RS Aggarwal Solutions for Class 9 Maths Chapter 9 – Congruence of Triangles and Inequalities in a Triangle

EXERCISE 9A

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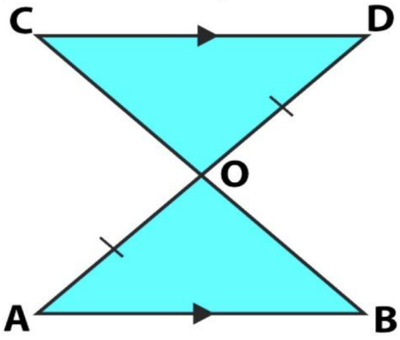
1. In the given figure, AB || CD and O is the midpoint of AD. Show that

(i) △ AOB ≅ △ DOC

(ii) O is the midpoint of BC.



Solution:

(i) From the figure △ AOB and △ DOC

We know that AB || CD and ∠ BAO and ∠ CDO are alternate angles

So we get

∠ BAO = ∠ CDO

From the figure we also know that O is the midpoint of the line AD

We can write it as AO = DO

According to the figure we know that ∠ AOB and ∠ DOC are vertically opposite angles.

So we get ∠ AOB = ∠ DOC

Therefore, by ASA congruence criterion we get

△ AOB ≅ △ DOC

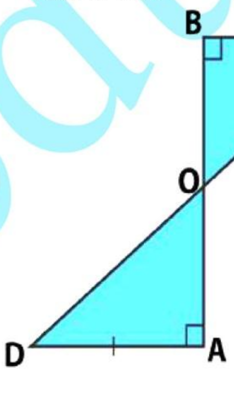
(ii) We know that △ AOB ≅ △ DOC

So we can write it as

BO = CO (c. p. c. t)

Therefore, it is proved that O is the midpoint of BC.

2. In the given figure, AD and BC are equal perpendiculars to a line segment AB. Show that CD bisects AB.

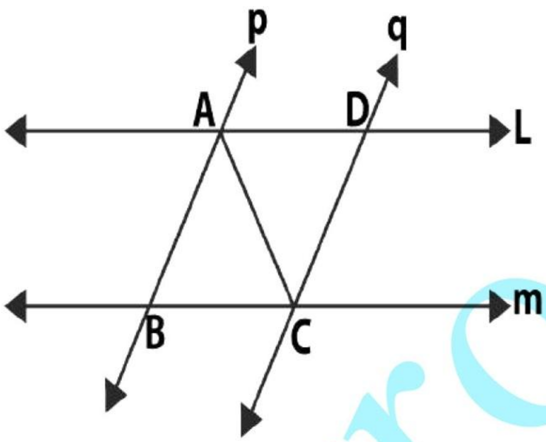


Solution:

Based on the △ AOD and △ BOC
From the figure we know that ∠ AOD and ∠ BOC are vertically opposite angles.
So we get
∠ AOD = ∠ BOC
We also know that
∠ DAO = ∠ CBO = 90°
It is given that AD = BC
Therefore, by AAS congruence criterion we get
△ AOD ≅ △ BOC

So we get AO = BO (c. p. c. t)
Therefore, it is proved that CD bisects AB.

3. In the given figure, two parallel lines l and m are intersected by two parallel lines p and q. Show that △ ABC ≅ △ CDA.



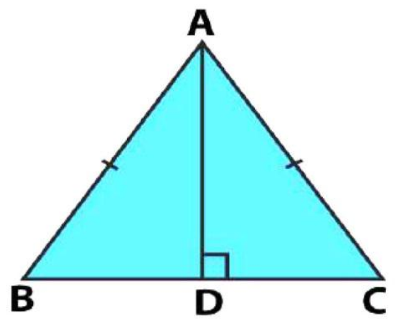
Solution:

Based on the △ ABC and △ CDA
We know that p || q and ∠ BAC and ∠ DCA are alternate interior angles
So we get
∠ BAC = ∠ DCA
We know that A and C are the common points for all the lines
So it can be written as
AC = CA

We know that l || m and ∠ BCA and ∠ DAC are alternate interior angles
So we get
∠ BCA = ∠ DAC

Therefore, by ASA congruence rule it is proved that
△ ABC ≅ △ CDA

4. AD is an altitude of an isosceles △ ABC in which AB = AC. Show that
(i) AD bisects BC,
(ii) AD bisects ∠ A.



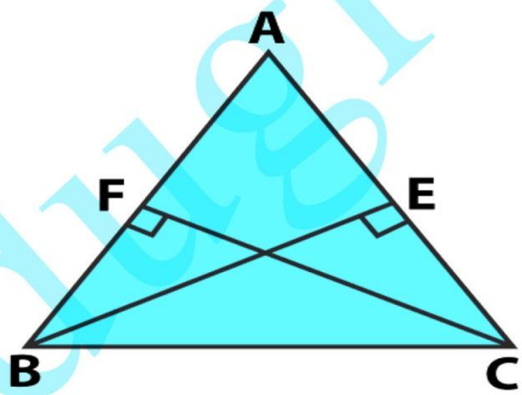
Solution:

(i) Based on the △ BAD and △ CAD
We know that AD is the altitude so the angle is 90°
So we get
∠ ADB = ∠ ADC = 90°
It is given that AB = AC and we know that AD is common
Based on the RHS Congruence Criterion we get
△ BAD ≅ △ CAD
So we get BD = CD (c. p. c. t)

Therefore, it is proved that AD bisects BC.

(ii) We also know that ∠ BAD = ∠ CAD (c. p. c. t)
Therefore, it is proved that AD bisects ∠ A.

5. In the given figure, BE and CF are two equal altitudes of △ ABC. Show that
(i) △ ABE ≅ △ ACF,
(ii) AB = AC.



Solution:

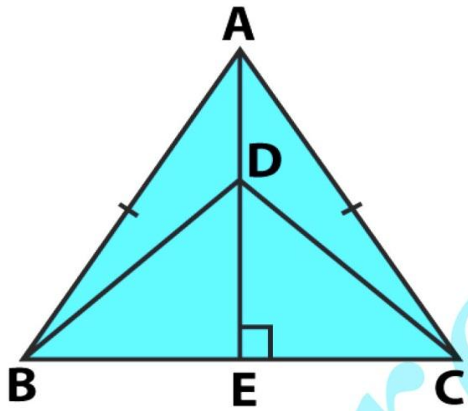
(i) Based on the △ ABE and △ ACF
We know that
∠ AEB = ∠ AFC = 90°
It is given that BE = CF
From the figure we know that ∠ A is common for both ∠ BAE and ∠ CAF

So we get
∠ BAE = ∠ CAF = ∠ A

Therefore, by ASA congruence criterion we get
△ ABE ≅ △ ACF

(ii) As we know that △ ABE ≅ △ ACF
It is proved that AB = AC.

6. △ ABC and △ DBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC. If AD is extended to interest BC at E, show that
(i) △ ABD ≅ △ ACD
(ii) △ ABE ≅ △ ACE
(iii) AE bisects ∠ A as well as ∠ D
(iv) AE is the perpendicular bisector of BC.



Solution:

(i) Based on the △ ABD and △ ACD
From △ ABC we know that AB and AC are equal sides of isosceles triangle
So we get
AB = AC
From △ DBC we know that DB and DC are equal sides of isosceles triangle
So we get
DB = DC
We also know that AD is common i.e. AD = AD

Therefore, by SSS congruence criterion we get
△ ABD ≅ △ ACD

(ii) We know that △ ABD ≅ △ ACD
We get ∠ BAD = ∠ CAD (c. p. c. t)
It can be written as
∠ BAE = ∠ CAE ……… (1)

Considering △ ABE and △ ACE
We know that AB and AC are the equal sides of isosceles △ ABC

AB = AC
So by using equation (1) we get
∠ BAE = ∠ CAE
We know that AE is common i.e. AE = AE

Therefore, by SAS congruence criterion we get
△ ABE ≅ △ ACE

(iii) We know that △ ABD ≅ △ ACD
We get ∠ BAD = ∠ CAD (c. p. c. t)
It can be written as
∠ BAE = ∠ CAE
Therefore, it is proved that AE bisects ∠ A.

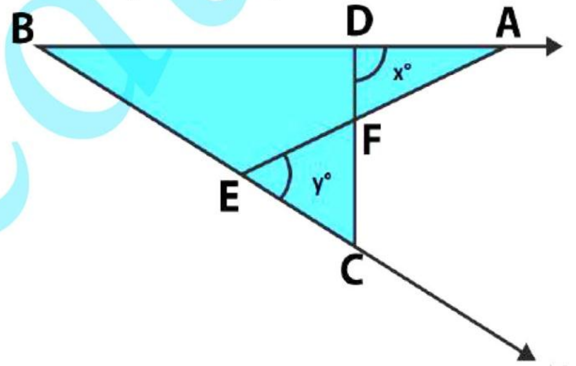
Considering △ BDE and △ CDE
We know that BD and CD are equal sides of isosceles △ ABC
Since △ ABE ≅ △ ACE
BE = CE (c. p. c. t)
We know that DE is common i.e. DE = DE

Therefore, by SSS congruence criterion we get
△ BDE ≅ △ CDE
We know that ∠ BDE = ∠ CDE (c. p. c. t)
So DE bisects ∠ D which means that AE bisects ∠ D
Hence it is proved that AE bisects ∠ A as well as ∠ D.

(iv) We know that △ BDE ≅ △ CDE
So we get
BE = CE and ∠ BED = ∠ CED (c. p. c. t)
From the figure we know that ∠ BED and ∠ CED form a linear pair of angles
So we get
∠ BED = ∠ CED = 90°
We know that DE is the perpendicular bisector of BC

Therefore, it is proved that AE is the perpendicular bisector of BC.

7. In the given figure, if x = y and AB = CB then prove that AE = CD.



Solution:

It is given that x = y and AB = CB

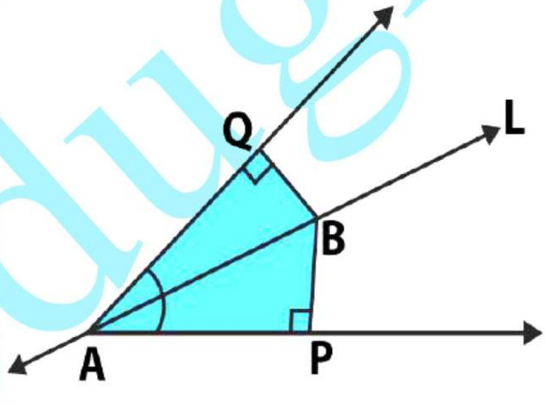
By considering the △ ABE
We know that
Exterior ∠ AEB = ∠ EBA + ∠ BAE
By substituting ∠ AEB as y we get
y = ∠ EBA + ∠ BAE

By considering the △ BCD
We know that
x = ∠ CBA + ∠ BCD
It is given that x = y
So we can write it as
∠ CBA + ∠ BCD = ∠ EBA + ∠ BAE
On further calculation we can write it as
∠ BCD = ∠ BAE

Based on both △ BCD and △ BAE
We know that B is the common point
It is given that AB = BC
It is proved that ∠ BCD = ∠ BAE

Therefore, by ASA congruence criterion we get
△ BCD ≅ △ BAE
We know that the corresponding sides of congruent triangles are equal
Therefore, it is proved that AE = CD.

8. In the given figure, line l is the bisector of an angle ∠ A and B is any point on l. If BP and BQ are perpendiculars from B to the arms of ∠ A, show that
(i) △ APB ≅ △ AQB
(ii) BP = BQ, i.e., B is equidistant from the arms of ∠ A.



Solution:

(i) Considering △ APB and △ AQB
We know that
∠ APB = ∠ AQC = 90º
From the figure we know that l is the bisector of ∠ A

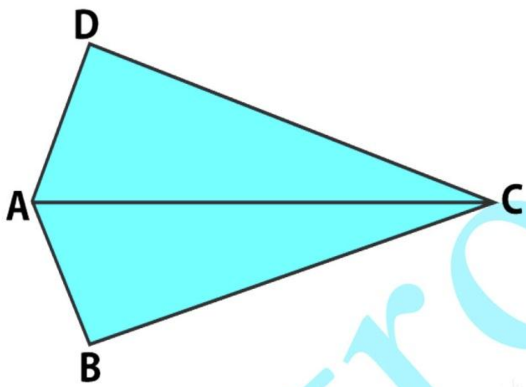
So we get
∠ BAP = ∠ BAQ
We know that AB is common i.e. AB = AB

Therefore, by AAS congruence criterion we get
△ APB ≅ △ AQB

(ii) We know that △ APB ≅ △ AQB
So it is proved that BP = BQ (c. p. c. t)

9. ABCD is a quadrilateral such that diagonal AC bisects the angles ∠ A and ∠ C. Prove that AB = AD and CB = CD.

Solution:



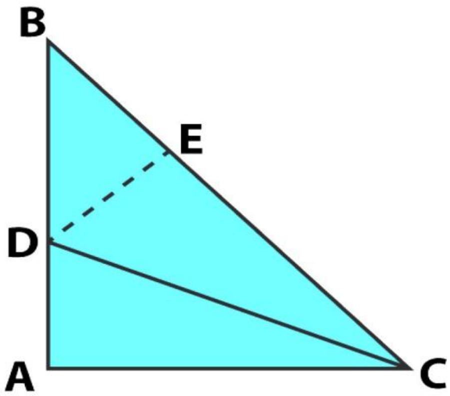
By considering △ ABC and △ ADC
We know that AC bisects at ∠ A
So we get
∠ BAC = ∠ DAC
We know that AC is common i.e. AC = AC
From the figure we know that AC bisects at ∠ C
∠ BCA = ∠ DCA

By ASA congruence criterion we get
△ ABC ≅ △ ADC

Therefore, it is proved that AB = AD and CB = CD (c. p. c. t)

10. △ ABC is a right triangle right angled at A such that AB = AC and bisector of ∠ C intersects the side AB at D. Prove that AC + AD = BC.

Solution:



Construct a triangle ABC where DE is perpendicular to BC
Consider the △ DAC and △ DEC
We know that
∠ BAC = ∠ DAC = 90°
From the figure we know that CD bisects ∠ C
So we get
∠ DCA = ∠ DCE
We know that CD is common i.e. CD = CD
By AAS congruence criterion
△ DAC ≅ △ DEC
So we know that DA = DE …… (1)
AC = EC (c. p. c. t) ….. (2)

It is given that AB = AC
We know that the angles opposite to equal sides are equal
∠ B = ∠ C

We know that the sum of angles of △ ABC is 180°.
∠ A + ∠ B + ∠ C = 180°
By substituting the values
90° + ∠ B + ∠ B = 180°
On further calculation
2 ∠ B = 180° - 90°
2 ∠ B = 90°
By division
∠ B = 45°

Considering the △ BED
We know that ∠ BED = 90°
So we can write it as
∠ BDE + ∠ B = 90°
By substituting the values
∠ BDE + 45° = 90°
On further calculation
∠ BDE = 90° - 45°
By subtraction

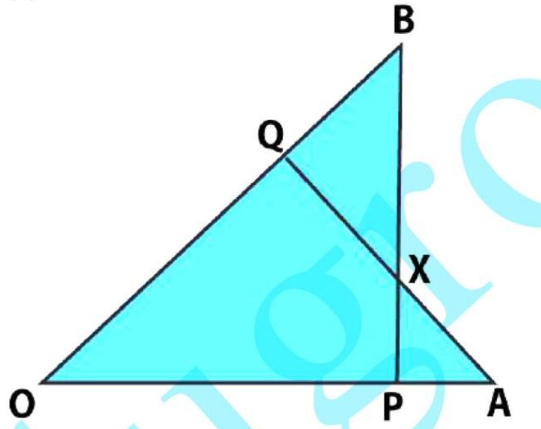
∠ BDE = 45°
It can be written as
∠ BDE = ∠ DBE = 45°
We know that DE and BE are the equal sides of isosceles triangle
DE = BE …….. (3)

By comparing the equations (1) and (3)
We get
DA = DE = BE …… (4)

We know that BC = BE + EC
By considering the equations (ii) and (iv)
We get
BC = DA + AC
We can also write it as
AC + AD = BC

Therefore, it is proved that AC + AD = BC.

11. In the given figure, OA = OB and OP = OQ. Prove that
(i) PX = QX,
(ii) AX = BX.



Solution:

It is given that OA = OB and OP = OQ
By considering the △ OAQ and △ OPB
Therefore, by SAS congruence criterion
△ OAQ = △ OPB

We know that the corresponding parts of congruent triangles are equal
So we get
∠ OBP = ∠ OAQ …….. (1)

Consider △ BXQ and △ PXA
We can write it as
BQ = OB – OQ and PA = OA - OP

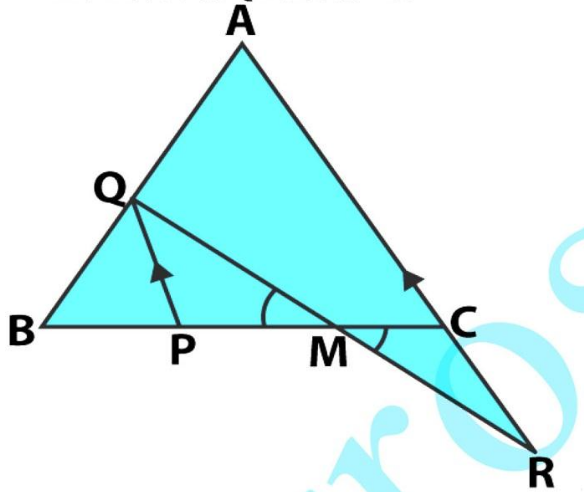
We know that OP = OQ and is given that OA = OB
So we get BQ = PA ……. (2)

In △ BXQ and △ PXA
We know that ∠ BXQ and ∠ PXA are vertically opposite angles
∠ BXQ = ∠ PXA

From (1) and (2) and AAS congruence criterion we get
△ BXQ ≅ △ PXA

So we get PX = QX and AX = BX (c. p. c. t)

12. In the given figure, ABC is an equilateral triangle; PQ || AC and AC is produced to R such that CR = BP. Prove that QR bisects PC.



Solution:

It is given that ABC is an equilateral triangle; PQ || AC and AC is produced to R such that CR = BP
Consider QR intersecting the line PC at point M
We know that △ ABC is an equilateral triangle
So we get ∠ A = ∠ ACB = 60°

From the figure we know that PQ || AC and ∠ BPQ and ∠ ACB are corresponding angles
So we get
∠ BPQ = ∠ ACB = 60°

Based on the △ BPQ we know that
∠ B = ∠ ACB = 60°
It can be written as
∠ BQP = 60°
According to the figure we know that △ BPQ is an equilateral triangle
So we get
PQ = BP = BQ

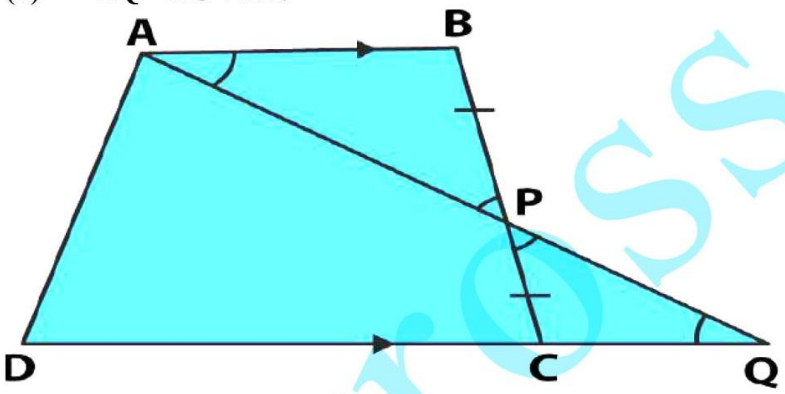
It is given that CR = BP so we get

PQ = CR ……. (1)

In the △ PMQ and △ CMR we know that PQ || AC and QR is the transversal
We know that ∠ PQM and ∠ CRM are alternate angles and ∠ PMQ and ∠ CMR are vertically opposite angles
∠ PQM = ∠ CRM
∠ PMQ = ∠ CMR
By considering equation (1) and AAS congruence criterion
△ PMQ ≅ △ CMR
We know that the corresponding parts of congruent triangles are equal
PM = MC

Therefore, it is proved that QR bisects PC.

13. In the given figure, ABCD is a quadrilateral in which AB || DC and P is the midpoint of BC. On producing, AP and DC meet at Q. Prove that
(i) AB = CQ,
(ii) DQ = DC + AB.



Solution:

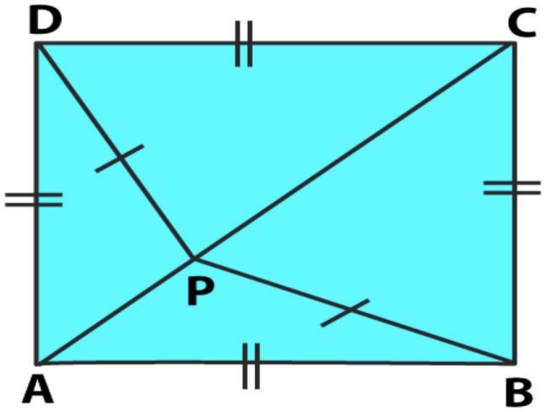
It is given that
ABCD is a quadrilateral in which AB || DC and P is the midpoint of BC
Considering △ ABP and △ PCQ
We know that ∠ PAB and ∠ PQC are alternate angles and ∠ APB and ∠ CPQ are vertically opposite angles
∠ PAB = ∠ PQC
∠ APB = ∠ CPQ
According to AAS congruence criterion
△ ABP ≅ △ PCQ
AB = CQ (c. p. c. t) …… (i)

We know that
DQ = DC + CQ
By substituting CQ as AB we get
DQ = DC + AB …… (ii)

Therefore, it is proved that AB = CQ and DQ = DC + AB.

14. In the given figure, ABCD is a square and P is a point inside it such that PB = PD. Prove that CPA

is a straight line.



Solution:

It is given that ABCD is a square and P is a point inside it such that PB = PD
Considering △ APD and △ APB
We know that all the sides are equal in a square
So we get DA = AB
AP is common i.e. AP = AP
According to SSS congruence criterion
△ APD ≅ △ APB
We get ∠ APD = ∠ APB (c. p. c. t)….. (1)

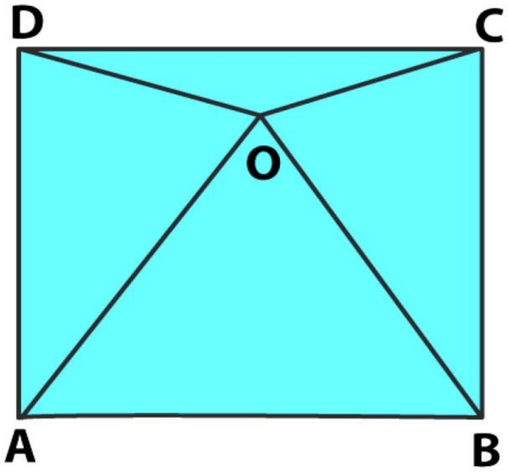
Considering △ CPD and △ CPB
We know that all the sides are equal in a square
So we get CD = CB
CP is common i.e. CP = CP
According to SSS congruence criterion
△ CPD ≅ △ CPB
We get ∠ CPD = ∠ CPB (c. p. c. t)….. (2)

By adding both the equation (1) and (2)
∠ APD + ∠ CPD = ∠ APB + ∠ CPB ……. (3)

From the figure we know that the angles surrounding the point P is 360^{o}
So we get
∠ APD + ∠ CPD + ∠ APB + ∠ CPB = 360^{o}
By grouping we get
∠ APB + ∠ CPB = 360^{o} – (∠ APD + ∠ CPD) …… (4)
Now by substitution of (4) in (3)
∠ APD + ∠ CPD = 360^{o} – (∠ APD + ∠ CPD)
On further calculation
2 (∠ APD + ∠ CPD) = 360^{o}
By division we get
∠ APD + ∠ CPD = 180^{o}

Therefore, it is proved that CPA is a straight line.

15. In the given figure, O is a point in the interior of square ABCD such that △ OAB is an equilateral triangle. Show that △ OCD is an isosceles triangle.



Solution:

We know that △ OAB is an equilateral triangle
So it can be written as
∠ OAB = ∠ OBA = AOB = 60°

From the figure we know that ABCD is a square
So we get
∠ A = ∠ B = ∠ C = ∠ D = 90°

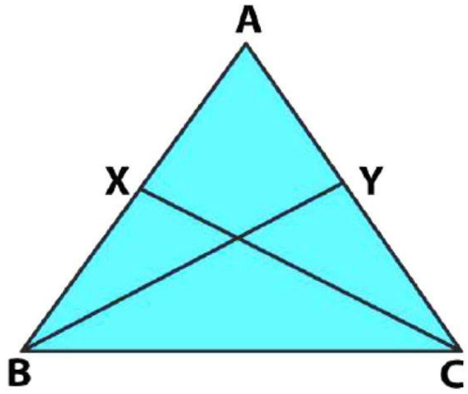
In order to find the value of ∠ DAO
We can write it as
∠ A = ∠ DAO + ∠ OAB
By substituting the values we get
90° = ∠ DAO + 60°
On further calculation
∠ DAO = 90° - 60°
By subtraction
∠ DAO = 30°

We also know that ∠ CBO = 30°

Considering the △ OAD and △ OBC
We know that the sides of a square are equal
AD = BC
We know that the sides of an equilateral triangle are equal
OA = OB
By SAS congruence criterion
△ OAD ≅ △ OBC
So we get OD = OC (c. p. c. t)

Therefore, it is proved that △ OCD is an isosceles triangle.

16. In the adjoining figure, X and Y are respectively two points on equal sides AB and AC of △ ABC such that AX = AY. Prove that CX = BY.

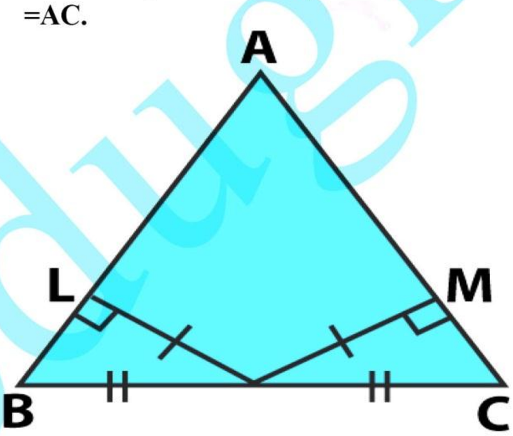


Solution:

It is given that AX = AY
Considering △ AXC and △ AYB
∠ A is common i.e. ∠ A = ∠ A
We know that the sides of equilateral triangle are equal so we get
AC = AB
By SAS congruence criterion
△ AXC ≅ △ AYB
XC = YB (c. p. c. t)

Therefore, it is proved that CX = BY.

17. In △ ABC, D is the midpoint of BC. If DL ⊥ AB and DM ⊥ AC such that DL = DM, prove that AB =AC.



Solution:

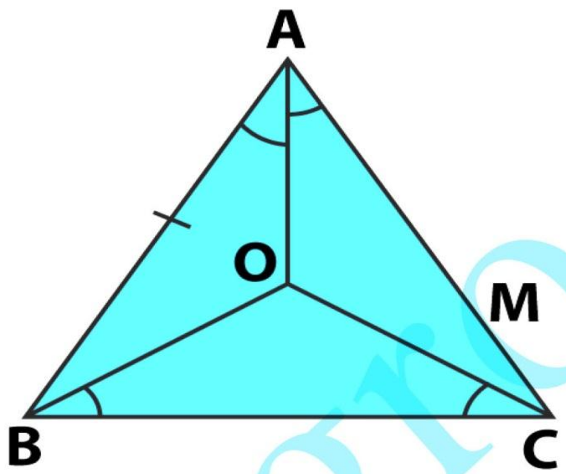
It is given that D is the midpoint of BC
DL ⊥ AB and DM ⊥ AC such that DL = DM
Considering △ BLD and △ CMD as right angled triangle

So we can write it as
∠ BLD = ∠ CMD = 90°
We know that BD = CD and DL = DM
By RHS congruence criterion
△ BLD = △ CMD
∠ ABD = ∠ ACD (c. p. c. t)

Now, in △ ABC
∠ ABD = ∠ ACD
We know that the sides opposite to equal angles are equal so we get
AB = AC

Therefore, it is proved that AB = AC.

18. In △ ABC, AB = AC and the bisectors ∠ B and ∠ C meet at a point O. Prove that BO = CO and the ray AO is the bisector of ∠ A.



Solution:

It is given that AB = AC and the bisectors ∠ B and ∠ C meet at a point O
Consider △ BOC
So we get
∠ BOC = ½ ∠ B and ∠ OCB = ½ ∠ C
It is given that AB = AC so we get ∠ B = ∠ C
So we get
∠ OBC = ∠ OCB
We know that if the base angles are equal even the sides are equal
So we get OB = OC ……. (1)

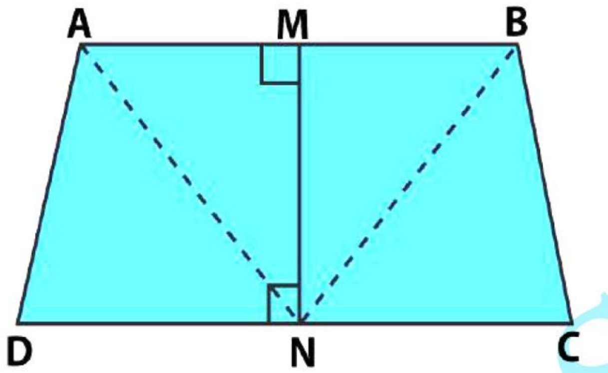
∠ B and ∠ C has the bisectors OB and OC so we get
∠ ABO = ½ ∠ B and ∠ ACO = ½ ∠ C
So we get
∠ ABO = ∠ ACO …….. (2)
Considering △ ABO and △ ACO and equation (1) and (2)
It is given that AB = AC

By SAS congruence criterion
△ ABO ≅ △ ACO
∠ BAO = ∠ CAO (c. p. c. t)

Therefore, it is proved that BO = CO and the ray AO is the bisector of ∠ A.

19. The line segments joining the midpoints M and N are parallel sides AB and DC respectively of a trapezium ABCD is perpendicular to both the sides AB and DC. Prove that AD = BC.
Solution:

Construct AN and BN at the point N



Consider △ ANM and △ BNM
We know that N is the midpoint of the line AB
So we get
AM = BM

From the figure we know that
∠ AMN = ∠ BMN = 90°
MN is common i.e. MN = MN
By SAS congruence criterion
△ ANM ≅ △ BNM
AN = BN (c. p. c. t) …… (1)

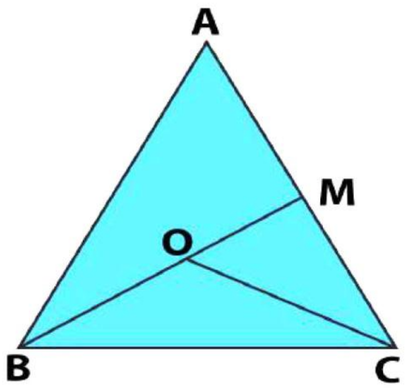
We know that
∠ ANM = ∠ BNM (c. p. c. t)
Subtracting LHS and RHS by 90°
90° - ∠ ANM = 90° - ∠ BNM
So we get
∠ AND = ∠ BNC …… (2)

Now, consider △ AND and △ BNC
AN = BN
∠ AND = ∠ BNC
We know that N is the midpoint of the line DC
DN = CN
By SAS congruence criterion
△ AND ≅ △ BNC
AD = BC (c. p. c. t)

Therefore, it is proved that AD = BC.

20. The bisectors ∠ B and ∠ C of and isosceles triangle with AB = AC intersect each other at a point O. BO is produced to meet AC at a point M. Prove that ∠ MOC = ∠ ABC.

Solution:



Consider △ ABC

It is given that AB = AC

So we get

∠ ABC = ∠ ACB

Dividing both sides by 2 we get

½ ∠ ABC = ½ ∠ ACB

So we get

∠ OBC = ∠ OCB

By using the exterior angle property

We get

∠ MOC = ∠ OBC + ∠ OCB

We know that ∠ OBC = ∠ OCB

So we get

∠ MOC = 2 ∠ OBC

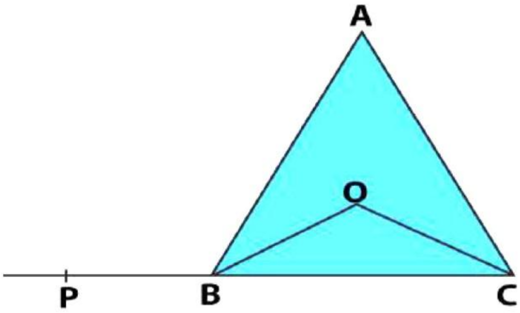
We know that OB is the bisector of ∠ ABC

∠ MOC = ∠ ABC

Therefore, it is proved that ∠ MOC = ∠ ABC.

21. The bisectors of ∠ B and ∠ C of an isosceles △ ABC with AB = AC intersect each other at a point O. Show that the exterior angle adjacent to ∠ ABC is equal to ∠ BOC.

Solution:



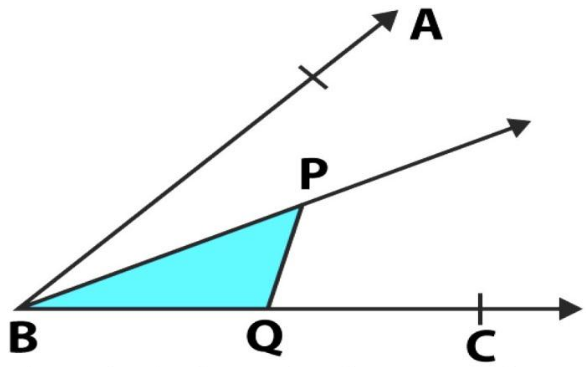
Considering the △ ABC
It is given that AB = AC
So we get
∠ ABC = ∠ ACB
Dividing by 2 both sides
½ ∠ ABC = ½ ∠ ACB
So we get
∠ OBC = ∠ OCB …… (1)

By using the angle sum property in △ BOC
∠ BOC + ∠ OBC + ∠ OCB = 180^{o}
Substituting equation (1)
∠ BOC + 2 ∠ OBC = 180^{o}
So we get
∠ BOC + ∠ ABC = 180^{o}
From the figure we know that ∠ ABC and ∠ ABP form a linear pair of angles so we get
∠ ABC + ∠ ABP = 180^{o}
∠ ABC = 180^{o} - ∠ ABP
By substituting the value in the above equation we get
∠ BOC + (180^{o} - ∠ ABP) = 180^{o}
On further calculation
∠ BOC + 180^{o} - ∠ ABP = 180^{o}
By subtraction
∠ BOC - ∠ ABP = 180^{o} - 180^{o}
∠ BOC - ∠ ABP = 0
∠ BOC = ∠ ABP

Therefore, it is proved that the exterior angle adjacent to ∠ ABC is equal to ∠ BOC.

22. P is a point on the bisector of ∠ ABC. If the line through P, parallel to BA meets BC at Q, prove that △ BPQ is an isosceles triangle.

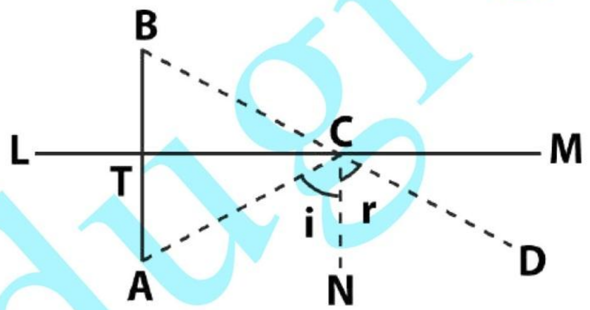
Solution:



We know that AB || PQ and BP is a transversal
From the figure we know that ∠ ABP and ∠ BPQ are alternate angles
So we get
∠ ABP = ∠ BPQ ….. (1)
We also know that BP is the bisector of ∠ ABC
So we get
∠ ABP = ∠ PBC and ∠ ABP = ∠ PBQ …. (2)
By considering equation (1) and (2) we get
∠ BPQ = ∠ PBQ
We know that the sides opposite to equal angles are equal
PQ = BQ

Therefore, it is proved that △ BPQ is an isosceles triangle.

23. The image of an object placed at a point A before a plane mirror LM is seen at the point B by an observer at D, as shown in the figure. Prove that the image is as far behind the mirror as the object is in front of the mirror.



Solution:

According to the figure we need to prove that AT = BT

We know that
Angle of incidence = Angle of reflection
So we get
∠ ACN = ∠ DCN ….. (1)

We know that AB || CN and AC is the transversal
From the figure we know that ∠ TAC and ∠ ACN are alternate angles

∠ TAC = ∠ CAN …… (2)

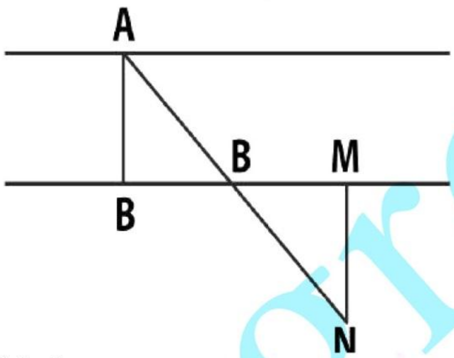
We know that AB || CN and BD is the transversal
From the figure we know that ∠ TBC and ∠ DCN are corresponding angles
∠ TBC = ∠ DCN ….. (3)

By considering the equation (1), (2) and (3)
We get
∠ TAC = ∠ TBC …… (4)

Now in △ ACT and △ BCT
∠ ATC = ∠ BTC = 90°
CT is common i.e. CT = CT
By AAS congruence criterion
△ ACT ≅ △ BCT
AT = BT (c. p. c. t)

Therefore, it is proved that the image is as far behind the mirror as the object is in front of the mirror.

24. In the adjoining figure, explain how one can find the breadth of the river without crossing it.



Solution:

Consider AB as the breadth of the river. Take a point M at a distance from B. Draw a perpendicular from the point M and name it as N so that it joins the point A as a straight line.

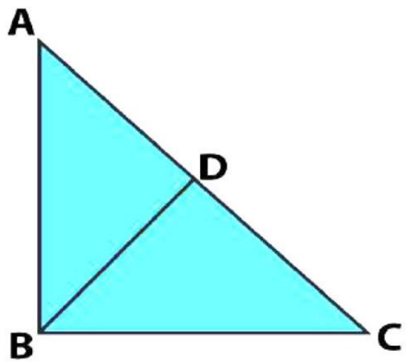
Now in △ ABO and △ NMO
We know that
∠ OBA = ∠ OMN = 90°
We know that O is the midpoint of the line BM
So we get
OB = OM

From the figure we know that ∠ BAO and ∠ MON are vertically opposite angles
∠ BAO = ∠ MON

By ASA congruence criterion
△ ABO ≅ △ NMO
AB = NM (c. p. c. t)

Therefore, MN is the width of the river.

25. In a △ ABC, D is the midpoint of side AC such that BD = ½ AC. Show that ∠ ABC is a right angle.
Solution:



From the figure we know that D is the midpoint of the line AC
So we get
AD = CD = ½ AC
It is given that BD = ½ AC
So we can write it as
AD = BD = CD

Let us consider AD = BD
We know that the angles opposite to equal sides are equal
So we get
∠ BAD = ∠ ABD …..(1)

Let us consider CD = BD
We know that the angles opposite to equal sides are equal
So we get
∠ BCD = ∠ CBD ….. (2)

By considering the angle sum property in △ ABC
We get
∠ ABC + ∠ BAC + ∠ BCA = 180°
So we can write it as
∠ ABC + ∠ BAD + ∠ BCD = 180°
By using equation (1) and (2) we get
∠ ABC + ∠ ABD + ∠ CBD = 180°
So we get
∠ ABC + ∠ ABC = 180°
By addition
2 ∠ ABC = 180°
By division
∠ ABC = 90°

Therefore, ∠ ABC is a right angle.

26. "If two sides and an angle of one triangle are equal to two sides and an angle of another triangle

then the two triangles must be congruent." Is the statement true? Why?
Solution:

The statement is false.
If two sides and the included angle of one triangle are equal to corresponding two sides and the included angle of another triangle then the two triangles must be congruent.

27. "If two angles and a side of one triangle are equal to two angles and a side of another triangle then the two triangles must be congruent." Is the statement true? Why?
Solution:

The statement is false.
If two angles and the corresponding side of one triangle are equal to the two included angles and the corresponding side of another triangle then the two triangles must be congruent.

EXERCISE 9B PAGE: 296

1. Is it possible to construct a triangle with lengths of its sides as given below? Give reason for your answer.

(i) 5cm, 4cm, 9cm
(ii) 8cm, 7cm, 4cm
(iii) 10cm, 5cm, 6cm
(iv) 2.5cm, 5cm, 7cm
(v) 3cm, 4cm, 8cm

Solution:

(i) No. It is not possible to construct a triangle with lengths of its sides 5cm, 4cm and 9cm because the sum of two sides is not greater than the third side i.e. 5 + 4 is not greater than 9.

(ii) Yes. It is possible to construct a triangle with lengths of its sides 8cm, 7cm and 4cm because the sum of two sides of a triangle is greater than the third side.

(iii) Yes. It is possible to construct a triangle with lengths of its sides 10cm, 5cm and 6cm because the sum of two sides of a triangle is greater than the third side.

(iv) Yes. It is possible to construct a triangle with lengths of its sides 2.5cm, 5cm and 7cm because the sum of two sides of a triangle is greater than the third side.

(v) No. It is not possible to construct a triangle with lengths of its sides 3cm, 4cm and 8cm because the sum of two sides is not greater than the third side.

2. In $\triangle ABC$, $\angle A = 50^\circ$ and $\angle B = 60^\circ$. Determine the longest and the shortest sides of the triangle.

Solution:

Consider $\triangle ABC$

Based on the sum property we can write it as

$\angle A + \angle B + \angle C = 180^\circ$

By substituting the values we get

$50^\circ + 60^\circ + \angle C = 180^\circ$

On further calculation

$\angle C = 180^\circ - 50^\circ - 60^\circ$

By subtraction

$\angle C = 180^\circ - 110^\circ$

$\angle C = 70^\circ$

So we have $\angle A < \angle B < \angle C$

We get

$BC < AC < AB$

Therefore, the longest side of the triangle is AB and the shortest side is BC.

3.

(i) In $\triangle ABC$, $\angle A = 90^\circ$. What is the longest side?
(ii) In $\triangle ABC$, $\angle A = \angle B = 45^\circ$. Which is its longest side?
(iii) In $\triangle ABC$, $\angle A = 100^\circ$ and $\angle C = 50^\circ$. Which is its shortest side?

Solution:

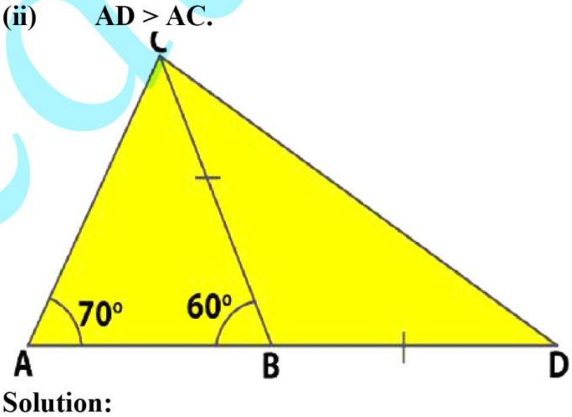
(i) It is given that ∠ A = 90^o
We know that in a right angled triangle the highest angle is 90^o and the sum of all the angles is 180^o
So we get that ∠ A is the greatest angle in △ ABC
Hence, BC is the longest side which is opposite to ∠ A

(ii) In △ ABC it is given that ∠ A = ∠ B = 45^o
Based on the sum property of the triangle
∠ A + ∠ B + ∠ C = 180^o
To find ∠ C
∠ C = 180^o - ∠ A - ∠ B
By substituting the values in the above equation
∠ C = 180^o - 45^o - 45^o
By subtraction
∠ C = 180^o - 90^o
∠ C = 90^o
So we get that ∠ C is the greatest angle in △ ABC
Hence, AB is the longest side which is opposite to ∠ C.

(iii) In △ ABC it is given that ∠ A = 100^o and ∠ C = 50^o
Based on the sum property of the triangle
∠ A + ∠ B + ∠ C = 180^o
To find ∠ B
∠ B = 180^o - ∠ A - ∠ C
By substituting the values in the above equation
∠ B = 180^o - 100^o - 50^o
By subtraction
∠ B = 180^o - 150^o
∠ B = 30^o

So we get ∠ B < ∠ C < ∠ A
i.e. AC < AB < BC
Hence, AC is the shortest side in △ ABC.

4. In △ ABC, side AB is produced to D such that BD = BC. If ∠ A = 70^o and ∠ B = 60^o, prove that
(i) AD > CD
(ii) AD > AC.



Solution:

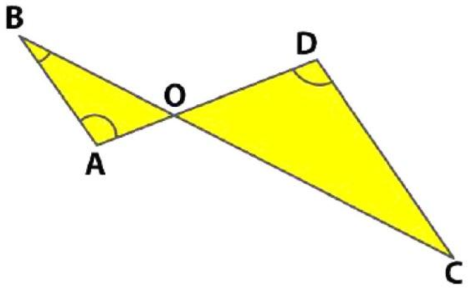
In △ ABC it is given that ∠ A = 70° and ∠ B = 60°
Based on the sum property of the triangle
∠ A + ∠ B + ∠ C = 180°
To find ∠ C
∠ C = 180° - ∠ A - ∠ B
By substituting the values in the above equation
∠ C = 180° - 70° - 60°
∠ C = 180° - 130°
By subtraction
∠ C = 50°
Consider △ BCD
We know that ∠ CBD is the exterior angle of ∠ ABC
So we get
∠ CBD = ∠ DAC + ∠ ACB
By substituting the values in the above equation
∠ CBD = 70° + 50°
By addition
∠ CBD = 120°

It is given that BC = BD
So we can write it as
∠ BCD = ∠ BDC
Based on the sum property of the triangle
∠ BCD + ∠ BDC + ∠ CBD = 180°
So we get
∠ BCD + ∠ BDC = 180° - ∠ CBD
By substituting values in the above equation
∠ BCD + ∠ BDC = 180° - 120°
∠ BCD + ∠ BDC = 60°
It can be written as
2 ∠ BCD = 60°
By division
∠ BCD = ∠ BDC = 30°

In △ ACD
It is given that ∠ A = 70° and ∠ B = 60°
We can write it as
∠ ACD = ∠ ACB + ∠ BCD
By substituting the values we get
∠ ACD = 50° + 30°
By addition
∠ ACD = 80°
So we get to know that ∠ ACD is the greatest angle and the side opposite to it i.e. AD is the longest side.
Therefore, it is proved that AD > CD

We know that ∠ BDC is the smallest angle and the side opposite to it i.e. AC is the shortest side.
Therefore, it is proved that AD > AC.

5. In the given figure, ∠ B < ∠ A and ∠ C < ∠ D. Show that AD < BC.



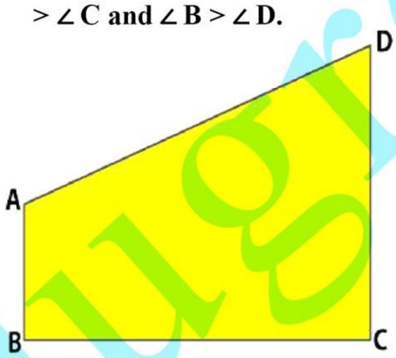
Solution:

In the figure it is given that ∠ B < ∠ A and ∠ C < ∠ D
Consider triangle AOB
Since ∠ B < ∠ A
We get
AO < BO …… (1)

Consider triangle COD
Since ∠ C < ∠ D
DO < CO ……. (2)
By adding both the equations we get
AO + DO < BO + CO
So we get
AD < BC

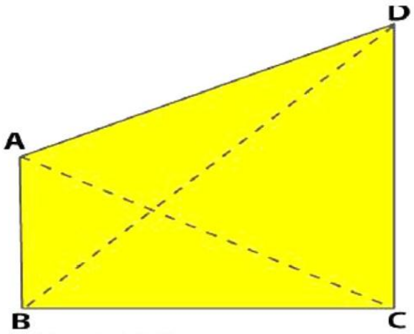
Therefore, it is proved that AD < BC.

6. **AB and CD are respectively the smallest and largest sides of a quadrilateral ABCD. Show that ∠ A > ∠ C and ∠ B > ∠ D.**



Solution:

Construct two lines AC and BD in the given quadrilateral



Consider △ ABC
We know that BC > AB
It can be written as
∠ BAC > ∠ ACB ……. (i)

Consider △ ACD
We know that CD > AD
It can be written as
∠ CAD > ∠ ACD ……. (ii)

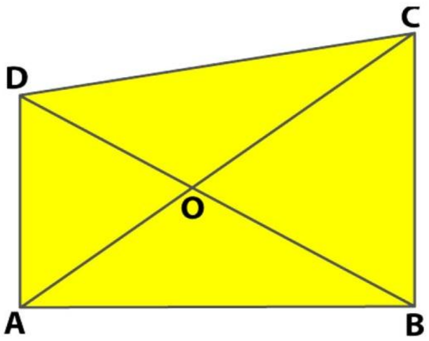
By adding both the equations we get
∠ BAC + ∠ CAD > ∠ ACB + ∠ ACD
So we get
∠ A > ∠ C

Consider △ ADB
We know that AD > AB
It can be written as
∠ ABD > ∠ ADB ……. (iii)

Consider △ BDC
We know that CD > BC
It can be written as
∠ CBD > ∠ BDC ……. (iv)

By adding both the equations we get
∠ ABD + ∠ CBD > ∠ ADB + ∠ BDC
So we get
∠ B > ∠ D

7. In a quadrilateral ABCD, show that (AB + BC + CD + DA) > (AC + BD).



Solution:

Consider △ ABC
We know that
AB + BC > AC ….. (1)

Consider △ ACD
We know that
DA + CD > AC ….. (2)

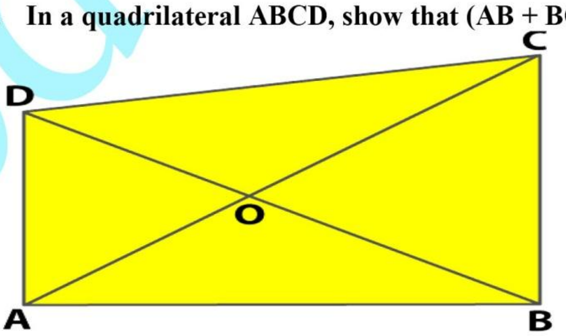
Consider △ ADB
We know that
DA + AB > BD ….. (3)

Consider △ BDC
We know that
BC + CD > BD ….. (4)

By adding all the equations
AB + BC + DA + CD + DA + AB + BC + CD > AC + AC + BD + BD
So we get
2 (AB + BD + CD + DA) > 2 (AC + BD)
Dividing by 2 both the sides
AB + BD + CD + DA > AC + BD

Therefore, it is proved that AB + BD + CD + DA > AC + BD.

8. In a quadrilateral ABCD, show that (AB + BC + CD + DA) < 2 (BD + AC).



Solution:

Consider △ AOB
We know that
AO + BO > AB ….. (1)

Consider △ BOC
We know that
BO + CO > BC ….. (2)

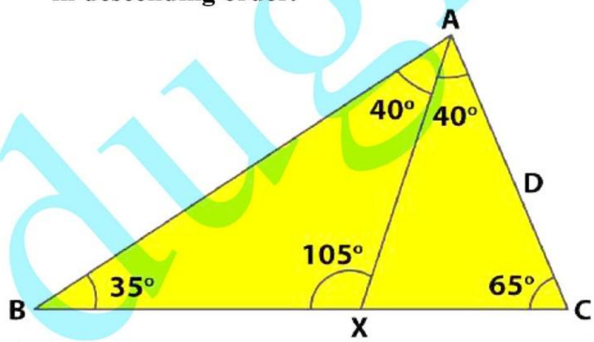
Consider △ COD
We know that
CO + DO > CD ….. (3)

Consider △ AOD
We know that
DO + AO > DA ….. (4)

By adding all the equations
AO + BO + BO + CO + CO + DO + DO + AO > AB + BC + CD + DA
So we get
2 (AO + CO) + 2 (BO + DO) > AB + BC + CD + DA
On further calculation
2AC + 2BD > AB + BC + CD + DA
By taking 2 as common
2 (AC + BD) > AB + BC + CD + DA
So we get
AB + BC + CD + DA < 2 (AC + BD)

Therefore, it is proved that AB + BC + CD + DA < 2 (AC + BD)

9. In △ ABC, ∠ B = 35°, ∠ C = 65° and the bisector of ∠ BAC meets BC in X. Arrange AX, BX and CX in descending order.



Solution:

Consider △ ABC
By sum property of a triangle
∠ A + ∠ B + ∠ C = 180°
To find ∠ A
∠ A = 180° - ∠ B - ∠ C
By substituting the values
∠ A = 180° - 35° - 65°

By subtraction
$\angle A = 180^\circ - 100^\circ$
$\angle A = 80^\circ$

We know that
$\angle BAX = ½ \angle A$
So we get
$\angle BAX = ½ (80^\circ)$
By division
$\angle BAX = 40^\circ$

Consider △ ABX
It is given that $\angle B = 35^\circ$ and $\angle BAX = 40^\circ$
By sum property of a triangle
$\angle BAX + \angle BXA + \angle XBA = 180^\circ$
To find $\angle BXA$
$\angle BXA = 180^\circ - \angle BAX - \angle XBA$
By substituting values
$\angle BXA = 180^\circ - 35^\circ - 40^\circ$
By subtraction
$\angle BXA = 180^\circ - 75^\circ$
$\angle BXA = 105^\circ$

We know that $\angle B$ is the smallest angle and the side opposite to it i.e. AX is the smallest side.
So we get AX < BX ….. (1)

Consider △ AXC
$\angle CAX = ½ \angle A$
So we get
$\angle CAX = ½ (80^\circ)$
By division
$\angle CAX = 40^\circ$

By sum property of a triangle
$\angle AXC + \angle CAX + \angle CXA = 180^\circ$
To find $\angle AXC$
$\angle AXC = 180^\circ - \angle CAX - \angle CXA$
By substituting values
$\angle AXC = 180^\circ - 40^\circ - 65^\circ$
So we get
$\angle AXC = 180^\circ - 105^\circ$
By subtraction
$\angle AXC = 75^\circ$

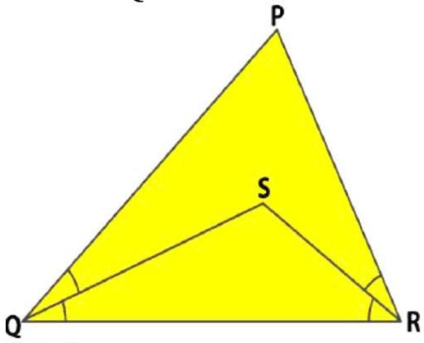
So we know that $\angle CAX$ is the smallest angle and the side opposite to it i.e. CX is the smallest side.
We get
CX < AX …… (2)

By considering equation (1) and (2)

BX > AX > CX

Therefore, BX > AX > CX is the descending order.

10. In the given figure, PQ > PR and QS and RS are the bisectors of ∠ Q and ∠ R respectively. Show that SQ > SR.

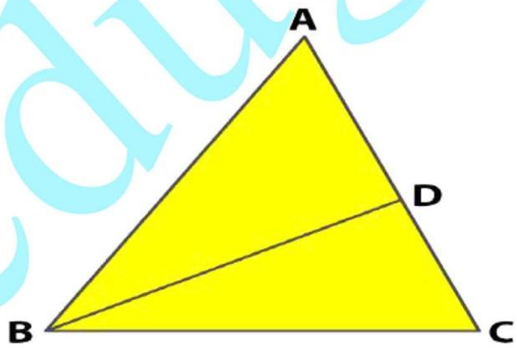


Solution:

Consider △ PQR
It is given that PQ > PR
So we get
∠ PRQ > ∠ PQR
Dividing both sides by 2 we get
½ ∠ PRQ > ½ ∠ PQR
From the figure we get
∠ SRQ > ∠ SQR
So we get
SQ > SR

Therefore, it is proved that SQ > SR.

11. D is any point on the side AC of △ ABC with AB = AC. Show that CD < BD.
Solution:

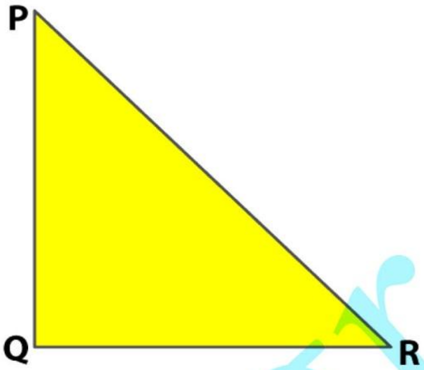


Consider △ ABC
It is given that AB = AC
So we get
∠ ABC = ∠ ACB …… (1)

From the figure we know that
∠ ABC = ∠ ADB + ∠ DBC
So we get
∠ ABC > ∠ DBC
From equation (1)
∠ ACB > ∠ DBC
i.e. ∠ DCB > ∠ DBC
It means that
BD > CD
So we get
CD < BD

Therefore, it is proved that CD < BD.

12. Prove that in a triangle, other than an equilateral triangle, angle opposite the longest side is greater 2/3 of a right angle.
Solution:



Consider △ PQR where PR is the longest side
So we get PR > PQ
i.e. ∠ Q > ∠ R ….. (1)

We also know that PR > QR
i.e. ∠ Q > ∠ P ….. (2)

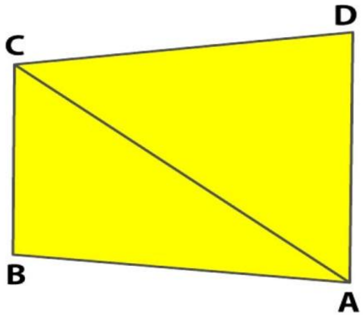
By adding both the equations
∠ Q + ∠ Q > ∠ R + ∠ P
So we get
2 ∠ Q > ∠ R + ∠ P
By adding ∠ Q on both LHS and RHS
2 ∠ Q + ∠ Q > ∠ R + ∠ P + ∠ Q
We know that ∠ R + ∠ P + ∠ Q = 180°
So we get
3 ∠ Q > 180°
By division
∠ Q > 60°

So we get

$\angle Q > 2/3\ (90^o)$
i.e. $\angle Q > 2/3$ of a right angle

Therefore, it is proved that in a triangle, other than an equilateral triangle, angle opposite the longest side is greater 2/3 of a right angle.

13. In the given figure, prove that
(i) CD + DA + AB > BC
(ii) CD + DA + AB + BC > 2AC.



Solution:

(i) Consider △ CDA
We know that CD + DA > AC ….. (1)

Consider △ ABC
We know that AC + AB > BC ….. 21)

By adding both the equations we get
CD + DA + AC + AB > AC + BC
By subtracting AC on both the sides
CD + DA + AC + AB – AC > AC + BC – AC
So we get
CD + DA + AB > BC

Therefore, it is proved that CD + DA + AB > BC.

(ii) Consider △ CDA
We know that CD + DA > AC ….. (1)

Consider △ ABC
We know that AB + BC > AC ….. (2)

By adding both the equations we get
CD + DA + AB + BC > AC + AC

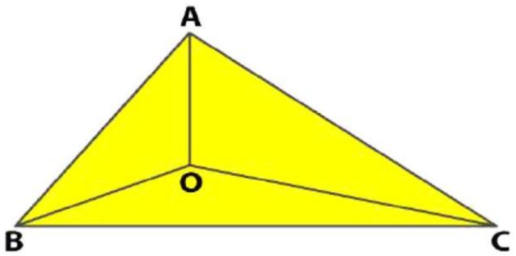
So we get CD + DA + AB + BC > 2AC

Therefore, it is proved that CD + DA + AB + BC > 2AC.

14. If O is a point within △ ABC, show that

(i) **AB + AC > OB + OC**
(ii) **AB + BC + CA > OA + OB + OC**
(iii) **OA + OB + OC > ½ (AB + BC + CA)**

Solution:



(i) It is given that O is a point within △ ABC
Consider △ ABC
We know that AB + AC > BC ….. (1)

Consider △ OBC
We know that OB + OC > BC ….. (2)

By subtracting both the equations we get
(AB + AC) – (OB + OC) > BC - BC
So we get
(AB + AC) – (OB + OC) > 0
AB + AC > OB + OC

Therefore, it is proved that AB + AC > OB + OC.

(ii) We know that AB + AC > OB + OC
In the same way we can write
AB + BC > OA + OC and AC + BC > OA + OB
By adding all the equations we get
AB + AC + AB + BC + AC + BC > OB + OC + OA + OC + OA + OB
So we get
2 (AB + BC + AC) > 2 (OA + OB + OC)
Dividing by 2 both sides
AB + BC + AC > OA + OB + OC

(iii) Consider △ OAB
We know that OA + OB > AB ….. (1)

Consider △ OBC
We know that OB + OC > BC ….. (2)

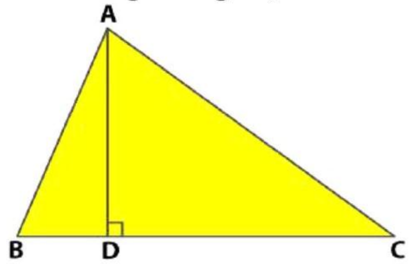
Consider △ OCA
OC + OA > CA …… (3)

By adding all the equations
OA + OB + OB + OC + OC + OA > AB + BC + CA

So we get
2 (OA + OB + OC) > AB + BC + CA
Dividing by 2
OA + OB + OC > ½ (AB + BC + CA)

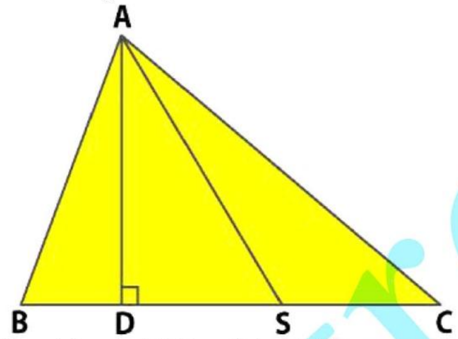
Therefore, it is proved that OA + OB + OC > ½ (AB + BC + CA).

15. In the given figure, AD ⊥ BC and CD > BD. Show that AC > AB.



Solution:

Consider point S on the line BC so that BD = SD and join AS.



Consider △ ADB and △ ADS
We know that SD = BD
Since AD is a perpendicular we know that
∠ ADB = ∠ ADS = 90°
AD is common i.e. AD = AD
By SAS congruence criterion
△ ADB ≅ △ ADS
AB = AS (c. p. c. t)

Consider △ ABS
We know that AB = AS
From the figure we know that ∠ ASB and ∠ ABS are angles opposite to the equal sides
∠ ASB = ∠ ABS …. (1)

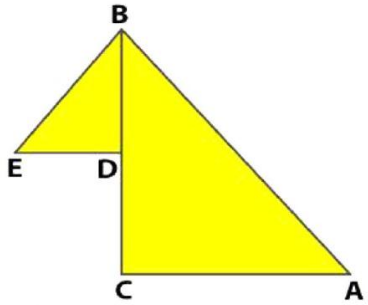
Consider △ ACS
From the figure we know that ∠ ASB and ∠ ACS are angles opposite to the equal sides
∠ ASB = ∠ ACS …. (2)

Considering the equations (1) and (2)
∠ ABS > ∠ ACS
It can be written as

$\angle ABC > \angle ACB$
So we get
AC > AB

Therefore, it is proved that AC > AB.

16. In the given figure, D is a point on side BC of a △ ABC and E is a point such that CD = DE. Prove that AB + AC > BE.



Solution:

Consider △ ABC
We know that
AB + AC > BC
It can be written as
AB + AC > BD + DC
We know that CD = DE
So we get
AB + AC > BD + DE ….. (1)

Consider △ BED
We know that
BD + DE > BE …… (2)

Considering both the equations we get
AB + AC > BE.