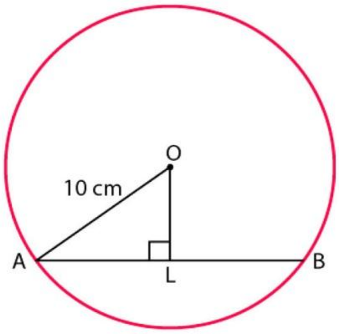
EXERCISE 12(A) PAGE: 436

1. A chord of length 16cm is drawn in a circle of radius 10cm. Find the distance of the chord from the centre of the circle.
Solution:

Consider AB as the chord with O as the centre and radius 10cm



So we get
OA = 10 cm and AB = 16cm
Construct OL ⊥ AB
Perpendicular from the centre of a circle to a chord bisects the chord
So we get
AL = ½ × AB
By substituting the values
AL = ½ × 16
So we get
AL = 8 cm

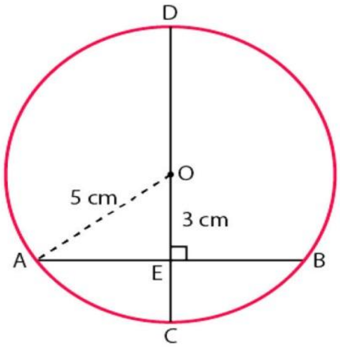
Consider △ OLA
Using the Pythagoras theorem it can be written as
$OA^2 = OL^2 + AL^2$
By substituting the values we get
$10^2 = OL^2 + 8^2$
On further calculation
$OL^2 = 10^2 - 8^2$
So we get
$OL^2 = 100 – 64$
By subtraction
$OL^2 = 36$
By taking the square root
OL = √36
So we get
OL = 6cm

Therefore, the distance of the chord from the centre of the circle is 6cm.

2. Find the length of a chord which is at a distance of 3cm from the centre of a circle of radius 5cm.
Solution:

Consider AB as the chord of the circle with O as the centre and radius 5cm.

Construct OL ⊥ AB



It is given that OA = 5cm and OL = 3cm
Perpendicular from the centre of a circle to a chord bisects the chord

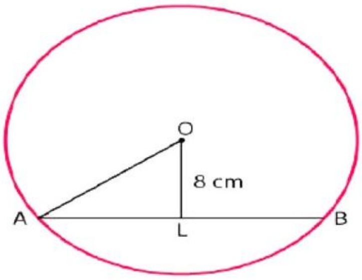
Consider △ OLA
Using the Pythagoras theorem it can be written as
$OA^2 = OL^2 + AL^2$
By substituting the values we get
$5^2 = OL^2 + 3^2$
On further calculation
$OL^2 = 5^2 - 3^2$
So we get
$OL^2 = 25 – 9$
By subtraction
$OL^2 = 16$
By taking the square root
$OL = \sqrt{16}$
So we get
OL = 4cm

We know that
AB = 2AL
By substituting the values
AB = 2 × 4
So we get
AB = 8cm

Therefore, length of the chord is 8cm.

3. A chord of length 30cm is drawn at a distance of 8cm from the centre of a circle. Find out the radius of the circle.
Solution:

Consider AB as as the chord of the circle with O as the centre
Construct OL ⊥ AB



From the figure we know that OL is the distance from the centre of chord
It is given that AB = 30cm and OL = 8cm

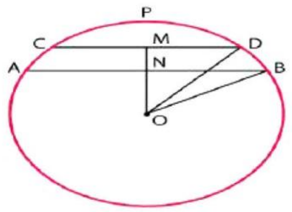
Perpendicular from the centre of a circle to a chord bisects the chord
So we get
AL = ½ × AB
By substituting the values
AL = ½ × 30
By division
AL = 15cm

Consider △ OLA
Using the Pythagoras theorem it can be written as
$OA^2 = OL^2 + AL^2$
By substituting the values we get
$OA^2 = 8^2 + 15^2$
On further calculation
$OA^2 = 64 + 225$
By addition
$OA^2 = 289$
By taking the square root
$OA = \sqrt{289}$
So we get
OA = 17cm

Therefore, the radius of the circle is 17cm.

4. In a circle of radius 5cm, AB and CD are two parallel chords of lengths 8cm and 6cm respectively. Calculate the distance between the chords if they are
(i) on the same side of the centre,
(ii) on the opposite sides of the centre.
Solution:

(i) Consider AB and CD as the two chords of the circle where AB || CD lying on the same side of the circle
It is given that AB = 8cm and OB = OD = 5cm
Draw a line to join the points OL and LM
Perpendicular from the centre of a circle to a chord bisects the chord



We know that
LB = ½ × AB
By substituting the values we get
LB = ½ × 8
So we get
LB = 4cm

We know that
MD = ½ × CD
By substituting the values we get
MD = ½ × 6
So we get
MD = 3cm

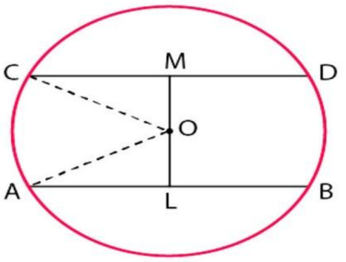
Consider △ BLO
Using the Pythagoras theorem it can be written as
$OB^2 = LB^2 + LO^2$
By substituting the values we get
$5^2 = 4^2 + LO^2$
On further calculation
$LO^2 = 5^2 - 4^2$
So we get
$LO^2 = 25 – 16$
By subtraction
$LO^2 = 9$
By taking the square root
LO = √9
LO = 3cm

Consider △ DMO
Using the Pythagoras theorem it can be written as
$OD^2 = MD^2 + MO^2$
By substituting the values we get
$5^2 = 3^2 + MO^2$
On further calculation
$MO^2 = 5^2 - 3^2$
So we get
$MO^2 = 25 – 9$
By subtraction
$MO^2 = 16$
By taking the square root
MO = √16
MO = 4cm

So the distance between the chords = MO – LO
By substituting the values
Distance between the chords = 4 – 3 = 1cm

Therefore, the distance between the chords on the same side of the centre is 1cm.

(ii) Consider AB and CD as the chords of the circle and AB || CD on the opposite sides of the centre
It is given that AB = 8cm and CD = 6cm.
Construct OL ⊥ AB and OM ⊥ CD
Join the diagonals OA and OC



We know that OA = OC = 5cm
Perpendicular from the centre of a circle to a chord bisects the chord

We know that AL = ½ × AB
By substituting the values we get
AL = ½ × 8
So we get
AL = 4cm

We know that CM = ½ × CD
By substituting the values we get
CM = ½ × 6
So we get
CM = 3cm

Consider △ OLA
Using the Pythagoras theorem it can be written as
$OA^2 = AL^2 + OL^2$
By substituting the values
$5^2 = 4^2 + OL^2$
On further calculation
$OL^2 = 5^2 - 4^2$
So we get
$OL^2 = 25 – 16$
By subtraction
$OL^2 = 9$
By taking the square root
OL = √9
OL = 3cm

Consider △ OMC
Using the Pythagoras theorem it can be written as

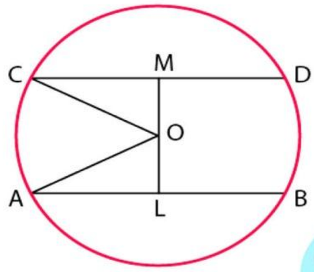
$OC^2 = OM^2 + CM^2$
By substituting the values
$5^2 = OM^2 + 3^2$
On further calculation
$OM^2 = 5^2 - 3^2$
$OM^2 = 25 - 9$
By subtraction
$OM^2 = 16$
By taking the square root
$OM = \sqrt{16}$
OM = 4cm

Distance between the chords = OM + OL
So we get
Distance between the chords = 4 + 3 = 7cm

Therefore, the distance between the chords on the opposite sides of the centre is 7 cm.

5. Two parallel chords of lengths 30cm and 16cm are drawn on the opposite sides of the centre of a circle of radius 17cm. Find the distance between the chords.
Solution:

Consider AB and CD as the chords of circle with centre O



It is given that AB = 30cm and CD = 16cm
Join the lines OA and OC
We know that AO = 17cm and CO = 17cm
Construct OM ⊥ CD and OL ⊥ AB
Perpendicular from the centre of a circle to a chord bisects the chord

We know that
AL = ½ × AB
By substituting the values
AL = ½ × 30
So we get
AL = 15cm

We know that
CM = ½ × CD
By substituting the values
CM = ½ × 16
So we get

CM = 8cm

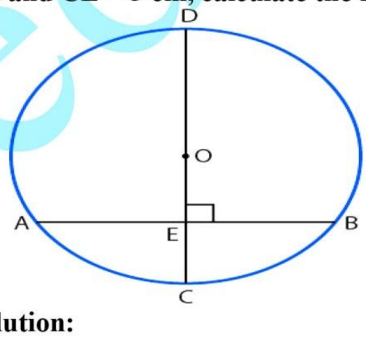
Consider △ ALO
Using the Pythagoras theorem it can be written as
$AO^2 = OL^2 + AL^2$
By substituting the values
$17^2 = OL^2 + 15^2$
So we get
$OL^2 = 17^2 - 15^2$
On further calculation
$OL^2 = 289 – 225$
By subtraction
$OL^2 = 64$
By taking the square root
$OL = \sqrt{64}$
OL = 8cm

Consider △ CMO
Using the Pythagoras theorem it can be written as
$CO^2 = CM^2 + OM^2$
By substituting the values
$17^2 = 8^2 + OM^2$
So we get
$OM^2 = 17^2 - 8^2$
On further calculation
$OM^2 = 289 – 64$
By subtraction
$OM^2 = 225$
By taking the square root
$OM = \sqrt{225}$
OM = 15cm

So the distance between the chords = OM + OL
By substituting the values
Distance between the chords = 8 + 15 = 23cm

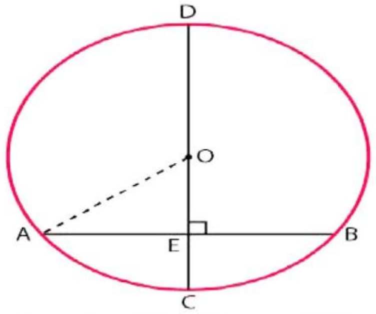
Therefore, the distance between the chords is 23 cm.

6. In the given figure, the diameter CD of a circle with centre O is perpendicular to chord AB. If AB = 12 cm and CE = 3 cm, calculate the radius of the circle.



Solution:

From the figure we know that CD is the diameter of the circle with centre O which is perpendicular to chord AB.
Draw the line OA.



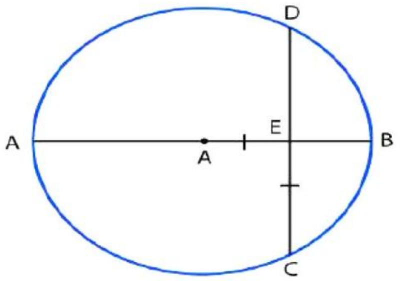
It is given that AB = 12cm and CE = 3cm
Consider OA = OC = r cm
It can be written as
OE = (r – 3) cm
Perpendicular from the centre of a circle to a chord bisects the chord

We know that
AE = ½ × AB
By substituting the values
AE = ½ × 12
So we get
AE = 6 cm

Consider △ OEA
By using the Pythagoras theorem
$OA^2 = OE^2 + AE^2$
By substituting the values
$r^2 = (r - 3)^2 + 6^2$
So we get
$r^2 = r^2 - 6r + 9 + 36$
On further calculation
$r^2 - r^2 + 6r = 45$
So we get
6r = 45
By division
r = 45/6
r = 7.5 cm

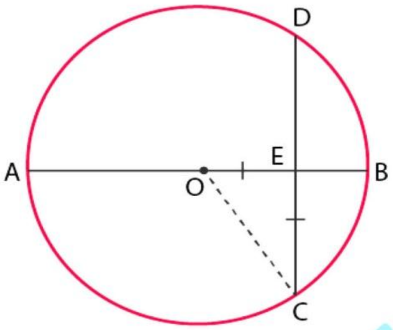
Therefore, the radius of the circle is 7.5 cm.

7. In the given figure, a circle with centre O is given in which a diameter AB bisects the chord CD at a point E such that CE = ED = 8cm and EB = 4cm. Find the radius of the circle.



Solution:

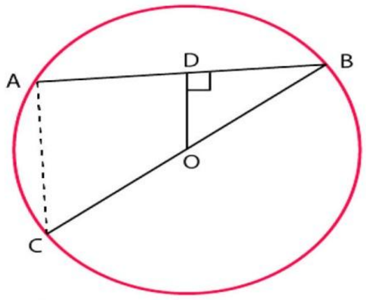
Consider AB as the diameter with centre O which bisects the chord CD at E
It is given that
CE = ED = 8cm and EB = 4cm
Join the diagonal OC
Consider OC = OB = r cm
It can be written as
OE = (r – 4) cm



Consider △ OEC
By using the Pythagoras theorem
$OC^2 = OE^2 + EC^2$
By substituting the values we get
$r^2 = (r - 4)^2 + 8^2$
On further calculation
$r^2 = r^2 - 8r + 16 + 64$
So we get
$r^2 = r^2 - 8r + 80$
It can be written as
$r^2 - r^2 + 8r = 80$
We get
$8r = 80$
By division
r = 10 cm

Therefore, the radius of the circle is 10 cm.

8. In the adjoining figure, OD is perpendicular to the chord AB of a circle with centre O. If BC is a diameter, show that AC || DO and AC = 2 × OD.



Solution:

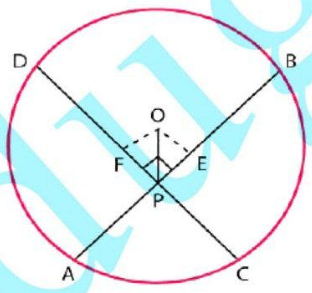
Perpendicular from the centre of a circle to a chord bisects the chord
We know that OD ⊥ AB
From the figure we know that D is the midpoint of AB
We get
AD = BD

We also know that O is the midpoint of BC
We get
OC = OB

Consider △ ABC
Using the midpoint theorem
We get OD || AC and
OD = ½ × AC
By cross multiplication
AC = 2 × OD

Therefore, it is proved that AC || DO and AC = 2 × OD.

9. In the given figure, O is the centre of a circle in which chords AB and CD intersect at P such that PO bisects ∠BPD. Prove that AB = CD.



Solution:

Consider △ OEP and △ OFP
We know that
∠OEP = ∠OFP = 90°
OP is common i.e. OP = OP
From the figure we know that OP bisects ∠BPD
It can be written as

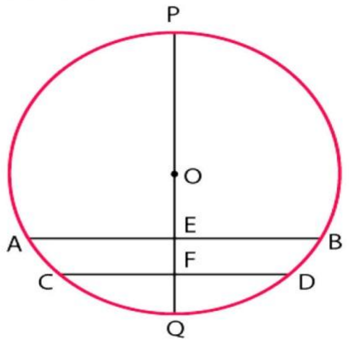
∠OPE = ∠OPF

By ASA congruence criterion
△ OEP ≅ △ OFP
OE = OF (c. p. c. t)

We know that AB and CD are equidistant from the centre
So we get
AB = CD

Therefore, it is proved that AB = CD.

10. Prove that the diameter of a circle perpendicular to one of the two parallel chords of a circle is perpendicular to the other and bisects it.
Solution:



Consider AB || CD and POQ as the diameter
It is given that ∠PEB = 90°

From the figure we know that AB || CD and ∠PFD and ∠PEB are corresponding angles
So we get
∠PFD = ∠PEB

It can be written as
PF ⊥ CD
In the same way
OF ⊥ CD

Perpendicular from the centre of a circle to a chord bisects the chord
So we get
CF = FD

Therefore, it is proved that the diameter of a circle perpendicular to one of the two parallel chords of a circle is perpendicular to the other and bisects it.

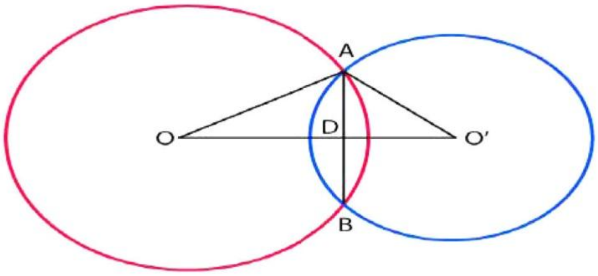
11. Prove that two different circles cannot intersect each other at more than two points.
Solution:

Consider two different circles intersecting at three point A, B and C
We know that these points are non collinear and a unique circle can be drawn using these points

This shows that our assumption is wrong

Therefore, it is proved that two different circles cannot intersect each other at more than two points.

12. Two circles of radii 10 cm and 8 cm intersect each other, and the length of the common chord is 12 cm. Find the distance between their centres.



Solution:

It is given that
OA = 10cm and AB = 12 cm

So we get
AD = ½ × AB
By substituting the values
AD = ½ × 12
By division
AD = 6cm

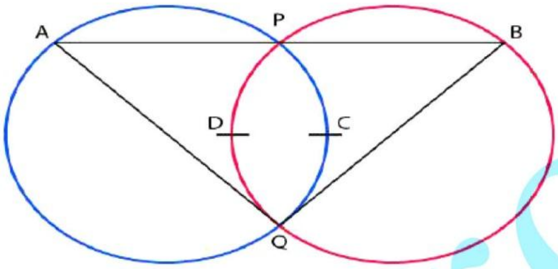
Consider △ ADO
Using the Pythagoras theorem
$OA^2 = AD^2 + OD^2$
By substituting the values we get
$10^2 = 6^2 + OD^2$
On further calculation
$OD^2 = 10^2 - 6^2$
So we get
$OD^2 = 100 - 36$
By subtraction
$OD^2 = 64$
By taking the square root
OD = √64
So we get
OD = 8cm
We know that O'A = 8cm
Consider △ ADO'
Using the Pythagoras theorem
$O'A^2 = AD^2 + O'D^2$
By substituting the values we get
$8^2 = 6^2 + O'D^2$
On further calculation
$O'D^2 = 8^2 - 6^2$

So we get
$O'D^2 = 64 - 36$
By subtraction
$O'D^2 = 28$
By taking the square root
$O'D = \sqrt{28}$
We get
$O'D = 2\sqrt{7}$

We know that
OO' = OD + O'D
By substituting the values
$OO' = (8 + 2\sqrt{7})$ cm

Therefore, the distance between their centres is $(8 + 2\sqrt{7})$ cm.

13. Two equal circles intersect in P and Q. A straight line through P meets the circles in A and B. Prove that QA = QB.



Solution:

We know that two circles will be congruent if they have equal radii
From the figure we know that if the two chords are equal then the corresponding arcs are congruent

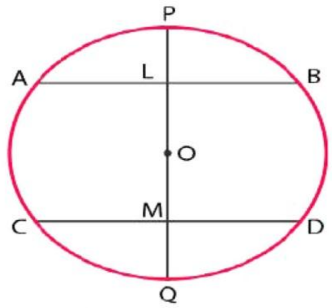
We know that PQ is the common chord in both the circles
So their corresponding arcs are equal
It can be written as
Arc PCQ = arc PDQ

We know that the congruent arcs have the same degree
So we get
∠QAP = ∠QBP

We know that the base angles of isosceles triangle are equal
So we get
QA = QB

Therefore, it is proved that QA = QB.

14. If a diameter of a circle bisects each of the two chords of a circle then prove that the chords are parallel.
Solution:

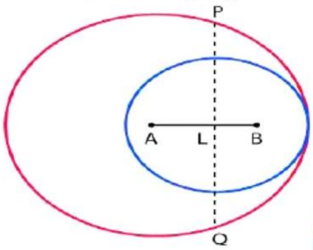


It is given that AB and CD are the two chords of the circle having O as the centre
We know that POQ bisects them at the points L and M

So we get
OL ⊥ AB and OM ⊥ CD
We know that the alternate angles are equal
∠ALM = ∠LMD
We get
AB || CD

Therefore, it is proved that the chords are parallel.

15. In the adjoining figure, two circles with centres at A and B, and of radii 5cm and 3cm touch each other internally. If the perpendicular bisector of AB meets the bigger circle in P and Q, find the length of PQ.

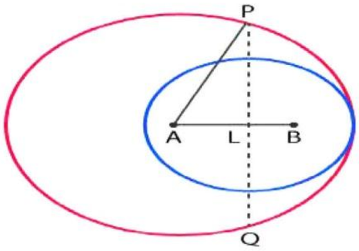


Solution:

It is given that two circles with centres at A and B, and of radii 5cm and 3cm touch each other internally.
The perpendicular bisector of AB meets the bigger circle in P
Join the line AP

From the figure we know that PQ intersects the line AB at L
So we get
AB = (5 – 3)
AB = 2cm

We know that that PQ is the perpendicular bisector of AB
So we get
AL = ½ × AB
Substituting the values
AL = ½ × 2
So we get
AL = 1 cm

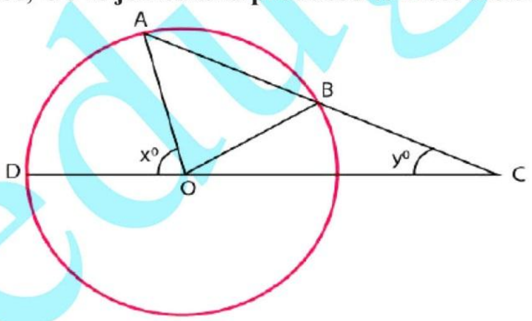


Consider △ PLA
Using the Pythagoras theorem
$AP^2 = AL^2 + PL^2$
By substituting the values we get
$5^2 = 1^2 + PL^2$
So we get
$PL^2 = 5^2 - 1^2$
On further calculation
$PL^2 = 25 – 1$
$PL^2 = 24$
By taking the square root
$PL = \sqrt{24}$
So we get
$PL = 2\sqrt{6}$ cm

We know that
$PQ = 2 \times PL$
By substituting the values
$PQ = 2 \times 2\sqrt{6}$
$PQ = 4\sqrt{6}$ cm

Therefore, the length of $PQ = 4\sqrt{6}$ cm.

16. In the given figure, AB is a chord of a circle with centre O and AB is produced to C such that BC = OB. Also, CO is joined and produced to meet the circle in D. If ∠ACD = y° and ∠AOD = x°, prove that x = 3y.



Solution:

It is given that AB is a chord of a circle with centre O and AB is produced to C such that BC = OB
We know that ∠BOC and ∠BCO form isosceles triangle
∠BOC = ∠BCO = y°

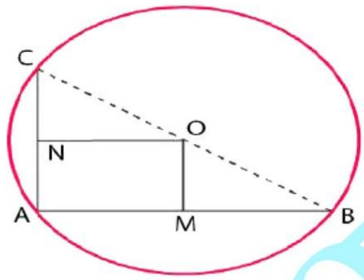
We know that OA and OB are the radii of same circle
OA = OB

We know that ∠OAB and ∠OBA form isosceles triangle
∠OAB = ∠OBA = 2y°

From the figure we know that
Exterior ∠AOD = ∠OAC + ∠ACO
It can be written as
Exterior ∠AOD = ∠OAB + ∠BCO
So we get
Exterior ∠AOD = 3y°
It is given that
∠AOD = x°
So we get
x° = 3y°

Therefore, it is proved that x = 3y.

17. AB and AC are two chords of a circle of radius r such that AB = 2AC. If p and q are the distances of AB and AC from the centre then prove that $4q^2 = p^2 + 3r^2$.
Solution:



Consider O as the centre of the circle with radius r
So we get
OB = OC = r

Consider AC = x and AB = 2x

We know that OM ⊥ AB
So we get
OM = p

We know that ON ⊥ AC
So we get
ON = q

Consider △ OMB
Using the Pythagoras theorem
$OB^2 = OM^2 + BM^2$

We know that the perpendicular from the centre of the circle bisects the chord

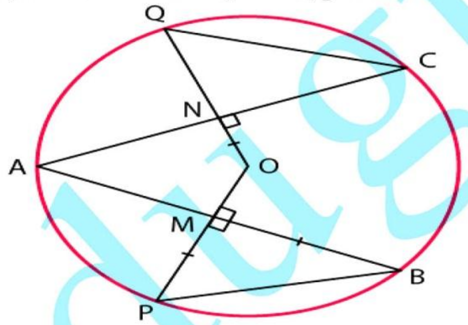
So we get
$r^2 = p^2 + ((1/2)\ AB)^2$
It can be written as
$r^2 = p^2 + \frac{1}{4} \times 4x^2$
So we get
$r^2 = p^2 + x^2$ ……… (1)

Consider △ ONC
Using the Pythagoras theorem
$OC^2 = ON^2 + CN^2$

We know that the perpendicular from the centre of the circle bisects the chord
So we get
$r^2 = q^2 + ((1/2)\ AC)^2$
It can be written as
$r^2 = q^2 + x^2/4$
We get
$q^2 = r^2 - x^2/4$
Multiplying the equation by 4
$4q^2 = 4r^2 - x^2$
Substituting equation (1)
$4q^2 = 4r^2 – (r^2 - p^2)$
So we get
$4q^2 = 3r^2 + p^2$

Therefore, it is proved that $4q^2 = p^2 + 3r^2$.

18. In the adjoining figure, O is the centre of a circle. If AB and AC are chords of the circle such that AB = AC, OP ⊥ AB and OQ ⊥ AC, prove that PB = QC.



Solution:

It is given that AB = AC
Dividing the equation by 2
We get
½ AB = ½ AC

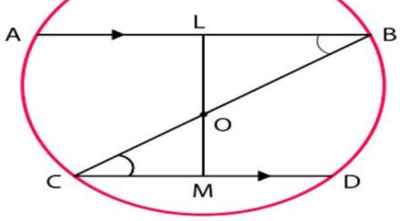
Perpendicular from the centre of a circle to a chord bisects the chord
MB = NC ……. (1)

We know that the equal chords are equidistant from the centre of the circle

OM = ON and OP = OQ
Subtracting both the equations
OP – OM = OQ - ON
So we get
PM = QN ……… (2)
Consider △ MPB and △ NQC
We know that
∠PMB = ∠QNC = 90°
By SAS congruence criterion
△ MPB ≅ △ NQC
PB = QC (c. p. c. t)

Therefore, it is proved that PB = QC.

19. In the adjoining figure, BC is a diameter of a circle with centre O. If AB and CD are two chords such that AB || CD, prove that AB = CD.



Solution:

Construct OL ⊥ AB and OM ⊥ CD
Consider △ OLB and △ OMC
We know that ∠OLB and ∠OMC are perpendicular bisector
∠OLB = ∠OMC = 90°

We know that AB || CD and BC is a transversal
From the figure we know that ∠OBL and ∠OCD are alternate interior angles
∠OBL = ∠OCD
So we get OB = OC which is the radii
By AAS congruence criterion
△ OLB ≅ △ OMC
OL = CM (c. p. c. t)

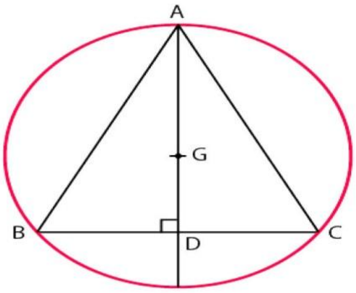
We know that the chords equidistant from the centre are equal
So we get
AB = CD

Therefore, it is proved that AB = CD.

20. An equilateral triangle of side 9cm is inscribed in a circle. Find the radius of the circle.
Solution:

Consider △ ABC as an equilateral triangle with side 9 cm

Take AD as one of its median
We know that
AD ⊥ BC
It can be written as
BD = ½ × BC
By substituting the values
BD = ½ × 9
So we get
BD = 4.5 cm

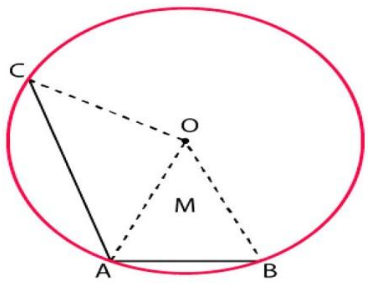


Consider △ ADB
Using the Pythagoras theorem
$AB^2 = AD^2 + BD^2$
Substituting the values
$9^2 = AD^2 + (9/2)^2$
On further calculation
$AD^2 = 9^2 – (9/2)^2$
So we get
$AD^2 = 81 – 81/4$
By taking out the square root
AD = 9√3/ 2 cm

We know that the centroid and circumcenter coincide in a equilateral triangle
AG: GD = 2: 1
The radius can be written as
AG = 2/3 AD
By substituting the values
AG = (2/3) × (9√3/ 2)
So we get
AG = 3√3 cm

Therefore, the radius of the circle is 3√3 cm.

21. In the adjoining figure, AB and AC are two equal chords of a circle with centre O. Show that O lies on the bisector of ∠BAC.



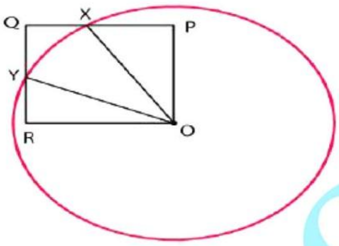
Solution:

Consider △ OAB and △ OAC
It is given that AB = AC
OA is common i.e. OA = OA
From the figure we know that OB = OC which is the radii

By SSS congruence criterion
△ OAB ≅ △ OAC
∠OAB = ∠OAC (c. p. c. t)

Therefore, it is proved that O lies on the bisector of ∠BAC.

22. In the adjoining figure, OPQR is a square. A circle drawn with centre O cuts the square in X and Y. Prove that QX = QY.



Solution:

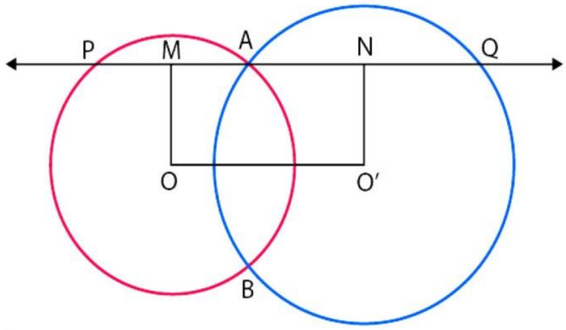
Consider △ OXP and △ OYR
We know that ∠OPX and ∠ORY are right angles
So we get
∠OPX = ∠ORY = 90°
We know that OX and OY are the radii
OX = OY
From the figure we know that the sides of a square are equal
OP = OR

By RHS congruence criterion
△ OXP ≅ △ OYR
PX = RY (c. p. c. t)
We know that PQ = QR
So we get
PQ – PX = QR – RY
QX = QY

Therefore, it is proved that QX = QY.

23. Two circles with centres O and O' intersect at two points A and B. A line PQ is drawn parallel to OO' through A or B, intersecting the circles at P and Q. Prove that PQ = 2OO'.
Solution:

Construct OM ⊥ PQ and O'N ⊥ PQ
So we get
OM ⊥ AP



We know that the perpendicular from the centre of a circle bisects the chord
AM = PM
It can be written as
AP = 2AM ………. (1)

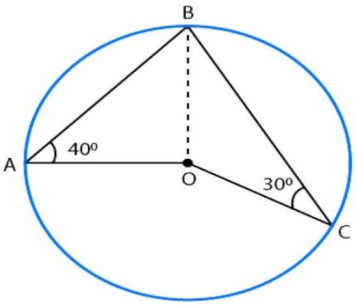
We know that
O'N ⊥ AQ
We know that the perpendicular from the centre of a circle bisects the chord
AN = QN
It can be written as
AQ = 2AN ……. (2)

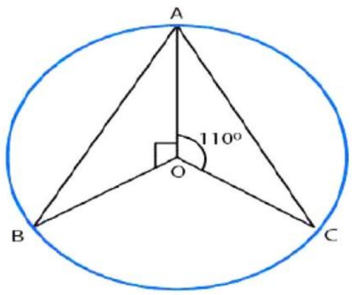
So we get
PQ = AP + PQ
By substituting the values
PQ = 2 (AM + AN)
We get
PQ = 2MN
From the figure we know that MNO'O is a rectangle
PQ = 2OO'

Therefore, it is proved that PQ = 2OO'.

EXERCISE 12(B) PAGE: 456

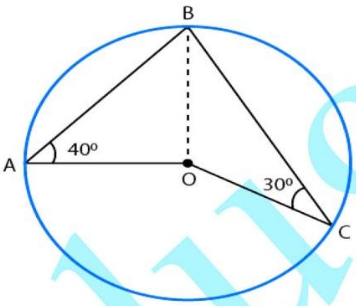
1. (i) In Figure (1), O is the centre of the circle. If ∠OAB = 40° and ∠OCB = 30°, find ∠AOC.
(ii) In Figure (2), A, B and C are three points on the circle with centre O such that ∠AOB = 90° and ∠AOC = 110°. Find ∠BAC.





Solution:

(i) Join the line BO



Consider △ BOC
We know that the sides are equal to the radius
So we get
OC = OB
From the figure we know that the base angles of an isosceles triangle are equal
∠OBC = ∠OCB

It is given that
∠OCB = 30°
So we get
∠OBC = ∠OCB = 30°

So we get ∠OBC = 30° ……. (1)

Consider △ BOA
We know that the sides are equal to the radius
So we get
OB = OC
From the figure we know that the base angles of an isosceles triangle are equal
∠OAB = ∠OBA

It is given that
∠OAB = 40°
So we get
∠OAB = ∠OBA = 40°

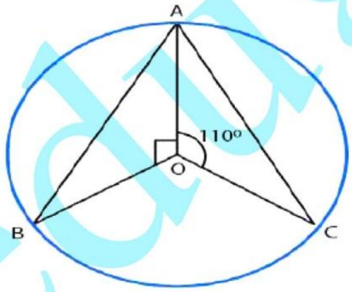
So we get ∠OBA = 40° ……. (2)

We know that
∠ABC = ∠OBC + ∠OBA
By substituting the values
∠ABC = 30° + 40°
So we get
∠ABC = 70°

We know that the angle subtended by an arc of a circle at the centre is double the angle subtended by the arc at any point on the circumference.
It can be written as
∠AOC = 2 × ∠ABC
So we get
∠AOC = 2 × 70°
By multiplication
∠AOC = 140°

Therefore, ∠AOC = 140°

(ii) From the figure we know that



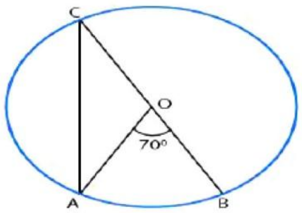
∠AOB + ∠AOC + ∠BOC = 360°
By substituting the values
90° + 110° + ∠BOC = 360°
On further calculation
∠BOC = 360° - 90° - 110°
By subtraction
∠BOC = 360° - 200°
So we get

∠BOC = 160°

We know that
∠BOC = 2 × ∠BAC
It is given that ∠BOC = 160°
∠BAC = 160°/2
By division
∠BAC = 80°

Therefore, ∠BAC = 80°.

2. In the given figure, O is the centre of the circle and ∠AOB = 70°. Calculate the values of
(i) ∠OCA,
(ii) ∠OAC.



Solution:

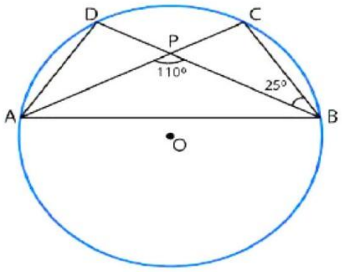
(i) We know that the angle subtended by an arc of a circle at the centre is double the angle subtended by the arc at any point on the circumference.
So we get
∠AOB = 2 ∠OCA
It is given that ∠AOB = 70°
We can write it as
∠OCA = 70°/2
By division
∠OCA = 35°

Therefore, ∠OCA = 35°.

(ii) From the figure we know that the radius is
OA = OC
We know that the base angles of an isosceles triangle are equal
∠OAC = ∠OCA
It is given that ∠OCA = 35°
So we get
∠OAC = 35°

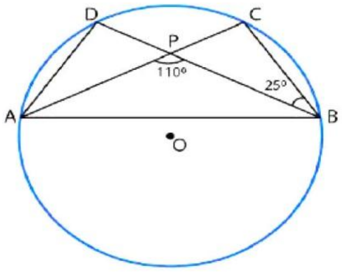
Therefore, ∠OAC = 35°.

3. In the given figure, O is the centre of the circle. If ∠PBC = 25° and ∠APB = 110°, find the value of ∠ADB.



Solution:

We know that ∠ACB = ∠PCB

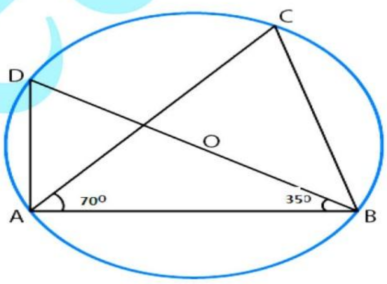


In △ PCB
Using the angle sum property
∠PCB + ∠BPC + ∠PBC = 180°
We know that ∠APB and ∠BPC are linear pair
By substituting the values
∠PCB + (180° – 110°) + 25° = 180°
On further calculation
∠PCB + 70° + 25° = 180°
∠PCB + 95° = 180°
By subtraction
∠PCB = 180° - 95°
So we get
∠PCB = 85°

We know that the angles in the same segment of a circle are equal
∠ADB = ∠ACB = 85°

Therefore, the value of ∠ADB is 85°.

4. In the given figure, O is the centre of the circle. If ∠ABD = 35° and ∠BAC = 70°, find ∠ACB.



Solution:

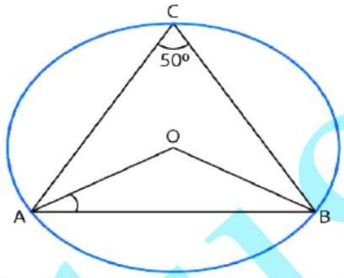
We know that BD is the diameter of the circle
Angle in a semicircle is a right angle
$\angle BAD = 90^o$

Consider $\triangle$ BAD
Using the angle sum property
$\angle ADB + \angle BAD + \angle ABD = 180^o$
By substituting the values
$\angle ADB + 90^o + 35^o = 180^o$
On further calculation
$\angle ADB = 180^o - 90^o - 35^o$
By subtraction
$\angle ADB = 180^o - 125^o$
So we get
$\angle ADB = 55^o$

We know that the angles in the same segment of a circle are equal
$\angle ACB = \angle ADB = 55^o$
So we get
$\angle ACB = 55^o$

Therefore, $\angle ACB = 55^o$.

5. In the given figure, O is the centre of the circle. If $\angle ACB = 50^o$, find $\angle OAB$.



Solution:

We know that the angle subtended by an arc of a circle at the centre is double the angle subtended by the arc at any point on the circumference.
So we get
$\angle AOB = 2 \angle ACB$
It is given that $\angle ACB = 50^o$
By substituting
$\angle AOB = 2 \times 50^o$
By multiplication
$\angle AOB = 100^o$ …… (1)

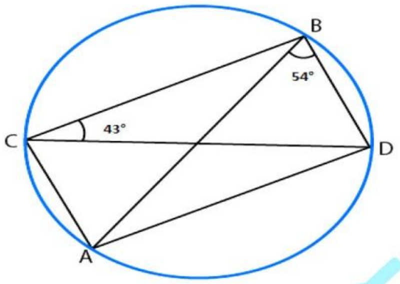
Consider $\triangle$ OAB
We know that the radius of the circle are equal
OA = OB

Base angles of an isosceles triangle are equal
So we get
∠OAB = ∠OBA ……. (2)

Using the angle sum property
∠AOB + ∠OAB + ∠OBA = 180°
Using equations (1) and (2) we get
100° + 2 ∠OAB = 180°
By subtraction
2 ∠OAB = 180° - 100°
So we get
2 ∠OAB = 80°
By division
∠OAB = 40°

Therefore, ∠OAB = 40°.

6. In the given figure, ∠ABD = 54° and ∠BCD = 43°, calculate
(i) ∠ACD,
(ii) ∠BAD,
(iii) ∠BDA.



Solution:

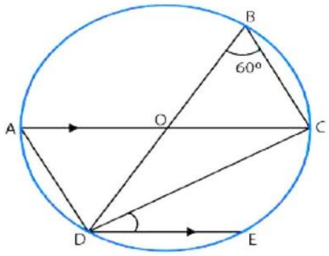
(i) We know that the angles in the same segment of a circle are equal
From the figure we know that ∠ABD and ∠ACD are in the segment AD
∠ABD = ∠ACD = 54°

(ii) We know that the angles in the same segment of a circle are equal
From the figure we know that ∠BAD and ∠BCD are in the segment BD
∠BAD = ∠BCD = 43°

(iii) In △ ABD
Using the angle sum property
∠BAD + ∠ADB + ∠DBA = 180°
By substituting the values
43° + ∠ADB + 54° = 180°
On further calculation
∠ADB = 180° - 43° - 54°
By subtraction
∠ADB = 180° - 97°
So we get

∠ADB = 83°
It can be written as
∠BDA = 83°

7. In the adjoining figure, DE is a chord parallel to diameter AC of the circle with centre O. If ∠CBD = 60°, calculate ∠CDE.



Solution:

We know that the angles in the same segment of a circle are equal
From the figure we know that ∠CAD and ∠CBD are in the segment CD
∠CAD = ∠CBD = 60°

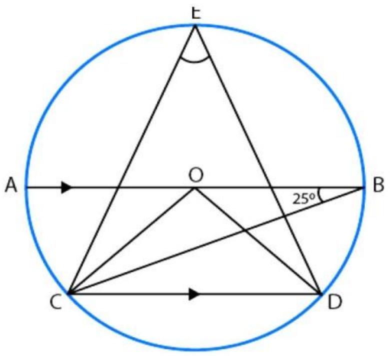
An angle in a semi-circle is a right angle
So we get
∠ADC = 90°

Using the angle sum property
∠ACD + ∠ADC + ∠CAD = 180°
By substituting the values
∠ACD + 90° + 60° = 180°
On further calculation
∠ACD = 180° - 90° - 60°
By subtraction
∠ACD = 180° - 150°
So we get
∠ACD = 30°

We know that AC || DE and CD is a transversal
From the figure we know that ∠CDE and ∠ACD are alternate angles
So we get
∠CDE = ∠ACD = 30°

Therefore, ∠CDE = 30°.

8. In the adjoining figure, O is the centre of a circle. Chord CD is parallel to diameter AB. If ∠ABC = 25°, calculate ∠CED.



Solution:

We know that ∠BCD and ∠ABC are alternate interior angles
∠BCD = ∠ABC = 25°

We know that the angle subtended by an arc of a circle at the centre is double the angle subtended by the arc at any point on the circumference.
∠BOD = 2 ∠BCD
It is given that ∠BCD = 25°
So we get
∠BOD = 2 (25°)
By multiplication
∠BOD = 50°

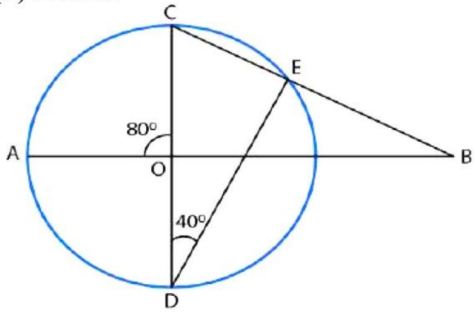
In the same way
∠AOC = 2 ∠ABC
So we get
∠AOC = 50°

From the figure we know that AB is a straight line passing through the centre
Using the angle sum property
∠AOC + ∠COD + ∠BOD = 180°
By substituting the values
50° + ∠COD + 50° = 180°
On further calculation
∠COD + 100° = 180°
By subtraction
∠COD = 180° - 100°
So we get
∠COD = 80°

We know that
∠CED = ½ ∠COD
So we get
∠CED = 80°/2
By division
∠CED = 40°

Therefore, ∠CED = 40°.

9. In the given figure, AB and CD are straight lines through the centre O of a circle. If ∠AOC = 80° and ∠CDE = 40°, find
(i) ∠DCE,
(ii) ∠ABC.

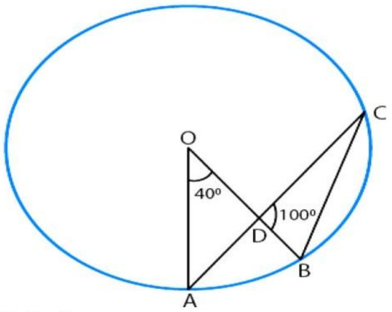


Solution:

(i) From the figure we know that ∠CED = 90°
Consider △ CED
Using the angle sum property
∠CED + ∠EDC + ∠DCE = 180°
By substituting the values
90° + 40° + ∠DCE = 180°
On further calculation
∠DCE = 180° - 90° - 40°
By subtraction
∠DCE = 180° - 130°
So we get
∠DCE = 50°
(ii) We know that ∠AOC and ∠BOC form a linear pair
It can be written as
∠BOC = 180° - 80°
By subtraction
∠BOC = 100°

Using the angle sum property
∠ABC + ∠BOC + ∠DCE = 180°
By substituting the values
∠ABC + 100° + 50° = 180°
On further calculation
∠ABC = 180° - 100° - 50°
By subtraction
∠ABC = 180° - 150°
So we get
∠ABC = 30°

10. In the given figure, O is the centre of a circle, ∠AOB = 40° and ∠BDC = 100°, find ∠OBC.



Solution:

We know that the angle subtended by an arc of a circle at the centre is double the angle subtended by the arc at any point on the circumference.
So we get
∠AOB = 2 ∠ACB
From the figure we know that
∠ACB = ∠DCB
It can be written as
∠AOB = 2 ∠DCB

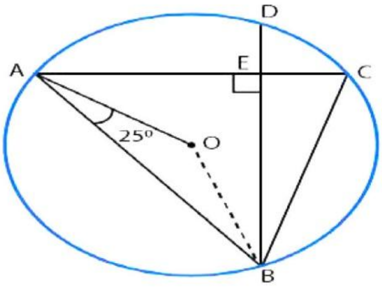
We also know that
∠DCB = ½ ∠AOB
By substituting the values
∠DCB = 40°/2
By division
∠DCB = 20°

In △ DBC
Using the angle sum property
∠BDC + ∠DCB + ∠DBC = 180°
By substituting the values we get
100° + 20° + ∠DBC = 180°
On further calculation
∠DBC = 180° - 100° - 20°
By subtraction
∠DBC = 180° - 120°
So we get
∠DBC = 60°

From the figure we know that
∠OBC = ∠DBC = 60°
So we get
∠OBC = 60°

Therefore, ∠OBC = 60°.

11. In the adjoining figure, chords AC and BD of a circle with centre O, intersect at right angles at E. If ∠OAB = 25°, calculate ∠EBC.



Solution:

We know that OA and OB are the radius
Base angles of an isosceles triangle are equal
So we get
$\angle OBA = \angle OAB = 25^\circ$

Consider $\triangle$ OAB
Using the angle sum property
$\angle OAB + \angle OBA + \angle AOB = 180^\circ$
By substituting the values
$25^\circ + 25^\circ + \angle AOB = 180^\circ$
On further calculation
$\angle AOB = 180^\circ - 25^\circ - 25^\circ$
By subtraction
$\angle AOB = 180^\circ - 50^\circ$
So we get
$\angle AOB = 130^\circ$

We know that the angle subtended by an arc of a circle at the centre is double the angle subtended by the arc at any point on the circumference
So we get
$\angle AOB = 2\ \angle ACB$
It can be written as
$\angle ACB = ½\ \angle AOB$
By substituting the values
$\angle ACB = 130/2$
By division
$\angle ACB = 65^\circ$

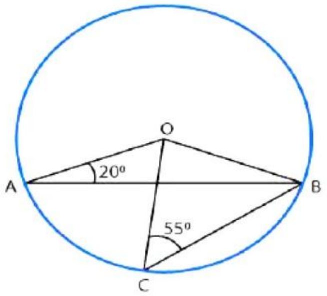
So we get
$\angle ECB = 65^\circ$

In $\triangle$ BEC
Using the angle sum property
$\angle EBC + \angle BEC + \angle ECB = 180^\circ$
By substituting the values
$\angle EBC + 90^\circ + 65^\circ = 180^\circ$
On further calculation
$\angle EBC = 180^\circ - 90^\circ - 65^\circ$
By subtraction

∠EBC = 180° - 155°
So we get
∠EBC = 25°

Therefore, ∠EBC = 25°.

12. In the given figure, O is the centre of a circle in which ∠OAB = 20° and ∠OCB = 55°. Find
(i) ∠BOC,
(ii) ∠AOC.



Solution:

(i) We know that OB = OC which is the radius
The base angles of an isosceles triangle are equal
So we get
∠OBC = ∠OCB = 55°

In △ BOC
Using the angle sum property
∠BOC + ∠OCB + ∠OBC = 180°
By substituting the values
∠BOC + 55° + 55° = 180°
On further calculation
∠BOC = 180° - 55° - 55°
By subtraction
∠BOC = 180° - 110°
So we get
∠BOC = 70°

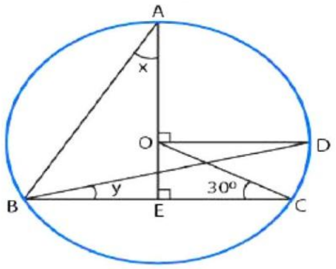
(ii) We know that OA = OB which is the radius
The base angles of an isosceles triangle are equal
So we get
∠OBA= ∠OAB = 20°

In △ AOB
Using the angle sum property
∠AOB + ∠OAB + ∠OBA = 180°
By substituting the values
∠AOB + 20° + 20° = 180°
On further calculation
∠AOB = 180° - 20° - 20°

By subtraction
∠AOB = 180° - 40°
So we get
∠AOB = 140°

We know that
∠AOC = ∠AOB - ∠BOC
By substituting the values
∠AOC = 140° - 70°
So we get
∠AOC = 70°

13. In the given figure, O is the centre of the circle and ∠BCO = 30°. Find x and y.



Solution:

From the figure we know that ∠AOD and ∠OEC form right angles
So we get
∠AOD = ∠OEC = 90°

We know that OD || BC and OC is a transversal
From the figure we know that ∠AOD and ∠OEC are corresponding angles
∠AOD = ∠OEC

We know that ∠DOC and ∠OCE are alternate angles
∠DOC = ∠OCE = 30°

Angle subtended by an arc of a circle at its centre is twice the angle subtended by the same arc at a point on the circumference.
So we get
∠DOC = 2 ∠DBC
It can be written as
∠DBC = ½ ∠DOC
By substituting the values
∠DBC = 30/2
By division
y = ∠DBC = 15°

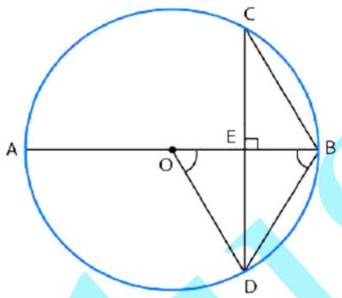
In the same way
∠ABD = ½ ∠AOD
By substituting the values
∠ABD = 90/2
By division

$\angle ABD = 45^\circ$

We know that
$\angle ABE = \angle ABC = \angle ABD + \angle DBC$
So we get
$\angle ABE = \angle ABC = 45^\circ + 15^\circ$
By addition
$\angle ABE = \angle ABC = 60^\circ$

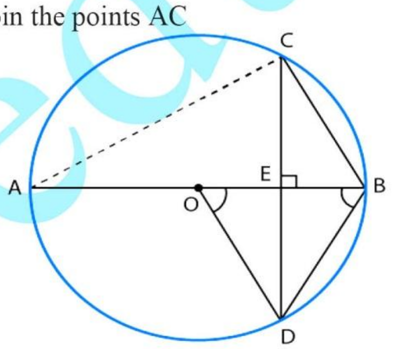
Consider $\triangle ABE$
Using the angle sum property
$\angle BAE + \angle AEB + \angle ABE = 180^\circ$
By substituting the values
$x + 90^\circ + 60^\circ = 180^\circ$
On further calculation
$x = 180^\circ - 90^\circ - 60^\circ$
By subtraction
$x = 180^\circ - 150^\circ$
So we get
$x = 30^\circ$
Therefore, the value of x is 30° and y is 15°.

14. In the given figure, O is the centre of the circle, BD = OD and CD ⊥ AB. Find ∠CAB.



Solution:

Join the points AC



It is given that BD = OD
We know that the radii of same circle are equal
OD = OB
It can be written as

BD = OD = OB

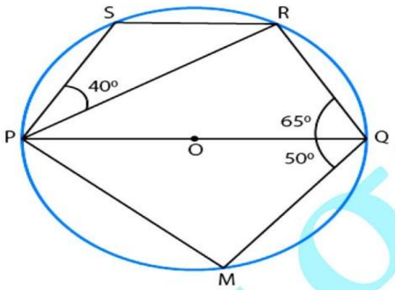
Consider △ ODB as equilateral triangle
We know that
∠ODB = 60°

The altitude of an equilateral triangle bisects the vertical angle
So we get
∠BDE = ∠ODE = ½ ∠ODB
By substituting the values
∠BDE = ∠ODE = 60/2
By division
∠BDE = ∠ODE = 30°

We know that the angles in the same segment are equal
∠CAB = ∠BDE = 30°

Therefore, ∠CAB = 30°.

15. In the given figure, PQ is a diameter of a circle with centre O. If ∠PQR = 65°, ∠SPR = 40° and ∠PQM = 50°, find ∠QPR, ∠QPM and ∠PRS.



Solution:

In △ PQR
We know that PQ is the diameter
So we get
∠PRQ = 90° as the angle in a semicircle is a right angle

Using the angle sum property
∠QPR + ∠PRQ + ∠PQR = 180°
By substituting the values
∠QPR + 90° + 65° = 180°
On further calculation
∠QPR = 180° - 90° - 65°
By subtraction
∠QPR = 180° - 155°
So we get
∠QPR = 25°

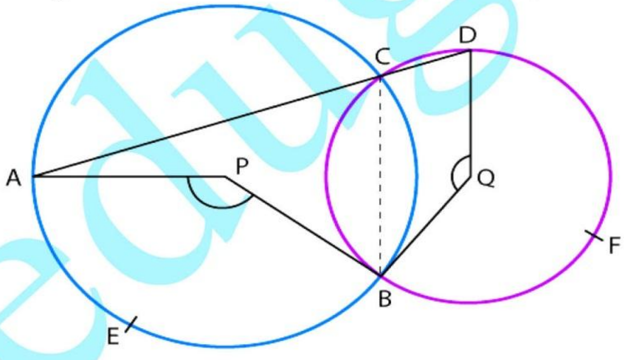
In △ PQM
We know that PQ is the diameter

So we get
∠PMQ = 90° as the angle in a semicircle is a right angle

Using the angle sum property
∠QPM + ∠PMQ + ∠PQM = 180°
By substituting the values
∠QPM + 90° + 50° = 180°
On further calculation
∠QPM = 180° - 90° - 50°
By subtraction
∠QPM = 180° - 140°
So we get
∠QPM = 40°

Consider the quadrilateral PQRS
We know that
∠QPS + ∠SRQ = 180°
It can be written as
∠QPR + ∠RPS + ∠PRQ + ∠PRS = 180°
By substituting the values
25° + 40° + 90° + ∠PRS = 180°
On further calculation
∠PRS = 180° - 25° - 40° - 90°
By subtraction
∠PRS = 180° - 155°
So we get
∠PRS = 25°

16. In the figure given below, P and Q are centres of two circles, intersecting at B and C, and ACD is a straight line. If ∠APB = 150° and ∠BQD = x°, find the value of x.



Solution:

The angle subtended by an arc of a circle at its centre is twice the angle subtended by the same arc at a point on the circumference
So we get
∠APB = 2 ∠ACB
It can be written as
∠ACB = ½ ∠APB
By substituting the values

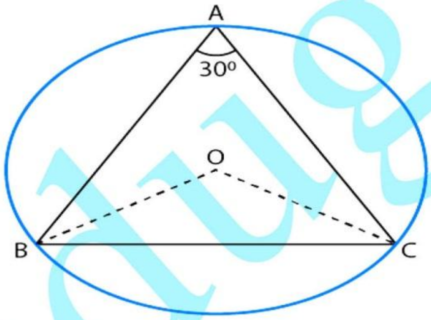
∠ACB = 150/2
So we get
∠ACB = 75°

We know that ACD is a straight line
It can be written as
∠ACB + ∠DCB = 180°
By substituting the values
75° + ∠DCB = 180°
On further calculation
∠DCB = 180° - 75°
By subtraction
∠DCB = 105°

We know that
∠DCB = ½ × reflex ∠BQD
By substituting the values
105° = ½ × (360° – x)
On further calculation
210° = 36° - x
By subtraction
x = 150°

Therefore, the value of x is 150°.

17. In the given figure, ∠BAC = 30°. Show that BC is equal to the radius of the circumcircle of △ ABC whose centre is O.



Solution:

Join the lines OB and OC
We know that the angle subtended by an arc of a circle at the centre is double the angle subtended by the arc at any point on the circumference
So we get
∠BOC = 2 ∠BAC
It is given that ∠BAC = 30°
∠BOC = 2 × 30°
By multiplication
∠BOC = 60°

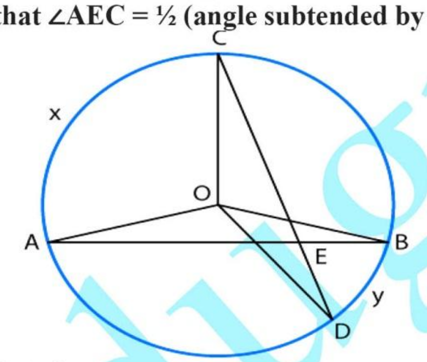
In △ BOC

We know that the radii are equal
OB = OC
The base angles of an isosceles triangle are equal
∠OBC = ∠OCB

Consider △ BOC
Using the angle sum property
∠BOC + ∠OBC + ∠OCB = 180°
By substituting the values we get
60° + ∠OCB + ∠OCB = 180°
So we get
2 ∠OCB = 180° - 60°
By subtraction
2 ∠OCB = 120°
By division
∠OBC = 60°

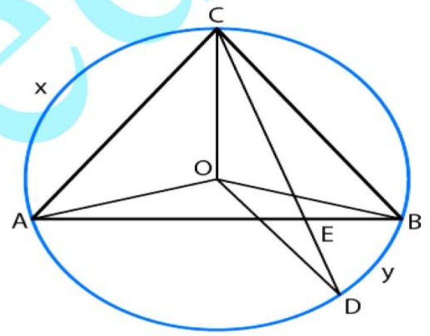
We get ∠OBC = ∠OCB = ∠BOC = 60°
We know that △ BOC is an equilateral triangle
So we get OB = OC = BC
Therefore, it is proved that BC is equal to the radius of the circumcircle of △ ABC whose centre is O.

18. In the given figure, AB and CD are two chords of a circle, intersecting each other at a point E. Prove that ∠AEC = ½ (angle subtended by arc CXA. At the centre + angle subtended by arc DYB at the centre).



Solution:

Construct the lines AC and BC



The angle subtended by an arc of a circle at its centre is twice the angle subtended by the same arc at a point on the circumference.

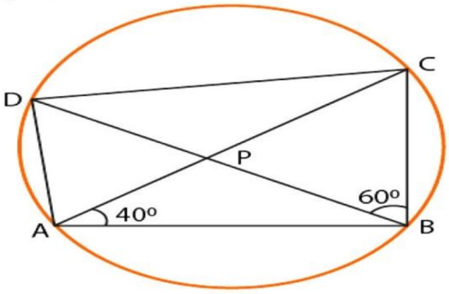
So we get
∠AOC = 2 ∠ABC …… (1)
In the same way
∠BOD = 2 ∠BCD …… (2)

By adding both the equations
∠AOC + ∠BOD = 2 ∠ABC + 2 ∠BCD
By taking 2 as common
∠AOC + ∠BOD = 2 (∠ABC + ∠BCD)
It can be written as
∠AOC + ∠BOD = 2 (∠EBC + ∠BCE)
So we get
∠AOC + ∠BOD = 2 (180° - ∠CEB)
We can write it as
∠AOC + ∠BOD = 2 (180° – (180° - ∠AEC))
We get
∠AOC + ∠BOD = 2 ∠AEC
Dividing the equation by 2
∠AEC = ½ (∠AOC + ∠BOD)
∠AEC = ½ (angle subtended by arc CXA at the centre + angle subtended by arc DYB at the centre)

Therefore, it is proved that ∠AEC = ½ (angle subtended by arc CXA. At the centre + angle subtended by arc DYB at the centre).

EXERCISE 12(C) PAGE: 482

1. In the given figure, ABCD is a cyclic quadrilateral whose diagonals intersect at P such that ∠DBC = 60° and ∠BAC = 40°. Find
(i) ∠BCD,
(ii) ∠CAD.



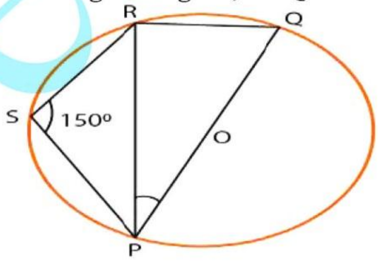
Solution:

(i) We know that the angles in the same segment are equal
So we get
∠BDC = ∠BAC = 40°

Consider △ BCD
Using the angle sum property
∠BCD + ∠BDC + ∠DBC = 180°
By substituting the values
∠BCD + 40° + 60° = 180°
On further calculation
∠BCD = 180° - 40° - 60°
By subtraction
∠BCD = 180° - 100°
So we get
∠BCD = 80°

(ii) We know that the angles in the same segment are equal
So we get
∠CAD = ∠CBD
So ∠CAD = 60°

2. In the given figure, POQ is a diameter and PQRS is a cyclic quadrilateral. If ∠PSR = 150°, find ∠RPQ.



Solution:

We know that PQRS is a cyclic quadrilateral

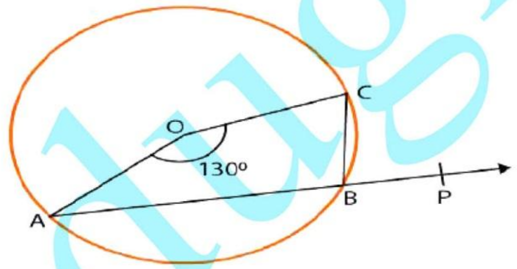
It can be written as
∠PSR+ ∠PQR = 180°
By substituting the values
150° + ∠PQR = 180°
On further calculation
∠PQR = 180° - 150°
By subtraction
∠PQR = 30°

We know that the angle in semi-circle is a right angle
∠PRQ = 90°

Consider △ PRQ
Using the angle sum property
∠PQR + ∠PRQ + ∠RPQ = 180°
By substituting the values
30° + 90° + ∠RPQ = 180°
On further calculation
∠RPQ = 180° - 30° - 90°
By subtraction
∠RPQ = 180° - 120°
So we get
∠RPQ = 60°

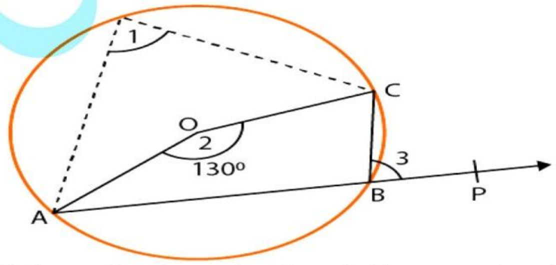
Therefore, ∠RPQ = 60°.

3. In the given figure, O is the centre of the circle and arc ABC subtends an angle of 130° at the centre. If AB is extended to P, find ∠PBC.



Solution:

Consider a point D on the arc CA and join DC and AD



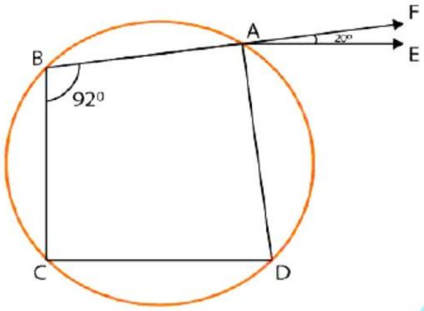
We know that the angle subtended by an arc is twice the angle subtended by it on the circumference in the alternate segment

So we get
$\angle 2 = 2\angle 1$
By substituting the values
$130^\circ = 2\angle 1$
So we get
$\angle 1 = 65^\circ$

From the figure we know that the exterior angle of a cyclic quadrilateral = interior opposite angle
$\angle PBC = \angle 1$
So we get $\angle PBC = 65^\circ$

Therefore, $\angle PBC = 65^\circ$.

4. In the given figure, ABCD is a cyclic quadrilateral in which AE is drawn parallel to CD, and BA is produced to F. If ∠ABC = 92° and ∠FAE = 20°, find ∠BCD.



Solution:

We know that ABCD is a cyclic quadrilateral
So we get
$\angle ABC + \angle ADC = 180^\circ$
By substituting the values
$92^\circ + \angle ADC = 180^\circ$
On further calculation
$\angle ADC = 180^\circ - 92^\circ$
By subtraction
$\angle ADC = 88^\circ$

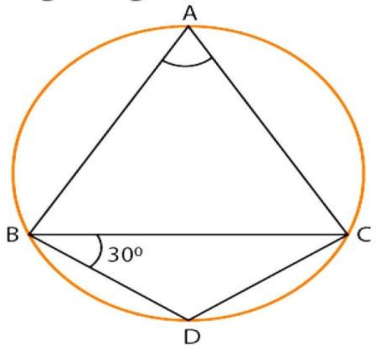
We know that AE || CD
From the figure we know that
$\angle EAD = \angle ADC = 88^\circ$
We know that the exterior angle of a cyclic quadrilateral = interior opposite angle
So we get
$\angle BCD = \angle DAF$

We know that
$\angle BCD = \angle EAD + \angle EAF$
It is given that $\angle FAE = 20^\circ$
By substituting the values
$\angle BCD = 88^\circ + 20^\circ$
By addition

$\angle BCD = 108^o$

Therefore, $\angle BCD = 108^o$.

5. In the given figure, BD = DC and ∠CBD = 30°, find ∠BAC.



Solution:

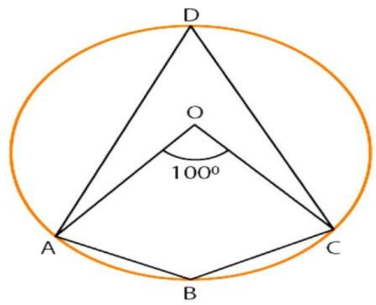
It is given that BD = DC
From the figure we know that
$\angle BCD = \angle CBD = 30^o$

Consider △ BCD
Using the angle sum property
$\angle BCD + \angle CBD + \angle CDB = 180^o$
By substituting the values
$30^o + 30^o + \angle CDB = 180^o$
On further calculation
$\angle CDB = 180^o - 30^o - 30^o$
By subtraction
$\angle CDB = 180^o - 60^o$
So we get
$\angle CDB = 120^o$

We know that the opposite angles of a cyclic quadrilateral are supplementary
It can be written as
$\angle CDB + \angle BAC = 180^o$
By substituting the values
$120^o + \angle BAC = 180^o$
On further calculation
$\angle BAC = 180^o - 120^o$
By subtraction
$\angle BAC = 60^o$

Therefore, $\angle BAC = 60^o$.

6. In the given figure, O is the centre of the given circle and measure of arc ABC is 100°. Determine ∠ADC and ∠ABC.



Solution:

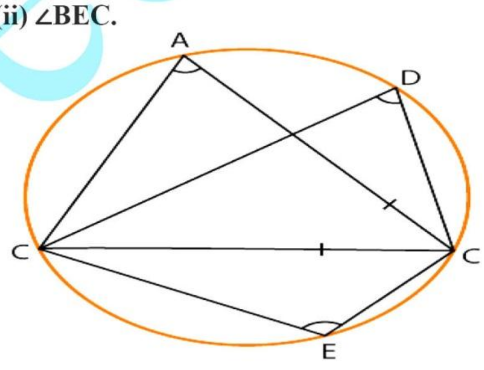
We know that the angle subtended by an arc is twice the angle subtended by it on the circumference
From the figure we know that
∠AOC = 100°

So we get
∠AOC = 2 ∠ADC
It can be written as
∠ADC = ½ ∠AOC
By substituting the values
∠ADC = ½ (100°)
So we get
∠ADC = 50°

We know that the opposite angles of a cyclic quadrilateral are supplementary
It can be written as
∠ADC + ∠ABC = 180°
By substituting the values
50° + ∠ABC = 180°
On further calculation
∠ABC = 180° - 50°
By subtraction
∠ABC = 130°

Therefore, ∠ADC = 50° and ∠ABC = 130°.

7. In the given figure, △ ABC is equilateral. Find
(i) ∠BDC,
(ii) ∠BEC.



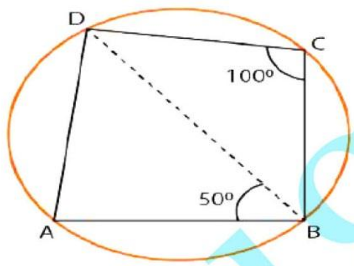
Solution:

It is given that △ ABC is equilateral
We know that
∠BAC = ∠ABC = ∠ACB = 60°

(i) We know that the angles in the same segment of a circle are equal
∠BDC = ∠BAC = 60°
So we get
∠BDC = 60°

(ii) We know that the opposite angles of a cyclic quadrilateral are supplementary
So we get
∠BAC + ∠BEC = 180°
By substituting the values
60° + ∠BEC = 180°
On further calculation
∠BEC = 180° - 60°
By subtraction
∠BEC = 120°

8. In the adjoining figure, ABCD is a cyclic quadrilateral in which ∠BCD = 100° and ∠ABD = 50°. Find ∠ADB.



Solution:

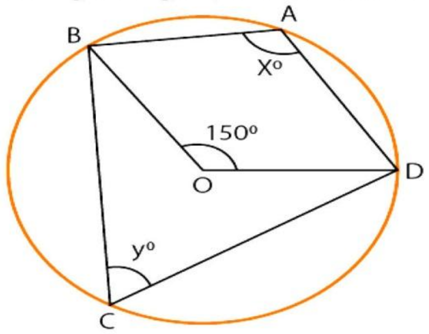
It is given that ABCD is a cyclic quadrilateral
We know that the opposite angles of a cyclic quadrilateral are supplementary
So we get
∠A + ∠C = 180°
By substituting the values
∠A + 100° = 180°
On further calculation
∠A = 180° - 100°
By subtraction
∠A = 80°

Consider △ ABD
Using the angle sum property
∠A + ∠ABD + ∠ADB = 180°
By substituting the values
80° + 50° + ∠ADB = 180°
On further calculation

$\angle ADB = 180^\circ - 80^\circ - 50^\circ$
By subtraction
$\angle ADB = 180^\circ - 130^\circ$
So we get
$\angle ADB = 50^\circ$

Therefore, $\angle ADB = 50^\circ$.

9. In the given figure, O is the centre of a circle and $\angle BOD = 150^\circ$. Find the values of x and y.



Solution:

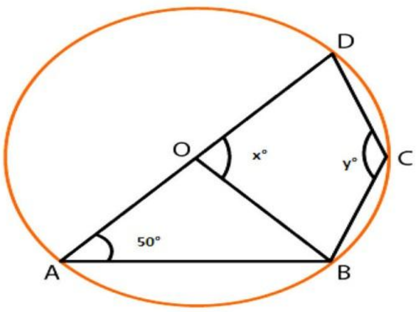
It is given that O is the centre of a circle and $\angle BOD = 150^\circ$
We know that
Reflex $\angle BOD = (360^\circ - \angle BOD)$
By substituting the values
Reflex $\angle BOD = (360^\circ – 150^\circ)$
By subtraction
Reflex $\angle BOD = 210^\circ$

Consider $x = ½$ (reflex $\angle BOD$)
By substituting the value
$x = 210/2$
So we get
$x = 105^\circ$

We know that
$x + y = 180^\circ$
By substituting the values
$105^\circ + y = 180^\circ$
On further calculation
$y = 180^\circ - 105^\circ$
By subtraction
$y = 75^\circ$

Therefore, the value of x is 105° and y is 75°.

10. In the given figure, O is the centre of the circle and $\angle DAB = 50^\circ$. Calculate the values of x and y.



Solution:

It is given that O is the centre of the circle and ∠DAB = 50°
We know that the radii of the circle are equal
OA = OB

From the figure we know that
∠OBA = ∠OAB = 50°

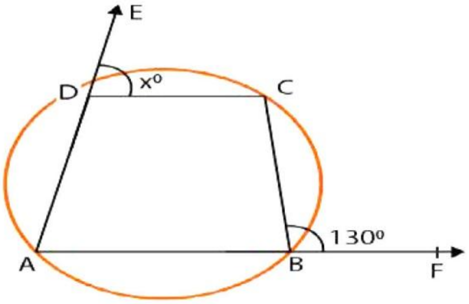
Consider △ OAB
Using the angle sum property
∠OAB + ∠OBA + ∠AOB = 180°
By substituting the values
50° + 50° + ∠AOB = 180°
On further calculation
∠AOB = 180° - 50° - 50°
By subtraction
∠AOB = 180° - 100°
So we get
∠AOB = 80°

From the figure we know that AOD is a straight line
It can be written as
x = 180° - ∠AOB
By substituting the values
x = 180° - 80°
By subtraction
x = 100°

We know that the opposite angles of a cyclic quadrilateral are supplementary
So we get
∠DAB + ∠BCD = 180°
By substituting the values
50° + ∠BCD = 180°
On further calculation
∠BCD = 180° - 50°
By subtraction
y = ∠BCD = 130°

Therefore, the value of x is 100° and y is 130°.

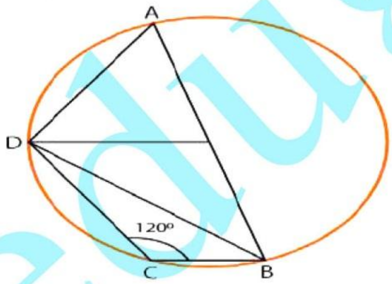
11. In the given figure, sides AD and AB of cyclic quadrilateral ABCD are produced to E and F respectively. If ∠CBF = 130° and ∠CDE = x°, find the value of x.



Solution:

In a cyclic quadrilateral we know that the exterior angle is equal to the interior opposite angle
So we get
∠CBF = ∠CDA
It can be written as
130° = 180° - x
On further calculation
x = 180° - 130°
By subtraction
x = 50°

12. In the given figure, AB is a diameter of a circle with centre O and DO || CB. If ∠BCD = 120°, calculate
(i) ∠BAD,
(ii) ∠ABD,
(iii) ∠CBD,
(iv) ∠ADC.
Also, show that △ AOD is an equilateral triangle.



Solution:

It is given that AB is a diameter of a circle with centre O and DO || CB

(i) We know that ABCD is a cyclic quadrilateral
It can be written as
∠BCD + ∠BAD = 180°
By substituting the values
120° + ∠BAD = 180°
On further calculation
∠BAD = 180° - 120°

By subtraction
∠BAD = 60°

(ii) We know that the angle in a semi-circle is right angle
∠BDA = 90°
Consider △ ABD
By using the angle sum property
∠BDA + ∠BAD + ∠ABD = 180°
By substituting the values
90° + 60° + ∠ABD = 180°
On further calculation
∠ABD = 180° - 90° - 60°
By subtraction
∠ABD = 180° - 150°
So we get
∠ABD = 30°

(iii) We know that OD = OA
So we get ∠ODA = ∠OAD = ∠BAD = 60°

From the figure we know that
∠ODB + ∠ODA = 90°
By substituting the values
∠ODB + 60° = 90°
On further calculation
∠ODB = 90° - 60°
By subtraction
∠ODB = 30°

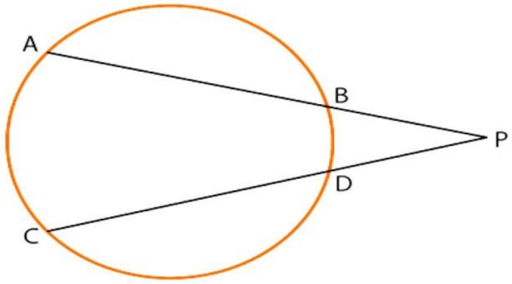
It is given that DO || CB
We know that the alternate angles are equal
∠CDB = ∠ODB = 30°

(iv) From the figure we know that
∠ADC = ∠ADB + ∠CDB
By substituting the values
∠ADC = 90° + 30°
By addition
∠ADC = 120°

Consider △ AOD
By using the angle sum property
∠ODA + ∠OAD + ∠AOD = 180°
By substituting the values
60° + 60° + ∠AOD = 180°
On further calculation
∠AOD = 180° - 60° - 60°
By subtraction
∠AOD = 180° - 120°
So we get

∠AOD = 60°
We know that all the angles of the △ AOD is 60°
Therefore, it is proved that △ AOD is an equilateral triangle.

13. Two chords AB and CD of a circle intersect each other at P outside the circle. If AB = 6cm, BP = 2cm and PD = 2.5cm, find CD.



Solution:

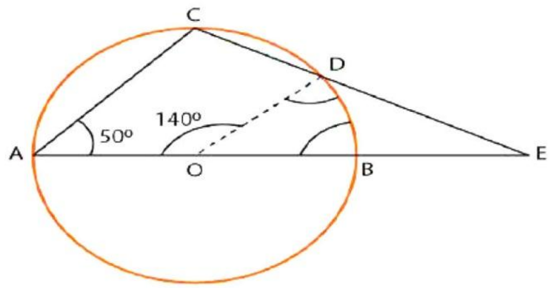
It is given that AB and CD of a circle intersect each other at P outside the circle. If AB = 6cm, BP = 2cm and PD = 2.5cm

So we get
AP × BP = CP × DP
From the figure we know that CP = CD + DP
By substituting the values
8 × 2 = (CD + 2.5) × 2.5 cm

Consider x = CD
So we get
8 × 2 = (x + 2.5) × 2.5
On further calculation
16 = 2.5x + 6.25
It can be written as
2.5x = 16 – 6.25
By subtraction
2.5x = 9.75
By division
x = 9.75/2.5
So we get
x = 3.9cm

Therefore, CD = 3.9cm.

14. In the given figure, O is the centre of a circle. If ∠AOD = 140° and ∠CAB = 50°, calculate
(i) ∠EDB,
(ii) ∠EBD.



Solution:

(i) We know that
∠BOD + ∠AOD = 180°
By substituting the values
∠BOD + 140° = 180°
On further calculation
∠BOD = 180° - 140°
By subtraction
∠BOD = 40°

We know that OB = OD
So we get ∠OBD = ∠ODB

Consider △ OBD
By using the angle sum property
∠BOD + ∠OBD + ∠ODB = 180°
We know that ∠OBD = ∠ODB
So we get
40° + 2 ∠OBD = 180°
On further calculation
2 ∠OBD = 180° - 40°
By subtraction
2 ∠OBD = 140°
By division
∠OBD = 70°

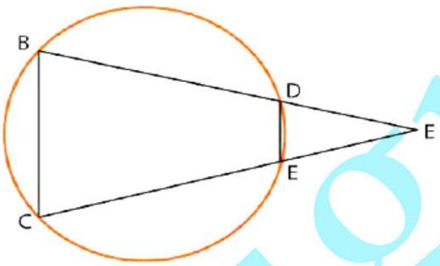
We know that ABCD is a cyclic quadrilateral
∠CAB + ∠BDC = 180°
∠CAB + ∠ODB + ∠ODC = 180°
By substituting the values
50° + 70° + ∠ODC = 180°
On further calculation
∠ODC = 180° - 50° - 70°
By subtraction
∠ODC = 180° - 120°
So we get
∠ODC = 60°

Using the angle sum property
∠EDB + ∠ODC + ∠ODB = 180°

By substituting the values
∠EDB + 60° + 70° = 180°
On further calculation
∠EDB = 180° - 60° - 70°
By subtraction
∠EDB = 180° - 130°
So we get
∠EDB = 50°

(ii) We know that
∠EDB + ∠OBD = 180°
By substituting the values
∠EDB + 70° = 180°
On further calculation
∠EDB = 180° - 70°
By subtraction
∠EDB = 110°

15. In the given figure, △ ABC is an isosceles triangle in which AB = AC and a circle passing through B and C intersects AB and AC at D and E respectively. Prove that DE || BC.

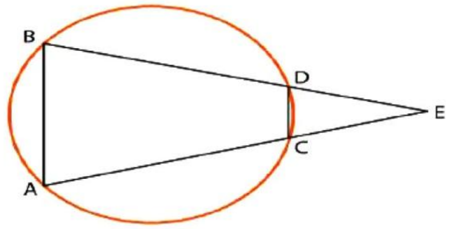


Solution:

We know that △ ABC is an isosceles triangle in which AB = AC and a circle passing through B and C intersects AB and AC at D and E
So AB = AC
We get
∠ACB = ∠ABC
It can be written as
∠ADE = ∠ACB = ∠ABC
So we get
∠ADE = ∠ABC
DE || BC

Therefore, it is proved that DE || BC.

16. In the given figure, AB and CD are two parallel chords of a circle. If BDE and ACE are straight lines, intersecting at E, prove that △ AEB is isosceles.



Solution:

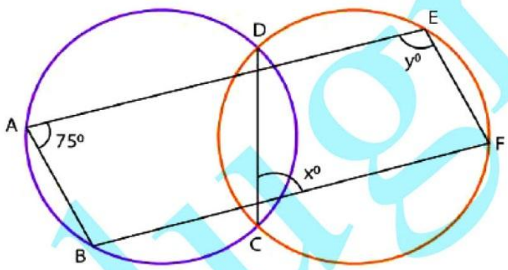
It is given that AB and CD are two parallel chords of a circle. If BDE and ACE are straight lines, intersecting at E

We know that in a cyclic quadrilateral exterior angle is equal to the interior opposite angle.
It can be written as
Exterior ∠EDC = ∠A
Exterior ∠DCE = ∠B

We know that AB || CD
So we get
∠EDC = ∠B and ∠DCE = ∠A
We get
∠A = ∠B

Therefore, it is proved that △ AEB is isosceles.

17. In the given figure, ∠BAD = 75°, ∠DCF = x° and ∠DEF = y°. Find the values of x and y.



Solution:

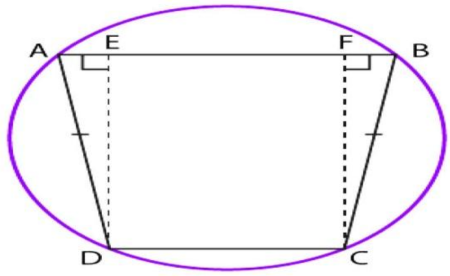
In a cyclic quadrilateral we know that the exterior angle is equal to the interior opposite angle
So we get
∠BAD = ∠DCF = 75°
It can be written as
∠DCF = x = 75°
We get x = 75°

We know that the opposite angles of a cyclic quadrilateral is 180°
So we get
∠DCF + ∠DEF = 180°
By substituting the values
75° + ∠DEF = 180°
On further calculation
∠DEF = 180° - 75°

By subtraction
∠DEF = y = 105°

Therefore, the values of x and y is 75° and 105°.

18. In the given figure, ABCD is a quadrilateral in which AD = BC and ∠ADC = ∠BCD. Show that the points A, B, C, D lie on a circle.



Solution:

It is given that ABCD is a quadrilateral in which AD = BC and ∠ADC = ∠BCD
Construct DE ⊥ AB and CF ⊥ AB

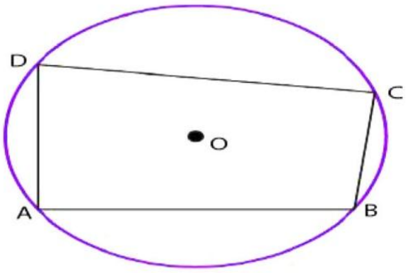
Consider △ ADE and △ BCF
We know that
∠AED + ∠BFC = 90°

From the figure it can be written as
∠ADE = ∠ADC - 90° = ∠BCD - 90° = ∠BCF
It is given that
AD = BC
By AAS congruence criterion
△ ADE ≅ △ BCF
∠A = ∠B (c. p. c. t)

We know that the sum of all the angles of a quadrilateral is 360°
∠A + ∠B + ∠C + ∠D = 360°
By substituting the values
2 ∠B + 2 ∠D = 360°
By taking 2 as common
2 (∠B + ∠D) = 360°
By division
∠B + ∠D = 360/2
So we get
∠B + ∠D = 180°
So, ABCD is a cyclic quadrilateral.

Therefore, it is proved that the points A, B, C and D lie on a circle.

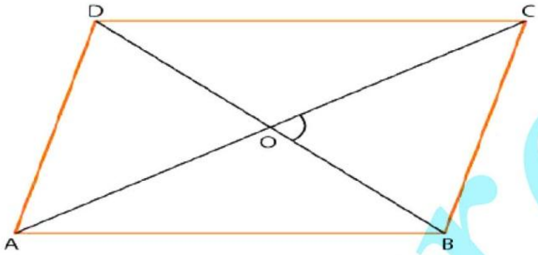
19. Prove that the perpendicular bisectors of the sides of a cyclic quadrilateral are concurrent.
Solution:



Consider ABCD as a cyclic quadrilateral with centre O passing through the points A, B, C, D
We know that AB, BC, CD and DA are the chords of the circle and its right bisector passing through the centre O
So we know that the right bisectors of AB, BC, CD and DA pass through the centre O and are concurrent.

Therefore, it is proved that the perpendicular bisectors of the sides of a cyclic quadrilateral are concurrent.

20. Prove that the circles described with the four sides of a rhombus as diameters pass through the point of intersection of its diagonals.
Solution:

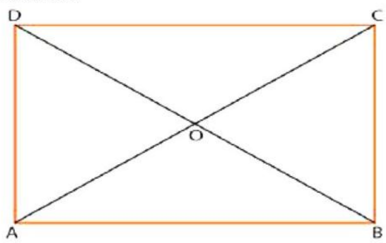


Consider ABCD as a rhombus
We know that the diagonals AC and BD intersect at the point O
From the figure we know that the diagonals of the rhombus bisect at right angles
It can be written as
$\angle BOC = 90^{\circ}$
Thus, $\angle BOC$ lies in the circle

We know that the circle can be drawn with BC as the diameter having the centre O
In the same way, all the circles with AB, AD and CD as diameters will pass through the centre O.

Therefore, it is proved that the circles described with the four sides of a rhombus as diameters pass through the point of intersection of its diagonals.

21. ABCD is a rectangle. Prove that the centre of the circle through A, B, C, D is the point of intersection of its diagonals.
Solution:



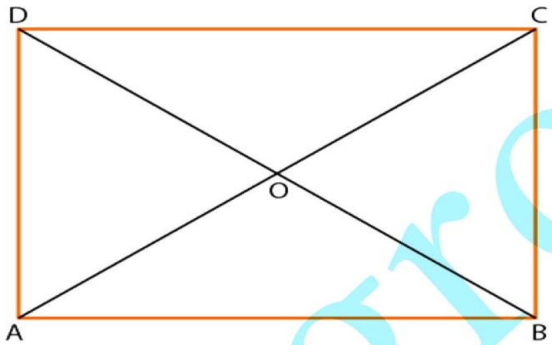
Consider ABCD as a rectangle
We know that O is the point of intersection of the diagonals AC and BD

The diagonals of a rectangle are equal and bisect each other
So we get
OA = OB = OC = OD
We get O as the centre of the circle through A, B, C and D

Therefore, it is proved that the centre of the circle through A, B, C, D is the point of intersection of its diagonals.

22. Give a geometrical construction for finding the fourth point lying on a circle passing through three given points, without finding the centre of the circle. Justify the construction.
Solution:

Consider A, B, C as the points



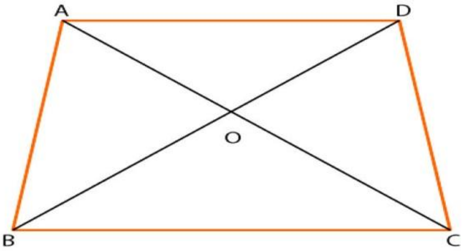
Taking B as the centre and radius equal to AC construct an arc
Taking C as the centre and radius equal to AB construct another arc which cuts the arc at D

So we get D as the required point BD and CD
Consider △ ABC and △ DCB
We know that
AB = DC and AC = DB
BC and CB are common i.e. BC = CB
By SSS congruence criterion
△ ABC ≅ △ DCB
∠BAC = ∠CDB (c. p. c. t)

We know that BC subtends equal angles ∠BAC and ∠CDB on the same side
So we get A, B, C, D are cyclic.

Therefore, the points A, B, C, D are cyclic.

23. In a cyclic quadrilateral ABCD, if (∠B - ∠D) = 60º, show that the smaller of the two is 60º.
Solution:



It is given that ABCD is a cyclic quadrilateral
(∠B - ∠D) = 60° …….. (1)
We know that
(∠B + ∠D) = 180° ……. (2)

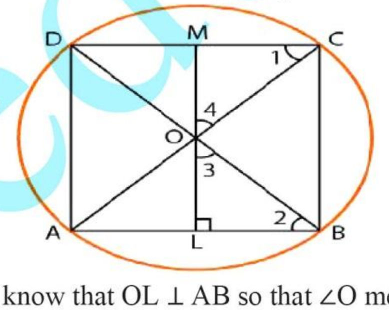
By adding both the equations
∠B - ∠D + ∠B + ∠D = 60° + 180°
So we get
2 ∠B = 240°
By division
∠B = 120°

By substituting equation it in equation (1)
(∠B - ∠D) = 60°
120° - ∠D = 60°
On further calculation
∠D = 120° - 60°
By subtraction
∠D = 60°

Therefore, the smaller of the two angles ∠D = 60°.

24. The diagonals of a cyclic quadrilateral are at right angles. Prove that the perpendicular from the point of their intersection on any side when produced backwards, bisects the opposite side.
Solution:

Consider ABCD as a cyclic quadrilateral with diagonals AC and BD intersecting at right angles



We know that OL ⊥ AB so that ∠O meets the line CD at the point M

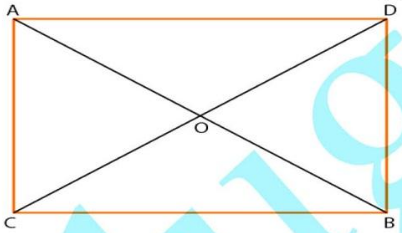
From the figure we know that the angles in the same segment are equal
∠1 = ∠2
We know that ∠OLB = 90° so we get
∠2 + ∠3 = 90° …… (1)

We know that OLM is a straight line and ∠BOC = 90°
So we get
∠3 + ∠4 = 90° ……… (2)
It can be written as
∠2 + ∠3 = ∠3 + ∠4
On further calculation
∠2 = ∠4
So we get
∠1 = ∠2 and ∠2 = ∠4
It can be written as
∠1 = ∠4
We get
OM = CM
In the same way
OM = MD
So CM = MD

Therefore, it is proved that the perpendicular from the point of their intersection on any side when produced backwards, bisects the opposite side.

25. On a common hypotenuse AB, two right triangles ACB and ADB are situated on opposite sides. Prove that ∠BAC = ∠BDC.
Solution:

We know that AB is the common hypotenuse of △ ACB and △ ADB

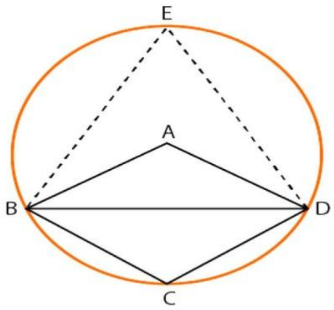


So we get
∠ACB = 90°
∠BDC = 90°
It can be written as
∠ACB + ∠BDC = 180°
We know that the opposite angles of a quadrilateral ABCD are supplementary
So we get ABCD as a cyclic quadrilateral which means that a circle passes through the points A, C, B and D
From the figure we know that the angles in the same segment are equal
∠BAC = ∠BDC

Therefore, it is proved that ∠BAC = ∠BDC.

26. ABCD is a quadrilateral such that A is the centre of the circle passing through B, C and D. Prove that ∠CBD + ∠CDB = ½ ∠BAD.
Solution:

Consider a point E on the circle and join BE, DE and BD



The angle subtended by an arc of a circle at its centre is twice the angle subtended by the same arc at a point on the circumference

So we get

$\angle BAD = 2 \angle BED$

It can be written as

$\angle BED = \frac{1}{2} \angle BAD$ …… (1)

Consider EBCD as a cyclic quadrilateral

So we get

$\angle BED + \angle BCD = 180^\circ$

We can write it as

$\angle BCD = 180^\circ - \angle BED$

Substituting equation (1)

$\angle BCD = 180^\circ - \frac{1}{2} \angle BAD$ …… (2)

Consider $\triangle$ BCD

Using the angle sum property

$\angle CBD + \angle CDB + \angle BCD = 180^\circ$

By using the equation (2)

$\angle CBD + \angle CDB + 180^\circ - \frac{1}{2} \angle BAD = 180^\circ$

So we get

$\angle CDB + \angle CDB - \frac{1}{2} \angle BAD = 180^\circ - 180^\circ$

On further calculation

$\angle CDB + \angle CDB = \frac{1}{2} \angle BAD$

Therefore, it is proved that $\angle CDB + \angle CDB = \frac{1}{2} \angle BAD$.