

EXERCISE 15(A)

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- 1. Find the volume, the lateral surface area and the total surface area of the cuboid whose dimensions are:
- (i) length = 12cm, breadth = 8cm and height = 4.5cm
- (ii) length = 26m, breadth = 14m and height = 6.5m
- (iii) length = 15m, breadth = 6m and height = 5dm
- (iv) length = 24m, breadth = 25cm and height = 6m.

Solution:

(i) It is given that length = 12cm, breadth = 8cm and height = 4.5cm

We know that

Volume of cuboid = $1 \times b \times h$

By substituting the values we get

Volume of cuboid = $12 \times 8 \times 4.5$

By multiplication

Volume of cuboid = 432 cm^3

We know that

Lateral surface area of a cuboid = $2(1 + b) \times h$

By substituting the values

Lateral surface area of a cuboid = $2(12 + 8) \times 4.5$

On further calculation

Lateral surface area of a cuboid = $2 \times 20 \times 4.5$

By multiplication

Lateral surface area of a cuboid = 180cm²

We know that

Total surface area of cuboid = 2 (lb + bh + lh)

By substituting the values

Total surface area of cuboid = $2(12 \times 8 + 8 \times 4.5 + 12 \times 4.5)$

On further calculation

Total surface area of cuboid = 2(96 + 36 + 54)

So we get

Total surface area of cuboid = 2×186

By multiplication

Total surface area of cuboid = 372 cm²

(ii) It is given that length = 26m, breadth = 14m and height = 6.5m

We know that

Volume of cuboid = $1 \times b \times h$

By substituting the values we get

Volume of cuboid = $26 \times 14 \times 6.5$

By multiplication

Volume of cuboid = 2366 m^3

We know that

Lateral surface area of a cuboid = $2(1 + b) \times h$

By substituting the values

Lateral surface area of a cuboid = $2(26 + 14) \times 6.5$

On further calculation



Lateral surface area of a cuboid = $2 \times 40 \times 6.5$

By multiplication

Lateral surface area of a cuboid = 520cm²

We know that

Total surface area of cuboid = 2 (lb + bh + lh)

By substituting the values

Total surface area of cuboid = $2(26 \times 14 + 14 \times 6.5 + 26 \times 6.5)$

On further calculation

Total surface area of cuboid = 2(364 + 91 + 169)

So we get

Total surface area of cuboid = 2×624

By multiplication

Total surface area of cuboid = 1248 m^2

(iii) It is given that length = 15m, breadth = 6m and height = 5dm = 0.5m

We know that

Volume of cuboid = $1 \times b \times h$

By substituting the values we get

Volume of cuboid = $15 \times 6 \times 0.5$

By multiplication

Volume of cuboid = 45 m^3

We know that

Lateral surface area of a cuboid = $2(1 + b) \times h$

By substituting the values

Lateral surface area of a cuboid = $2(15 + 6) \times 0.5$

On further calculation

Lateral surface area of a cuboid = $2 \times 21 \times 0.5$

By multiplication

Lateral surface area of a cuboid = 21 m^2

We know that

Total surface area of cuboid = 2 (lb + bh + lh)

By substituting the values

Total surface area of cuboid = $2(15 \times 6 + 6 \times 0.5 + 15 \times 0.5)$

On further calculation

Total surface area of cuboid = 2(90 + 3 + 7.5)

So we get

Total surface area of cuboid = 2×100.5

By multiplication

Total surface area of cuboid = 201 m^2

(iv) It is given that length = 24m, breadth = 25cm = 0.25m and height = 6m

We know that

Volume of cuboid = $1 \times b \times h$

By substituting the values we get

Volume of cuboid = $24 \times 0.25 \times 6$

By multiplication

Volume of cuboid = 36 m^3



We know that

Lateral surface area of a cuboid = $2(1 + b) \times h$

By substituting the values

Lateral surface area of a cuboid = $2(24 + 0.25) \times 6$

On further calculation

Lateral surface area of a cuboid = $2 \times 24.25 \times 6$

By multiplication

Lateral surface area of a cuboid = 291 m^2

We know that

Total surface area of cuboid = 2 (lb + bh + lh)

By substituting the values

Total surface area of cuboid = $2(24 \times 0.25 + 0.25 \times 6 + 24 \times 6)$

On further calculation

Total surface area of cuboid = 2(6 + 1.5 + 144)

So we get

Total surface area of cuboid = 2×151.5

By multiplication

Total surface area of cuboid = 303 m^2

2. A matchbox measure 4cm × 2.5cm × 1.5 cm. What is the volume of a packet containing 12 such matchboxes?

Solution:

It is given that

Length of the matchbox = 4cm

Breadth of the matchbox = 2.5cm

Height of the matchbox = 1.5cm

We know that

Volume of one matchbox = volume of cuboid = $1 \times b \times h$

By substituting the values

Volume of one matchbox = $4 \times 2.5 \times 1.5$

By multiplication

Volume of one matchbox = 15 cm^3

So the volume of 12 such matchboxes = $12 \times 15 = 180 \text{ cm}^3$

Therefore, the volume of a packet containing 12 such matchboxes is 180 cm³.

3. A cuboidal water tank is 6m long, 5m wide and 4.5m deep. How many litres of water can it hold? (Given, $1m^3 = 1000$ litres.)

Solution:

It is given that

Length of the cuboidal water tank = 6m

Breadth of the cuboidal water tank = 5m

Height of the cuboidal water tank = 4.5m

We know that



Volume of a cuboidal water tank = $1 \times b \times h$ By substituting the values Volume of a cuboidal water tank = $6 \times 5 \times 4.5$ By multiplication Volume of a cuboidal water tank = 135 m^3 We know that $1\text{m}^3 = 1000$ litres So we get Volume of a cuboidal water tank = $135 \times 1000 = 135000$ litres

Therefore, the cuboidal water tank can hold 135000 litres of water.

4. The capacity of a cuboidal tank is 50000 litres of water. Find the breadth of the tank if its length and depth are respectively 10m and 2.5m. (Given, 1000 litres = 1m³.) Solution:

It is given that Length of the cuboidal tank = 10mDepth of the cuboidal tank = 2.5mVolume of the cuboidal tank = 5000 litres = 50 m^3

We know that Volume of a cuboidal tank = $1 \times b \times h$ By substituting the values $50 = 10 \times b \times 2.5$ On further calculation b = 2m

Therefore, the breadth of the cuboidal tank is 2m.

5. A godown measure 40m × 25m × 15m. Find the maximum number of wooden crates, each measuring 1.5m × 1.25m × 0.5m that can be stored in the godown. Solution:

It is given that Length of the godown = 40m Breadth of the godown = 25m Height of the godown = 15m

We know that Volume of godown = $1 \times b \times h$ By substituting the values Volume of godown = $40 \times 25 \times 15$ On further calculation Volume of godown = 15000 m^3

It is given that Length of wooden crate = 1.5m Breadth of wooden crate = 1.25m Height of wooden crate = 0.5m



We know that

Volume of each wooden crate = $1 \times b \times h$

By substituting the values

Volume of each wooden crate = $1.5 \times 1.25 \times 0.5$

On further calculation

Volume of each wooden crate = 0.9375 m^3

So we get

Number of wooden crates that can be stored in godown = Volume of godown/ Volume of each wooden crate

By substituting the values

Number of wooden crates that can be stored in godown = 15000/0.9375

So we get

Number of wooden crates that can be stored in godown = 16000

Therefore, the number of wooden crates that can be stored in the godown are 16000.

6. How many planks of dimensions (5m × 25cm × 10cm) can be stored in a pit which is 20m long, 6m wide and 80cm deep?

Solution:

The dimensions of the plank are Length = 5m = 500cmBreadth = 25cmHeight = 10cm

We know that

Volume of the plank = $1 \times b \times h$

By substituting the values

Volume of the plank = $500 \times 25 \times 10$

So we get

Volume of the plank = 125000 cm^3

The dimensions of the pit are

Length = 20m = 2000 cm

Breadth = 6m = 600 cm

Height = 80cm

We know that

Volume of one pit = $1 \times b \times h$

By substituting the values

Volume of one pit = $2000 \times 600 \times 80$

So we get

Volume of one pit = 96000000cm³

So the number of planks that can be stored = Volume of one pit/ Volume of plank

By substituting the values

Number of planks that can be stored = 96000000/125000

So we get

Number of planks that can be stored = 768



Therefore, the number of planks that can be stored is 768.

7. How many bricks will be required to construct a wall 8m long, 6m high and 22.5cm thick if each brick measures (25cm × 11.25cm × 6cm)? Solution:

The dimensions of the wall are Length = 8m = 800 cm Breadth = 6m = 600 cm Height = 22.5 cm

We know that Volume of wall = $1 \times b \times h$ By substituting the values Volume of wall = $800 \times 600 \times 22.5$ By multiplication Volume of wall = 108000000 cm^3

The dimensions of brick are Length = 25cm Breadth = 11.25cm Height = 6cm

We know that Volume of brick = $1 \times b \times h$ By substituting the values Volume of brick = $25 \times 11.25 \times 6$ By multiplication Volume of brick = 1687.5 cm^3

So the number of bricks required = Volume of wall/ Volume of brick By substituting the values Number of bricks required = 10800000/1687.5 By division Number of bricks required = 6400

Therefore, the number of bricks required to construct a wall is 6400.

8. Find the capacity of a closed rectangular cistern whose length is 8m, breadth 6m and depth 2.5m. Also, find the area of the iron sheet required to make the cistern.

Solution:

The dimensions of the cistern are Length = 8m Breadth = 6m Height = 2.5m

We know that Capacity of cistern = volume of cistern Volume of cistern = $1 \times b \times h$



By substituting the values Volume of cistern = $8 \times 6 \times 2.5$ By multiplication Volume of cistern = 120m^3

We know that the area of iron sheet required is equal to the total surface area of the cistern

So we get

Total surface area = 2 (lb + bh + lh)

By substituting the values

Total surface area = $2(8 \times 6 + 6 \times 2.5 + 2.5 \times 8)$

On further calculation

Total surface area = 2(48 + 15 + 20)

So we get

Total surface area = $2 \times 83 = 166 \text{ m}^2$

Therefore, the capacity of the cistern is 120 m³ and the area of the iron sheet required to make the cistern is 166m².

9. The dimensions of a room are (9m × 8m × 6.5m). It has one door of dimensions (2m × 1.5m) and two windows, each of dimensions (1.5m × 1m). Find the cost of whitewashing the walls at ₹ 25 per square metre. Solution:

The dimensions of the room is

Length = 9m

Breadth = 8m

Height = 6.5m

We know that

Area of four walls of the room = $2(1 + b) \times h$

By substituting the values

Area of the four walls of the room = $2(9+8) \times 6.5$

On further calculation

Area of the four walls of the room = 34×6.5

So we get

Area of the four walls of the room = 221 m^2

The dimensions of the door are

Length = 2m

Breadth = 1.5 m

We know that

Area of one door = $1 \times b$

By substituting the values

Area of one door = 2×1.5

So we get

Area of one door = $3m^2$

The dimensions of the window are

Length = 1.5m

Breadth = 1m



We know that

Area of two windows = $2(1 \times b)$

By substituting the values

Area of two windows = $2(1.5 \times 1)$

On further calculation

Area of two windows = $2 \times 1.5 = 3\text{m}^2$

So the area to be whitewashed = Area of four walls of the room - Area of one door - Area of two windows

By substituting the values

Area to be whitewashed = (221 - 3 - 3)

So we get

Area to be whitewashed = $215m^2$

It is given that the cost of whitewashing = ₹ 25 per square metre

So the cost of whitewashing $215m^2 = ₹ (25 × 215)$

Cost of whitewashing 215m² = ₹ 5375

Therefore, the cost of whitewashing 215m² is ₹ 5375.

10. A wall 15m long, 30cm wide and 4m high is made of bricks, each measuring (22cm × 12.5cm × 7.5cm). If 1/12 of the total volume of the wall consists of mortar, how many bricks are there in the wall? Solution:

The dimensions of the wall are

Length = 15m

Breadth = 0.3 m

Height = 4m

We know that

Volume of the wall = $1 \times b \times h$

By substituting the values

Volume of the wall = $15 \times 0.3 \times 4$

So we get

Volume of the wall = 18 m^3

It is given that the wall consists of 1/12 mortar

So we get

Volume of mortar = $1/12 \times 18$

By division

Volume of mortar = 1.5 m^3

So the volume of wall = Volume of wall – Volume of mortar

By substituting the values

Volume of wall = 18 - 1.5

By subtraction

Volume of wall = 16.5 m^3

The dimensions of the brick are

Length = 22cm = 0.22 m

Breadth = 12.5 cm = 0.125 m



Height = 7.5cm = 0.075 m

We know that Volume of one brick = $1 \times b \times h$ By substituting the values Volume of one brick = $0.22 \times 0.125 \times 0.075$ So we get Volume of one brick = 0.0020625 m^3

So the number of bricks = Volume of bricks/ Volume of one brick By substituting the values Number of bricks = 16.5/0.0020625 So we get Number of bricks = 8000

Therefore, the number of bricks in the wall are 8000.

11. How many cubic centimetres of iron are there in an open box whose external dimensions are 36cm, 25cm and 16.5cm, the iron being 1.5cm thick throughout? If 1cm³ of iron weighs 15g, find the weight of the empty box in kilograms.

Solution:

The external dimensions of the box are Length = 36cm Breadth = 25cm Height = 16.5cm

We know that External volume of the box = $1 \times b \times h$ By substituting the values External volume of the box = $36 \times 25 \times 16.5$ So we get External volume of the box = 14850 cm^3

It is given that the box is 1.5cm thick throughout Internal length of the box = $(36 - (1.5 \times 2))$ So we get Internal length of the box = 33 cm

Internal breadth of the box = $(25 - (1.5 \times 2))$ So we get

Internal breadth of the box = 22cm

Internal height of the box = (16.5 - 1.5)So we get Internal height of the box = 15cm

We know that Internal Volume of the box = $1 \times b \times h$ By substituting the values



Internal Volume of the box = $33 \times 22 \times 15$ By multiplication Internal volume of the box = 10890 cm^3

So the volume of iron used in the box = External volume of box - internal volume of box

By substituting the values

Volume of iron used in the box = $14850 - 10890 = 3960 \text{ cm}^3$

It is given that

Weight of 1cm^3 of iron = 15g = 15/1000 kg

So the weight of 3960 cm³ of iron = $3960 \times (15/1000)$

We get

Weight of $3960 \text{ cm}^3 \text{ of iron} = 59.4 \text{ kg}$

Therefore, the volume of iron used in the box is 3960 cm³ and the weight of the empty box is 59.4 kg.

12. A box made of sheet metal costs ₹ 6480 at ₹ 120 per square metre. If the box is 5m long and 3m wide, find its height.

Solution:

We know that

Area of sheet metal = Total cost/ Cost per m²

By substituting the values

Area of sheet metal = 6480/120

So we get

Area of sheet metal = 54 m^2

So we get

Area of sheet metal = 2 (lb + bh + hl)

By substituting the values

$$54 = 2 (5 \times 3 + 3 \times h + h \times 5)$$

On further calculation

27 = 15 + 3h + 5h

So we get

8h = 12

By division

h = 1.5 m

Therefore, the height of sheet metal is 1.5m.

13. The volume of a cuboid is 1536m³. Its length is 16m, and its breadth and height are in the ratio 3:2. Find the breadth and height of the cuboid.

Solution:

It is given that

Volume of cuboid = 1536 m³

Length of cuboid = 16m

Consider breath as 3x and height as 2x

We know that



Volume of cuboid = $1 \times b \times h$ By substituting the values $1536 = 16 \times 3x \times 2x$ On further calculation $1536 = 96 \times 2$ So we get $x^2 = 1536/96$ $x^2 = 16$ By taking the square root $x = \sqrt{16}$ We get x = 4m

Substituting the value of x Breadth of cuboid = 3x = 3(4) = 12mHeight of cuboid = 2x = 2(4) = 8m

Therefore, the breadth and height of the cuboid are 12m and 8m.

14. How many persons can be accommodated in a dining hall of dimensions (20m × 16m × 4.5m), assuming that each person requires 5 cubic metres of air? Solution:

The dimensions of hall are Length = 20m Breadth = 16m Height = 4.5m

We know that Volume of hall = $1 \times b \times h$ By substituting the values Volume of hall = $20 \times 16 \times 4.5$ So we get Volume of hall = 1440 m^3

It is given that volume of air for each person = 5 cubic metres

So the number of persons = volume of hall/volume of air needed per person By substituting the values
Number of persons = 1440/5
So we get
Number of persons = 288

15. A classroom is 10m long, 6.4m wide and 5m high. If each student be given 1.6m² of the floor area, how many students can be accommodated in the room? How many cubic metres of air would each student get? Solution:

The dimensions of classroom are Length = 10mBreadth = 6.4m



Height = 5m

It is given that the floor area for each student = $1.6m^2$

So the number of students = area of room/floor area for each student By substituting the values Number of students = $(10 \times 6.4)/1.6$ So we get

Number of students = 40

So the air required by each student = Volume of room/ number of students

We know that Volume of room = $1 \times b \times h$

By substituting the values

Volume of room = $10 \times 6.4 \times 5 = 320 \text{ m}^3$

So we get

Air required by each student = $320/40 = 8m^3$

Therefore, the number of students that can be accommodated in the room is 40 and the air required by each student is 8m³.

16. The surface area of a cuboid is 758cm². Its length and breadth are 14cm and 11cm respectively. Find its height.

Solution:

It is given that

Surface area of cuboid = 758 cm^2

The dimensions of cuboid are

Length = 14cm

Breadth = 11cm

Consider h as the height of cuboid

We know that

Surface area of cuboid = 2 (lb + bh + lh)

By substituting the values

 $758 = 2 (14 \times 11 + 11 \times h + 14 \times h)$

On further calculation

758 = 2 (154 + 11h + 14h)

So we get

758 = 2 (154 + 25h)

By multiplication

758 = 308 + 50h

It can be written as

50h = 758 - 308

By subtraction

50h = 450

By division

h = 9cm



Therefore, the height of cuboid is 9cm.

17. In a shower, 5cm of rain falls. Find the volume of water that falls on 2 hectares of ground. Solution:

We know that 1 hectare = 10000 m^2

So we get

2 hectares = $2 \times 10000 = 20000 \text{ m}^2$

It is given that

Depth of ground = 5 cm = 0.05 m

We know that

Volume of water = area \times depth

By substituting the values

Volume of water = $20000 \times 0.05 = 1000 \text{ m}^3$

Therefore, the volume of water that falls is 1000 m³.

18. Find the volume, the lateral surface area, the total surface area and the diagonal of a cube, each of whose edges measures 9m. (Take $\sqrt{3} = 1.73$) Solution:

It is given that each edge of a cube = 9m

We know that

Volume of cube = a^3

By substituting the values

Volume of cube = 9^3

So we get

Volume of cube = 729 m^3

We know that

Lateral surface area of cube = $4a^2$

By substituting the values

Lateral surface area of cube = 4×9^2

So we get

Lateral surface area of cube = $4 \times 81 = 324 \text{ m}^2$

We know that

Total surface area of cube = $6a^2$

By substituting the values

Total surface area of cube = 6×9^2

So we get

Total surface area of cube = $6 \times 81 = 486 \text{ m}^2$

We know that

Diagonal of cube = $\sqrt{3}$ a

By substituting the values

Diagonal of cube = $1.73 \times 9 = 15.57 \text{ m}$



Therefore, the volume is 729 m³, lateral surface area is 324 m², total surface area is 486 m² and the diagonal of cube is 15.57 m.

19. The total surface area of a cube is 1176 cm². Find its volume. Solution:

Consider a cm as each side of the cube We know that Total surface area of the cube = $6a^2$ By substituting the values $6a^2 = 1176$ On further calculation $a^2 = 1176/6$ So we get $a^2 = 196$ By taking square root $a = \sqrt{196} = 14$ cm

We know that Volume of cube = a^3 By substituting the values Volume of cube = 14^3 So we get Volume of cube = 2744 cm³

Therefore, the volume of cube is 2744 cm³.

20. The lateral surface area of a cube is 900 cm². Find its volume. Solution:

Consider a cm as each side of the cube We know that
Lateral surface area of cube = $4a^2$ By substituting the values $4a^2 = 900$ On further calculation $a^2 = 900/4$ By division $a^2 = 225$ By taking square root $a = \sqrt{225} = 15$ cm

We know that Volume of cube = a^3 By substituting the values Volume of cube = 15^3 So we get Volume of cube = 3375 cm^3

Therefore, the volume of cube is 3375 cm³.



21. The volume of a cube is 512cm³. Find its surface area. Solution:

It is given that Volume of a cube = 512 cm^3 We know that Volume of cube = a^3

So we get Each edge of the cube = $\sqrt[3]{512} = 8$ cm

We know that Surface area of cube = $6a^2$ By substituting the values Surface area of cube = $6 \times (8)^2$ So we get Surface area of cube = $6 \times 64 = 384$ cm²

Therefore, the surface area of cube is 384 cm².

22. Three cubes of metal with edges 3cm, 4cm and 5cm respectively are melted to form a single cube. Find the lateral surface area of the new cube formed. Solution:

We know that

Volume of new cube = $(3^3 + 4^3 + 5^3)$

So we get

Volume of new cube = $27 + 64 + 125 = 216 \text{ cm}^2$

Consider a cm as the edge of the cube

So we get

 $a^3 = 216$

By taking cube root

 $a = \sqrt[3]{216} = 6cm$

We know that

Lateral surface area of the new cube = $4a^2$

By substituting the values

Lateral surface area of the new cube = $4 \times (6)^2$

So we get

Lateral surface area of the new cube = $4 \times 36 = 144 \text{ cm}^2$

Therefore, the lateral surface area of the new cube formed is 144 cm².

23. Find the length of the longest pole that can be put in a room of dimensions ($10m \times 10m \times 5m$). Solution:

The dimensions of the room are

Length = 10m

Breadth = 10m



Height = 5m

We know that

Length of the longest pole = length of diagonal = $\sqrt{(1^2 + b^2 + h^2)}$

By substituting the values

Length of the longest pole = $\sqrt{(10^2 + 10^2 + 5^2)}$

So we get

Length of the longest pole = $\sqrt{(100 + 100 + 25)}$

By addition

Length of the longest pole = $\sqrt{225} = 15$ m

Therefore, the length of the longest pole that can be put in the room is 15m.

24. The sum of length, breadth and depth of a cuboid is 19cm and the length of its diagonal is 11cm. Find the surface area of the cuboid.

Solution:

It is given that

$$1 + b + h = 19$$
cm(1)

Diagonal =
$$\sqrt{(1^2 + b^2 + h^2)} = 11 \text{cm}.....(2)$$

Squaring on both sides of equation (1)

$$(1 + b + h)^2 = 19^2$$

So we get

$$(1^2 + b^2 + h^2) + 2 (1b + bh + hl) = 361 \dots (3)$$

Squaring on both sides of equation (2)

$$1^2 + b^2 + h^2 = 11^2 = 121$$
 (4)

Substituting equation (4) in (3)

$$121 + 2 (lb + bh + hl) = 361$$

So we get

$$2 (lb + bh + hl) = 361 - 121$$

By subtraction

$$2 (lb + bh + hl) = 240 cm^2$$

Therefore, the surface area of the cuboid = 240 cm^2 .

25. Each edge of a cube is increased by 50%. Find the percentage increase in the surface area of the cube. Solution:

Consider a cm as the edge of the cube

We know that

Surface area of cube = $6 a^2$

So we get

New edge = a + 50% of a

It can be written as

New edge = a + 50/100 a

By LCM



New edge = 150/100 a

We get

New edge = 3/2 a cm

So the new surface area = $6(3/2 \text{ a})^2$

We ge

New surface area = $6 \times 9/4$ a² = 27/2 a² cm²

Increased surface area = new surface area – surface area

So we get

Increased surface area = 27/2 a² - $6a^2$ = 15/2 a² cm²

So the percentage increase in surface area = (increased surface area/original surface area) × 100

By substituting the values

Percentage increase in surface area = $(15/2 \text{ a}^2/6\text{a}^2) \times 100$

It can be written as

Percentage increase in surface area = $15/2 \text{ a}^2 \times 1/6 \text{a}^2 \times 100$

So we get

Percentage increase in surface area = 125%

Therefore, the percentage increase in the surface area of the cube is 125%.

26. If V is the volume of a cuboid of dimensions a, b, c and S is its surface area then prove that 1/V = 2/S (1/a + 1/b + 1/c).

Solution:

We know that

Volume of a cuboid = $a \times b \times c$

Surface area of cuboid = 2 (ab + bc + ac)

So we get

2/s (1/a + 1/b + 1/c) = 2/s ((bc + ac + ab)/abc)

It can be written as

2/s (1/a + 1/b + 1/c) = 2/s (s/2V)

On further calculation

2/s (1/a + 1/b + 1/c) = 1/V

We get

1/V = 2/S (1/a + 1/b + 1/c)

Therefore, it is proved that 1/V = 2/S (1/a + 1/b + 1/c).

27. Water in a canal, 30dm wide and 12dm deep, is flowing with a velocity of 20km per hour. How much area will it irrigate, if 9cm of standing water is desired? Solution:

We know that water in a canal forms a cuboid

The dimensions are

Breadth = 30dm = 3m

Height = 12dm = 1.2m



We know that

Length = distance covered by water in 3 minutes = velocity of water in m/hr × time in hours

By substituting the values

Length = $20000 \times (30/60)$

So we get

Length = 10000m

We know that

Volume of water flown in 30 minutes = $1 \times b \times h$

By substituting the values

Volume of water flown in 30 minutes = $10000 \times 3 \times 1.2 = 36000 \text{ m}^3$

Consider A m² as the area irrigated

So we get

 $A \times (9/100) = 36000$

On further calculation

 $A = 400000 \text{ m}^2$

Therefore, the area to be irrigated is 400000 m².

28. A solid metallic cuboid of dimensions (9m × 8m × 2m) is melted and recast into solid cubes of edge 2m. Find the number of cubes so formed.

Solution:

The dimensions of cuboid are

Length = 9m

Breadth = 8m

Height = 2m

We know that

Volume of cuboid = $1 \times b \times h$

By substituting the values

Volume of cuboid = $9 \times 8 \times 2$

So we get

Volume of cuboid = 144 m³

We know that

Volume of each cube of edge $2m = a^3$

So we get

Volume of each cube of edge $2m = 2^3 = 8 \text{ m}^3$

So the number of cubes formed = volume of cuboid / volume of each cube

By substituting the values

Number of cubes formed = 144/8 = 18

Therefore, the number of cubes formed is 18.



EXERCISE 15(B)

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1. The diameter of a cylinder is 28cm and its height is 40cm. Find the curved surface area, total surface area and the volume of the cylinder. Solution:

It is given that
Diameter of a cylinder = 28cm
We know that radius = diameter/2 = 28/2 = 14cm
Height of a cylinder = 40cm

We know that Curved surface area = $2 \pi rh$ By substituting the values Curved surface area = $2 \times (22/7) \times 14 \times 40$ So we get Curved surface area = 3520 cm^2

We know that Total surface area = $2 \pi rh + 2 \pi r^2$ By substituting the values Total surface area = $(2 \times (22/7) \times 14 \times 40) + (2 \times (22/7) \times 14^2)$ On further calculation Total surface area = $3520 + 1232 = 4752 \text{ cm}^2$

We know that Volume of cylinder = $\pi r^2 h$ By substituting the values Volume of cylinder = $(22/7) \times 14^2 \times 40$ So we get Volume of cylinder = 24640cm^3

Therefore, the curved surface area, total surface area and the volume of cylinder are 3520 cm², 4752 cm² and 24640cm³.

2. A patient in a hospital is given soup daily in a cylindrical bowl of diameter 7cm. If the bowl is filled with soup to a height of 4cm, how much soup the hospital has to prepare daily to serve 250 patients? Solution:

It is given that Diameter of the bowl = 7cm We know that Radius of the bowl = 7/2 = 3.5cm Height = 4cm

We know that Volume of soup in one bowl = $\pi r^2 h$ By substituting the values Volume of soup in one bowl = $(22/7) \times (3.5)^2 \times 4$ So we get



Volume of soup in one bowl = $154cm^3$

So the volume of soup in 250 bowls = 250×154 On further calculation Volume of soup in 250 bowls = $38500 \text{ cm}^3 = 38.5 \text{ litres}$

Therefore, the hospital must prepare 38.5 litres of soup daily to serve 250 patients.

3. The pillars of a temple are cylindrically shaped. Each pillar has a circular base of radius 20cm and height 10m. How much concrete mixture would be required to build 14 such pillars? **Solution:**

It is given that Radius of pillar = 20cm = 0.2mHeight of pillar = 10m

We know that Volume of one pillar = $\pi r^2 h$ By substituting the values Volume of one pillar = $(22/7) \times (0.2)^2 \times 10$ So we get Volume of one pillar = 1.2571 m^3

So the volume of concrete mixture in 14 pillars = $14 \times 1.2571 = 17.6$ m³

Therefore, the volume of concrete mixture required in 14 pillars is 17.6m³.

4. A soft drink is available in two packs:

(i) a tin can with a rectangular base of length 5cm, breadth 4cm and height 15cm, and (ii) a plastic cylinder with circular base of diameter 7cm and height 10cm. Which container has greater capacity and by how much? Solution:

(i) The dimensions for a tin can with a rectangular base is

Length = 5cmBreadth = 4cmHeight = 15cm

We know that Volume of tin can = $1 \times b \times h$ By substituting the values Volume of tin can = $5 \times 4 \times 15$ So we get Volume of tin can = 300 cm^3

(ii) The dimensions for cylinder with circular base Diameter = 7cm

We know that radius = 7/2 = 3.5 cm Height = 10cm



We know that Volume of cylinder = $\pi r^2 h$ By substituting the values Volume of cylinder = $(22/7) \times (3.5)^2 \times 10$ So we get Volume of cylinder = 385 cm^3

We know that the volume of plastic cylinder is greater than volume of tin can So difference in volume = $385 - 300 = 85 \text{ cm}^3$

Therefore, a plastic cylinder has greater capacity than a tin can by 85 cm³.

5. There are 20 cylindrical pillars in a building, each having a diameter of 50cm and height 4m. Find the cost of cleaning them at ₹ 14 per m². Solution:

It is given that Diameter of one pillar = 50cm = 0.5mSo the radius of one pillar = 0.5/2 = 0.25mHeight of one pillar = 4m

We know that Lateral surface area of one pillar = $2 \pi rh$ By substituting the values Lateral surface area of one pillar = $2 \times (22/7) \times 0.25 \times 4$ So we get Lateral surface area of one pillar = 6.285 m^2

So the lateral surface area of 20 pillars = $20 \times 6.285 = 125.714 \text{ m}^2$

It is given that the cost of cleaning = ₹ 14 per m² So the cost of cleaning 125.714 m² = ₹ (14 × 125.714) We get Cost of cleaning 125.714 m² = ₹ 1760

Therefore, the cost of cleaning 125.714 m^2 is ₹ 1760.

- 6. The curved surface area of a right circular cylinder is 4.4 m². If the radius of its base is 0.7m, find its
- (i) height and
- (ii) volume.

Solution:

It is given that Curved surface area of a cylinder = 4.4 m^2 Radius of the cylinder = 0.7m

(i) We know that Curved surface area of a cylinder = $2 \pi rh$ By substituting the values $4.4 = 2 \times (22/7) \times 0.7 \times h$



On further calculation $4.4 = 2 \times 22 \times 0.1 \times h$ So we get $h = 4.4/(2 \times 22 \times 0.1)$ By division h = 1m

(ii) We know that Volume of a cylinder = $\pi r^2 h$ By substituting the values Volume of a cylinder = $(22/7) \times (0.7)^2 \times 1$ So we get Volume of a cylinder = 1.54 m³

7. The lateral surface area of a cylinder is 94.2 cm² and its height is 5cm. Find

(i) the radius of its base and

(ii) its volume. (Take $\pi = 3.14$)

Solution:

It is given that Lateral surface area of a cylinder = 94.2 cm² Height = 5cm

(i) We know that Lateral surface area of cylinder = $2 \pi rh$ By substituting the values $94.2 = 2 \times 3.14 \times r \times 5$ On further calculation $r = 94.2/(2 \times 3.14 \times 5)$ So we get r = 3 cm

(ii) We know that Volume of a cylinder = $\pi r^2 h$ By substituting the values Volume of a cylinder = $(3.14) \times (3)^2 \times 5$ So we get Volume of a cylinder = 141.3 cm^3

8. The capacity of a closed cylindrical vessel of height 1m is 15.4 litres. Find the area of the metal sheet needed to make it.

Solution:

It is given that Volume of the cylindrical vessel = 15.4 litres = 15400 cm³ Height of the cylindrical vessel = 1m = 100cm

We know that Volume of a cylinder = $\pi r^2 h$ By substituting the values



 $15400 = (22/7) \times r^2 \times 100$ On further calculation $r^2 = (15400 \times 7)/(22 \times 100)$ So we get $r^2 = 49$ By taking square root $r = \sqrt{49} = 7$ cm

So area of metal sheet needed = total surface area of cylinder It can be written as Area of metal sheet needed = $2 \pi r (h + r)$ By substituting the values Area of metal sheet needed = $2 \times (22/7) \times 7 (100 + 7)$ On further calculation Area of metal sheet needed = $2 \times 22 \times 107$ So we get Area of metal sheet needed = 4708 cm^2

Therefore, the area of metal sheet needed is 4708 cm².

9. The inner diameter of a cylindrical wooden pipe is 24cm and its outer diameter is 28cm. The length of the pipe is 35cm. Find the mass of the pipe, if 1cm³ of wood has a mass of 0.6g. Solution:

The dimensions of a cylinder are Internal diameter = 24cm
Internal radius = 24/12 = 12cm
External diameter = 28cm
External radius = 28/2 = 14cm
Length = 35cm

We know that Volume of pipe = Volume of cylinder = π (R² – r²) h By substituting the values Volume of pipe = $((22/7) \times (14^2 - 12^2) \times 35)$ On further calculation Volume of pipe = $22 \times (196 - 144) \times 5$ So we get Volume of pipe = $22 \times 52 \times 5$ By multiplication

It is given that 1cm^3 of wood has a mass of 0.6gSo the mass of pipe = $5720 \times 0.6 = 3432g = 3.432kg$

Therefore, the mass of pipe is 3.432kg.

Volume of pipe = 5720 cm^3

10. In a water heating system, there is a cylindrical pipe of length 28m and diameter 5cm. Find the total radiating surface in the system. Solution:



The dimensions of cylindrical pipe Diameter = 5cm Radius = 5/2 = 2.5cm Height = 28m = 2800cm

We know that

Total radiating surface in the system = curved surface area of cylindrical pipe = $2 \pi rh$

By substituting the values

Total radiating surface in the system = $2 \times (22/7) \times 2.5 \times 2800$

So we get

Total radiating surface in the system = 44000 cm^2

Therefore, the total radiating surface in the system is 44000 cm².

11. Find the weight of a solid cylinder of radius 10.5cm and height 60cm if the material of the cylinder weighs 5g per cm³.

Solution:

It is given that Radius of cylinder = 10.5cm Height of cylinder = 60cm

We know that Volume of cylinder = $\pi r^2 h$ By substituting the values Volume of cylinder = $(22/7) \times (10.5)^2 \times 60$ So we get

Volume of cylinder = 20790 cm^3

So the weight of the cylinder if the material weighs 5 g per cm³ = $20790 \times 5 = 103950$ g We know that 1000g = 1kg

Weight of the cylinder = 103950/1000 = 103.95kg

Therefore, the weight of solid cylinder is 103.95kg.

12. The curved surface area of a cylinder is 1210 cm² and its diameter is 20cm. Find its height and volume. Solution:

It is given that Curved surface area = 1210 cm² Diameter of the cylinder = 20cm Radius of the cylinder = 20/2 = 10cm

We know that Curved surface area of the cylinder = $2 \pi rh$ By substituting the values $1210 = 2 \times (22/7) \times 10 \times h$ So we get $h = (1210 \times 7) / (2 \times 22 \times 10) = 19.25 cm$



We know that Volume of cylinder = $\pi r^2 h$ By substituting the values Volume of cylinder = $(22/7) \times (10)^2 \times 19.25$ So we get Volume of cylinder = 6050 cm^3

Therefore, the height of cylinder is 19.25cm and the volume is 6050 cm³.

13. The curved surface area of a cylinder is 4400 cm² and the circumference of its base is 110cm. Find the height and the volume of the cylinder. Solution:

Consider r as the radius as h as the height of cylinder It is given that Surface area of cylinder = $2 \pi rh$ By substituting the values $2 \pi rh = 4400 \dots (1)$

It is given that circumferences of its base = $2 \pi r$ So we get $2 \pi r = 110$

We know that $2 \pi rh/2 \pi r = 4400/110$ On further calculation h = 40 cm

Substituting the value of h in (1) We get $2 \times (22/7) \times r \times 40 = 4400$ On further calculation $r = (4400 \times 7)/(44 \times 40)$ So we get r = 17.5cm

We know that Volume of cylinder = $\pi r^2 h$ By substituting the values Volume of cylinder = $(22/7) \times (17.5)^2 \times 40$ So we get Volume of cylinder = 38500 cm^3

Therefore, the height of the cylinder is 40cm and the volume is 38500 cm³.

14. The radius of the base and the height of a cylinder are in the ratio 2: 3. If its volume is 1617cm³, find the total surface area of the cylinder. Solution:

Consider radius as 2x cm and height as 3x cm



We know that Volume of cylinder = $\pi r^2 h$ By substituting the values Volume of cylinder = $(22/7) \times (2x)^2 \times 3x$ On further calculation Volume of cylinder = $(22/7) \times 4x^2 \times 3x$

So we get

Volume of cylinder = $(22/7) \times 12x^3$ It can be written as $1617 = (22/7) \times 12x^3$ On further calculation $12x^3 = (1617 \times 7)/22$ So we get $x^3 = (1617 \times 7)/(22 \times 12)$ $x^3 = 42.875$ By taking cube root $x = \sqrt[3]{42.865}$ We get

By substituting the value of x Radius = 2x = 2(3.5) = 7cm Height = 3x = 3(3.5) = 10.5cm

x = 3.5

We know that Total surface area = $2 \pi r (h + r)$ By substituting the values Total surface area = $2 \times (22/7) \times 7 (10.5 + 7)$ On further calculation Total surface area = 44×17.5 So we get Total surface area = 770 cm^2

Therefore, the total surface area of the cylinder is 770 cm².

15. The total surface area of a cylinder is 462cm². Its curved surface area is one third of its total surface area. Find the volume of the cylinder. Solution:

We know that Curved surface area = $1/3 \times \text{Total}$ surface area By substituting the values Curved surface area = $1/3 \times 462$ So we get Curved surface area = 154 cm^2

So the total surface area – curved surface area = $462 - 154 = 308 \text{cm}^2$ We know that $2 \pi \text{r}^2 = 308$

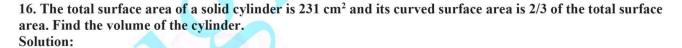


By substituting the values $2 \times (22/7) \times r^2 = 308$ On further calculation $r^2 = (308 \times 7)/44$ So we get $r^2 = 49$ By taking square root $r = \sqrt{49} = 7$ cm

We know that Curved surface area = $2 \pi rh$ By substituting the values $154 = 2 \times (22/7) \times 7 \times h$ So we get h = 154/44By division h = 3.5 cm

So we get r = 7 cm and h = 3.5 cmWe know that Volume of the cylinder = $\pi r^2 h$ By substituting the values Volume of cylinder = $(22/7) \times (7)^2 \times 3.5$ So we get Volume of cylinder = 539 cm^3

Therefore, volume of the cylinder is 539 cm³.



We know that Curved surface area = $2/3 \times \text{Total}$ surface area By substituting the values Curved surface area = $2/3 \times 231$ So we get Curved surface area = 154 cm^2

So the total surface area – curved surface area = $231 - 154 = 77 \text{cm}^2$ We know that $2 \pi r^2 = 77$ By substituting the values $2 \times (22/7) \times r^2 = 77$ On further calculation $r^2 = (77 \times 7)/44$ So we get $r^2 = 49/4$ By taking square root $r = \sqrt{49/4} = 7/2 \text{cm}$



We know that Curved surface area = $2 \pi rh$ By substituting the values $154 = 2 \times (22/7) \times (7/2) \times h$ So we get h = 154/22By division h = 7 cm

So we get r = 7/2cm and h = 7 cm We know that Volume of the cylinder = $\pi r^2 h$ By substituting the values Volume of cylinder = $(22/7) \times (7/2)^2 \times 7$ So we get Volume of cylinder = 269.5 cm^3

Therefore, volume of the cylinder is 269.5 cm³.

17. The ratio between the curved surface area and the total surface area of a right circular cylinder is 1: 2. Find the volume of the cylinder if its total surface area is 616 cm². **Solution:**

We know that

Curved surface area = $2 \pi rh$

It is given that the ratio of curved surface area and total surface area is 1: 2

So we get

 $2 \pi rh/2 \pi r(h+r) = \frac{1}{2}$

On further calculation

 $h/(h+r) = \frac{1}{2}$

It can be written as

2h = h + r

So we get

2h - h = r

h = r

By substituting h = r we get

 $2 \pi r (h + r) = 616$

So we get

 $4 \pi r^2 = 616$

It can be written as

 $4 \times (22/7) \times r^2 = 616$

On further calculation

 $r^2 = (616 \times 7)/88$

We get

 $r^2 = 49$

By taking out square root

 $r = \sqrt{49} = 7cm$

We know that



Volume = $\pi r^2 h$ By substituting the values Volume of cylinder = $(22/7) \times (7)^2 \times 7$ So we get Volume of cylinder = 1078 cm^3

Therefore, volume of the cylinder is 1078 cm³.

18. A cylindrical bucket, 28cm in diameter and 72cm high, is full of water. The water is emptied into a rectangular tank, 66cm long and 28cm wide. Find the height of the water level in the tank. Solution:

It is given that Diameter of the bucket = 28cm Radius = 28/2 = 14cm Height of the bucket = 72cm

Length of the tank = 66cm Breadth of tank = 28cm

We know that Volume of tank = volume of cylindrical bucket $1 \times b \times h = \pi r^2 h$ By substituting the values $66 \times 28 \times h = (22/7) \times (14)^2 \times 72$ On further calculation $h = (22 \times 2 \times 14 \times 72)/(66 \times 28)$ So we get h = 24cm

Therefore, the height of the water level in the tank is 24cm.

19. The barrel of a fountain pen, cylindrical in shape, is 7cm long and 5mm in diameter. A full barrel of ink in the pen will be used up on writing 330 words on an average. How many words would use up a bottle of ink containing one fifth of a litre? Solution:

The dimensions of barrel are Length = 7cm Diameter = 5mm Radius = 5/2 = 2.5 mm = 0.25cm

We know that Volume of barrel = $\pi r^2 h$ By substituting the values Volume of cylinder = $(22/7) \times (0.25)^2 \times 7$ So we get Volume of cylinder = 1.375 cm³

So 1.375 cm³ is used for writing 330 words



So the bottle containing one fifth of a litre ink would write = $330 \times (1/1.375) \times (1/5) \times 1000 = 48000$ words

Therefore, a bottle of ink containing one fifth of a litre would write 48000 words.

20. 1cm³ of gold is drawn into a wire 0.1mm in diameter. Find the length of the wire. Solution:

We know that

 $1 \text{cm}^3 = 1 \text{cm} \times 1 \text{cm} \times 1 \text{cm}$

1 cm = 0.01 m

So the volume of gold = $0.01 \text{m} \times 0.01 \text{m} \times 0.01 \text{m}$

We get

Volume of gold = $0.000001 \text{m}^3 \dots (1)$

It is given that

Diameter of the wire drawn = 0.1mm

So the radius = 0.1/2 = 0.05mm = 0.00005 m(2)

Consider length of the wire = $h m \dots (3)$

We know that

Volume of wire drawn = Volume of gold

By substituting the values using (1), (2) and (3)

 $\pi r^2 h = 0.000001$

On further calculation

 $\Pi \times 0.00005 \times 0.00005 \times h = 0.000001$

So we get

 $h = (0.000001 \times 7) / (0.00005 \times 0.00005 \times 22) = 127.27m$

Therefore, the length of the wire is 127.27m.

21. If 1cm³ of cast iron weighs 21g, find the weight of a cast iron pipe of length 1m with a bore of 3cm in which the thickness of the metal is 1cm. Solution:

We know that

Internal radius = 3/2 = 1.5cm

External radius = 1.5 + 1 = 2.5cm

We know that

Volume of cast iron = $(\pi \times (2.5)^2 \times 100 - \pi \times (1.5)^2 \times 100)$

Taking the common terms out

Volume of cast iron = $\pi \times 100 \times (2.5^2 - 1.5^2)$

On further calculation

Volume of cast iron = $(22/7) \times 100 \times (6.25 - 2.25)$

So we get

Volume of cast iron = $(22/7) \times 100 \times 4 = 1257.142 \text{ cm}^3$

It is given that 1cm³ of cast iron weighs 21g

We know that 1 kg = 1000 g



So the weight of cast iron pipe = $1257.142 \times (21/1000) = 26.4 \text{kg}$

Therefore, the weight of cast iron pipe is 26.4kg.

22. A cylindrical tube, open at both ends, is made of metal. The internal diameter of the tube is 10.4cm and its length is 25cm. The thickness of the metal is 8mm everywhere. Calculate the volume of the metal. Solution:

It is given that

Internal diameter of the tube = 10.4cm

Internal radius of the tube = 10.4/2 = 5.2 cm

Length = 25cm

We know that

External radius = 5.2 + 0.8 = 6cm

We know that

Required volume = $(\pi \times 6^2 \times 25 - \pi \times 5.2^2 \times 25)$

Taking the common terms out

Required volume = $\pi \times 25 \times (6^2 - 5.2^2)$

On further calculation

Required volume = $(22/7) \times 25 \times (36 - 27.04)$

So we get

Required volume = $(22/7) \times 25 \times 8.96 = 704 \text{ cm}^3$

Therefore, the volume of the metal is 704 cm³.

23. It is required to make a closed cylindrical tank of height 1m and base diameter 140cm from a metal sheet. How many square metres of the sheet are required for the same? Solution:

It is given that

Diameter of the cylinder = 140cm

Radius of the cylinder = 140/2 = 70cm

Height of the cylinder = 1m = 100cm

We know that

Area of sheet required = Total surface area of cylinder = $2 \pi r (h + r)$

By substituting the values

Area of sheet required = $2 \times (22/7) \times 70 (100 + 70)$

On further calculation

Area of sheet required = $2 \times 22 \times 10 \times 170$

So we get

Area of sheet required = $74800 \text{ cm}^2 = 7.48 \text{ m}^2$

Therefore, the area of sheet required is 7.48 m².

24. A juice seller has a large cylindrical vessel of base radius 15cm filled up to a height of 32cm with orange juice. The juice is filled in small cylindrical glasses of radius 3cm up to a height of 8cm, and sold for ₹ 15 each. How much money does he receive by selling the juice completely? Solution:



The dimensions of the cylindrical vessel

Radius = 15cm

Height = 32cm

We know that

Volume of cylindrical vessel = $\pi r^2 h$

By substituting the values

Volume of cylindrical vessel = $(22/7) \times (15)^2 \times 32$

So we get

Volume of cylindrical vessel = 22628.571 cm³

The dimensions of cylindrical glass are

Radius = 3cm

Height = 8cm

We know that

Volume of each small cylindrical glass = $\pi r^2 h$

By substituting the values

Volume of each small cylindrical glass = $(22/7) \times (3)^2 \times 8$

So we get

Volume of each small cylindrical glass = 226.28 cm³

So the number of small glasses filled = volume of cylindrical vessel/volume of each glass

By substituting the values

Number of small glasses filled = 22628.571/226.28 = 100

It is given that

Cost of 1 glass = ₹ 15

So the cost of 100 glasses = $\mathbf{\xi}$ (15 × 100) = $\mathbf{\xi}$ 1500

Therefore, the juice seller receives ₹ 1500 by selling 100 glasses of orange juice.

25. A well with inside diameter 10m is dug 8.4m deep. Earth taken out of it is spread all around it to a width of 7.5m to form an embankment. Find the height of the embankment. Solution:

The dimensions of the well are

Radius = 5 m

Depth = 8.4m

We know that

Volume of the earth dug out = Volume of well = $\pi r^2 h$

By substituting the values

Volume of the earth dug out = $(22/7) \times (5)^2 \times 8.4$

So we get

Volume of the earth dug out = 660 m^2

It is given that

Width of embankment = 7.5m

We know that



External radius of embankment R = 5 + 7.5 = 12.5m Internal radius of embankment r = 5m

We know that

Area of embankment = $\pi (R^2 - r^2)$

By substituting the values

Area of embankment = $(22/7) \times (12.5^2 - 5^2)$

On further calculation

Area of embankment = $(22/7) \times (156.25 - 25)$

So we get

Area of embankment = $(22/7) \times 131.25 = 412.5 \text{ m}^2$

We know that

Volume of embankment = volume of earth dug out = 660m^2

So the height of embankment = volume of embankment/ area of embankment

By substituting the values

Height of embankment = 660/412.5 = 1.6m

Therefore, the height of the embankment is 1.6m.

26. How many litres of water flows out of a pipe having an area of cross section of 5cm² in 1 minute, if the speed of water in the pipe is 30cm/sec? Solution:

It is given that

Speed of water in the pipe = 30 cm/sec

We know that

Volume of water that flows out of the pipe in one second = area of cross section × length of water flown in one second

By substituting the values

Volume of water that flows out of the pipe in one second = $5(30) = 150 \text{cm}^3$

So the volume of water that flows out of the pipe in one minute = $150 (60) = 9000 \text{ cm}^3 = 9 \text{ litres}$

Therefore, 9 litres of water flows out of the pipe in one minute.

27. A cylindrical water tank of diameter 1.4m and height 2.1m is being fed by a pipe of diameter 3.5cm through which water flows at the rate of 2m per second. In how much time will the tank be filled? Solution:

Consider the tank to be filled in x minutes

We know that

Volume of water that flows through the pipe in x minutes = Volume of tank

By substituting the values

 $\Pi \times (3.5/(2 \times 100))^2 \times (2 \times 60x) = \Pi \times (0.7)^2 \times 2.1$

On further calculation

0.115395x = 3.23106

So we get

x = 28



Therefore, the tank will be filled in 28 minutes.

28. A cylindrical container with diameter of base 56cm contains sufficient water to submerge a rectangular solid of iron with dimensions (32cm × 22cm × 14cm). Find the rise in the level of water when the solid is completely submerged.

Solution:

Consider h cm as the rise in level of water

We know that

Volume of cylinder of height h and base radius 28cm = volume of rectangular iron solid

By substituting the values

 $(22/7) \times 28^2 \times h = 32 \times 22 \times 14$

On further calculation

 $22 \times 28 \times 4 \times h = 32 \times 22 \times 14$

So we get

 $h = (32 \times 22 \times 14)/(22 \times 28 \times 4)$

By division

h = 4cm

Therefore, the rise in the level of water when the solid is completely submerged is 4cm.

29. Find the cost of sinking a tube-well 280m deep, having a diameter 3m at the rate of ₹ 15 per cubic metre. Find also the cost of cementing its inner curved surface at ₹ 10 per square metre. Solution:

It is given that

Radius = 1.5m

Height = 280m

We know that

Volume of the tube well = $\pi r^2 h$

By substituting the values

Volume of the tube well = $(22/7) \times (1.5)^2 \times 280$

So we get

Volume of the tube well = 1980 m^3

It is given that

Cost of sinking the tube well = ξ (15 × 1980) = ξ 29700

We know that

Curved surface area of tube well = $2 \pi rh$

By substituting the values

Curved surface area of tube well = $2 \times (22/7) \times 1.5 \times 280$

So we get

Curved surface area of tube well = 2640 m^2

It is given that

So the cost of cementing = $\mathbf{\xi}$ (10 × 2640) = $\mathbf{\xi}$ 26400

Therefore, cost of sinking the tube well is ₹ 29700 and the cost of cementing is ₹ 26400.

30. Find the length of 13.2 kg of copper wire of diameter 4mm, when 1 cubic centimeter of copper weighs



8.4kg. **Solution:**

Consider h m as the length of the wire We know that Volume of the wire \times 8.4 g = (13.2 \times 1000) g By substituting the values $(22/7) \times (2/10)^2 \times h \times 8.4 = 13200$ On further calculation $22 \times (1/5)^2 \times h \times 8.4 = 13200$ So we get $h = (13200 \times 5 \times 5)/(22 \times 1.2)$ By simplification h = 12500cm = 125m

Therefore, the length of wire is 125m.

- 31. It costs ₹ 3300 to paint the inner curved surface of a cylindrical vessel 10m deep at the rate of ₹ 30 per m². Find the
- (i) inner curved surface area of the vessel,
- (ii) inner radius of the base, and
- (iii) capacity of the vessel.

Solution:

(i) We know that

Cost of painting inner curved surface of the vessel = $\cos t$ of painting per m² × inner curved surface of vessel By substituting the values

 $3300 = 30 \times Inner curved surface of vessel$

On further calculation

Inner curved surface of vessel = 110 m^2

(ii) Consider r as the inner radius of the base

It is given that depth = 10m

We know that

Inner curved surface of vessel = $2 \pi rh$

By substituting the values

$$110 = 2 \times (22/7) \times r \times 10$$

So we get

$$r = (110 \times 7)/(2 \times 22 \times 10)$$

r = 1.75 m

(iii) We know that

Capacity of the vessel = $\pi r^2 h$

By substituting the values

Capacity of the vessel = $(22/7) \times (1.75)^2 \times 10$

So we get

Capacity of the vessel = 96.25 m^3

32. The difference between inside and outside surfaces of a cylindrical tube 14cm long, is 88cm². If the volume of the tube is 176 cm³, find the inner and outer radii of the tube.



Solution:

Consider R cm as the outer radius and r cm as the inner radius of the cylindrical tube It is given that length = 14 cm

We know that

Outside surface area – Inner surface area = 88

So we get

 $2 \pi Rh - 2 \pi rh = 88$

It can be written as

 $2 \pi (R-r) h = 88$

By substituting the values

 $2 \times (22/7) \times (R - r) \times 14 = 88$

On further calculation

 $2 \times 22 \times (R-r) \times 2 = 88$

We get

 $R - r = 88/(2 \times 22 \times 2) = 1 \dots (1)$

We know that

Volume of tube = 176 cm^3

It can be written as

External volume – Internal volume = 176

So we get

 $\pi R^2 h - \pi r^2 h = 176$

By taking common out

$$\Pi (R^2 - r^2) h = 176$$

By substituting the values

$$(22/7) \times (R - r)(R + r) \times 14 = 176$$

Substituting equation (1)

$$22 \times 1 \times (R + r) \times 2 = 176$$

We ge

$$R + r = 176/(22 \times 2) = 4....(2)$$

By adding both the equations

2R = 5

So we get R = 2.5 cm

By substituting r

$$2.5 - r = 1$$

So we get r = 1.5cm

Therefore, the inner and outer radii of the tube are 1.5cm and 2.5cm.

33. A rectangular sheet of paper 30cm × 18cm can be transformed into the curved surface of a right circular cylinder in two ways namely, either by rolling the paper along its length or by rolling it along its breadth. Find the ratio of the volumes of the two cylinders, thus formed. Solution:

We know that

If the sheet is folded along its length it forms a cylinder of height $h_1 = 18$ cm and perimeter = 30cm Consider r_1 as the radius and V_1 as the volume



So we get $2 \pi r_1 = 30$

It can be written as

 $r_1 = 30/2 \ \pi = 15/\pi$

We know that

 $V_1 = \pi r_1^2 h_1$

By substituting the values

 $V_1 = \pi \times (15/\pi)^2 \times 18$

We get

 $V_1 = (225/\pi) \times 18 \text{ cm}^3$

We know that

If the sheet is folded along its breadth it forms a cylinder of height $h_2 = 30$ cm and perimeter 18cm Consider r_2 as the radius and V_2 as the volume

So we get

 $2 \pi r_2 = 18$

It can be written as

 $r_2 = 18/2 \ \pi = 9/\pi$

We know that

 $V_2 = \pi r_2^2 h_2$

By substituting the values

 $V_2 = \pi \times (9/\pi)^2 \times 3$

We get

 $V_2 = (81/\pi) \times 30 \text{ cm}^3$

So we get

 $V_1/V_2 = \{(225/\pi) \times 18\}/\{(81/\pi) \times 30\}$

On further calculation

 $V_1/V_2 = (225 \times 18)/(81 \times 30)$

We get

 $V_1/V_2 = 5/3$

We can write it as

 $V_1:V_2=5:3$

Therefore, the ratio of the volumes of the two cylinders thus formed is 5:3.



EXERCISE 15(C)

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1. Find the curved surface area of a cone with base radius 5.25cm and slant height 10cm. Solution:

It is given that Radius of the cone = 5.25cm Slant height of the cone = 10cm

We know that Curved surface area of the cone = π rl By substituting the values Curved surface area of the cone = $(22/7) \times 5.25 \times 10$ So we get Curved surface area of the cone = 165 cm^2

Therefore, the curved surface area of a cone is 165 cm².

2. Find the total surface area of a cone, if its slant height is 21m and diameter of its base is 24m. Solution:

It is given that Diameter of the cone = 24mRadius of the cone = 24/2 = 12mSlant height of the cone = 21m

We know that Total surface area of a cone = $\pi r (1 + r)$ By substituting the values Total surface area of a cone = $(22/7) \times 12 (21+12)$ On further calculation Total surface area of a cone = $(22/7) \times 12 \times 33$ So we get Total surface area of a cone = 1244.57 m^2

Therefore, the total surface area of a cone is 1244.57 m².

3. A joker's cap is in the form of a right circular cone of base radius 7cm and height 24cm. Find the area of the sheet required to make 10 such caps. Solution:

It is given that Radius of the cap = 7cm Height of the cap = 24cm

We know that Slant height of the cap $1 = \sqrt{(r^2 + h^2)}$ By substituting the values $1 = \sqrt{(7^2 + 24^2)}$



On further calculation $1 = \sqrt{(49 + 576)} = \sqrt{625}$ So we get 1 = 25cm

We know that Curved surface area of one cap = π rl By substituting the values Curved surface area of one cap = $(22/7) \times 7 \times 25$ So we get Curved surface area of one cap = 550 cm^2

So the curved surface area of 10 conical caps = $10 \times 550 = 5500 \text{ cm}^2$

Therefore, the area of the sheet required to make 10 such caps is 5500 cm².

4. The curved surface area of a cone is 308cm² and its slant height is 14cm. Find the radius of the base and total surface area of the cone.

Solution:

Consider r as the radius of the cone
It is given that
Slant height of the cone = 14cm
Curved surface area of the cone = 308cm^2 It can be written as $\Pi rl = 308$ By substituting the values $(22/7) \times r \times 14 = 308$ On further calculation $22 \times r \times 2 = 308$ So we get $r = 308/(22 \times 2)$

We know that

r = 7cm

Total surface area of a cone = $\Pi r (1 + r)$

By substituting the values

Total surface area of a cone = $(22/7) \times 7 \times (14 + 7)$

On further calculation

Total surface area of a cone = $22 \times 21 = 462 \text{ cm}^2$

Therefore, the base radius of the cone is 7cm and the total surface area is 462 cm².

5. The slant height and base diameter of a conical tomb are 25m and 14m respectively. Find the cost of whitewashing its curved surface at the rate of ₹ 12 per m². Solution:

It is given that Radius of the cone = 7m Slant height of the cone = 25m



We know that Curved surface area of the cone = π rl By substituting the values Curved surface area of the cone = $(22/7) \times 7 \times 25$ So we get Curved surface area of the cone = 550 m^2

It is given that the cost of whitewashing = $₹ 12 \text{ per m}^2$ So the cost of whitewashing 550 m² area = $₹ 12 \times 550 = ₹ 6600$

Therefore, the cost of whitewashing its curved surface area is ₹ 6600.

6. A conical tent is 10m high and the radius of its base is 24m. Find the slant height of the tent. If the cost of 1m² canvas is ₹ 70, find the cost of canvas required to make the tent. Solution:

It is given that Radius of the conical tent = 24m Height of conical tent = 10m

We know that Slant height of conical tent can be written as $1 = \sqrt{(r^2 + h^2)}$ By substituting the values $1 = \sqrt{(24^2 + 10^2)}$ On further calculation $1 = \sqrt{(576 + 100)} = \sqrt{676}$ So we get 1 = 26m

We know that Curved surface area of conical tent = π rl By substituting the values Curved surface area of conical tent = $(22/7) \times 24 \times 26$ So we get Curved surface area of conical tent = (13728/7) m²

It is given that the cost of 1m^2 canvas = $\stackrel{?}{=} 70$ So the cost of (13728/7) m² canvas = $\stackrel{?}{=} 70 \times (13728/7) = \stackrel{?}{=} 137280$

Therefore, the slant height of the tent is 26m and the cost of canvas required to make the tent is ₹ 137280.

7. A bus stop is barricaded from the remaining part of the road by using 50 hollow cones made of recycled cardboard. Each one has a base diameter of 40cm and height 1m. If the outer side of each of the cones is to be painted and the cost of painting is $\stackrel{?}{\underset{1}{}}$ 25 per m², what will be the cost of painting all these cones? (Use $\pi = 3.14$ and $\sqrt{1.04} = 1.02$.)

Solution:

It is given that Radius of the cone = 20cm = 0.2m



Height of the cone = 1 m

We know that Slant height $1 = \sqrt{(r^2 + h^2)}$ By substituting the values $1 = \sqrt{(0.2^2 + 1^2)}$ On further calculation $1 = \sqrt{(0.04 + 1)} = \sqrt{1.04}$ So we get 1 = 1.02m

We know that Curved surface area of cone = π rl By substituting the values Curved surface area of cone = $3.14 \times 0.2 \times 1.02$ On further calculation

Curved surface area of cone = 0.64056 m^2

So the curved surface area of 50 cones = $50 \times 0.64056 = 32.028 \text{ m}^2$

It is given that Cost of painting = $₹25 \text{ per m}^2$ So the cost of painting 32.028 m² area = $₹25 \times 32.028 = ₹800.70$

Therefore, the cost of painting all these cones is ₹ 800.70.

8. Find the volume, curved surface area and the total surface area of a cone having base radius 35cm and height 12cm.

Solution:

It is given that
Radius of the cone = 35cm
Height of the cone = 12cm

We know that Volume of the cone = $1/3 \pi r^2 h$ By substituting the values Volume of the cone = $1/3 \times (22/7) \times 35^2 \times 12$ On further calculation Volume of the cone = 15400 cm^3

We know that Slant height $1 = \sqrt{(r^2 + h^2)}$ By substituting the values $1 = \sqrt{(35^2 + 12^2)}$ On further calculation $1 = \sqrt{1369}$ So we get 1 = 37 cm



We know that

Curved surface area of a cone = πrl

By substituting the values

Curved surface area of a cone = $(22/7) \times 35 \times 37$

So we get

Curved surface area of a cone = 4070 cm^2

We know that

Total surface area of cone = $\pi r (1 + r)$

By substituting the values

Total surface area of cone = $(22/7) \times 35 \times (37 + 35)$

On further calculation

Total surface area of cone = $22 \times 5 \times 72$

So we get

Total surface area of cone = 7920 cm^2

9. Find the volume, curved surface area and the total surface area of a cone whose height is 6cm and slant height 100cm. (Take $\pi = 3.14$.)

Solution:

It is given that

Height of the cone = 6 cm

Slant height of the cone b= 10cm

We know that

Radius of the cone = $\sqrt{(1^2 - h^2)}$

By substituting the values

Radius of the cone = $\sqrt{(10^2 - 6^2)}$

On further calculation

Radius of the cone = $\sqrt{(100-36)} = \sqrt{64}$

So we get

Radius of the cone = 8 cm

We know that

Volume of the cone = $1/3 \pi r^2 h$

By substituting the values

Volume of the cone = $1/3 \times 3.14 \times 8^2 \times 6$

On further calculation

Volume of the cone = 401.92 cm^3

We know that

Curved surface area of a cone = π rl

By substituting the values

Curved surface area of a cone = $3.14 \times 8 \times 10$

So we get

Curved surface area of a cone = 251.2 cm^2

We know that

Total surface area of cone = $\pi r (1 + r)$

By substituting the values



Total surface area of cone = $3.14 \times 8 \times (10 + 8)$ On further calculation Total surface area of cone = $3.14 \times 8 \times 18$ So we get Total surface area of cone = 452.16 cm^2

10. A conical pit of diameter 3.5m is 12m deep. What is its capacity in kilolitres? Solution:

It is given that Diameter of the conical pit = 3.5m Radius of the conical pit = 3.5/2 = 1.75m Depth of the conical pit = 12m

We know that Volume of the conical pit = $1/3 \pi r^2 h$ By substituting the values Volume of the conical pit = $1/3 \times (22/7) \times 1.75^2 \times 12$ On further calculation Volume of the conical pit = $38.5 \text{ m}^3 = 38.5 \text{ kilolitres}$

Therefore, the capacity of the conical pit is 38.5 kilolitres.

11. A heap of wheat is in the form of a cone of diameter 9m and height 3.5m. Find its volume. How much canvas cloth is required to just cover the heap? (Use $\pi = 3.14$.) Solution:

It is given that
Diameter of the conical heap = 9m
Radius of the conical heap = 9/2 = 4.5m
Height of the conical heap = 3.5m

We know that Volume of the conical heap = $1/3 \pi r^2 h$ By substituting the values Volume of the conical heap = $1/3 \times 3.14 \times 4.5^2 \times 3.5$ On further calculation Volume of the conical heap = $3.14 \times 1.5 \times 4.5 \times 3.5$ So we get Volume of the conical heap = 74.1825 m^3

We know that Slant height $1 = \sqrt{(r^2 + h^2)}$ By substituting the values $1 = \sqrt{(4.5^2 + 3.5^2)}$ On further calculation $1 = \sqrt{32.5}$ So we get 1 = 5.7 m



We know that

Curved surface area of the conical heap = πrl

By substituting the values

Curved surface area of the conical heap = $3.14 \times 4.5 \times 5.7$

On further calculation

Curved surface area of the conical heap = 80.54 m^2

Therefore, 80.54 m² of canvas is required to cover the heap of wheat.

12. A man uses a piece of canvas having an area of 551m², to make a conical tent of base radius 7m. Assuming that all the stitching margins and wastage incurred while cutting, amount to approximately 1m², find the volume of the tent that can be made with it. Solution:

It is given that

Radius of the conical tent = 7m

So the area of canvas required to make the conical tent = $551 - 1 = 550 \text{ m}^2$

We know that

Curved surface area of a conical tent = 550

So we get

 $\Pi rl = 550$

By substituting the values

 $(22/7) \times 7 \times 1 = 550$

On further calculation

1 = 550/22 = 25m

We know that

Height $h = \sqrt{(1^2 - r^2)}$

By substituting the values

 $h = \sqrt{(25^2 - 7^2)}$

On further calculation

 $h = \sqrt{(625 - 49)} = \sqrt{576}$

So we get

h = 24m

We know that

Volume of the conical tent = $1/3 \pi r^2 h$

By substituting the values

Volume of the conical tent = $1/3 \times (22/7) \times 7^2 \times 24$

On further calculation

Volume of the conical tent = 1232 m^3

Therefore, the volume of the conical tent is 1232 m³.

13. How many metres of cloth, 2.5m wide, will be required to make a conical tent whose base radius is 7m and height 24m?

Solution:

It is given that



Radius of the conical tent = 7mHeight of the conical tent = 24m

We know that Slant height $1 = \sqrt{(r^2 + h^2)}$ By substituting the values $1 = \sqrt{(7^2 + 24^2)}$ On further calculation $1 = \sqrt{(49 + 576)} = \sqrt{625}$ So we get 1 = 25 m

We know that Area of the cloth = π rl By substituting the values Area of the cloth = $(22/7) \times 7 \times 25$ On further calculation Area of the cloth = 550 m^2

We know that Length of the cloth = area/ width By substituting the values Length of the cloth = 550/2.5 = 220m

Therefore, 220m of cloth is required to make the conical tent.

14. Two cones have their heights in the ratio 1:3 and the radii of their bases in the ratio 3:1. Show that their volumes are in the ratio 3:1. Solution:

Consider the heights as h and 3h and radii as 3r and r So we get $V_1 = 1/3 \pi (3r)^2 h$ and $V_2 = 1/3 \pi r^2 \times 3h$ By dividing both we get $V_1/V_2 = (1/3 \pi (3r)^2 h)/(1/3 \pi r^2 \times 3h)$ On further calculation $V_1/V_2 = 3/1$ It can be written as $V_1:V_2 = 3:1$

Therefore, it is proved that their volumes are in the ratio 3:1.

15. A cylinder and a cone have equal radii of their bases and equal heights. If their curved surface areas are in the ratio 8:5, show that the radius and height of each has the ratio 3:4. Solution:

Consider the curved surface area of cylinder and cone as 8x and 5x. So we get

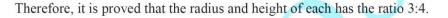
2
$$\pi$$
rh = 8x(1)
 Π r $\sqrt{(h^2 + r^2)} = 5$ x(2)



By squaring equation (1) $(2 \pi rh)^2 = (8x)^2$ So we get $4 \pi^2 r^2 h^2 = 64 x^2 \dots (3)$

By squaring equation (2) $\Pi^2 r^2 (h^2 + r^2) = 25x^2 \dots$ (4)

Dividing equation (3) by (4) $4 \pi^2 r^2 h^2 / \Pi^2 r^2 (h^2 + r^2) = 64 x^2 / 25 x^2$ On further calculation $h^2 / (h^2 + r^2) = 16 / 25$ It can be written as $9 h^2 = 16 r^2$ So we get $r^2 / h^2 = 9 / 16$ By taking square root $r / h = \frac{3}{4}$ We get r + h = 3 / 4



16. A right circular cone is 3.6cm high and radius of its base is 1.6cm. It is melted and recast into a right circular cone having base radius 1.2cm. Find its height. Solution:

It is given that Height of the cone = 3.6cm Radius of the cone = 1.6cm

Radius after melting = 1.2cm

We know that

Volume of original cone = Volume of cone after melting

By substituting the values

 $1/3 \pi \times 1.6^2 \times 3.6 = 1/3 \pi \times 1.2^2 \times h$

It can be written as

 $h = (1/3 \pi \times 1.6^2 \times 3.6)/(1/3 \pi \times 1.2^2)$

On further calculation

h = 6.4cm

Therefore, the height of the right circular cone is 6.4cm.

17. A circus tent is cylindrical to a height of 3 metres and conical above it. If its diameter is 15m and the slant height of the conical portion is 53m, calculate the length of the canvas 5m wide to make the required tent.

Solution:

It is given that

Diameter of the cylinder = 105m



Radius of the cylinder = 105/2 = 52.5m Height of the cylinder = 3m Slant height of the cylinder = 53m

We know that Area of canvas = $2 \pi RH + \pi RI$ By substituting the values Area of canvas = $(2 \times (22/7) \times 52.5 \times 3) + ((22/7) \times 52.5 \times 53)$ On further calculation Area of canvas = $990 + 8745 = 9735 \text{m}^2$

We know that Length of canvas = area/ width = 9735/5 = 1947m

Therefore, the length of canvas required to make the tent is 1947m.

18. An iron pillar consists of a cylindrical portion 2.8m high and 20cm in diameter and a cone 42cm high is surmounting it. Find the weight of the pillar, given that 1cm³ of iron weighs 7.5g. Solution:

It is given that Height of the cylinder = 2.8m = 280cm Diameter of the cylinder = 20cm Radius of the cylinder = 20/2 = 10cm Height of the cone = 42cm

We know that Volume of the pillar = $\pi r^2 h + 1/3 \pi r^2 H$ It can be written as Volume of the pillar = $\pi r^2 (h + 1/3 H)$ By substituting the values Volume of the pillar = $(22/7) \times 10^2 (280 + (1/3 \times 42))$ On further calculation Volume of the pillar = $2200/7 \times (280 + 14) = 92400 \text{ cm}3$

So the weight of pillar = $(92400 \times 7.5)/1000 = 693$ kg

Therefore, the weight of the pillar is 693kg.

19. From a solid right circular cylinder with height 10cm and radius of the base 6cm, a right circular cone of the same height and base is removed. Find the volume of the remaining solid. (Take $\pi = 3.14$.) Solution:

It is given that Height of the cylinder = 10cm Radius of the cylinder = 6cm

We know that Volume of the remaining solid = $\pi r^2 h - 1/3 \pi r^2 h$ By substituting the values



Volume of the remaining solid = $(\pi \times 6^2 \times 10) - (1/3 \pi \times 6^2 \times 10)$

It can be written as

Volume of the remaining solid = $2/3 \pi \times 6^2 \times 10$

So we get

Volume of the remaining solid = $2/3 \times 3.14 \times 360 = 753.6 \text{ cm}^3$

Therefore, the volume of the remaining solid is 753.6 cm³.

20. Water flows at the rate of 10 metres per minute through a cylindrical pipe 5mm in diameter. How long would it take to fill a conical vessel whose diameter at the surface is 40cm and depth 24cm? Solution:

It is given that

Diameter of the pipe = 5 mm = 0.5 cm

Radius of the pipe = 0.5/2 = 0.25cm

Length of the pipe = 10m = 1000cm

We know that

Volume of water that flows in 1 minute = $\pi \times 0.25^2 \times 1000$

So the volume of conical flask = $1/3 \pi \times 20^2 \times 24$

The time required to fill the conical vessel = volume of the conical vessel/ volume that flows in 1 minute

By substituting the values

Required time = $(1/3 \pi \times 20^2 \times 24)/(\pi \times 0.25^2 \times 1000)$

On further calculation

Required time = $(1/3 \pi \times 400 \times 24)/(\pi \times 0.0625 \times 1000)$

So we get

Required time = 51.12 minutes = 51 minutes 12 seconds

Therefore, the time required to fill the conical vessel is 51 minutes 12 seconds.

- 21. A cloth having an area of 165 m² is shaped into the form of a conical tent of radius 5m.
- (i) How many students can sit in the tent if a student, on an average, occupies 5/7 m² on the ground?
- (ii) Find the volume of the cone.

Solution:

(i) We know that

Area of the floor of the tent = πr^2

By substituting the values

Area of the floor of the tent = $(22/7) \times 5^2 = 550/7 \text{ m}^2$

We know that the area required by one student is 5/7 m²

So the required number of students = (550/7)/(5/7) = 110

(ii) We know that

Curved surface area of the tent = area of the cloth = 165 m^2

So we get

 Π rl = 165

By substituting the values

 $(22/7) \times 5 \times 1 = 165$

On further calculation



 $1 = (165 \times 7)/(22 \times 5) = 21/2 \text{ m}$

We know that $h = \sqrt{(l^2 - r^2)}$ By substituting the values $h = \sqrt{((21/2)^2 - 5^2)}$ On further calculation $h = \sqrt{((441/4) - 25)} = \sqrt{(341/4)}$ So we get h = 9.23m

We know that Volume of the tent = $1/3 \pi r^2 h$ By substituting the values Volume of the tent = $1/3 \times (22/7) \times 5^2 \times 9.23$ On further calculation Volume of the tent = 241.7 m^3



EXERCISE 15(D)

1. Find the volume and surface area of a sphere whose radius is:

(i) 3.5 cm

(ii) 4.2 cm

(iii) 5m

Solution:

(i) It is given that

Radius of the sphere = 3.5cm

We know that

Volume of the sphere = $4/3 \pi r^3$

By substituting the values

Volume of the sphere = $4/3 \times (22/7) \times 3.5^3$

So we get

Volume of the sphere = 179.67 cm^3

We know that

Surface area of the sphere = $4 \pi r^2$

By substituting the values

Surface area of the sphere = $4 \times (22/7) \times 3.5^2$

So we get

Surface area of the sphere = 154 cm^2

(ii) It is given that

Radius of the sphere = 4.2 cm

We know that

Volume of the sphere = $4/3 \pi r^3$

By substituting the values

Volume of the sphere = $4/3 \times (22/7) \times 4.2^3$

So we get

Volume of the sphere = 310.464 cm^3

We know that

Surface area of the sphere = $4 \pi r^2$

By substituting the values

Surface area of the sphere = $4 \times (22/7) \times 4.2^2$

So we get

Surface area of the sphere = 221.76 cm^2

(iii) It is given that

Radius of the sphere = 5 cm

We know that

Volume of the sphere = $4/3 \pi r^3$

By substituting the values

Volume of the sphere = $4/3 \times (22/7) \times 5^3$

So we get

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Volume of the sphere = 523.81 m^3

We know that Surface area of the sphere = $4 \pi r^2$ By substituting the values Surface area of the sphere = $4 \times (22/7) \times 5^2$ So we get Surface area of the sphere = 314.28 m^2

2. The volume of a sphere is 38808 cm³. Find its radius and hence its surface area. Solution:

We know that Volume of the sphere = $4/3 \pi r^3$ By substituting the values $38808 = 4/3 \times (22/7) \times r^3$ On further calculation $r^3 = (38808 \times 3 \times 7)/88$ So we get $r^3 = 9261$ By taking cube root r = 21 cm

We know that Surface area of the sphere = $4 \pi r^2$ By substituting the values Surface area of the sphere = $4 \times (22/7) \times 21^2$ So we get Surface area of the sphere = 5544 cm^2

Therefore, the radius of the sphere is 21cm and the surface area is 5544 cm².

3. Find the surface area of a sphere whose volume is 606.375m³. Solution:

We know that Volume of the sphere = $4/3 \pi r^3$ By substituting the values $606.375 = 4/3 \times (22/7) \times r^3$ On further calculation $r^3 = (606.375 \times 3 \times 7)/88$ So we get $r^3 = 144.703125$ By taking cube root r = 5.25m

We know that Surface area of the sphere = $4 \pi r^2$ By substituting the values Surface area of the sphere = $4 \times (22/7) \times 5.25^2$



So we get

Surface area of the sphere = 346.5 m^2

Therefore, the surface area of the sphere is 346.5 m².

4. Find the volume of a sphere whose surface area is 154 cm². Solution:

We know that Surface area of the sphere = $4 \pi r^2$ By substituting the values $4 \pi r^2 = 154$ On further calculation $4 \times (22/7) \times r^2 = 154$ So we get $r^2 = (154 \times 7)/(4 \times 22) = 49/4$ By taking the square root r = 7/2 cm

We know that Volume of the sphere = $4/3 \text{ mr}^3$ By substituting the values Volume of the sphere = $4/3 \times (22/7) \times (7/2)^3$ So we get Volume of the sphere = 179.67 cm^3

Therefore, the volume of the sphere is 179.67 cm³.

5. The surface area of sphere is (576π) cm². Find its volume. Solution:

We know that Surface area of the sphere = $4 \pi r^2$ By substituting the values $4 \pi r^2 = 576\pi$ On further calculation $r^2 = 576/4 = 144$ By taking square root r = 12 cm

We know that

Volume of the sphere = $4/3 \pi r^3$ By substituting the values Volume of the sphere = $4/3 \times \pi \times (12)^3$ So we get Volume of the sphere = $2304 \pi \text{ cm}^3$

Therefore, the volume of the sphere is $2304 \pi \text{ cm}^3$.

6. How many lead shots, each 3mm in diameter, can be made from a cuboid with dimensions (12cm × 11cm



× 9cm)? Solution:

It is given that Diameter = 3 mm = 0.3 cmRadius = 0.3/2 cm

We know that

Number of lead shots = volume of cuboid/ volume of 1 lead shot

By substituting the values

Number of lead shots = $(12 \times 11 \times 9)/(4/3 \times (22/7) \times (0.3/2)^3)$

On further calculation

Number of lead shots = $(12 \times 11 \times 9)/(4/3 \times (22/7) \times (0.027/8))$

So we get

Number of lead shots = 84000

Therefore, the number of lead shots are 84000.

7. How many lead balls, each of radius 1cm, can be made from a sphere of radius 8cm? Solution:

It is given that

Radius of lead ball = 1 cm

Radius of sphere = 8cm

We know that

Number of lead balls = volume of sphere/volume of one lead ball

So we get

Number of lead balls = $(4/3 \pi R^3)/(4/3 \pi r^3)$

By substituting the values

Number of lead balls = $(4/3 \times (22/7) \times 8^3)/(4/3 \times (22/7) \times 1^3)$

On further calculation

Number of lead balls = $(4/3 \times (22/7) \times 512)/(4/3 \times (22/7) \times 1)$

We get

Number of lead balls = 512

Therefore, 512 lead balls can be made from the sphere.

8. A solid sphere of radius 3cm is melted and then cast into smaller spherical balls, each of diameter 6cm. Find the number of small balls thus obtained.

Solution:

It is given that

Radius of sphere = 3 cm

Diameter of spherical ball = 0.6cm

Radius of spherical ball = 0.6/2 = 0.3cm

We know that

Number of balls = Volume of sphere/ Volume of one small ball

So we get



Number of balls = $(4/3 \times (22/7) \times 3^3)/(4/3 \times (22/7) \times 0.3^3)$ On further calculation Number of balls = $(4/3 \times (22/7) \times 27)/(4/3 \times (22/7) \times 0.027)$ We get Number of balls = 1000

Therefore, 1000 balls are obtained from the solid sphere.

9. A metallic sphere of radius 10.5cm is melted and then recast into smaller cones, each of radius 3.5cm and height 3cm. How many cones are obtained? Solution:

It is given that Radius of the sphere = 10.5cm Radius of smaller cone = 3.5cm

Height = 3cm

We know that

Number of cones = Volume of the sphere/ Volume of one small cone

So we get

Number of cones = $(4/3 \times (22/7) \times 10.5^3)/(1/3 \times (22/7) \times 3.5^2 \times 3)$

On further calculation

Number of cones = 4851/38.5 = 126

Therefore, 126 cones are obtained from the metallic sphere.

10. How many spheres 12cm in diameter can be made from a metallic cylinder of diameter 8cm and height 90cm?

Solution:

It is given that
Diameter of the sphere = 12cm
Radius of the sphere = 12/2 = 6cm

We know that Volume of the sphere = $4/3 \pi r^3$ By substituting the values Volume of the sphere = $4/3 \times (22/7) \times 6^3$ So we get

Volume of the sphere = 905.142 cm^3

It is given that Diameter of the cylinder = 8cm Radius of the cylinder = 8/2 = 4cm Height of the cylinder = 90cm

We know that Volume of the cylinder = $\pi r^2 h$ By substituting the values Volume of the cylinder = $(22/7) \times 4^2 \times 90$



So we get

Volume of the cylinder = 4525.714 cm^3

We know that

Number of spheres = Volume of cylinder/ Volume of sphere

By substituting the values

Number of spheres = 4525.714/905.142 = 5

Therefore, 5 spheres can be made from a metallic cylinder.

11. The diameter of a sphere is 6cm. It is melted and drawn into a wire of diameter 2mm. Find the length of the wire.

Solution:

It is given that Diameter of the sphere = 6 cmRadius of the sphere = 6/2 = 3 cmDiameter of the wire = 2 mm = 0.2 cmRadius of the wire = 2/2 = 1 mm = 0.1 cm

Consider h cm as the required length So we get $\pi r^2 h = 4/3 \pi R^3$ By substituting the values $(22/7) \times 0.1^2 \times h = 4/3 \times (22/7) \times 3^3$ On further calculation $h = (4/3 \times (22/7) \times 27)/((22/7) \times 0.1^2)$ So we get h = 36/0.01 = 3600 cm = 36 m

Therefore, the length of the wire is 36m.

12. The diameter of a copper sphere is 18cm. It is melted and drawn into a long wire of uniform cross section. If the length of the wire is 108m, find its diameter. Solution:

It is given that
Diameter of the sphere = 18cm
Radius of the sphere = 18/2 = 9cm
Length of the wire = 108m = 10800 cm

We know that $\pi r^2 h = 4/3 \pi r^3$ By substituting the values $(22/7) \times r^2 \times 10800 = 4/3 \times (22/7) \times 9^3$ On further calculation $r^2 = (4/3 \times (22/7) \times 729)/((22/7) \times 10800)$ So we get $r^2 = (4 \times 243)/10800 = 9/100$ By taking square root on the RHS



$$r = 3/10 = 0.3$$
 cm
Diameter = 2 (0.3) = 0.6 cm

Therefore, the diameter of the wire is 0.6cm.

13. A sphere of diameter 15.6cm is melted and cast into a right circular cone of height 31.2cm. Find the diameter of the base of the cone.

Solution:

It is given that Diameter of the sphere = 15.6 cmRadius of the sphere = 15.6/2 = 7.8 cmHeight of the cone = 31.2 cm

We know that $4/3 \pi R^3 = 1/3 \pi r^2 h$ So we get $4/3 \times (22/7) \times 7.8^3 = 1/3 \times (22/7) \times r^2 \times 31.2$ On further calculation $r^2 = (4/3 \times (22/7) \times 7.8^3)/(1/3 \times (22/7) \times 31.2)$ So we get $r^2 = (4 \times 474.552)/(31.2) = 60.84$ By taking square root on the RHS r = 7.8 cmDiameter = 2 (7.8) = 15.6 cm

Therefore, the diameter of the base of the cone is 15.6cm.

14. A spherical cannonball 28cm in diameter is melted and cast into a right circular cone mould, whose base is 35cm in diameter. Find the height of the cone. Solution:

It is given that Diameter of the sphere = 28cmRadius of the sphere = 28/2 = 14cmDiameter of the cone = 35cmRadius of the cone = 35/2 = 17.5cm

We know that $4/3 \pi R^3 = 1/3 \pi r^2 h$ So we get $4/3 \times (22/7) \times 14^3 = 1/3 \times (22/7) \times 17.5^2 \times h$ On further calculation $h = (4/3 \times (22/7) \times 14^3)/(1/3 \times (22/7) \times 17.5^2)$ We get h = 10976/306.25h = 35.84 cm

Therefore, the height of the cone is 35.84cm.



15. A spherical ball of radius 3cm is melted and recast into three spherical balls. The radii of two of these balls are 1.5cm and 2cm. Find the radius of the third ball. Solution:

Consider r cm as the radius of the third ball

We know that

 $4/3 \pi (3)^3 = 4/3 \pi (3/2)^3 + 4/3 \pi (2)^3 + 4/3 \pi r^3$

It can be written as

 $4/3 \pi (27) = 4/3 \pi (27/8) + 4/3 \pi (8) + 4/3 \pi r^3$

Dividing the entire equation by $4/3 \pi$

 $27 = 27/8 + 8 + r^3$

On further calculation

 $r^3 = 27 - (27/8 + 8)$

By taking LCM

 $r^3 = 27 - ((27 + 64)/8)$

So we get

 $r^3 = 27 - (91/8)$

By taking LCM

 $r^3 = (216 - 91)/8 = 125/8$

By taking cube root

r = 5/2 = 2.5 cm

Therefore, the radius of the third ball is 2.5cm.

16. The radii of two spheres are in the ratio 1:2. Find the ratio of their surface areas. Solution:

Consider x and 2x as the radius of two spheres and S1 and S2 as the surface areas

It can be written as

 $S_1/S_2 = 4\pi x^2/4\pi (2x)^2$

On further calculation

 $S_1/S_2 = x^2/4x^2$

So we get

 $S_1/S_2 = \frac{1}{4}$

Therefore, the ratio of their surface areas is 1:4.

17. The surface areas of two spheres are in the ratio 1:4. Find the ratio of their volumes. Solution:

Consider r and R as the radii of two spheres

We know that

 $4\pi r^2 / 4\pi R^2 = \frac{1}{4}$

So we get

 $(r/R)^2 = (1/2)^2$

It can be written as

 $r/R = \frac{1}{2}$

Consider V_1 and V_2 as the volumes of the spheres

So we get



$$V_1/V_2 = (4/3 \text{ mr}^3)/(4/3 \text{ mR}^3)$$

We can write it as
 $(r/R)^3 = (1/2)^3 = 1/8$

Therefore, the ratio of their volumes is 1: 8.

18. A cylindrical tub of radius 12cm contains water to a depth of 20cm. A spherical iron ball is dropped into the tub and thus the level of water is raised by 6.75cm. What is the radius of the ball? Solution:

Consider r cm as the radius of ball and R cm as the radius of cylindrical tub

So we get

 $4/3 \pi r^3 = \pi R^2 h$

By substituting the values

 $4/3 \times \pi \times r^3 = \pi \times 12^2 \times 6.75$

On further calculation

 $r^3 = (\pi \times 12^2 \times 6.75)/4/3 \times \pi$

So we get

 $r^3 = 2916/4 = 729$

By taking cube root

r = 9cm

Therefore, the radius of the ball is 9cm.

19. A cylindrical bucket with base radius 15cm is filled with water up to a height of 20cm. A heavy iron spherical ball of radius 9cm is dropped into the bucket to submerge completely in the water. Find the increase in the level of water.

Solution:

It is given that

Radius of the cylindrical bucket = 15cm

Height of the cylindrical bucket = 2cm

We know that

Volume of water in bucket = π r²h

By substituting the values

Volume of water in bucket = $(22/7) \times 15^2 \times 20$

So we get

Volume of water in bucket = 14142.8571 cm³

It is given that

Radius of spherical ball = 9cm

We know that

Volume of spherical ball = $4/3 \pi r^3$

By substituting the values

Volume of spherical ball = $4/3 \times (22/7) \times 9^3$

So we get

Volume of spherical ball = $3054.8571 \text{ cm}^3 \dots (1)$



Consider h cm as the increase in water level

So we get

Volume of increased water level = π r²h

By substituting the values

Volume of increased water level = $(22/7) \times 15^2 \times h \dots (2)$

By equating both the equations

 $3054.8571 = (22/7) \times 15^2 \times h$

On further calculation

 $h = 3054.8571/((22/7) \times 15^2) = 4.32 \text{ cm}$

Therefore, the increase in the level of water is 4.32cm.

20. The outer diameter of a spherical shell is 12cm and its inner diameter is 8cm. Find the volume of metal contained in the shell. Also, find its outer surface area. Solution:

It is given that

Outer diameter of spherical shell = 12cm

Radius of spherical shell = 12/2 = 6cm

Inner diameter of spherical shell = 8cm

Radius of spherical shell = 8/4 = 2cm

We know that

Volume of outer shell = $4/3 \pi r^3$

By substituting the values

Volume of outer shell = $4/3 \times (22/7) \times 6^3$

So we get

Volume of outer shell = 905.15 cm^3

Volume of inner shell = $4/3 \pi r^3$

By substituting the values

Volume of outer shell = $4/3 \times (22/7) \times 4^3$

So we get

Volume of outer shell = 268.20 cm^3

So the volume of metal contained in the shell = Volume of outer shell - Volume of inner shell

By substituting the values

Volume of metal contained in the shell = 905.15 - 268.20 = 636.95 cm³

We know that

Outer surface area = $4\pi r^2$

By substituting the values

Outer surface area = $4 \times (22/7) \times 6^2$

On further calculation

Outer surface area = 452.57 cm^2

Therefore, the volume of metal contained in the shell is 636.95 cm³ and the outer surface area is 452.57 cm².



21. A hollow spherical shell is made of a metal of density 4.5g per cm³. If its internal and external radii are 8cm and 9cm respectively, find the weight of the shell. Solution:

It is given that Internal radius of the spherical shell = 8cm External radius of the spherical shell = 9cm Density of metal = 4.5g per cm³

We know that Weight of shell = $4/3 \pi [R^3 - r^3] \times Density$ By substituting the values Weight of shell = $4/3 \times (22/7) \times [9^3 - 8^3] \times (4.5/1000)$ On further calculation Weight of shell = $4/3 \times (22/7) \times [729 - 512] \times (4.5/1000)$ So we get Weight of shell = $4/3 \times (22/7) \times 217 \times (4.5/1000)$ We get Weight of shell = 4.092 kg

Therefore, the weight of the shell is 4.092kg.

22. A hemisphere of lead of radius 9cm is cast into a right circular cone of height 72cm. Find the radius of the base of the cone.

Solution:

It is given that Radius of hemisphere = 9cm Height of cone = 72cm

Consider r cm as the radius of the base of cone

We know that $1/3 \pi r^2 h = 2/3 \pi R^3$

By substituting the values

 $1/3 \times \pi \times r^2 \times 72 = 2/3 \times \pi \times 9^3$

On further calculation

 $r^2 = (2/3 \times \pi \times 729)/(1/3 \times \pi \times 72)$

So we get

 $r^2 = 20.25$

By taking square root

r = 4.5 cm

Therefore, the radius of the base of the cone is 4.5 cm.

23. A hemispherical bowl of internal radius 9cm contains a liquid. This liquid is to be filled into cylindrical shaped small bottles of diameter 3cm and height 4cm. How many bottles are required to empty the bowl? Solution:

It is given that

Internal radius of the hemispherical bowl = 9cm



Diameter of the hemispherical bowl = 9/2 = 4.5 cm

Diameter of the bottle = 3 cm

Radius of the bottle = 3/2 = 1.5cm

Height of the bottle = 4cm

We know that

Number of bottles = Volume of bowl/ Volume of each bottle

So we get

Number of bottles = $(2/3 \pi R^3)/(\pi r^2 h)$

By substituting the values

Number of bottles = $(2/3 \pi (9)^3)/(\pi (3/2)^2h)$

On further calculation

Number of bottles = $(2/3 (9)^3)/((3/2)^2h)$

So we get

Number of bottles = 54

Therefore, 54 bottles are required to empty the bowl.

24. A hemispherical bowl is made of steel 0.5cm thick. The inside radius of the bowl is 4cm. Find the volume of steel used in making the bowl. Solution:

It is given that

Internal radius of the hemispherical bowl = 4cm

Thickness of the hemispherical bowl = 0.5cm

We know that

External radius = 4 + 0.5 = 4.5 cm

We know that

Volume of steel used in making the hemispherical bowl = volume of the shell

So we get

Volume of steel used in making the hemispherical bowl = $2/3 \pi (4.5^3 - 4^3)$

On further calculation

Volume of steel used in making the hemispherical bowl = $2/3 \times (22/7) \times [91.125 - 64]$

We get

Volume of steel used in making the hemispherical bowl = $2/3 \times (22/7) \times 27.125 = 56.83$ cm³

Therefore, the volume of steel used in making the bowl is 56.83 cm³.

25. A hemispherical bowl is made of steel 0.25cm thick. The inner radius of the bowl is 5cm. Find the outer curved surface area of the bowl.

Solution:

It is given that

Inner radius of the bowl = 5 cm

Thickness of the bowl = 0.25cm

External radius = 5 + 0.25 = 5.25cm

We know that

Outer curved surface area of the bowl = $2 \pi r^2$



By substituting the values

Outer curved surface area of the bowl = $2 \times (22/7) \times 5.25^2$

On further calculation

Outer curved surface area of the bowl = $2 \times (22/7) \times 27.5625$

So we get

Outer curved surface area of the bowl = 173.25 cm^2

Therefore, the outer curved surface area of the bowl is 173.25 cm².

26. A hemispherical bowl made of brass has inner diameter 10.5cm. Find the cost of tin-plating it on the inside at the rate of ₹ 32 per 100cm².

Solution:

It is given that

Inner diameter of the hemispherical bowl = 10.5cm

Inner radius of the hemispherical bowl = 10.5/2 = 5.25cm

We know that

Inner curved surface area of the bowl = $2 \pi r^2$

By substituting the values

Inner curved surface area of the bowl = $2 \times (22/7) \times 5.25^2$

On further calculation

Inner curved surface area of the bowl = $2 \times (22/7) \times 27.5625$

So we get

Inner curved surface area of the bowl = 173.25 cm^2

It is given that

Cost of tin plating inside = ₹ 32 per 100cm²

So the cost of tin plating $173.25 \text{ cm}^2 = \text{ } \{(32 \times 173.25)/100 = \text{ } \text{ } \text{ } 55.44 \}$

Therefore, the cost of tin plating it on the inside is ₹ 55.44.

27. The diameter of the moon is approximately one fourth of the diameter of the earth. What fraction of the volume of the earth is the volume of the moon?

Solution:

Consider d as the diameter of the earth

So radius of the earth = d/2

Consider d/4 as the diameter of the moon

So radius of the moon = d/8

We know that

Volume of moon = $4/3 \pi (d/8)^3$

On further calculation

Volume of moon = $1/512 \times 4/3 \pi d^3$

We know that

Volume of earth = $4/3 \pi (d/2)^3$

On further calculation

Volume of earth = $1/8 \times 4/3 \pi d^3$



So we get Volume of moon/ Volume of earth = $(1/512 \times 4/3 \pi d^3)/(1/8 \times 4/3 \pi d^3)$ On further calculation Volume of moon/ Volume of earth = 1/64

Therefore, the volume of moon is 1/64 of volume of earth.

28. Volume and surface area of a solid hemisphere are numerically equal. What is the diameter of the hemisphere?

Solution:

We know that Volume of solid hemisphere = Surface area of solid hemisphere So we get $2/3 \pi r^3 = 3 \pi r^2$ It can be written as $r^3/r^2 = (3 \times \pi \times 3)/(2 \times \pi)$ We get r = 9/2 units

So the diameter = 2(9/2) = 9 units

Therefore, the diameter of the hemisphere is 9units.