p(2) and q(2).

To find a,
Let us take
p(2) = q(2)

$2x^3 + x^2 - ax + 2 = 2x^3 - 3x^2 - 3x + a$

By substituting 2 in the place of x

$2(2)^3 + (2)^2 - a(2) + 2 = 2(2)^3 - 3(2)^2 - 3(2) + a$

On further calculation

2(8) + 4 – 2a + 2 = 2(8) – 3(4) – 6 + a

16 + 4 – 2a + 2 = 16 – 12 – 6 + a

So we get

22 – 2a = -2 + a

22 + 2 = a + 2a

24 = 3a

By dividing

a = 8

Thus, the value of a is 8.

14. The polynomial p(x) = $x^4 - 2x^3 + 3x^2 - ax + b$ when divided by (x-1) and (x+1) leaves the remainders 5 and 19 respectively. Find the values of 'a' and 'b'. Hence find the remainder when p(x) is divided by (x-2).
Solution:

Given,
p(x) = $x^4 - 2x^3 + 3x^2 - ax + b$

Consider (x-1) = 0 where x = 1 and the remainder is 5

p(1) = 5

By substituting 1 in the place of x

$1^4 - 2(1)^3 + 3(1)^2 - a + b = 5$

So we get

1 – 2 + 3 – a + b = 5

2 – a + b = 5 ……. (1)

Consider (x+1) = 0 where x = -1 and the remainder is 19

p(-1) = 19

By substituting -1 in the place of x

$(-1)^4 - 2(-1)^3 + 3(-1)^2 - a + b = 19$

So we get

1 + 2 + 3 – a + b = 19

6 – a + b = 19 ……… (2)

By adding equation (1) and (2)
8 + 2b = 24

2b = 24 – 8

2b = 16

Dividing 16 by 2 we get

b = 8 ……… (3)

Now applying (3) in (1)
2 – a + 8 = 5

So we get

10 – a = 5

On subtraction

10 – 5 = a

a = 5

Substituting the value of a and b in p(x) when divided by (x-2)
p(x) = $x^4 - 2x^3 + 3x^2 - ax + b$

By substituting 2 in the place of x

p(2) = $2^4 - 2(2)^3 + 3(2)^2 - (5)(2) + 8$

On further calculation

p(2) = 16 – 16 + 12 – 10 + 8

p(2) = 10

Therefore, the remainder when p(x) is divided by (x-2) is 10.

15. If p(x) = $x^3 - 5x^2 + 4x - 3$ and g(x) = x-2, show that p(x) is not a multiple of g(x).
Solution:

Consider,
g(x) = 0

Which means
x – 2 = 0

x = 2
Now applying x=2 in p(x), we obtain
p(x) = $x^3 - 5x^2 + 4x - 3$

By substituting the value 2 in the place of x p(2) = $2^3 - 5(2)^2 + 4(2) - 3$

On further calculation

p(2) = 8 – 20 + 8 – 3

So we get

p(2) = - 4 – 3

p(2) = -7 ≠ 0

Therefore, it is proved that p(x) is not a multiple of g(x).

16. If p(x) = $2x^3 - 11x^2 - 4x + 5$ and g(x) = 2x+1, show that g(x) is not a factor of p(x).
Solution:

Consider,
g(x) = 0

2x + 1 = 0

So we get

2x = -1

x = $\frac{-1}{2}$

Now apply x = $\frac{-1}{2}$ in p(x)
p(x) = $2x^3 - 11x^2 - 4x + 5$

By substituting $\frac{-1}{2}$ in the place of x

$p(\frac{-1}{2}) = 2(\frac{-1}{2})^3 - 11\left(\frac{-1}{2}\right)^2 - 4(\frac{-1}{2}) + 5$

On further calculation

$p(\frac{-1}{2}) = 2\left(\frac{-1}{8}\right) - 11 \times \frac{1}{4} + 2 + 5$

So we get

$p(\frac{-1}{2}) = \frac{-1}{4} - \frac{11}{4} + 7$

$p(\frac{-1}{2}) = \frac{-1-11+28}{4}$

By dividing 16 by 4

$p(\frac{-1}{2}) = \frac{16}{4}$

So we get

$p(\frac{-1}{2}) = 4 \neq 0$

Hence, it is shown that g(x) is not a factor of p(x).

EXERCISE 2(D)

Using factor theorem, show that g(x) is a factor of p(x), when

1. p(x) = $x^3 - 8$, g(x) = x-2

Solution:

Given,
p(x) = $x^3 - 8$

Based on the factor theorem,
x-2 will be a factor of p(x) if p(2) = 0

So we get,
p(2) = $2^3 - 8$

On subtraction

p(2) = 8 – 8

p(2) = 0

Hence x-2 is a factor of $x^3 - 8$.

2. p(x) = $2x^3 + 7x^2 - 24x - 45$, g(x) = x-3

Solution:

Given,
p(x) = $2x^3 + 7x^2 - 24x - 45$

Based on the factor theorem,
x-3 will be a factor of p(x) if p(3) = 0

So we get,
p(3) = $2(3)^3 + 7(3)^2 - 24(3) - 45$

On further calculation

p(3) = 2(27) + 7(9) – 72 – 45

p(3) = 54 + 63 – 72 – 45

So we get

p(3) = 117 - 117

p(3) = 0

Hence x-3 is a factor of $2x^3 + 7x^2 - 24x - 45$.

3. p(x) = $2x^4 + 9x^3 + 6x^2 - 11x - 6$, g(x) = x-1

Solution:

Given,
p(x) = $2x^4 + 9x^3 + 6x^2 - 11x - 6$

Based on the factor theorem,
x-1 will be a factor of p(x) if p(1) = 0

So we get,
p(1) = $2(1)^4 + 9(1)^3 + 6(1)^2 - 11(1) - 6$

On further calculation

p(1) = 2(1) + 9(1) + 6(1) – 11 - 6

p(1) = 2 + 9 + 6 – 11 – 6

So we get

p(1) = 11 + 6 – 11 – 6

p(1) = 0

Hence x-1 is a factor of $2x^4 + 9x^3 + 6x^2 - 11x - 6$.

4. p(x) = $x^4 - x^2 - 12$, g(x) = x + 2
Solution:

Given,
p(x) = $x^4 - x^2 - 12$

Based on the factor theorem,
x + 2 will be a factor of p(x) if p(-2) = 0

So we get,
p(-2) = $(-2)^4 - (-2)^2 - 12$

On further calculation

p(-2) = 16 – 4 – 12

On subtraction

p(-2) = 16 - 16

p(-2) = 0

Hence x+2 is a factor of $x^4 - x^2 - 12$.

5. p(x) = 69 + 11x - $x^2 + x^3$, g(x) = x+3

Solution:

Given,
p(x) = 69 + 11x - x^2 + x^3

Based on the factor theorem,
x + 3 will be a factor of p(x) if p(-3) = 0

So we get,
p(-3) = 69 + 11(-3) - $(-3)^2$ + $(-3)^3$

On further calculation

p(-3) = 69 – 33 – 9 – 27

On subtraction

p(-3) = 69 - 69

p(-3) = 0
Hence x+3 is a factor of $69 + 11x - x^2 + x^3$.

6. p(x) = $2x^3 + 9x^2 - 11x - 30$, g(x) = x+5
Solution:

Given,
p(x) = $2x^3 + 9x^2 - 11x - 30$

Based on the factor theorem,
x + 5 will be a factor of p(x) if p(-5) = 0

So we get,
p(-5) = $2(-5)^3 + 9(-5)^2 - 11(-5) - 30$

On further calculation

p(-5) = 2(-125) + 9(25) + 55 – 30

p(-5) = - 250 + 225 + 55 – 30

On subtraction

p(-5) = -280 + 280

p(-5) = 0

Hence x + 5 is a factor of $2x^3 + 9x^2 - 11x - 30$.

7. p(x) = $2x^4 + x^3 - 8x^2 - x + 6$, g(x) = 2x – 3
Solution:

Given,
p(x) = $2x^4 + x^3 - 8x^2 - x + 6$

Based on the factor theorem,
2x - 3 will be a factor of p(x) if p($\frac{3}{2}$) = 0

So we get,
p($\frac{3}{2}$) = $2(\frac{3}{2})^4 + (\frac{3}{2})^3 - 8(\frac{3}{2})^2 - \frac{3}{2} + 6$

By further calculation

p($\frac{3}{2}$) = 2($\frac{81}{16}$) + $\frac{27}{8}$ + 8($\frac{9}{4}$) $-\frac{3}{2}$ + 6

p($\frac{3}{2}$) = $\frac{81}{8}$ + $\frac{27}{8}$ + 18 $-\frac{3}{2}$ + 6

So we get

p($\frac{3}{2}$) = $\frac{81+27-144-12+48}{8}$

By subtraction we get

p($\frac{3}{2}$) = $\frac{156-156}{8}$

p($\frac{3}{2}$) = 0

Hence 2x - 3 is a factor of $2x^4 + x^3 - 8x^2 - x + 6$.

8. p(x) = $3x^3 + x^2 - 20x + 12$, g(x) = 3x – 2
Solution:

Given,
p(x) = $3x^3 + x^2 - 20x + 12$

Based on the factor theorem,
3x – 2 will be a factor of p(x) if p($\frac{2}{3}$) = 0

So we get,
p($\frac{2}{3}$) = $3(\frac{2}{3})^3 + (\frac{2}{3})^2 - 20(\frac{2}{3}) + 12$

On further calculation

p($\frac{2}{3}$) = 3($\frac{8}{27}$) + ($\frac{4}{9}$) - $\frac{40}{3}$ + 12

So we get

$p(\frac{2}{3}) = \frac{8}{9} + \frac{4}{9} - \frac{40}{3} + 12$

$p(\frac{2}{3}) = \frac{8+4-120+108}{9}$

By subtraction we get

$p(\frac{2}{3}) = \frac{0}{9}$

$p(\frac{2}{3}) = 0$

Hence 3x – 2 is a factor of $3x^3 + x^2 - 20x + 12$.

9. p(x) = $7x^2 - 4\sqrt{2}x - 6$, g(x) = x - $\sqrt{2}$
Solution:

Given,
p(x) = $7x^2 - 4\sqrt{2}x - 6$

Based on the factor theorem,
x - $\sqrt{2}$ will be a factor of p(x) if p($\sqrt{2}$) = 0

So we get,
p($\sqrt{2}$) = $7(\sqrt{2})^2 - 4\sqrt{2}\sqrt{2} - 6$

On further calculation

p($\sqrt{2}$) = 7 (2) – 8 – 6

So we get

p($\sqrt{2}$) = $14 - 14$

p($\sqrt{2}$) = 0

Hence x - $\sqrt{2}$ is a factor of $7x^2 - 4\sqrt{2}x - 6$.

10. p(x) = $2\sqrt{2}x^2 + 5x + \sqrt{2}$, g(x) = x+$\sqrt{2}$
Solution:

Given,
p(x) = $2\sqrt{2}x^2 + 5x + \sqrt{2}$

Based on the factor theorem,
x + $\sqrt{2}$ will be a factor of p(x) if p($-\sqrt{2}$) = 0

So we get,

$p(-\sqrt{2}) = 2\sqrt{2}(-\sqrt{2})^2 + 5(-\sqrt{2}) + \sqrt{2}$

On further calculation

$p(-\sqrt{2}) = 2\sqrt{2}(2) - 5\sqrt{2} + \sqrt{2}$

So we get

$p(-\sqrt{2}) = 5\sqrt{2} - 5\sqrt{2}$

$p(-\sqrt{2}) = 0$

Hence x + $\sqrt{2}$ is a factor of $2\sqrt{2}x^2 + 5x + \sqrt{2}$.

11. Show that (p-1) is a factor of $(p^{10} - 1)$ and also of $(p^{11} - 1)$.
Solution:

Let us consider
q(p) = $(p^{10} - 1)$
f(p) = $(p^{11} - 1)$

Based on the factor theorem,
(p-1) will be a factor of q(p) and f(p) if q(1)and f(1)= 0

We know that,
q(p) = $(p^{10} - 1)$

where,
q(1)

By substituting 1 in the place of p

= $(1^{10} - 1)$

So we get

= 1-1

= 0

We know that,
f(p) = $(p^{11} - 1)$

where,
f(1)

By substituting 1 in the place of p

= $(1^{11} - 1)$

So we get

= 1 -1

= 0

Therefore, (p-1) is a factor of $(p^{10} - 1)$ and also of $(p^{11} - 1)$.

12. Find the value of k for which (x-1) is a factor of $(2x^3 + 9x^2 + x + k)$.
Solution:

Let us consider,
f(x) = $(2x^3 + 9x^2 + x + k)$

Given,
x – 1 = 0
x = 1

So we get,
f(1)

= $(2(1)^3 + 9(1)^2 + 1 + k)$

On further calculation

= 2 + 9 + 1 + k

= 12 + k

It is given that (x-1) is a factor of $(2x^3 + 9x^2 + x + k)$

Based on the factor theorem,
(x-1) is a factor of f(x) if f(1) = 0

We know that,
f(1) = 0

In order to find the value of k

12 + k = 0

k = -12

Hence, the value of k is -12.

13. Find the value of a for which (x-4) is a factor of $(2x^3 - 3x^2 - 18x + a)$.
Solution:

Let us consider,
f(x) = $(2x^3 - 3x^2 - 18x + a)$

Given,
x – 4 = 0
x = 4

So we get,
f(4)

= $(2(4)^3 - 3(4)^2 - 18(4) + a)$

On further calculation

= 2 (64) – 3 (16) – 18 (4) + a

In order to find the value of a

= 128 – 48 – 72 + a

= 8 + a

It is given that (x-4) is a factor of $(2x^3 - 3x^2 - 18x + a)$

Based on the factor theorem,
(x – 4) is a factor of f(x) if f(4) = 0

We know that,
f(4) = 0

So we get

8 + a = 0

a = -8

Hence, the value of 'a' is -8.

14. Find the value of a for which (x+1) is a factor of (a$x^3 + x^2 - 2x + 4a - 9$).
Solution:

Let us consider,
f(x) = (a$x^3 + x^2 - 2x + 4a - 9$)

Given,
x + 1 = 0
x = -1

So we get,
f(-1)

= (a$(-1)^3 + (-1)^2 - 2(-1) + 4a - 9$)

On further calculation

= - a + 1 + 2 + 4a – 9

So we get

= 3a - 6

It is given that (x+1) is a factor of (a$x^3 + x^2 - 2x + 4a - 9$)

Based on the factor theorem,
(x+1) is a factor of f(x) if f(-1) = 0

We know that,
f(-1) = 0

3a – 6 = 0

So we get

3a = 6

By division we get

a = 2

Hence, the value of ‘a’ is 2.

15. Find the value of a for which (x+2a) is a factor of ($x^5 - 4a^2x^3 + 2x + 2a + 3$).
Solution:

Let us consider,
f(x) = ($x^5 - 4a^2x^3 + 2x + 2a + 3$)

Given,
x +2a = 0

So we get

x = -2a

So we get,
f(-2a)

= $((-2a)^5 - 4a^2(-2a)^3 + 2(-2a) + 2a + 3)$

On further calculation

= -32a^5+ 32a^5- 4a + 2a + 3

= -2a+3

It is given that (x+2a) is a factor of $(x^5 - 4a^2x^3 + 2x + 2a + 3)$

Based on the factor theorem,
(x+2a) is a factor of f(x) if f(-2a) = 0

We know that,
f(-2a) = 0

-2a + 3= 0

So we get

2a = 3

On division

a = $\frac{3}{2}$

Hence, the value of 'a' is $\frac{3}{2}$.

16. Find the value of m for which (2x-1) is a factor of $(8x^4 + 4x^3 - 16x^2 + 10x + m)$.
Solution:

Let us consider,
f(x) = $(8x^4 + 4x^3 - 16x^2 + 10x + m)$
Given,
2x – 1 = 0

2x = 1

So we get

x = $\frac{1}{2}$

So we get,
f($\frac{1}{2}$)

= $(8(\frac{1}{2})^4 + 4(\frac{1}{2})^3 - 16\left(\frac{1}{2}\right)^2 + 10(\frac{1}{2}) + m)$

On further calculation

= 8 ($\frac{1}{16}$) + 4($\frac{1}{8}$) – 16($\frac{1}{4}$) + 5 + m

So we get

$= \frac{1}{2} + \frac{1}{2} - 4 + 5 + m$

By addition

= 1 + 1 + m

= 2 + m

It is given that (2x-1) is a factor of $(8x^4 + 4x^3 - 16x^2 + 10x + m)$.

Based on the factor theorem,
(2x - 1) is a factor of f(x) if $f(\frac{1}{2}) = 0$

We know that,
$f(\frac{1}{2}) = 0$

So we get

2 + m = 0

m = -2

Hence, the value of m is -2.

17. Find the value of 'a' for which the polynomial $(x^4 - x^3 - 11x^2 - x + a)$is divisible by (x+3).
Solution:

Let us consider,
f(x) = $(x^4 - x^3 - 11x^2 - x + a)$

Given,
x + 3 = 0
x = -3

So we get,
f(-3)

$= ((-3)^4 - (-3)^3 - 11(-3)^2 - (-3) + a)$

On further calculation

= 81 + 27 – 99 + 3 + a

So we get

= 111 – 99 + a

= 12 + a

It is given that $(x^4 - x^3 - 11x^2 - x + a)$is divisible by (x+3).

Based on the factor theorem,
(x+3) divides f(x) if f(-3) = 0

We know that,
f(-3) = 0

12 + a = 0

So we get

a = -12

Hence, the value of 'a' is -12.

18. Without actual division, show that $(x^3 - 3x^2 - 13x + 15)$is exactly divisible by (x^2+2x-3).
Solution:

Consider,
f(x) = $(x^3 - 3x^2 - 13x + 15)$

We can write x^2+2x-3 as

= x^2+3x- x – 3

By taking the common terms out

= x (x+3) – 1(x+3)

So we get

= (x+3) (x-1)

It is given that $(x^3 - 3x^2 - 13x + 15)$is exactly divisible by (x^2+2x-3).

Based on the factor theorem,
(x+3) and (x-1) divides f(x) if f(-3) and f(1) = 0

So we get,
f(-3)

= $((-3)^3 - 3(-3)^2 - 13(-3) + 15)$

On further calculation

= - 27 – 27 + 39 + 15

So we get

= - 54 + 54

= 0

f(1)
= $(1^3 - 3(1)^2 - 13(1) + 15)$

On further calculation

= 1 – 3 – 13 + 15

So we get

= 16 – 16

= 0

So we get, f(-3) and f(1) = 0

Hence, it is shown that $(x^3 - 3x^2 - 13x + 15)$is exactly divisible by (x^2+2x-3).

19. If $(x^3 + ax^2 + bx + 6)$ has (x-2) as a factor and leaves a remainder 3 when divided by (x-3), find the values of 'a' and 'b'.
Solution:

Consider
f(x) = $(x^3 + ax^2 + bx + 6)$

It is given that $(x^3 + ax^2 + bx + 6)$ when divided by (x-3) leaves a remainder f(3)
So we get,

f(3) = 3

$(3^3 + a(3)^2 + 3b + 6)$ = 3

So we get

27 + 9a + 3b + 6 = 3

On further calculation

9a + 3b + 33 = 3

By subtracting we get

9a + 3b = 3 – 33

9a + 3b = -30

3a + b = -10 ……. (1)

Based on the factor theorem,
(x-2) is a factor of f(x) if f(2) = 0

f(2) = 0

$2^3 + a(2)^2 + 2b + 6 = 0$

On further calculation

8 + 4a + 2b + 6 = 0

So we get

4a + 2b + 14 = 0

4a + 2b = -14

Dividing the equation by 2

2a + b = -7 ……. (2)
Now subtracting both the equations,

3a – 2a + b – b = - 10 + 7

So we get

a = - 3

Substitute a = -3 in (2) we get

2(-3) + b = -7

-6 + b = -7

By subtraction

b = -7 + 6

So we get

b = -1

Therefore, the values of 'a' and 'b' is -3 and -1.

20. Find the values of 'a' and 'b' so that the polynomial $(x^3 - 10x^2 + ax + b)$ is exactly divisible by (x-1) as well as (x-2).
Solution:

Consider,
f(x) = $(x^3 - 10x^2 + ax + b)$

Based on the factor theorem,
(x-1) and (x-2) is a factor of f(x) if f(1) and f(2) = 0

f(1) = 0

$(1^3 - 10(1)^2 + a + b) = 0$

On further calculation

1 – 10 + a + b = 0

So we get

a + b = 9 …… (1)

f(2) = 0

$(2^3 - 10(2)^2 + 2a + b) = 0$

On further calculation

8 – 40 + 2a + b = 0

So we get

2a + b = 32 …… (2)

Now subtracting both the equations,
a – 2a + b – b = 9 – 32

-a = -23
Dividing by -1 on both sides we get

a = 23

Substitute a = 23 in (2)
2(23) + b = 32

So we get

46 + b = 32

By subtraction

b = 32 – 46

b = -14

Therefore, the values of ‘a’ and ‘b’ are 23 and – 14.

21. Find the values of a and b so that the polynomial $(x^4 + ax^3 - 7x^2 - 8x + b)$ is exactly divisible by (x+2) as well as(x+3).
Solution:

Consider,
f(x) = $(x^4 + ax^3 - 7x^2 - 8x + b)$

Based on factor theorem,
(x+2) and (x+3) is a factor of f(x) if f(-2) and f(-3) = 0

f(-2) = 0

$((-2)^4 + a(-2)^3 - 7(-2)^2 - 8(-2) + b) = 0$

On further calculation

16 – 8a – 28 + 16 + b = 0

So we get

4 – 8a + b = 0

8a – b = 4 …… (1)

f(-3) = 0

$((-3)^4 + a(-3)^3 - 7(-3)^2 - 8(-3) + b) = 0$

On further calculation

81 – 27a – 63 + 24 + b = 0

So we get

42 – 27a + b = 0

27a – b = 42 ……. (2)

Now subtracting both the equations,
8a – 27a – b + b = 4 – 42

19a = 38

Dividing 38 by 19 we get

a = 2

Substitute a = 2 in (2)
27 (2) – b = 42

54 – b = 42

By subtraction we get

b = 54 – 42

b = 12

Therefore, the values of ‘a’ and ‘b’ are 2 and 12.

22. If both (x-2) and (x - $\frac{1}{2}$) are factors of px^2+ 5x + r, prove that p = r.
Solution:

Consider,
f(x) = px^2+ 5x + r

Based on factor theorem,
(x-2) and (x - $\frac{1}{2}$) are factors of f(x) if f(2) and f($\frac{1}{2}$) = 0

f(2) = 0

p$(2)^2$+ 5(2) + r = 0

On further calculation

4p + 10 + r = 0

So we get

4p + r = -10 …… (1)

f($\frac{1}{2}$) = 0

p$(\frac{1}{2})^2$+ 5($\frac{1}{2}$) + r = 0

On further calculation

$\frac{p}{4}+\frac{5}{2}$ + r = 0

So we get

$\frac{p+10+4r}{4}$ = 0

Multiplying 4 on both sides

p + 10 + 4r = 0

So we get

p + 4r = -10 ……. (2)

We have to prove that p = r

So we get,

4p + r = p + 4r

Rearranging the terms

4p – p = 4r – r

3p = 3r

Dividing both the sides by 3

p = r

Hence, it is proved that p = r.

23. Without actual division, prove that $2x^4 - 5x^3 + 2x^2 - x + 2$ is divisible by x^2- 3x + 2.
Solution:

Consider,
f(x) = $2x^4 - 5x^3 + 2x^2 - x + 2$

We can write x^2- 3x + 2 as

= x^2- 2x – x + 2

Taking out the common terms

= x (x-2) – 1(x-2)

So we get

= (x-2) (x-1)

Based on the factor theorem,
(x-2) and (x-1) are the factors of f(x) if f(2) and f(1) = 0

f(2)

= $2(2)^4 - 5(2)^3 + 2(2)^2 - 2 + 2$

On further calculation
= 2(16) – 5(8) + 2(4) – 2 + 2

So we get

= 32 – 40 + 8 – 2 + 2

= - 8 + 8 – 2 + 2

= 0

f(1)

= $2(1)^4 - 5(1)^3 + 2(1)^2 - 1 + 2$

On further calculation

= 2 – 5 + 2 – 1 + 2

So we get

= - 3 + 2 – 1 + 2

= - 3 + 3

= 0

Therefore, $2x^4 - 5x^3 + 2x^2 - x + 2$ is divisible by x^2- 3x + 2.

24. What must be added to $2x^4 - 5x^3 + 2x^2 - x - 3$ so that the result is exactly divisible by (x-2)?
Solution:

Consider,
f(x) = $2x^4 - 5x^3 + 2x^2 - x - 3$
g(x) = x-2

Let us add a to f(x)
So it becomes
f(x) = $2x^4 - 5x^3 + 2x^2 - x - 3$ + a

Let g(x) = 0
x – 2 = 0
x = 2

In order to find the value of a
Consider f(2) = 0

$2x^4 - 5x^3 + 2x^2 - x - 3$ + a = 0

By substituting 2 in the place of x
$2(2)^4 - 5(2)^3 + 2(2)^2 - 2 - 3$ + a = 0

On further calculation

2(16) – 5(8) + 2(4) – 2 – 3 + a = 0

So we get

32 – 40 + 8 – 2 – 3 + a = 0

-8 + 8 – 5 + a = 0

-5 + a = 0

We get,

a = 5

Therefore, the value of the number which should be added is 5.

25. What must be subtracted from $(x^4 + 2x^3 - 2x^2 + 4x + 6)$ so that the result is exactly divisible by $(x^2$+2x – 3)?
Solution:

Consider,
p(x) = $x^4 + 2x^3 - 2x^2 + 4x + 6$
q(x) = x^2+2x – 3

Assume r(x) = ax + b

Let p(x) be subtracted from r(x) and divided by q(x)

f(x)

= p(x) – r(x)

By substituting (ax + b) in the place of r we get

= p(x) – (ax + b)

On further calculation

= $(x^4 + 2x^3 - 2x^2 + 4x + 6)$ – (ax + b)

= $x^4 + 2x^3 - 2x^2 + (4 - a)x + 6$ - b

q(x)

= x^2+2x – 3

We can further write it as

= x^2+ 3x - x – 3

By taking the common terms out

= x(x+3) – 1(x+3)

So we get

= (x-1) (x+3)

(x-1) and (x+3) are the factors of f(x) if f(1) and f(-3) = 0

f(1) = 0

By substituting 1 in the place of x

$1^4 + 2(1)^3 - 2(1)^2 + (4 - a)(1) + 6$ - b= 0

On further calculation

1 + 2 – 2 + 4 – a + 6 – b = 0

So we get

11 – a – b = 0

-a - b = -11 ……. (1)

f(-3) = 0

By substituting -3 in the place of x

$(-3)^4 + 2(-3)^3 - 2(-3)^2 + (4 - a)(-3) + 6$ – b = 0

So we get

81 – 54 – 18 – 12 + 3a + 6 – b = 0

3 + 3a – b = 0

3a – b = -3 …….. (2)

By subtracting both the equations (1) and (2)
-a – 3a – b + b = -11 + 3

-4a = - 8

Dividing both sides by -4

a = 2

Substitute a = 2 in (2)

3(2) – b = -3

6 – b = -3

On further calculation

b = 6 + 3

So we get

b = 9

Substituting a and b values in r(x) = ax + b
r(x) = 2x + 9

Therefore, $(x^4 + 2x^3 - 2x^2 + 4x + 6)$ is divisible by x^2+2x – 3 if 2x + 9 is subtracting from it.

26. Use factor theorem to prove that (x + a) is a factor of $(x^n + a^n)$ for any odd positive integer n.
Solution:

Consider,
f(x) = $(x^n + a^n)$

Based on the factor theorem,
(x + a) is a factor of $(x^n + a^n)$ if f(-a) = 0

So we get,
f(-a)

= $(x^n + a^n)$

By substituting –a in the place of x

= $(-a^n + a^n)$

So we get

= $(-1)^n\ a^n + a^n$

= $[(-1)^n + 1^n]\ a^n$

We know that n is an odd integer
So we get, $(-1)^n$ = -1

= [-1+1] a^n

On subtraction

$= 0 \times a^n$

We get

= 0

Therefore, (x + a) is a factor of $(x^n + a^n)$for any odd positive integer n.