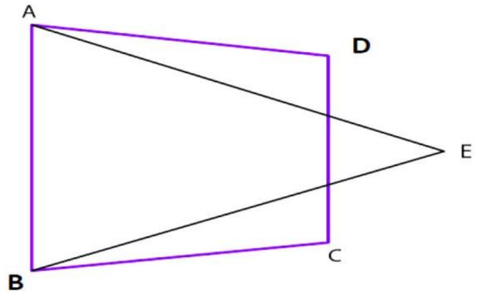
RS Aggarwal Solutions for Class 9 Maths Chapter 11 –
Areas of Parallelograms and Triangles

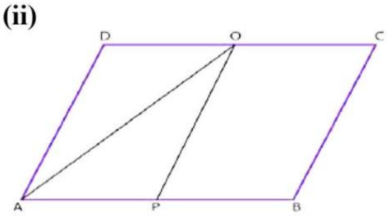
EXERCISE 11 PAGE: 387

1. Which of the following figures lie on the same base and between the same parallels. In such a case, write the common base and the two parallels.

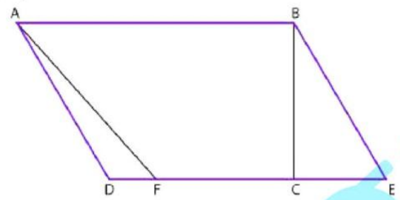
(i)



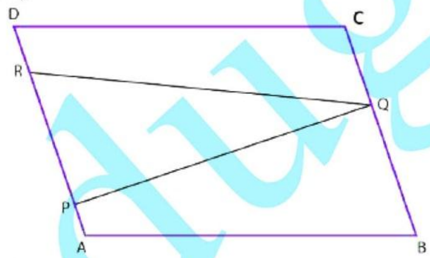
(ii)



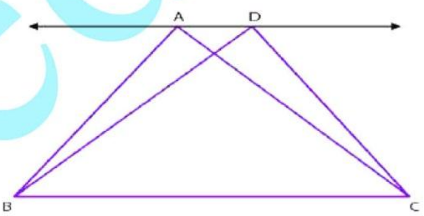
(iii)



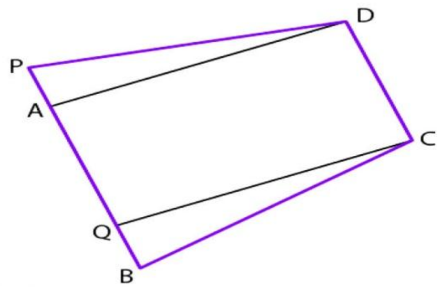
(iv)



(v)



(vi)



Solution:

(i) It does not lie on the same base and between the same parallels.

(ii) It does not lie on the same base and between the same parallels.

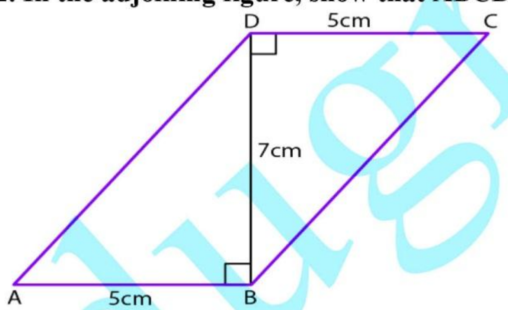
(iii) The given figure, lies on the same base and between the same parallels. The common base is AB and the two parallels are AB and DE.

(iv) It does not lie on the same base and between the same parallels.

(v) The given figure, lies on the same base and between the same parallels. The common base is BC and the two parallels are BC and AD.

(vi) The given figure lies on the same base and between the same parallels. The common base is CD and the two parallels are CD and BP.

2. In the adjoining figure, show that ABCD is a parallelogram. Calculate the area of parallelogram ABCD.



Solution:

Construct AM ⊥ DC and CL ⊥ AB. Extend the lines DC and AB and join AC.
We know that

Area of the quadrilateral ABCD = area of triangle ABD + area of triangle DCB
From the figure we know that
Area of △ ABD = Area of △ DCB
So it can be written as
Area of the quadrilateral ABCD = 2 (Area of △ ABD)
We get
Area of △ ABD = ½ (area of quadrilateral ABCD) ……… (1)

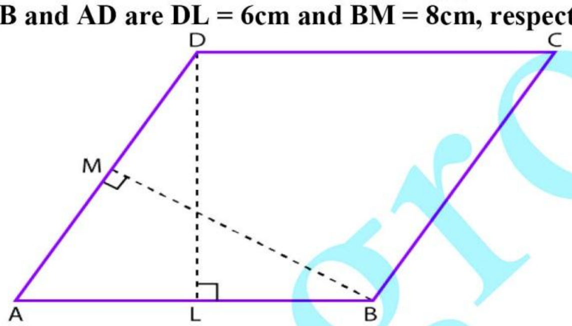
Area of quadrilateral ABCD = area of △ ABC + area of △ CDA
From the figure we know that
Area of △ ABC = Area of △ CDA
So it can be written as
Area of the quadrilateral ABCD = 2 (Area of △ ABC)
We get
Area of △ ABC = ½ (area of quadrilateral ABCD) ……… (2)

Using the equations (1) and (2)
Area of △ ABD = Area of △ ABC = ½ BD = ½ CL
It can be written as
CL = BD
In the same way we get
DC = AB and AD = BC
Hence, ABCD is a parallelogram

Area of parallelogram ABCD = base × height
= 5 × 7
= 35 cm^2

Therefore, area of parallelogram ABCD is 35 cm^2.

3. In a parallelogram ABCD, it is being given that AB = 10cm and the altitudes corresponding to the sides AB and AD are DL = 6cm and BM = 8cm, respectively. Find AD.



Solution:

From the figure we know that
Area of the parallelogram ABCD = base ×height
So we get
AB × DL = AD × BM
By substituting the values in the above equation
10 × 6 = AD × BM
It is given that BM = 8cm
AD × BM = 60 cm^2
AD × 8 = 60
By division
AD = 60/8
So we get
AD = 7.5 cm

Therefore, AD = 7.5cm.

4. Find the area of a figure formed by joining the midpoints of the adjacent sides of a rhombus with diagonals 12 cm and 16 cm.
Solution:

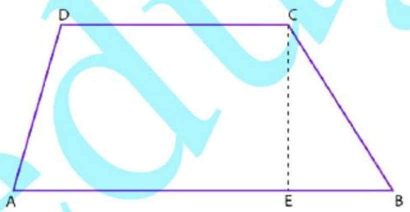
We know that ABCD is a rhombus with P, Q, R and S as the midpoints of AB, BC, CD and DA.
AC and BD diagonals are joined.
Using the midpoint theorem
We know that
PQ = ½ AC
By substituting the values we get
PQ = ½ (16)
By division
PQ = 8cm

In △ DAC we know that S and R are the midpoints of AD and DC
Using the midpoint theorem
SR = ½ AC
By substituting the values
SR = ½ (12)
By division
SR = 6cm

Consider the rectangle PQRS
Area of the rectangle PQRS = length × breadth
So we get
Area of the rectangle PQRS = 6 × 8
By multiplication
Area of the rectangle PQRS = 48 cm^2

Therefore, area of the figure is 48 cm^2.

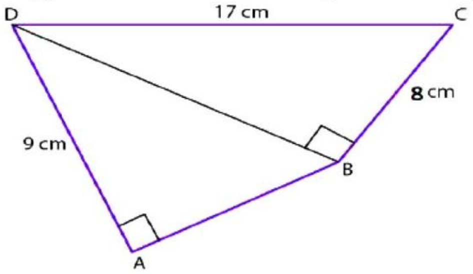
5. Find the area of a trapezium whose parallel sides are 9cm and 6cm respectively and the distance between these sides is 8cm.
Solution:



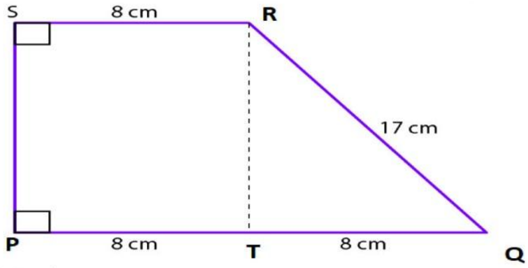
We know that
Area of trapezium = ½ (sum of parallel sides × distance between them)
By substituting the values
Area of trapezium = ½ ((9 + 6) × 8)
On further calculation
Area of trapezium = ½ (120)
So we get
Area of trapezium = 60 cm^2

Therefore, area of the trapezium is 60 cm^2.

6. (i) Calculate the area of quad. ABCD, given in Fig. (i).



(ii) Calculate the area of trap. PQRS, given in Fig. (ii).



Solution:

(i)Consider △ BCD
Using the Pythagoras theorem
We can write it as
$DB^2 + BC^2 = DC^2$
By substituting the values
$DB^2 + 8^2 = 17^2$
By subtraction
$DB^2 = 17^2 – 8^2$
$DB^2 = 289 – 64$
By subtraction
$DB^2 = 225$
By taking the square root
DB = √ 225
So we get
DB = 15cm

We can find
Area of △ BCD = ½ × b × h
By substituting the values
Area of △ BCD = ½ × 8 × 15
On further calculation
Area of △ BCD = 60 cm^2

Consider △ BAD
Using the Pythagoras theorem
We can write it as
$DA^2 + AB^2 = DB^2$

By substituting the values
$AB^2 + 9^2 = 15^2$
By subtraction
$AB^2 = 15^2 – 9^2$
$AB^2 = 225 – 81$
By subtraction
$AB^2 = 144$
By taking the square root
$AB = \sqrt{144}$
So we get
AB = 12cm

We can find
Area of △ DAB = ½ × b × h
By substituting the values
Area of △ DAB = ½ × 9 × 12
On further calculation
Area of △ DAB = 54 cm^2

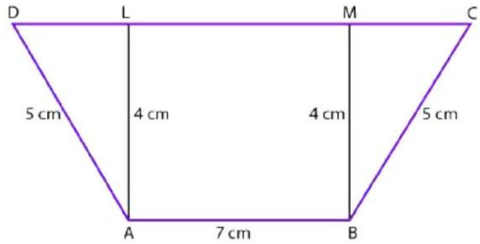
So we get
Area of quadrilateral ABCD = area of △ DAB + area of △ BCD
By substituting the values
Area of quadrilateral ABCD = 54 + 60
By addition
Area of quadrilateral ABCD = 114 cm^2

Therefore, the area of quadrilateral ABCD is 114 cm^2.

(ii) From the figure
Using the Pythagoras theorem in △ RTQ
We get
$RT^2 + TQ^2 = RQ^2$
By substituting the values
$RT^2 + 8^2 = 17^2$
On further calculation
$RT^2 = 17^2 - 8^2$
So we get
$RT^2 = 289 – 64$
By subtraction
$RT^2 = 225$
By taking square root
$RT = \sqrt{225}$
RT = 15cm
We can find the area of trapezium
Area of trapezium PQRS = ½ (sum of parallel sides × distance between them)
So we get
Area of trapezium PQRS = ½ ((8 + 16) × 15)
On further calculation we get
Area of trapezium PQRS = 180 cm^2

Therefore, the area of trapezium PQRS is 180 cm^2.

7. In the adjoining figure, ABCD is a trapezium in which AB || DC; AB = 7cm; AD = BC = 5cm and the distance between AB and DC is 4cm. Find the length of DC and hence, find the area of trap. ABCD.



Solution:

Consider △ ALD
Based on the Pythagoras theorem
$AL^2 + DL^2 = AD^2$
By substituting the values
$4^2 + DL^2 = 5^2$
So we get
$DL^2 = 5^2 - 4^2$
$DL^2 = 25 – 16$
By subtraction
$DL^2 = 9$
By taking square root
$DL = \sqrt{9}$
So we get
DL = 3 cm

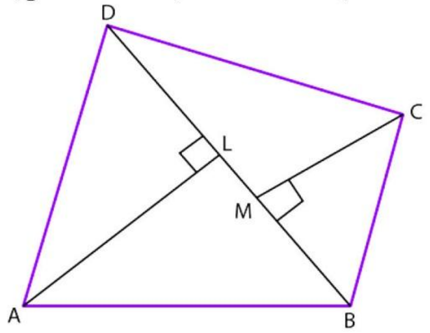
Consider △ BMC
Based on the Pythagoras theorem
$MC^2 + MB^2 = CB^2$
By substituting the values
$MC^2 + 4^2 = 5^2$
So we get
$MC^2 = 5^2 - 4^2$
$MC^2 = 25 – 16$
By subtraction
$MC^2 = 9$
By taking square root
$MC = \sqrt{9}$
So we get
MC = 3 cm

From the figure we know that LM = AB = 7cm
So we know that CD = DL + LM + MC
By substituting the values
CD = 3 + 7 + 3
By addition
CD = 13cm

Area of Trapezium ABCD = ½ (sum of parallel sides × distance between them)
So we get
Area of Trapezium ABCD = ½ × (CD + AB) × AL
By substituting the values
Area of Trapezium ABCD = ½ × (13 + 7) × 4
On further calculation
Area of Trapezium ABCD = 20 × 2
By multiplication
Area of Trapezium ABCD = 40 cm^2

Therefore, length of DC = 13cm and area of trapezium ABCD = 40 cm^2.

8. BD is one of the diagonals of a quad. ABCD. If AL ⊥ BD and CM ⊥ BD, show that Ar (quad. ABCD) = ½ × BD × (AL + CM).



Solution:

From the figure we know that
Area of △ ABD = ½ × b × h
It can be written as
Area of △ ABD = ½ × BD × AL

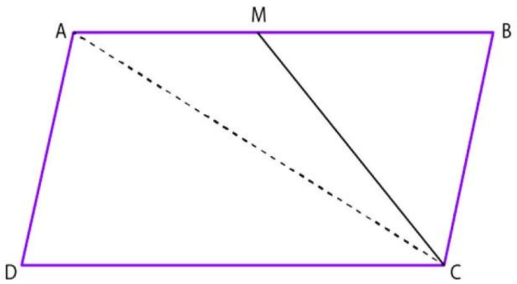
Area of △ CBD = ½ × b × h
It can be written as
Area of △ CBD = ½ × BD × CM

We know that
Area of quadrilateral ABCD = Area of △ ABD + Area of △ CBD
So we get
Area of quadrilateral ABCD = ½ × BD × AL + ½ × BD × CM
It can be written as
Area of quadrilateral ABCD = ½ × BD (AL + CM)

Therefore, it is proved that Area of quadrilateral ABCD = ½ × BD (AL + CM).

9. M is the midpoint of the side AB of a parallelogram ABCD. If ar (AMCD) = 24 cm^2, find ar (△ ABC).
Solution:

Join the diagonal AC



From the figure we know that the diagonal AC divides the parallelogram ABCD into two triangles having the same area
It can be written as
Area of △ ADC = Area of △ ABC ……. (1)

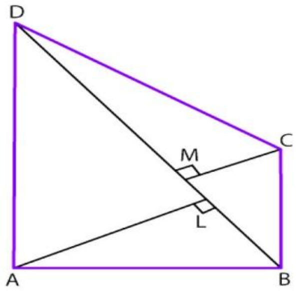
We know that △ ADC and parallelogram ABCD are on the same base CD and between the same parallel lines DC and AM.
It can be written as
Area of △ ADC = Area of △ ABC = ½ (Area of parallelogram ABCD)

From the figure we know that M is the midpoint of AB
So we get
Area of △ AMC = Area of △ BMC = ½ (Area of △ ABC) = ½ (Area of △ ADC)
It can be written as
Area of AMCD = Area of △ ADC + Area of △ AMC
By substituting the values
24 = Area of △ ADC + ½ (Area of △ ADC)
It can be written as
24 = 3/2 (Area of △ ADC)
By cross multiplication
24 × 2 = 3 × (Area of △ ADC)
On further calculation
48 = 3 × (Area of △ ADC)
So we get
Area of △ ADC = 48/3
By division
Area of △ ADC = 16 cm^2

From equation (1)
Area of △ ADC = Area of △ ABC = 16 cm^2

Therefore, Area of △ ABC = 16 cm^2.

10. In the adjoining figure, ABCD is a quadrilateral in which diag. BD = 14cm. If AL ⊥ BD and CM ⊥ BD such that AL = 8cm and CM = 6cm, find the area of quad. ABCD.



Solution:

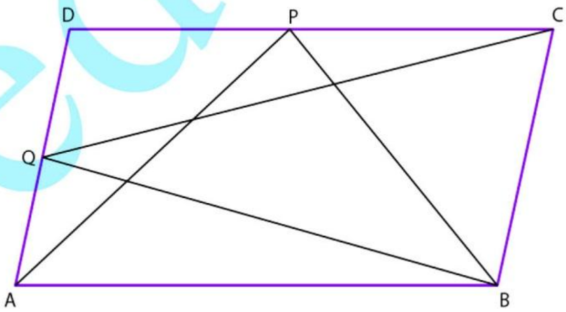
We know that
Area of △ BAD = ½ × BD × AL
By substituting the values in the above equation
Area of △ BAD = ½ × 14 × 8
By multiplication
Area of △ BAD = 56 cm^2

We know that
Area of △ CBD = ½ × BD × CM
By substituting the values in the above equation
Area of △ CBD = ½ × 14 × 6
By multiplication
Area of △ CBD = 42 cm^2

So we get
Area of quadrilateral ABCD = Area of △ ABD + Area of △ CBD
By substituting the values
Area of quadrilateral ABCD = 56 + 42
By addition
Area of quadrilateral ABCD = 98 cm^2

Therefore, area of quadrilateral ABCD is 98 cm^2.

11. If P and Q are any two points lying respectively on the sides DC and AD of a parallelogram ABCD then show that ar (△ APB) = ar (△ BQC).
Solution:



From the figure we know that △ APB and parallelogram ABCD lie on the same base AB and between the same parallels AB and DC

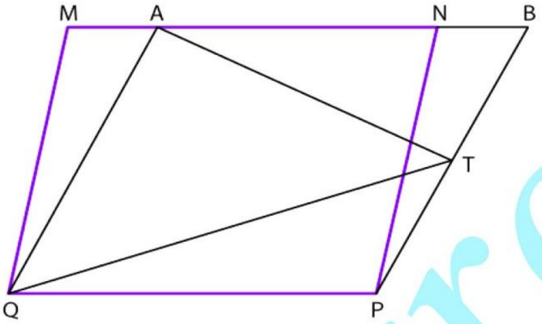
So we get
Area of △ APB = ½ (Area of parallelogram ABCD) …….. (1)

In the same way △ BQC and parallelogram ABCD lie on the same base BC and between the same parallels BC and AD
So we get
Area of △ BQC = ½ (Area of parallelogram ABCD) …….. (1)

Using the equations (1) and (2)
We get
Area of △ APB = Area of △ BQC

Therefore, it is proved that Area of △ APB = Area of △ BQC.

12. In the adjoining figure, MNPQ and ABPQ are parallelograms and T is any point on the side BP. Prove that
(i) ar (MNPQ) = ar (ABPQ)
(ii) ar (△ ATQ) = ½ ar (MNPQ).



Solution:

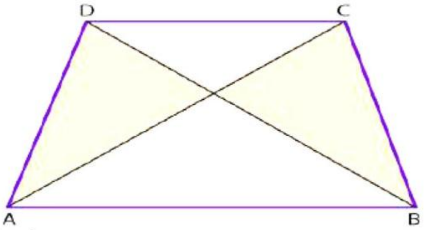
(i)From the figure we know that the parallelograms MNPQ and ABPQ are on the same base PQ and lie between the same parallels PQ and MB.
So we get
Area of parallelogram MNPQ = Area of parallelogram ABPQ

Therefore, it is proved that ar (MNPQ) = ar (ABPQ).

(ii) From the figure we know that △ ATQ and parallelogram ABPQ lie on the same base AQ and between the same parallels AQ and BP.
So we get
Area of △ ATQ = ½ (Area of parallelogram ABPQ)
It can be written as
Area of △ ATQ = ½ (Area of parallelogram MNPQ)

Therefore, it is proved that ar (△ ATQ) = ½ ar (MNPQ).

13. In the adjoining figure, ABCD is a trapezium in which AB || DC and its diagonals AC and BD intersect at O. Prove that ar (△ AOD) = ar (△ BOC).



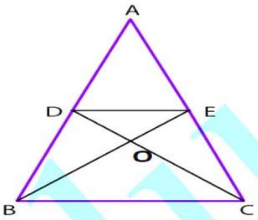
Solution:

From the figure we know that △ AOD and △ DCB lie on the same base CD and between two parallel lines DC and AB.
We know that the triangles lying on the same base and parallels have equal area.

Consider △ CDA and △ CDB
It can be written as
Area of △ CDA = Area of △ CDB
So we get
Area of △ AOD = Area of △ ADC – Area of △ OCD
In the same way
Area of △ BOC = Area of △ CDB – Area of △ OCD
So we get
Area of △ BOC = Area of △ ADC – Area of △ OCD
Area of △ AOD = Area of △ BOC

Therefore, it is proved that ar (△ AOD) = ar (△ BOC).

14. In the adjoining figure, DE || BC. Prove that
(i) ar (△ ACD) = ar (△ ABE)
(ii) ar (△ OCE) = ar (△ OBD).



Solution:

(i)From the figure we know that △ DBE and △ DCE lie on the same base DE between the parallel lines BC and DE.
So we get
Area of △ DBE = Area of △ DCE ……. (1)
By adding △ ADE both sides
Area of △ DBE + Area of △ ADE = Area of △ DCE + Area of △ ADE
We get
Area of △ ABE = Area of △ ACD

Therefore, it is proved that ar (△ ACD) = ar (△ ABE).

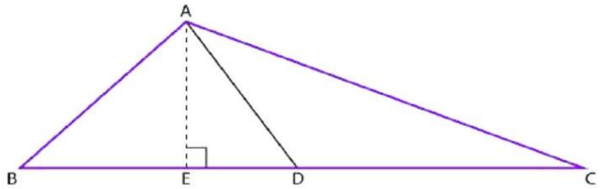
(ii) Subtracting △ ODE from equation (1)

We get
Area of △ DBE – Area of △ ODE = Area of △ DCE - Area of △ ODE
We get
Area of △ OBD = Area of △ OCE

Therefore, it is proved that ar (△ OCE) = ar (△ OBD).

15. Prove that a median divides a triangle into two triangles of equal area.
Solution:

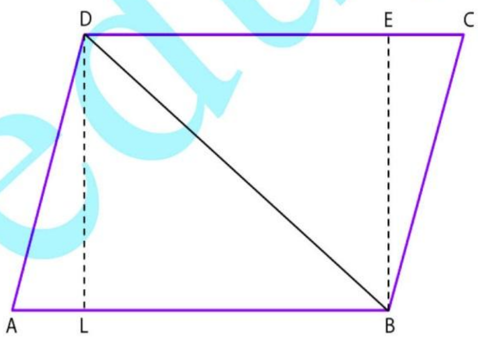
Consider △ ABD



We know that
Area of △ ABD = ½ × BD × AE
In the same way
Area of △ ADC = ½ × DC × AE
From the figure we know that D is the median
So we get
BD = DC
We know that
Area of △ ABD = ½ × BD × AE
It can be written as
Area of △ ABD = ½ × DC × AE
So we get
Area of △ ABD = Area of △ ADC

Therefore, it is proved that a median divides a triangle into two triangles of equal area.

16. Show that a diagonal divides a parallelogram into two triangles of equal area.
Solution:



We know that
Area of △ ABD = ½ × AB × DL …….. (1)
Area of △ CBD = ½ × CD × BE …….. (2)

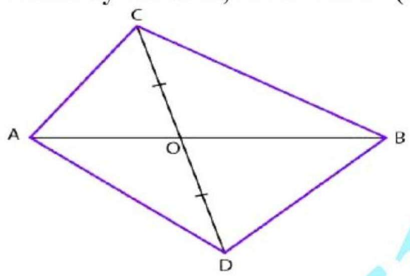
Hence, ABCD is a parallelogram.
It can be written as AB || CD and
AB = CD …… (3)

We know that the distance between two parallel lines is constant t,
So we get
DL = BE …….. (4)

Using the equations (1), (2), (3) and (4)
We get
Area of △ ABD = ½ × AB × DL
It can be written as
Area of △ ABD = ½ × CD × BE
So we get
Area of △ ABD = Area of △ CBD

Therefore, it is proved that a diagonal divides a parallelogram into two triangles of equal area.

17. In the adjoining figure, ABC and ABD are two triangles on the same base AB. If line segment CD is bisected by AB at O, show that ar (△ ABC) = ar (△ ABD).



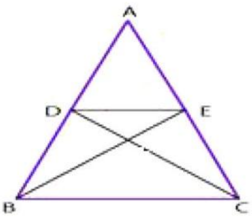
Solution:

From the figure we know that median of a triangle divides it into triangles of equal area
We know that AO is the median of △ ACD
It can be written as
Area of △ COA = Area of △ DOA ……. (1)

We know that BO is the median of △ BCD
It can be written as
Area of △ COB = Area of △ DOB ……. (2)

By adding both the equations
Area of △ COA + Area of △ COB = Area of △ DOA + Area of △ DOB
So we get
Area of △ ABC = Area of △ ABD
Therefore, it is proved that ar (△ ABC) = ar (△ ABD).

18. D and E are points on sides AB and AC respectively of △ ABC such that ar (△ BCD) = ar (△ BCE). Prove that DE || BC.
Solution:

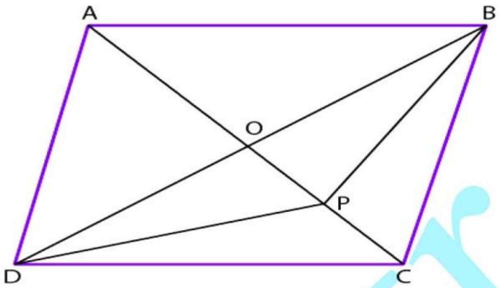


From the figure we know that △ BCD and △ BCE have equal area and lie on the same base BC.
We know that
Altitude from D of △ BCD = Altitude from E of △ BCE
So we know that △ BCD and △ BCE lie between the same parallel lines
We get
DE || BC

Therefore, it is proved that DE || BC.

19. P is any point on the diagonal AC of a parallelogram ABCD. Prove that ar (△ ADP) = ar (△ ABP).
Solution:

Join the diagonal BD.



From the figure we know that AC and BD are the diagonals intersecting at point O.
We know that the diagonals of a parallelogram bisect each other.
Thus, we get O as the midpoint of AC and BD.

Median of triangle divides it into two triangles having equal area.

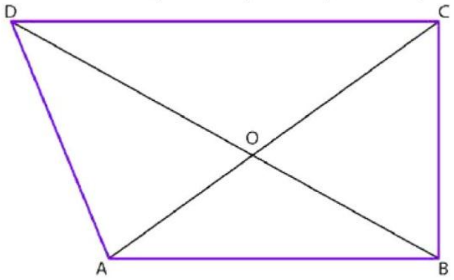
Consider △ ABD
We know that OA is the median
So we get
Area of △ AOD = Area of △ AOB ……. (1)

Consider △ BPD
We know that OP is the median
So we get
Area of △ OPD = Area of △ OPB …….. (2)

By adding both the equations we get
Area of △ AOD + Area of △ OPD = Area of △ AOB + Area of △ OPB
So we get
Area of △ ADP = Area of △ ABP

Therefore, it is proved that ar (△ ADP) = ar (△ ABP).

20. In the adjoining figure, the diagonals AC and BD of a quadrilateral ABCD intersect at O. If BO = OD, prove that ar (△ ABC) = ar (△ ADC).



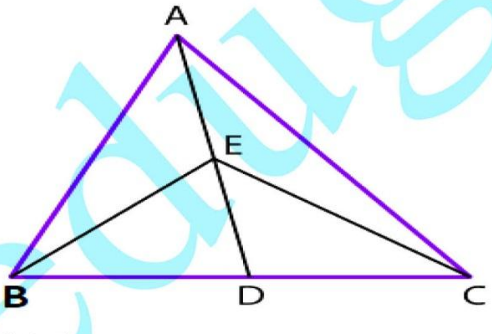
Solution:

It is given that BO = OD
From the figure we know that AO is the median of △ ABD
So we get
Area of △ AOD = Area of △ AOB ……. (1)
We know that OC is the median of △ CBD
So we get
Area of △ DOC = Area of △ BOC ……. (2)

By adding both the equations
Area of △ AOD + Area of △ DOC = Area of △ AOB + Area of △ BOC
So we get
Area of △ ADC = Area of △ ABC

Therefore, it is proved that ar (△ ABC) = ar (△ ADC).

21. The vertex A of △ ABC is joined to a point D on the side BC. The midpoint of AD is E. Prove that ar (△ BEC) = ½ ar (△ ABC).



Solution:

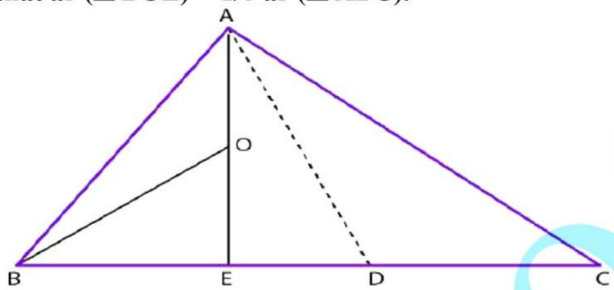
From the figure we know that BE is the median of △ ABD
So we get
Area of △ BDE = Area of △ ABE
It can be written as
Area of △ BDE = ½ (Area of △ ABD) …… (1)

From the figure we know that CE is the median of △ ADC
So we get
Area of △ CDE = Area of △ ACE
It can be written as
Area of △ CDE = ½ (Area of △ ACD) …….. (2)

By adding both the equations
Area of △ BDE + Area of △ CDE = ½ (Area of △ ABD) + ½ (Area of △ ACD)
By taking ½ as common
Area of △ BEC = ½ (Area of △ ABD + Area of △ ACD)
So we get
Area of △ BEC = ½ (Area of △ ABC)

Therefore, it is proved that ar (△ BEC) = ½ ar (△ ABC).

22. D is the midpoint of side BC of △ ABC and E is the midpoint of BD. If O is the midpoint of AE, prove that ar (△ BOE) = 1/8 ar (△ ABC).



Solution:

From the figure we know that O is the midpoint of AE.
We know that BO is the median of △ BAE
So we get
ar (△ BOE) = ½ ar (△ ABE) ……… (1)

We know that E is the midpoint of BD
AE divides the △ ABD into two triangles of equal area
So we get
ar (△ ABE) = ½ ar (△ ABD) ……… (2)

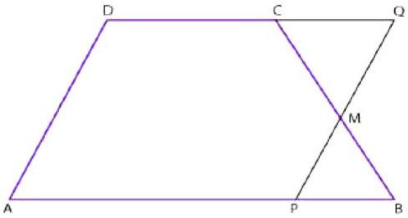
We know that D is the midpoint of BC
So we get
ar (△ ABD) = ½ ar (△ ABC) ……… (3)

We know that
ar (△ BOE) = ½ ar (△ ABE) using equation (1)
Substituting equation (2) in (1)
ar (△ BOE) = ½ (½ ar (△ ABD))
So we get
ar (△ BOE) = ¼ ar (△ ABD)
By substituting equation (3)
ar (△ BOE) = ¼ (½ ar (△ ABC))

So we get
ar (△ BOE) = 1/8 ar (△ ABC)

Therefore, it is proved that ar (△ BOE) = 1/8 ar (△ ABC).

23. In a trapezium ABCD, AB || DC and M is the midpoint of BC. Through M, a line PQ || AD has been drawn which meets AB in P and DC produced in Q, as shown in the adjoining figure. Prove that ar (ABCD) = ar (APQD).



Solution:

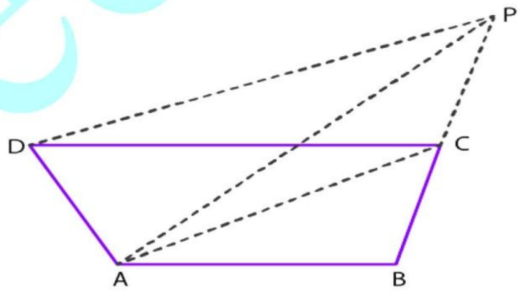
Consider △ MCQ and △ MPB
From the figure we know that
∠ QCM and ∠ PBM are alternate angles
So we get
∠ QCM = ∠ PBM
We know that M is the midpoint of BC
CM = BM

∠ CMQ and ∠ PBM are vertically opposite angles
∠ CMQ = ∠ PBM
By ASA congruence criterion
△ MCQ ≅ △ MPB
So we get
Area of △ MCQ = Area of △ MPB

We know that
Area of ABCD = Area of APQD + Area of DMPB – Area of △ MCQ
So we get
Area of ABCD = Area of APQD

Therefore, it is proved that ar (ABCD) = ar (APQD).

24. In the adjoining figure, ABCD is a quadrilateral. A line through D, parallel to AC, meets BC produced in P. Prove that ar (△ ABP) = ar (quad. ABCD).



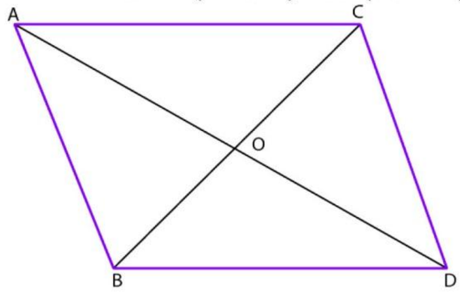
Solution:

From the figure we know that △ ACP and △ ACD lie on the same base AC between parallel lines AC and DP
It can be written as
Area of △ ACP = Area of △ ACD

By adding △ ABC to both LHS and RHS
Area of △ ACP + Area of △ ABC = Area of △ ACD + Area of △ ABC
So we get
Area of △ ABP = Area of quadrilateral ABCD

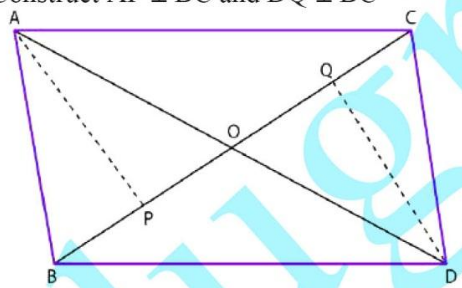
Therefore, it is proved that ar (△ ABP) = ar (quad. ABCD).

25. In the adjoining figure, △ ABC and △ DBC are on the same base BC with A and D on opposite sides of BC such that ar (△ ABC) = ar (△ DBC). Show that BC bisects AD.



Solution:

Construct AP ⊥ BC and DQ ⊥ BC



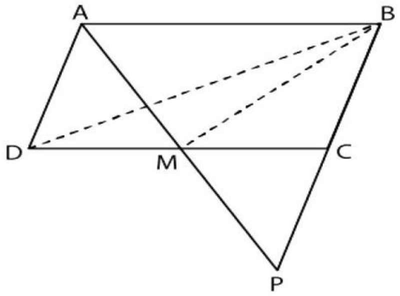
We know that
Area of △ ABC = ½ × BC × AP
In the same way
Area of △ BCD = ½ × BC × DQ
Equating both we get
½ × BC × AP = ½ × BC × DQ
So we get
AP = DQ ……. (1)

Consider △ AOP and △ QOD
From the figure we know that
∠ APO = ∠ DQO = 90°
We know that ∠ AOP and ∠ DOQ are vertically opposite angles
∠ AOP = ∠ DOQ

By AAS congruence criterion
△ AOP ≅ △ QOD
OA = OD (c. p. c. t)

Therefore, it is proved that BC bisects AD.

26. ABCD is a parallelogram in which BC is produced to P such that CP = BC, as shown in the adjoining figure. AP intersects CD at M. If ar (DMB) = 7 cm², find the area of parallelogram ABCD.



Solution:

Consider △ ADM and △ PCM
From the figure we know that ∠ ADM and ∠ PCM are alternate angles
∠ ADM = ∠ PCM
We know that AD = BC = CP
It can be written as
AD = CP

∠ AMD and ∠ PMC are vertically opposite angles
∠ AMD = ∠ PMC

By ASA congruence criterion
△ ADM ≅ △ PCM
So we get
Area of △ ADM = Area of △ PCM
DM = CM (c. p. c. t)

We know that BM is the median of △ BDC
So we get
Area of △ DMB = Area of △ CMB

We get
Area of △ BDC = 2 (Area of △ DMB)
By substituting the value
Area of △ BDC = 2 × 7
By multiplication
Area of △ BDC = 14 cm²

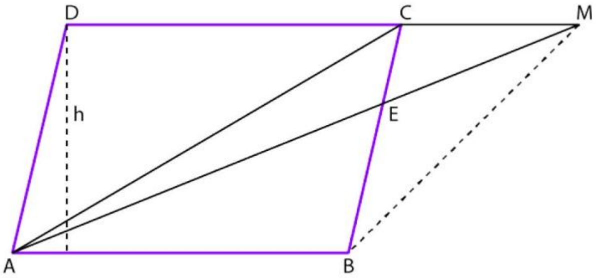
We know that
Area of parallelogram ABCD = 2 (Area of △ BDC)
So we get
Area of parallelogram ABCD = 2 × 14

By multiplication
Area of parallelogram ABCD = 28 cm^2

Therefore, area of parallelogram ABCD is 28 cm^2.

27. In a parallelogram ABCD, any point E is taken on the side BC. AE and DC when produced to meet at a point M. Prove that ar (△ ADM) = ar (ABMC).
Solution:

Construct lines AC and BM.
Let us take h as the distance between AB and CD

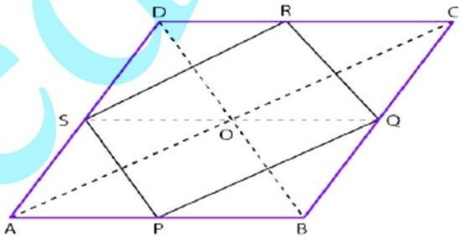


We know that
Area of △ ACD = ½ × CD × h
In the same way
Area of △ ABM = ½ × AB × h
From the figure we know that AB = CD
So it can be written as
Area of △ ABM = ½ × CD × h

So we get
Area of △ ABM = Area of △ ACD
Let us add △ ACM on both sides
Area of △ ABM + Area of △ ACM = Area of △ ACD + Area of △ ACM
So we get
Area of ABMC = Area of △ ADM

Therefore, it is proved that ar (△ ADM) = ar (ABMC).

28. P, Q, R, S are respectively the midpoints of the sides AB, BC, CD and DA of parallelogram ABCD. Show that PQRS is a parallelogram and also show that ar (||gm PQRS) = ½ × ar (||gm ABCD).



Solution:

We know that
Area of parallelogram PQRS = ½ (Area of parallelogram ABCD)

Construct diagonals AC, BD and SQ

From the figure we know that S and R are the midpoints of AD and CD
Consider △ ADC
Using the midpoint theorem
SR || AC
From the figure we know that P and Q are the midpoints of AB and BC
Consider △ ABC
Using the midpoint theorem
PQ || AC

It can be written as
PQ || AC || SR
So we get
PQ || SR
In the same way we get SP || RQ

Consider △ ABD
We know that O is the midpoint of AC and S is the midpoint of AD
Using the midpoint theorem
OS || AB

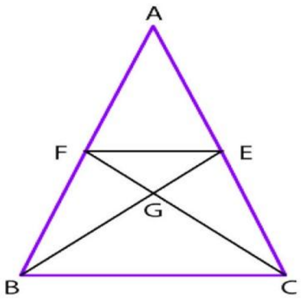
Consider △ ABC
Using the midpoint theorem
OQ || AB
It can be written as
SQ || AB

We know that ABQS is a parallelogram
We get
Area of △ SPQ = ½ (Area of parallelogram ABQS) ……. (1)
In the same way we get
Area of △ SRQ = ½ (Area of parallelogram SQCD) …….. (2)

By adding both the equations
Area of △ SPQ + Area of △ SRQ = ½ (Area of parallelogram ABQS + Area of parallelogram SQCD)
So we get
Area of parallelogram PQRS = ½ (Area of parallelogram ABCD)

Therefore, it is proved that ar (||gm PQRS) = ½ × ar (||gm ABCD).

29. In a triangle ABC, the medians BE and CF intersect at G. Prove that ar (△ BCG) = ar (AFGE).
Solution:



Draw a line EF.
We know that the line segment joining the midpoint of two sides of a triangle is parallel to the third side.
So we get
FE || BC

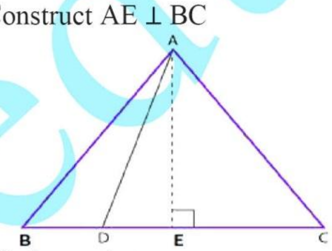
From the figure we know that △ BEF and △ CEF lie on the same base EF between the same parallel lines
So we get
Area of △ BEF = Area of △ CEF
By subtracting △ GEF both the sides
Area of △ BEF – Area of △ GEF = Area of △ CEF – Area of △ GEF
We get
Area of △ BFG = Area of △ CEG …… (1)

Median of a triangle divides it into two triangles having equal area
Area of △ BEC = Area of △ ABE
It can be written as
Area of △ BGC + Area of △ CEG = Area of quadrilateral AFGE + Area of Area of △ BFG
Using the equation (1) we get
Area of △ BGC + Area of △ BFG = Area of quadrilateral AFGE + Area of Area of △ BFG
We get
Area of △ BGC = Area of quadrilateral AFGE

Therefore, it is proved that ar (△ BCG) = ar (AFGE).

30. The base BC of △ ABC is divided at D such that BD = ½ DC. Prove that ar (△ ABD) = 1/3 × ar (△ ABC).
Solution:

Construct AE ⊥ BC



We know that
Area of △ ABD = ½ × BD × AE ……. (1)
Area of △ ABC = ½ × BC × AE …….. (2)

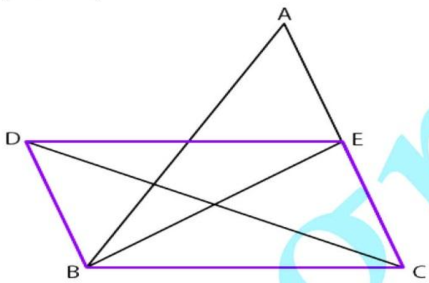
It is given that

BD = ½ BC

We get
BC = BC + DC
It can be written as
BC = BD + 2BD
So we get
BC = 3BD
By division
BD = 1/3 BC …….. (3)
Using equation (1) we get
Area of △ ABD = ½ × BD × AE
By substituting (3)
Area of △ ABD = ½ × 1/3 × BC × AE
So we get
Area of △ ABD = 1/3 (1/2 × BC × AE)
Substituting equation (2)
Area of △ ABD = 1/3 × Area of △ ABC

Therefore, it is proved that ar (△ ABD) = 1/3 × ar (△ ABC).

31. In the adjoining figure, BD || CA, E is the midpoint of CA and BD = ½ CA. Prove that ar (△ ABC) = 2 ar (△ DBC).



Solution:

It is given that E is the midpoint of CA and BD = ½ CA
So we get
BD = CE

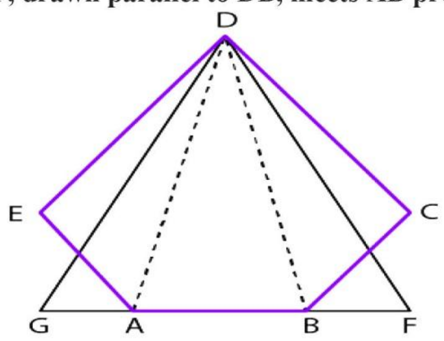
We know that BD || CA
BD = CE
In the same way BD || CE
BD = CE
Hence, BCED is a parallelogram.

We know that △ DBC and △ EBC lie on the same base and between the same parallel lines
So we get
Area of △ DBC = Area of △ EBC ….. (1)
From the figure we know that BE is the median of △ ABC
We get
Area of △ BEC = ½ (Area of △ ABC)

Using equation (1) we get
Area of △ DBC = ½ (Area of △ ABC)
So we get
Area of △ ABC = 2 (Area of △ DBC)

Therefore, it is proved that ar (△ ABC) = 2 ar (△ DBC).

32. The given figure shows a pentagon ABCDE. EG, drawn parallel to DA, meets BA produced to G, and CF, drawn parallel to DB, meets AB produced at F. Show that ar (pentagon ABCDE) = ar (△ DGF).



Solution:

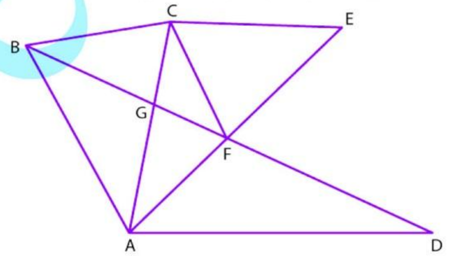
Consider △ DGA and △ AED
We know that both the triangles have the same base AD and lie between the parallel lines AD and EG.
So we get
Area of △ DGA = Area of △ AED ……. (1)

Consider △ DBC and △ BFD
We know that both the triangles have the same base DB and lie between the parallel lines BD and CF.
So we get
Area of △ DBF = Area of △ BCD ……. (2)

By adding both the equations
Area of △ DGA + Area of △ DBF = Area of △ AED + Area of △ BCD
By adding △ ABD both sides
Area of △ DGA + Area of △ DBF + Area of △ ABD = Area of △ AED + Area of △ BCD + Area of △ ABD
So we get
Area of △ DGF = Area of pentagon ABCDE

Therefore, it is proved that ar (pentagon ABCDE) = ar (△ DGF).

33. In the adjoining figure, CE || AD and CF || BA. Prove that ar (△ CBG) = ar (△ AFG).

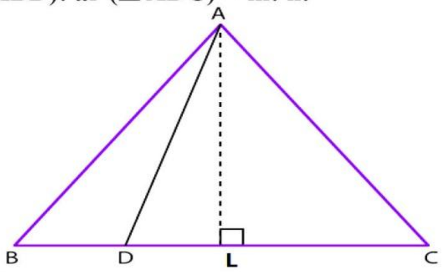


Solution:

From the figure we know that △ BCF and △ ACF lie on the same base CF between the same parallels CF and BA
So we get
Area of △ BCF = Area of △ ACF
By subtracting △ CGF both sides
Area of △ BCF – Area of △ CGF = Area of △ ACF – Area of △ CGF
So we get
Area of △ CBG = Area of △ AFG

Therefore, it is proved that ar (△ CBG) = ar (△ AFG).

34. In the adjoining figure, the point D divides the side BC of △ ABC in the ratio m: n. Prove that ar (△ ABD): ar (△ ADC) = m: n.



Solution:

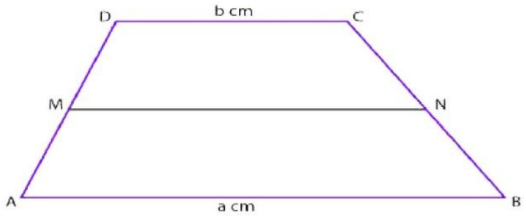
We know that
Area of △ ABD = ½ × BD × AL
Area of △ ADC = ½ × DC × AL
It is given that BD: DC = m: n
It can be written as
BD = DC × m/n

We know that
Area of △ ABD = ½ × BD × AL
By substituting BD
Area of △ ABD = ½ × (DC × m/n) × AL
So we get
Area of △ ABD = m/n × (1/2 × DC × AL)
It can be written as
Area of △ ABD = m/n × (Area of △ ADC))

We know that
Area of △ ABD/ Area of △ ADC = m/n
We can write it as
Area of △ ABD: Area of △ ADC = m: n

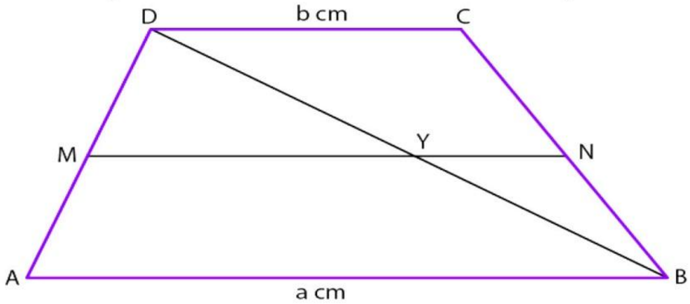
Therefore, it is proved that ar (△ ABD): ar (△ ADC) = m: n.

35. In a trapezium ABCD, AB || DC, AB = a cm, and DC = b cm. If M and N are the midpoints of the nonparallel sides, AD and BC respectively then find the ratio of ar (DCNM) and ar (MNBA).



Solution:

Join the diagonal DB which cuts the line MN at the point Y.



We know that M and N are the midpoints of AD and BC
So we get
MN || AB || CD

Consider △ ADB
We know that M is the midpoint of AD and MY || AB
So we get Y as the midpoint of DB
It can be written as
MY = ½ AB

Consider △ BDC
We know that
NY = ½ CD

We get
MN = MY + YN
By substituting the values
MN = ½ AB + ½ CD
By taking out ½ as common
MN = ½ (AB + CD)
It is given that AB = a and CD = b
So we get
MN = (a + b)/2

Construct DQ ⊥ AB where DQ cuts the line MN at the point P
So we know that P is the midpoint of DQ
It can be written as
DP = PQ = h

We get
Area of trapezium DCNM = ½ × (MN + CD) × DP

By substituting

Area of trapezium DCNM = $\frac{1}{2}(\frac{a+b}{2}+b)h$

So we get

Area of trapezium DCNM = h/4 (a + 3b) ……. (1)

Area of trapezium MNBA = ½ × (MN + AB) × PQ

By substituting

Area of trapezium MNBA = $\frac{1}{2}(\frac{a+b}{2}+a)h$

So we get

Area of trapezium MNBA = h/4 (3a + b) ……. (2)

By dividing both the equations

Area of trapezium DCNM / Area of trapezium MNBA = {h/4 (a + 3b)}/ {h/4 (3a + b)}

By cancelling the similar terms

Area of trapezium DCNM / Area of trapezium MNBA = (a + 3b)/ (3a + b)

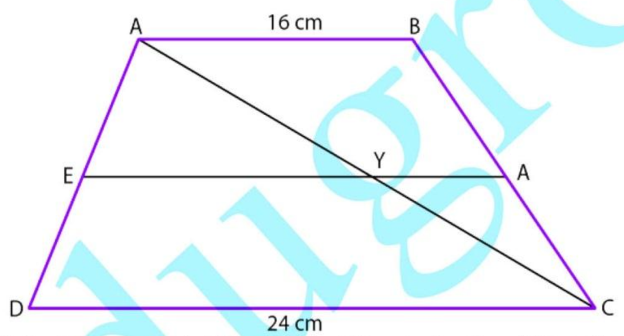
It can be written as

Area of trapezium DCNM: Area of trapezium MNBA = (a + 3b): (3a + b)

Therefore, ratio of ar (DCNM) and ar (MNBA) is (a + 3b): (3a + b)

36. ABCD is a trapezium in which AB || DC, AB = 16cm and DC = 24cm. If E and F are respectively the midpoints of AD and BC, prove that ar (ABFE) = 9/11 ar (EFCD).

Solution:



Join the diagonal AC such that it cuts the line EF at the point Y

From the figure we know that E and F are the midpoints of AD and BC

So we get

EF || AB || CD

Consider △ ACF

We know that E is the midpoint of AD and EY || CD

So we get Y as the midpoint of AC

It can be written as

EY = ½ CD

Consider △ ABC

We get

FY = ½ AB

We know that
EF = EY + YF
By substituting the values
EF = ½ CD + ½ AB
By taking ½ as common
EF = ½ (CD + AB)
By substituting the values
EF = (24 + 16)/2
So we get
EF = 20cm

Construct AQ ⊥ DC such that AQ cuts EF at P
We know that P is the midpoint of AQ
So we get
AP = PQ = h

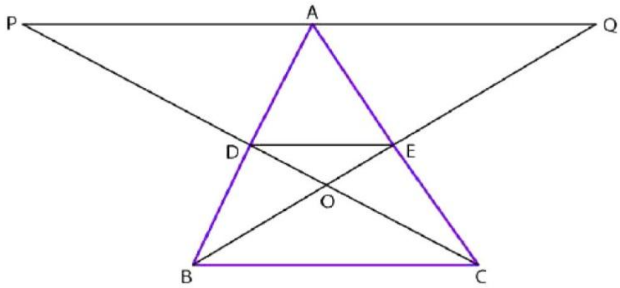
We get
Area of trapezium ABFE = 1/2 × (EF + AB) × AP
By substituting the values
Area of trapezium ABFE = ½ × (20 + 16) × h
On further calculation
Area of trapezium ABFE = 18h cm^2

Area of trapezium EFCD = ½ × (EF + CD) × PQ
By substituting the values
Area of trapezium EFCD = ½ × (20 + 24) × h
On further calculation
Area of trapezium EFCD = 22h cm^2

By division
Area of trapezium ABFE/ Area of trapezium EFCD = 18h/ 22h
So we get
Area of trapezium ABFE/ Area of trapezium EFCD = 9/11
It can be written as
Area of trapezium ABFE = 9/11 × (Area of trapezium EFCD)

Therefore, it is proved that ar (ABFE) = 9/11 ar (EFCD).

37. In the adjoining figure, D and E are respectively the midpoints of sides AB and AC of △ ABC. If PQ || BC and CDP and BEQ are straight lines then prove that ar (△ ABQ) = ar (△ ACP).



Solution:

From the figure we know that D and E are the midpoints of AB and AC
So we get
DE || BC || PQ

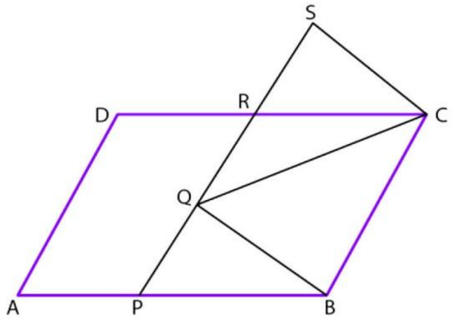
Consider △ ACP
We know that AP || DE and E is the midpoint of AC
Using the midpoint theorem we know that D is the midpoint of PC
So we get
DE = ½ AP
It can be written as
AP = 2DE …… (1)

Consider △ ABQ
We know that AQ || DE and D is the midpoint of AB
Using the midpoint theorem we know that E is the midpoint of BQ
So we get
DE = ½ AQ
It can be written as
AQ = 2DE ……. (2)

Using equations (1) and (2)
We get
AP = AQ
We know that △ ACP and △ ABQ lie on the bases AP and AQ between the same parallels BC and PQ
So we get
Area of △ ACP = Area of △ ABQ

Therefore, it is proved that ar (△ ABQ) = ar (△ ACP).

38. In the adjoining figure, ABCD and BQSC are two parallelograms. Prove that ar (△ RSC) = ar (△ PQB).



Solution:

Consider △ RSC and △ PQB
From the figure we know that RC || PB and ∠ CRS and ∠ BPQ and ∠ RSC and ∠ PQB are corresponding angles
It can be written as
∠ CRS = ∠ BPQ and ∠ RSC = ∠ PQB
We know that the opposite sides of parallelogram are equal
SC = QB
By AAS congruence criterion
△ RSC ≅ △ PQB
So we get
Area of △ RSC = Area of △ PQB

Therefore, it is proved that ar (△ RSC) = ar (△ PQB).