EXERCISE 1(A) PAGE: 9

1. Is zero a rational number? Justify.

Solution:

Yes, zero is a rational number.

For example p and q can be written as $\frac{p}{q}$ which are integers and q≠0.

2. Represent each of the following rational numbers on the number line:

(i) $\frac{5}{7}$

(ii) $\frac{8}{3}$

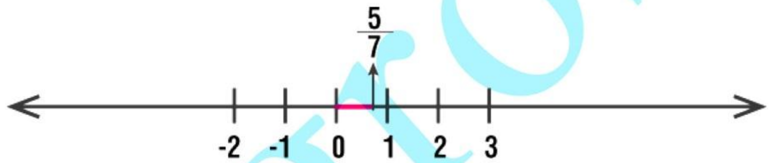
(iii) $-\frac{23}{6}$

(iv) 1.3

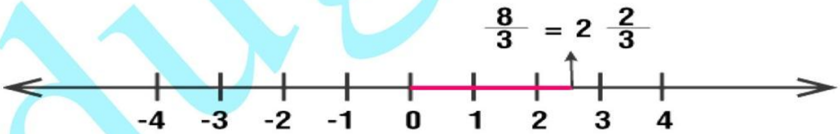
(v) -2.4

Solution:

(i) Since it is a positive fraction it lies between the numbers 0 and 1.

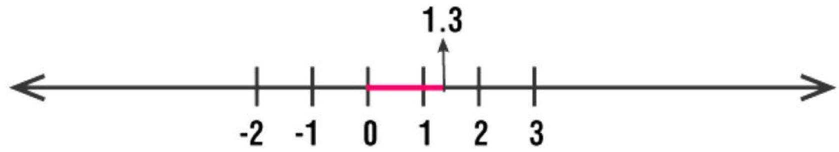


(ii) We can write $\frac{8}{3}$ as $2\frac{2}{3}$.

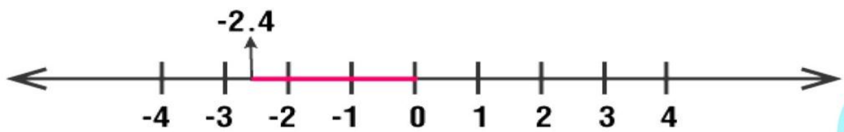


(iii) We can write $-\frac{23}{6}$ as $-3\frac{5}{6}$

(iv) Since 1.3 is a positive decimal it lies between 1 and 2.



(v) Since it is a negative decimal it lies between -2 and -3.



3. Find a rational number between

(i) $\frac{3}{8}$ **and** $\frac{2}{5}$

(ii) 1.3 ***and*** 1.4

(iii) **-1 and** $\frac{1}{2}$

(iv) $-\frac{3}{4}$ **and** $-\frac{2}{5}$

(v) $\frac{1}{9}$ **and** $\frac{2}{9}$

Solution:

(i) Consider $x = \frac{3}{8}$ and $y = \frac{2}{5}$

Then $\frac{3}{8} < \frac{2}{5}$

Rational number which lies in-between x and y

$= \frac{1}{2}(x+y)$

Substituting the value of x and y

$= \frac{1}{2}(\frac{3}{8} + \frac{2}{5})$

On further calculation

$= \frac{1}{2}(\frac{15+16}{40})$

So we get

$= \frac{1}{2} \times \frac{31}{40} = \frac{31}{80}$

Thus the rational number which lies in-between $\frac{3}{8}$ and $\frac{2}{5}$ is $\frac{31}{80}$

Consider x = 1.3 and y = 1.4
Then 1.3 < 1.4
Rational number in-between x and y

$= \frac{1}{2}(1.3+1.4)$

So we get

$= \frac{1}{2} \times 2.7 = 1.35$

Thus the rational number which lies in-between 1.3 and 1.4 is 1.35.

(ii) Consider x = -1 and y = $\frac{1}{2}$

Then $-1 < \frac{1}{2}$

Rational number in-between x and y

$= \frac{1}{2}(-1+\frac{1}{2})$

So on further calculation we get

$= \frac{1}{2}(\frac{-2+1}{2})$

So we get,

$= \frac{1}{2} \times -\frac{1}{2} = -\frac{1}{4}$

Thus the rational number which lies in-between -1 and $\frac{1}{2}$ is $-\frac{1}{4}$

(iii) Consider x = $-\frac{3}{4}$ and y = $-\frac{2}{5}$

Then $-\frac{3}{4} < -\frac{2}{5}$

Rational number in-between x and y
$= \frac{1}{2}((-\frac{3}{4}) + (-\frac{2}{5}))$

On further calculation

$=\frac{1}{2}(\frac{-15-18}{20})$

So we get

$=\frac{1}{2}x-\frac{23}{20}=-\frac{23}{40}$

Thus the rational number which lies in-between $-\frac{3}{4}$ and $-\frac{2}{5}$ is $-\frac{23}{40}$

(iv) Consider $x=\frac{1}{9}$ and $y=\frac{2}{9}$

Then $\frac{1}{9}<\frac{2}{9}$

Rational number in-between x and y

$=\frac{1}{2}(\frac{1}{9}+\frac{2}{9})$

So we get,

$=\frac{1}{2}x\frac{3}{9}=\frac{1}{6}$

Thus the rational number which lies in-between $\frac{1}{9}$ and $\frac{2}{9}$ is $\frac{1}{6}$

4. Find three rational number lying between $\frac{3}{5}$ and $\frac{7}{8}$.

How many rational numbers can be determined between these two numbers?

Solution:

Consider $\frac{3}{5}<\frac{7}{8}$

Then $x=\frac{3}{5}$, $y=\frac{7}{8}$ and n = 3

$d=\frac{y-x}{n+1}=\frac{\frac{7}{8}-\frac{3}{5}}{3+1}=\frac{\frac{35-24}{40}}{4}=\frac{11}{160}$

Rational number in-between x and y are:
x +d, x+2d and x+3d

So we get,
$=\frac{3}{5}+\frac{11}{160}, \frac{3}{5}+2\ x\frac{11}{160}$ and $\frac{3}{5}+3\ x\ \frac{11}{160}$

$=\frac{96+11}{160}, \frac{3}{5}+\frac{11}{80}$ and $\frac{3}{5}+\frac{33}{160}$

On further calculation we get

$=\frac{107}{160}, \frac{48+11}{80}$ and $\frac{96+33}{160}$

$=\frac{107}{160}, \frac{59}{80}$ and $\frac{129}{160}$

Many rational numbers can be determined in-between the given numbers.

5. Find four rational numbers between $\frac{3}{7}$ and $\frac{5}{7}$.

Solution:

Let us consider $\frac{3}{7}=\frac{3\times3}{7\times3}=\frac{9}{21}$

And $\frac{5}{7}=\frac{5\times3}{7\times3}=\frac{15}{21}$

As we know $\frac{9}{21}<\frac{10}{21}<\frac{11}{21}<\frac{12}{21}<\frac{13}{21}<\frac{14}{21}<\frac{15}{21}$

Thus the four rational numbers which lies in-between $\frac{3}{7}=\frac{9}{21}$ and $\frac{5}{7}=\frac{15}{21}$ are $\frac{10}{21}, \frac{11}{21}, \frac{12}{21}$ and $\frac{13}{21}$.

6. Find the six rational numbers between 2 and 3.

Solution:

2 can be written as $\frac{14}{7}$ and 3 can be written as $\frac{21}{7}$

Thus the six rational numbers which lies in-between 2 and 3 are $\frac{15}{7}, \frac{16}{7}, \frac{17}{7}, \frac{18}{7}, \frac{19}{7}, \frac{20}{7}$

7. Find five rational numbers between $\frac{3}{5}$ and $\frac{2}{3}$.

Solution:

Consider $\frac{3}{5}<\frac{2}{3}$

Then x $=\frac{3}{5}$, y $=\frac{2}{3}$ and n = 5

$$d=\frac{y-x}{n+1}=\frac{\frac{2}{3}-\frac{3}{5}}{5+1}=\frac{\frac{10-9}{15}}{6}=\frac{1}{90}$$

Rational numbers in-between x and y are
x+d, x+2d, x+3d, x+4d and x+5d

So we get,

$=\frac{3}{5}+\frac{1}{90}, \frac{3}{5}+2(\frac{1}{90}), \frac{3}{5}+3(\frac{1}{90}), \frac{3}{5}+4(\frac{1}{90})$ and $\frac{3}{5}+5(\frac{1}{90})$

$=\frac{54+1}{90}, \frac{3}{5}+\frac{1}{45}, \frac{3}{5}+\frac{1}{30}, \frac{3}{5}+\frac{2}{45}$ and $\frac{3}{5}+\frac{1}{18}$

$=\frac{55}{90}, \frac{27+1}{45}, \frac{18+1}{30}, \frac{27+2}{45}$ and $\frac{54+5}{90}$

$=\frac{11}{18}, \frac{28}{45}, \frac{19}{30}, \frac{29}{45}$ and $\frac{59}{90}$

8. Insert 16 rational numbers between 2.1 and 2.2.

Solution:

Let us consider x=2.1 and y=2.2
We know that x<y as 2.1<2.2

We can also write 2.1 as $\frac{21}{10}$ and 2.2 as $\frac{22}{10}$

It can also be written as $\frac{21*100}{10*100} < \frac{22*100}{10*100}$

As we know
2100<2105< 2110< 2115< 2120< 2125< 2130< 2135< 2140< 2145< 2150< 2155< 2160< 2165 <2170< 2175< 2180< 2185< 2190< 2195< 2200

We can also write that as

$\frac{2100}{1000}<\frac{2105}{1000}<\frac{2110}{1000}<\frac{2115}{1000}<\frac{2120}{1000}<\frac{2125}{1000}<\frac{2130}{1000}<\frac{2135}{1000}<\frac{2140}{1000}<\frac{2145}{1000}<\frac{2150}{1000}<\frac{2155}{1000}<\frac{2160}{1000}<\frac{2165}{1000}<\frac{2170}{1000}<\frac{2175}{1000}<\frac{2180}{1000}<\frac{2185}{1000}<\frac{2190}{1000}<\frac{2195}{1000}<\frac{2200}{1000}$

Thus the 16 rational numbers which lies between 2.1 and 2.2 are

$\frac{2105}{1000}, \frac{2110}{1000}, \frac{2115}{1000}, \frac{2120}{1000}, \frac{2125}{1000}, \frac{2130}{1000}, \frac{2135}{1000}, \frac{2140}{1000}, \frac{2145}{1000}, \frac{2150}{1000}, \frac{2155}{1000}, \frac{2160}{1000}, \frac{2165}{1000}, \frac{2170}{1000}, \frac{2175}{1000}, \frac{2180}{1000}$

On the other hand it can also be written as
2.105, 2.11, 2.115, 2.12, 2.125, 2.13, 2.135, 2.14, 2.145, 2.15, 2.155, 2.16, 2.165, 2.17, 2.175, 2.18

9. State whether the following statements are true or false. Give reasons for your answer.

(i) Every natural number is a whole number.

(ii) Every whole number is a natural number.

(iii) Every integer is a whole number.

(iv) Every integer is a rational number.
(v) Every rational number is an integer.
(vi) Every rational number is a whole number.

Solution:

(i) True. The group of natural numbers is mainly a sub collection of whole numbers. Thus each and every element of natural number is also a whole number.

(ii) False. Even though zero is a whole number we cannot consider it as a natural number.

(iii) False. Positive integers are whole numbers and negative integers like -1, -2……..etc. are not whole numbers.

(iv) True. Integers can be represented as $\frac{p}{q}$ and $q \neq 0$ which means it can be represented in the form of fraction having the denominator as 1.

(v) False. The numbers in the form of fraction i.e. $\frac{p}{q}$ are not integers.

(vi) False. The division of whole numbers i.e. $\frac{p}{q}$ and $q \neq 0$ the result will not be a whole number.

EXERCISE 1(B) PAGE: 18

1. Without actual division, find which of the following rational numbers are terminating decimals.

(i) $\frac{13}{80}$

(ii) $\frac{7}{24}$

(iii) $\frac{5}{12}$

(iv) $\frac{31}{375}$

(v) $\frac{16}{125}$

Solution:

(i) We can write $\frac{13}{80}$ as

$$\frac{13}{80} = \frac{13}{2\times2\times2\times2\times5} = \frac{13}{2^4\times5}$$

If the denominator has prime factors as 2 or 5 then we can consider the rational number as a terminating Decimal.

$\frac{13}{80}$ Is considered a terminating decimal as we have 2 and 5 as prime factors for 80.

(ii) $\frac{7}{24} = \frac{7}{2\times2\times2\times3} = \frac{7}{2^3\times3}$

The denominator has the prime factors 2 and 3 then $\frac{7}{24}$ is not a terminating decimal.

(iii) $\frac{5}{12} = \frac{5}{2\times2\times3} = \frac{5}{2^2\times3}$

$\frac{5}{12}$ Has the prime factors in the denominator as 2 and 3.

Thus $\frac{5}{12}$ is not a terminating decimal.

(iv) $\frac{31}{375} = \frac{31}{3\times5\times5\times5} = \frac{3}{3\times5^3}$

The denominator is not in the form of 2^p then $\frac{31}{375}$ is not a terminating decimal.

(v) $\frac{16}{125} = \frac{16}{5\times5\times5} = \frac{16}{5^3}$

The denominator prime factor has 5 thus we can consider $\frac{16}{125}$ as a terminating decimal.

2. Write each of the following in decimal form and say what kind of decimal expansion each has.

(i) $\frac{5}{8}$

(ii) $\frac{7}{25}$

(iii) $\frac{3}{11}$

(iv) $\frac{5}{13}$

(v) $\frac{11}{24}$

(vi) $\frac{261}{400}$

(vii) $\frac{231}{625}$

(viii) $2\frac{5}{12}$

Solution:

(i) $\frac{5}{8}$

```
     0. 6 2 5
 8 | 5. 0 0 0
 - 0
   5 0
 - 4 8
     2 0
   - 1 6
       4 0
     - 4 0
         0
```

(ii) $\frac{7}{25}$

```
      0. 2 8
 2 5 | 7. 0 0
   - 0
     7 0
   - 5 0
     2 0 0
   - 2 0 0
         0
```

(iii) $\frac{3}{11}$

```
         0. 2 7 2 7
  1 1 | 3. 0 0 0 0
      - 0
        3 0
      - 2 2
          8 0
        - 7 7
            3 0
          - 2 2
              8 0
            - 7 7
                3
```

(iv) $\frac{5}{13}$

```
         0. 3 8 4 6 1 5
  1 3 | 5. 0 0 0 0 0 0
      - 0
        5 0
      - 3 9
        1 1 0
      - 1 0 4
            6 0
          - 5 2
              8 0
            - 7 8
                2 0
              - 1 3
                  7 0
                - 6 5
                    5
```

(v) $\frac{11}{24}$

```
         0 0. 4 5 8 3 3
  2 4 | 1 1. 0 0 0 0 0
      - 0
        1 1
      -   0
        1 1 0
      -   9 6
          1 4 0
        - 1 2 0
            2 0 0
          - 1 9 2
                8 0
              - 7 2
                  8 0
                - 7 2
                    8
```

(vi) $\frac{261}{400}$

```
          0 0 0. 6 5 2 5
    4 0 0|2 6 1. 0 0 0 0
        - 0
         ---
          2 6
        -   0
         -----
          2 6 1
        -     0
         -------
          2 6 1 0
        - 2 4 0 0
         ---------
            2 1 0 0
          - 2 0 0 0
           ---------
              1 0 0 0
            -   8 0 0
             ---------
                2 0 0 0
              - 2 0 0 0
               ---------
                      0
```

(vii) $\frac{231}{625}$

```
          0 0 0. 3 6 9
    6 2 5|2 3 1. 0 0 0
        - 0
         ---
          2 3
        -   0
         -----
          2 3 1
        -     0
         -------
          2 3 1 0
        - 1 8 7 5
         ---------
            4 3 5 0
          - 3 7 5 0
           ---------
              6 0 0 0
            - 5 6 2 5
             ---------
                3 7 5
```

(viii) $2\frac{5}{12} = \frac{29}{12}$

```
        0 2. 4 1 6 6
   1 2 | 2 9. 0 0 0 0
       - 0
         2 9
       - 2 4
           5 0
         - 4 8
             2 0
           - 1 2
               8 0
             - 7 2
                 8 0
               - 7 2
                     8
```

3. Express each of the following decimals in the form $\frac{p}{q}$, where p, q are integers and q≠0.

(i) $0.\overline{2}$
(ii) $0.\overline{53}$
(iii) $2.\overline{93}$
(iv) $18.\overline{48}$
(v) $0.\overline{235}$
(vi) $0.00\overline{32}$
(vii) $1.\overline{323}$
(viii) $0.3\overline{178}$
(ix) $32.12\overline{35}$
(x) $0.40\overline{7}$

Solution:

(i) Take $x = 0.\overline{2}$

Which means x = 0.2222 consider it as equation (1)
Then 10x = 2.2222 consider it as equation (2)

By subtracting equation (1) from (2)

9x = 2

$x = \frac{2}{9}$

By substituting the value of x we get

$0.\overline{2} = \frac{2}{9}$

(ii) Take $x = 0.\overline{53}$

Which means x = 0.5353 consider it as equation (1)
Then 100x = 53.535353 consider it as equation (2)

By subtracting equation (1) from (2)

99x = 53

So we get

$x = \frac{53}{99}$

By substituting the value of x we get

$0.\overline{53} = \frac{53}{99}$

(iii) Take $x = 2.\overline{93}$

Which means x = 2.9393 which is equation (1)
Consider 100x = 293.939 which is equation (2)

By subtracting the equation (1) and (2)

99x = 291

On further calculation

$x = \frac{291}{99} = \frac{97}{33}$

By substituting the value of x

$2.\overline{93} = \frac{97}{33}$

(iv) Let us take $x = 18.\overline{48}$

Where x = 18.4848 is the equation (1)
And 100x = 1848.48 is the equation (2)

(2) – (1)
We get,

99x = 1830

On further calculation we get

$x = \frac{1830}{99} = \frac{610}{33}$

By substituting the value of x

$18.\overline{48} = \frac{610}{33}$

(v) Take $x = 0.\overline{235}$

We know that x = 0.235235 which is eq. (1)
1000x = 235. 235235 which is eq. (2)

By subtracting both

999x = 235

On further calculation

$x = \frac{235}{99}$

By substituting the value of x

i.e. $0.\overline{235} = \frac{235}{99}$

(vi) Consider $x = 0.00\overline{32}$

Which means x = 0.003232
Then 100 x = 0.323232 which is eq. (1)
And 10000x = 32.3232 which is eq. (2)

By subtracting both

9900x = 32

On further calculation

$x = \frac{32}{9900} = \frac{8}{2475}$

By substituting the value of x

$0.00\overline{32} = \frac{8}{2475}$

(vii) Consider $x = 1.\overline{323}$

Where x = 1.3232323 which is eq. (1)
100x = 132.323232 which is eq. (2)

On subtraction we obtain

99x = 131

On further calculation

x = $\frac{131}{99}$

So we get

$1.\overline{323} = \frac{131}{99}$

(viii) As x = $0.3\overline{178}$

Then x = 0.3178178
10x = 3.17878 ……….. (1)
10000x = 3178.178 …………. (2)

(2) – (1) we obtain
9990x = 3175

On further calculation

$x = \frac{3175}{9990} = \frac{635}{1998}$

So we get,

$0.3\overline{178} = \frac{635}{1998}$

(ix) Take x = $32.12\overline{35}$

Where x = 32.123535
100x = 3212.3535 …….. (1)
10000x = 321235.3535 ………. (2)

By subtraction we obtain

9900x = 318023

On further calculation

x = $\frac{318023}{9900}$

So we get,

$32.12\overline{35} = \frac{318023}{9900}$

(x) Consider x = $0.40\overline{7}$

Where x = 0.40777
100x = 40.777 …….. (1)
1000x = 407.777 …….. (2)

By subtracting both

900x = 367

On further calculation

$x = \frac{367}{900}$

So we get,

$0.40\overline{7} = \frac{367}{900}$

4. Express $2.\overline{36} + 0.\overline{23}$ as a fraction in simplest form.
Solution:

Consider x as $2.\overline{36}$ and y as $0.\overline{23}$

Taking $x = 2.\overline{36}$
x = 2.3636 ……… (1)
100x = 236.3636 …….. (2)

By subtracting both

99x = 234

On further calculation

$x = \frac{234}{99} = \frac{26}{11}$

Taking $y = 0.\overline{23}$
y = 0.2323 ……. (3)
100y = 23.2323 …… (4)

On subtraction

99y =23

On further calculation

$y = \frac{23}{99}$

$2.\overline{36} + 0.\overline{23}$ Can also be written as x+y
Where x+y

$= \frac{26}{11} + \frac{23}{99}$

By taking LCM

$= \frac{26\times9+23}{99}$

On further calculation

$= \frac{234+23}{99}$

So we get,

$= \frac{257}{99}$

5. Express in the form of $\frac{p}{q}$: $0.\overline{38} + 1.\overline{27}$

Solution:

Consider x as $0.\overline{38}$ and y as $1.\overline{27}$

Taking $x = 0.\overline{38}$
x = 0.3838 ….. (1)
100x = 38.3838 ……. (2)

On subtraction we obtain
99x = 38

On further calculation

$x = \frac{38}{99}$

Taking $y = 1.\overline{27}$
y = 1.2727 …… (1)
100y = 127.2727 …….. (2)

By subtracting

99y = 126

On further calculation

$y = \frac{126}{99}$

$0.\overline{38} + 1.\overline{27}$ can be written as x + y
Where x + y

$= \frac{38}{99} + \frac{126}{99}$

By taking LCM

$= \frac{38+126}{99}$

So we get

$= \frac{164}{99}$

EXERCISE 1 (C) PAGE: 23

1. What are irrational numbers? How do they differ from rational numbers? Give examples.
Solution:

Irrational numbers are those which cannot be indicated as a repeating decimal or as a terminating decimal.

For example,
0.212112111211112 is neither a repeating nor a terminating decimal. It is the best example for irrational numbers.

2. Classify the following numbers as rational and irrational. Give reasons to support your answer.

(i) $\sqrt{\frac{3}{81}}$
(ii) $\sqrt{361}$
(iii) $\sqrt{21}$
(iv) $\sqrt{1.44}$
(v) $\frac{2}{3}\sqrt{6}$
(vi) 4.1276
(vii) $\frac{22}{7}$
(viii) 1.232332333 …
(ix) 3.040040004 …
(x) 2.356565656 …
(xi) 6.834834 …

Solution:

(i) $\sqrt{\frac{3}{81}} = \frac{\sqrt{3}}{9} = \frac{1}{3\sqrt{3}}$

It is an irrational number.

(ii) $\sqrt{361} = 19 = \frac{19}{1}$

It is an irrational number.

(iii) As we know 21 is not a perfect square.

Therefore, $\sqrt{21}$ is irrational.

(iv) $\sqrt{1.44} = 1.2 = \frac{12}{10}$

It is an irrational number.

(v) We have a rational number $\frac{2}{3}$ and irrational number$\sqrt{6}$.

As per the theorem the product of both rational and irrational number is an irrational number.
Thus, $\frac{2}{3}\sqrt{6}$ is irrational.

(vi) 4.1276 is basically a terminating decimal.

Therefore, it is rational.

(vii) $\frac{22}{7} = 3.\overline{142857}$

$\frac{22}{7}$ is a non-terminating decimal and is considered as a rational number.

(viii) 1.232332333 … is a non-recurring and non-terminating decimal.

Thus, 1.232332333 … is irrational.

(ix) 3.040040004 … is a non-terminating and non-recurring decimal and hence considered as an irrational number.

(x) 2.356565656 … can be written as $2.3\overline{56}$ is a non-terminating recurring decimal.

Thus, 2.356565656 … is rational.

(xi) 6.834834 … = $6.\overline{834}$

The number 6.834834 … is a non-terminating recurring decimal and hence taken as a rational number.

3. Let x be a rational number and y be an irrational number. Is x+y necessarily an irrational number? Give an example in support of your answer.

Solution:

As we know before,
The sum of rational and irrational number is mainly an irrational number.
Therefore, if we consider m as rational and n as irrational
Then the sum of m + n is an irrational number

Example:
m = 1 and n = $\sqrt{2}$
m + n = 1 + $\sqrt{2}$ is an irrational number

4. Let 'a' be a rational number and b be an irrational number. Is ab necessarily an irrational number? Justify your answer with an example.

Solution:

As we know before,

The product of rational and irrational number is an irrational number.
If a is a rational number and b is an irrational number
Then ab is irrational.

Example:
a = 2 and b = $\sqrt{3}$
ab = 2 $\sqrt{3}$ is an irrational number

5. Is the product of two irrationals always irrational? Justify your answer.
Solution:

No.
The product of two irrational numbers will not be irrational.

Example:
Consider two irrational numbers (4 - $\sqrt{3}$) and (4 +$\sqrt{3}$)
(4 - $\sqrt{3}$) (4 +$\sqrt{3}$) = 16 – 3 = 13 is not irrational.

6. Give an example of two irrational numbers whose
(i) Difference is an irrational number.
(ii) Difference is a rational number.
(iii) Sum is an irrational number.
(iv) Sum is a rational number.
(v) Product is an irrational number.
(vi) Product is a rational number.
(vii) Quotient is an irrational number.
(viii) Quotient is a rational number.

Solution:

(i) Difference is an irrational number.

(7 $\sqrt{3}$ + 1) – (5 $\sqrt{3}$ – 8) = 2 $\sqrt{3}$ + 9

(ii) Difference is a rational number.

(6 - $\sqrt{5}$) – (2 - $\sqrt{5}$) = 4

(iii) Sum is an irrational number.

($\sqrt{7}$- 5) + ($\sqrt{4}$ + 5) = $\sqrt{7}$ + $\sqrt{4}$

(iv) Sum is a rational number.

(5 +$\sqrt{3}$) + (2 - $\sqrt{3}$) = 7

(v) Product is an irrational number.

(2 +$\sqrt{7}$) (2 - $\sqrt{3}$) = 4 – 2 $\sqrt{3}$ + 2 $\sqrt{7}$ - $\sqrt{21}$

(vi) Product is a rational number.

$(1 + \sqrt{7})(1 - \sqrt{7}) = 1 - 7 = -6$

(vii) Quotient is an irrational number.

$\frac{\sqrt{27}}{\sqrt{2}} = \frac{3\sqrt{3}}{\sqrt{2}}$

(viii) Quotient is a rational number.

$\frac{\sqrt{32}}{\sqrt{2}} = \frac{4\sqrt{2}}{\sqrt{2}} = 4$

7. Examine whether the following numbers are rational or irrational.

(i) $3 + \sqrt{3}$

(ii) $\sqrt{7} - 2$

(iii) $\sqrt[3]{5} \times \sqrt[3]{25}$

(iv) $\sqrt{7} \times \sqrt{343}$

(v) $\sqrt{\frac{13}{117}}$

(vi) $\sqrt{8} \times \sqrt{2}$

Solution:

(i) We know that the sum of rational and irrational number is an irrational number.

Thus, $3 + \sqrt{3}$ is an irrational number.

(ii) As we know that the subtraction of a rational and irrational number is irrational then $\sqrt{7} - 2$ is irrational.

(iii) $\sqrt[3]{5} \times \sqrt[3]{25} = 5^{\frac{1}{3}} \times 5^{2 \times \frac{1}{3}}$

$= 5^{\frac{1}{3} + \frac{2}{3}} = 5$

Thus, it is a rational number.

(iv) $\sqrt{7} \times \sqrt{343} = \sqrt{7} \times \sqrt{7^3}$

$= 7^{\frac{1}{2}} \times 7^{3 \times \frac{1}{2}}$

$= 7^2 = 49$

It is a rational number.

(v) $\sqrt{\frac{13}{117}} = \frac{\sqrt{13}}{\sqrt{9 \times 13}} = \frac{\sqrt{13}}{3\sqrt{13}} = \frac{1}{3}$

It is a rational number.

(vi) $\sqrt{8} \times \sqrt{2} = \sqrt{4 \times 2} \times \sqrt{2}$
$= 2\sqrt{2} \times \sqrt{2}$
$= 4$

It is a rational number.

8. Insert a rational and an irrational number between 2 and 2.5.
Solution:

Rational number which lies between 2 and 2.5
$= \frac{2+2.5}{2}$

On further calculation

$= \frac{4.5}{2}$

On division

$= 2.25$

Irrational number which lies between 2 and 2.5

$= \sqrt{2 \times 2.5}$

On further calculation

$= \sqrt{5}$

9. How many irrational numbers lie between $\sqrt{2}$ and $\sqrt{3}$? Find any three irrational numbers lying between $\sqrt{2}$ and $\sqrt{3}$.
Solution:

We know that
$\sqrt{2} = 1.414213562 \ldots$
$\sqrt{3} = 1.7320508075 \ldots$

Thus, the three irrational numbers which lies between $\sqrt{2}$ and $\sqrt{3}$ are
1.5010010001……., 1.6010010001…….. and 1.7010010001…..

10. Find two rational and two irrational numbers between 0.5 and 0.55.
Solution:

We know that,
0.5 < 0.55

Consider x = 0.5, y = 0.55 and n = 2
$d= \frac{y-x}{n+1} = \frac{0.55-0.5}{2+1} = \frac{0.05}{3}$

Two rational and two irrational numbers which lies between 0.5 and 0.55 are
x + d and x + 2d
so we get,

$= 0.5 + \frac{0.05}{3}$ and $0.5 + 2 \times \frac{0.05}{3}$

On further calculation

$= \frac{1.5+0.05}{3}$ and $\frac{1.5+0.1}{3}$

So we get

$= \frac{1.55}{3}$ and $\frac{1.6}{3}$

By division

= 0.51 and 0.53

Two irrational numbers which lies between 0.5 and 0.55 are
0.5151151115 ……. And 0.5353553555 ……

11. Find the three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.
Solution:

$\frac{5}{7}$ can be written as $0.\overline{714285}$

$\frac{9}{11}$ can be written as $0.\overline{81}$

Hence the three different irrational numbers that lies between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$ are:

0.727227222…, 0.757557555…… and 0.808008000…..

12. Find two rational numbers of the form $\frac{p}{q}$ between the numbers 0.2121121112 …. And 0.2020020002 …. .

Solution:

Consider x and y as the two rational numbers lying between 0.2121121112 … and 0.2020020002 ….

We know that,
0.2020020002 …… < 0.2121121112 ……

We get,
0.2020020002 ….. < x < y < 0.2121121112 ……

$x = \frac{51}{250} = 0.204$

$y = \frac{103}{500} = 0.206$

13. Find two irrational numbers between 0.16 and 0.17.

Solution:

We know that the two irrational numbers lying between 0.16 and 0.17 is
0.1611161111611111611111 …….. and 0.169669666 ……..

14. State, in each case, whether the given statement is true or false.

(i) The sum of two rational numbers is rational.
(ii) The sum of two irrational numbers is irrational.
(iii) The product of two rational numbers is rational.
(iv) The product of two irrational numbers is irrational.
(v) The sum of a rational number and an irrational number is irrational.
(vi) The product of a non-zero rational number and an irrational number is a rational number.
(vii) Every real number is rational.
(viii) Every real number is either rational or irrational.
(ix) π is irrational and $\frac{22}{7}$ is rational.

Solution:

(i) True.

Example $\frac{2}{3} + \frac{1}{3} = \frac{3}{3} = 1$

(ii) False.

Sum of two irrational numbers is rational as the irrational parts will have the sum as zero so the result is rational.

(iii) True.

Example $\frac{8}{2} \times \frac{2}{4} = 2$

(iv) False.

Example (1 +$\sqrt{5}$) (1 - $\sqrt{7}$) = 1 – 1 $\sqrt{7}$ + 1 $\sqrt{5}$ - $\sqrt{35}$

(v) True.

Example 1 + $\sqrt{5}$ = 1 + 2.236067977 = 3.236067977 ……

(vi) False.

The product rational number with denominator not equal to zero and an irrational number is irrational.

(vii) False.

Rational number is mainly represented in the form of a fraction where the denominator will not be equal to zero which means q is equal to 1. This proves that real numbers are irrational.

(viii) True.

Real numbers with denominator equal to zero are rational and the denominator not equal to zero are irrational numbers.

(ix) True.

Π is irrational where as $\frac{22}{7}$ is rational where both the numerator and denominator are integers.

EXERCISE 1(D)

PAGE: 27

1. Add

(i) $(2\sqrt{3} - 5\sqrt{2})$ and $(\sqrt{3} + 2\sqrt{2})$

(ii) $(2\sqrt{2} + 5\sqrt{3} - 7\sqrt{5})$ and $(3\sqrt{3} - \sqrt{2} + \sqrt{5})$

(iii) $(\frac{2}{3}\sqrt{7} - \frac{1}{2}\sqrt{2} + 6\sqrt{11})$ and $(\frac{1}{3}\sqrt{7} + \frac{3}{2}\sqrt{2} - \sqrt{11})$

Solution:

(i) $(2\sqrt{3} - 5\sqrt{2})$ and $(\sqrt{3} + 2\sqrt{2})$

By adding both

$= (2\sqrt{3} - 5\sqrt{2}) + (\sqrt{3} + 2\sqrt{2})$

On further calculation

$= (2\sqrt{3} + \sqrt{3}) + (-5\sqrt{2} + 2\sqrt{2})$

So we get

$= (2+1)\sqrt{3} + (-5 + 2)\sqrt{2}$

By simplification

$= 3\sqrt{3} - 3\sqrt{2}$

(ii) $(2\sqrt{2} + 5\sqrt{3} - 7\sqrt{5})$ and $(3\sqrt{3} - \sqrt{2} + \sqrt{5})$

By adding both

$= (2\sqrt{2} + 5\sqrt{3} - 7\sqrt{5}) + (3\sqrt{3} - \sqrt{2} + \sqrt{5})$

On further calculation

$= (2\sqrt{2} - \sqrt{2}) + (5\sqrt{3} + 3\sqrt{3}) + (-7\sqrt{5} + \sqrt{5})$

So we get

$= (2 - 1)\sqrt{2} + (5 + 3)\sqrt{3} + (-7 + 1)\sqrt{5}$

By simplification

$= \sqrt{2} + 8\sqrt{3} - 6\sqrt{5}$

(iii) $(\frac{2}{3}\sqrt{7} - \frac{1}{2}\sqrt{2} + 6\sqrt{11})$ and $(\frac{1}{3}\sqrt{7} + \frac{3}{2}\sqrt{2} - \sqrt{11})$

By adding both

$= (\frac{2}{3}\sqrt{7} - \frac{1}{2}\sqrt{2} + 6\sqrt{11}) + (\frac{1}{3}\sqrt{7} + \frac{3}{2}\sqrt{2} - \sqrt{11})$

On further calculation

$= (\frac{2}{3}\sqrt{7} + \frac{1}{3}\sqrt{7}) + (-\frac{1}{2}\sqrt{2} + \frac{3}{2}\sqrt{2}) + (6\sqrt{11} - \sqrt{11})$

So we get

$= (\frac{2}{3} + \frac{1}{3})\sqrt{7} + (-\frac{1}{2} + \frac{3}{2})\sqrt{2} + (6-1)\sqrt{11}$

By simplification

$= \sqrt{7} + \sqrt{2} + 5\sqrt{11}$

2. Multiply

(i) $3\sqrt{5}$ **by** $2\sqrt{5}$
(ii) $6\sqrt{15}$ **by** $4\sqrt{3}$
(iii) $2\sqrt{6}$ **by** $3\sqrt{3}$
(iv) $3\sqrt{8}$ **by** $3\sqrt{2}$
(v) $\sqrt{10}$ **by** $\sqrt{40}$
(vi) $3\sqrt{28}$ **by** $2\sqrt{7}$

Solution:

(i) $3\sqrt{5} \times 2\sqrt{5}$

It can also be written as
$= 3 \times 2 \times \sqrt{5} \times \sqrt{5}$

So we get

$= (3 \times 2 \times 5)$

$= 30$

(ii) $6\sqrt{15} \times 4\sqrt{3}$

It can also be written as
$= 6 \times 4 \times \sqrt{15} \times \sqrt{3}$

$= 24 \times \sqrt{15 \times 3}$

So we get

$= 24 \times \sqrt{3 \times 5 \times 3}$

$= 24 \times 3\sqrt{5}$

$= 72\sqrt{5}$

(iii) $2\sqrt{6} \times 3\sqrt{3}$

It can also be written as
$= 2 \times 3 \times \sqrt{6} \times \sqrt{3}$

$= 6 \times \sqrt{6 \times 3}$

So we get

$= 6 \times \sqrt{2 \times 3 \times 3}$

By multiplication

$= 6 \times 3\sqrt{2}$

$= 18\sqrt{2}$

(iv) $3\sqrt{8} \times 3\sqrt{2}$

It can also be written as
$= 3 \times 3 \times \sqrt{8} \times \sqrt{2}$

$= 9 \times \sqrt{8 \times 2}$

So we get

$= 9 \times \sqrt{2 \times 2 \times 2 \times 2}$

By multiplication

$= (9 \times 2 \times 2)$

$= 36$

(v) $\sqrt{10} \times \sqrt{40}$

It can also be written as
$= \sqrt{10 \times 40}$

$= \sqrt{2 \times 5 \times 2 \times 2 \times 2 \times 5}$

So we get

$= (2 \times 2 \times 5)$

By multiplication

= 20

(vi) $3\sqrt{28} \times 2\sqrt{7}$

It can also be written as
$= 3 \times 2 \times \sqrt{28} \times \sqrt{7}$

So we get

$= 6 \times \sqrt{28 \times 7}$

By multiplication

$= 6 \times \sqrt{2 \times 2 \times 7 \times 7}$

= (6 × 2 × 7)

= 84

3. Divide

(i) $\mathbf{16\sqrt{6}}$ **by** $\mathbf{4\sqrt{2}}$
(ii) $\mathbf{12\sqrt{15}}$ **by** $\mathbf{4\sqrt{3}}$
(iii) $\mathbf{18\sqrt{21}}$ **by** $\mathbf{6\sqrt{7}}$

Solution:

(i) $16\sqrt{6} \div 4\sqrt{2}$

By dividing both numerator and denominator
$= \frac{16\sqrt{6}}{4\sqrt{2}}$

Since 16 is a multiple of 4

$= \frac{4\sqrt{6}}{\sqrt{2}}$

It can also be written as

$= \frac{4\sqrt{6} \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}}$

On further multiplication

$= \frac{4\sqrt{6} \times 2}{2}$

$= \frac{4\sqrt{2 \times 3 \times 2}}{2}$

So we get

$= \frac{4 \times 2\sqrt{3}}{2}$

$= 4\sqrt{3}$

(ii) $12\sqrt{15} \div 4\sqrt{3}$

By dividing both numerator and denominator

$= \frac{12\sqrt{15}}{4\sqrt{3}}$

Since 12 is a multiple of 4

$= \frac{3\sqrt{15}}{\sqrt{3}}$

It can also be written as

$= \frac{3\sqrt{15} \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$

$= \frac{3\sqrt{15 \times 3}}{3}$

So we get

$= \sqrt{3 \times 5 \times 3}$

$= 3\sqrt{5}$

(iii) $18\sqrt{21} \div 6\sqrt{7}$

By dividing both numerator and denominator

$= \frac{18\sqrt{21}}{6\sqrt{7}}$

Since 18 is a multiple of 6

$= \frac{3\sqrt{21}}{\sqrt{7}}$

It can also be written as

$= \frac{3\sqrt{21} \times \sqrt{7}}{\sqrt{7} \times \sqrt{7}}$

By multiplication

$= \frac{3\sqrt{3\times7\times7}}{7}$

So we get

$= \frac{3 \times 7\sqrt{3}}{7}$

$= 3\sqrt{3}$

4. Simplify

(i) $(3 - \sqrt{11})(3+\sqrt{11})$

(ii) $(-3 + \sqrt{5})(-3 - \sqrt{5})$

(iii) $(3 - \sqrt{3})^2$

(iv) $(\sqrt{5} - \sqrt{3})^2$

(v) $(5 +\sqrt{7})(2+\sqrt{5})$

(vi) $(\sqrt{5} - \sqrt{2})(\sqrt{2} - \sqrt{3})$

Solution:

(i) $(3 - \sqrt{11})(3+\sqrt{11})$

According to the formula $a^2 - b^2 = (a+b)(a-b)$

$= (3)^2 - (\sqrt{11})^2$

So we get

$= 9 – 11$

$= - 2$

(ii) $(-3 + \sqrt{5})(-3 - \sqrt{5})$

According to the formula $a^2 - b^2 = (a+b)(a-b)$

$= (-3)^2 - (\sqrt{5})^2$

So we get

$= 9 – 5$

$= 4$

(iii) $(3 - \sqrt{3})^2$

According to the formula $(a-b)^2 = a^2 + b^2 - 2ab$
$= (3)^2 + (\sqrt{3})^2 - 2 \times 3\sqrt{3}$

So we get

$= 9 + 3 - 6\sqrt{3}$

$= 12 - 6\sqrt{3}$

(iv) $(\sqrt{5} - \sqrt{3})^2$

According to the formula $(a-b)^2 = a^2 + b^2 - 2ab$
$= (\sqrt{5})^2 + (\sqrt{3})^2 - 2\sqrt{5}\sqrt{3}$

So we get

$= 5 + 3 - 2\sqrt{15}$

$= 8 - 2\sqrt{15}$

(v) $(5 + \sqrt{7})(2 + \sqrt{5})$

On further calculation
$= 5 \times 2 + 5 \times \sqrt{5} + 2 \times \sqrt{7} + \sqrt{5} \times \sqrt{7}$

So we get

$= 10 + 5\sqrt{5} + 2\sqrt{7} + \sqrt{35}$

(vi) $(\sqrt{5} - \sqrt{2})(\sqrt{2} - \sqrt{3})$

On further calculation
$= \sqrt{5}(\sqrt{2} - \sqrt{3}) - \sqrt{2}(\sqrt{2} - \sqrt{3})$

So we get

$= (\sqrt{10} - \sqrt{15} - 2 + \sqrt{6})$

5. Simplify $(3 + \sqrt{3})(2 + \sqrt{2})^2$.
Solution:

$(3 + \sqrt{3})(2 + \sqrt{2})^2$

According to the formula $(a+b)^2 = a^2 + b^2 + 2ab$

$= (3 + \sqrt{3})[(2)^2 + 2 \times 2 \times \sqrt{2} + (\sqrt{2})^2]$

On further calculation

= (3 +√3) [4 + 4√2 + 2]

= (3 +√3) (6 + 4√2)

By multiplying the terms

= 3 × 6 + 3 × 4√2 + 6√3 + 4√2 × √3

So we get

= 18 + 12√2 + 6√3 + 5√6

6. Examine whether the following numbers are rational or irrational:

(i) **(5 - √5) (5 +√5)**

(ii) $(\sqrt{\mathbf{3}} + \mathbf{2})^{\mathbf{2}}$

(iii) $\frac{2\sqrt{13}}{3\sqrt{52}-4\sqrt{117}}$

(iv) $\sqrt{8} + 4\sqrt{32} - 6\sqrt{2}$

Solution:

(i) (5 - √5) (5 +√5)

According to the formula $a^2 - b^2 = (a + b)(a - b)$

= $(5)^2 – (\sqrt{5})^2$

So we get

= 25 – 5

= 20

Hence (5 - √5) (5 +√5) is rational.

(ii) $(\sqrt{3} + 2)^{2}$

According to the formula $(a + b)^2 = a^2 + b^2 + 2ab$

= $(\sqrt{3})^2 + 2 \times 2 \times \sqrt{3} + (2)^2$

On further calculation

= 3 + 4√3 + 4

So we get

= 7 + 4√3

Thus $(\sqrt{3} + 2)^{2}$ is irrational.

(iii) $\frac{2\sqrt{13}}{3\sqrt{52}-4\sqrt{117}}$

We can also write it as

$= \frac{2\sqrt{13}}{3\sqrt{4\times13}-4\sqrt{9\times13}}$

On further calculation

$= \frac{2\sqrt{13}}{3\times2\sqrt{13}-4\times3\sqrt{13}}$

So we get

$= \frac{2\sqrt{13}}{6\sqrt{13}-12\sqrt{13}}$

By taking out the common terms

$= \frac{2\sqrt{13}}{-6\sqrt{13}}$

By division

$= -\frac{1}{3}$

Therefore $\frac{2\sqrt{13}}{3\sqrt{52}-4\sqrt{117}}$ is rational.

(iv) $\sqrt{8}+4\sqrt{32}-6\sqrt{2}$

We can also write it as

$= \sqrt{4\times2} + 4\sqrt{16\times2} - 6\sqrt{2}$

So we get

$= 2\sqrt{2} + 16\sqrt{2} - 6\sqrt{2}$

On further calculation

$= 12\sqrt{2}$

Thus $\sqrt{8}+4\sqrt{32}-6\sqrt{2}$ is irrational.

7. On her birthday Reema distributed chocolates in an orphanage. The total number of chocolates she distributed is given by $(5+\sqrt{11})(5-\sqrt{11})$.

(i) Find the number of chocolates distributed by her.

(ii) Write the moral values depicted here by Reema.
Solution:

(i) Number of chocolates distributed by Reema

$= (5 + \sqrt{11})\ (5 - \sqrt{11})$
According to the formula $a^2 - b^2 = (a + b)(a - b)$
$= (5)^2 - (\sqrt{11})^2$
On further calculation
$= 25 - 11$
$= 14$

(ii) The moral values depicted by Reema is to help the needy and poor and to make the children satisfied and happy.

8. Simplify
(i) $3\sqrt{45} - \sqrt{125} + \sqrt{200} - \sqrt{50}$
(ii) $\frac{2\sqrt{30}}{\sqrt{6}} - \frac{3\sqrt{140}}{\sqrt{28}} + \frac{\sqrt{55}}{\sqrt{99}}$
(iii) $\sqrt{72} + \sqrt{800} - \sqrt{18}$

Solution:

(i) $3\sqrt{45} - \sqrt{125} + \sqrt{200} - \sqrt{50}$

It can also be written as
$= 3\sqrt{9 \times 5} - \sqrt{25 \times 5} + \sqrt{100 \times 2} - \sqrt{25 \times 2}$

On further calculation

$= 3 \times 3\sqrt{5} - 5\sqrt{5} + 10\sqrt{2} - 5\sqrt{2}$

So we get

$= 9\sqrt{5} - 5\sqrt{5} + 10\sqrt{2} - 5\sqrt{2}$

$= 4\sqrt{5} + 5\sqrt{2}$

(ii) $\frac{2\sqrt{30}}{\sqrt{6}} - \frac{3\sqrt{140}}{\sqrt{28}} + \frac{\sqrt{55}}{\sqrt{99}}$

It can also be written as

$= \frac{2\sqrt{6 \times 5}}{\sqrt{6}} - \frac{3\sqrt{28 \times 5}}{\sqrt{28}} + \frac{\sqrt{11 \times 5}}{\sqrt{11 \times 9}}$

$= \frac{2\sqrt{6} \times \sqrt{5}}{\sqrt{6}} - \frac{3\sqrt{28} \times \sqrt{5}}{\sqrt{28}} + \frac{\sqrt{11} \times \sqrt{5}}{3\sqrt{11}}$

On further calculation

$= 2\sqrt{5} - 3\sqrt{5} + \frac{\sqrt{5}}{3}$

$= -\sqrt{5} + \frac{\sqrt{5}}{3}$

So we get

$= \frac{-3\sqrt{5} + \sqrt{5}}{3}$

$= \frac{-2\sqrt{5}}{3}$

(iii) $\sqrt{72} + \sqrt{800} - \sqrt{18}$

It can also be written as

$= \sqrt{36 \times 2} + \sqrt{400 \times 2} - \sqrt{9 \times 2}$

On further calculation

$= 6\sqrt{2} + 20\sqrt{2} - 3\sqrt{2}$

So we get

$= 23\sqrt{2}$

EXERCISE 1(E) PAGE: 35

1. Represent $\sqrt{5}$ on the number line.

Solution:

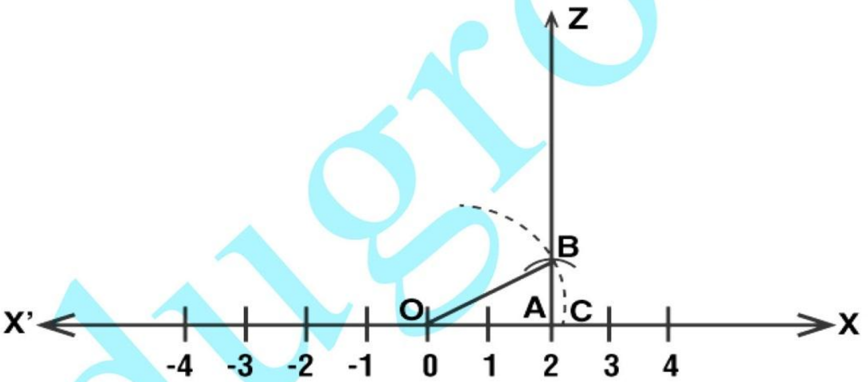
Draw a number line.
Take a point O which is corresponding to zero.
Consider a point A so that OA = 2 units

Construct a perpendicular at A and name it as AZ
Draw a cut off arc AB = 1unit

On the basis of Pythagoras Theorem,
We know that
$OB^2 = OA^2 + AB^2$
$= 2^2 + 1^2$
$= 4 + 1$
$= 5$
OB $= \sqrt{5}$

Consider O as the centre and OB = $\sqrt{5}$ as the radius construct an arc which cuts the line at the point C.
Thus, OB = OC = $\sqrt{5}$

$\sqrt{5}$ is represented by point C on the number line.



2. Locate $\sqrt{3}$ on the number line.

Solution:

Draw a number line.
Take a point O which is corresponding to zero.
Consider a point A so that OA = 1 unit.

Construct a perpendicular at A and name it as AZ
Draw a cut off arc AB = 1unit

On the basis of Pythagoras Theorem,

We know that
$OB^2 = OA^2 + AB^2$
$= 1^2 + 1^2$
$= 1 + 1$
$= 2$
OB = $\sqrt{2}$

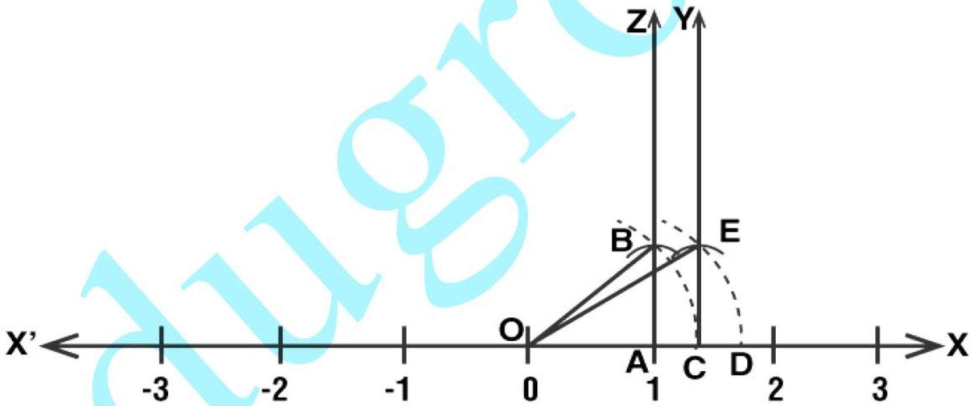
Consider O as the centre and OB = $\sqrt{2}$ as the radius construct an arc which cuts the line at the point C.
Thus, OB = OC = $\sqrt{2}$

Draw a perpendicular at C and name it as CY along with a cut off arc CE = 1 unit.

On the basis of Pythagoras Theorem,
We know that
$OE^2 = OC^2 + CE^2$
$= \sqrt{2}^2 + 1^2$
$= 2 + 1$
$= 3$
OE = $\sqrt{2}$

Consider O as the centre and OE = $\sqrt{3}$ as the radius construct an arc which cuts the line at the point D.
Thus, OD = OE = $\sqrt{2}$

$\sqrt{3}$ is represented by point C on the number line.



3. Locate $\sqrt{10}$ on the number line.
Solution:

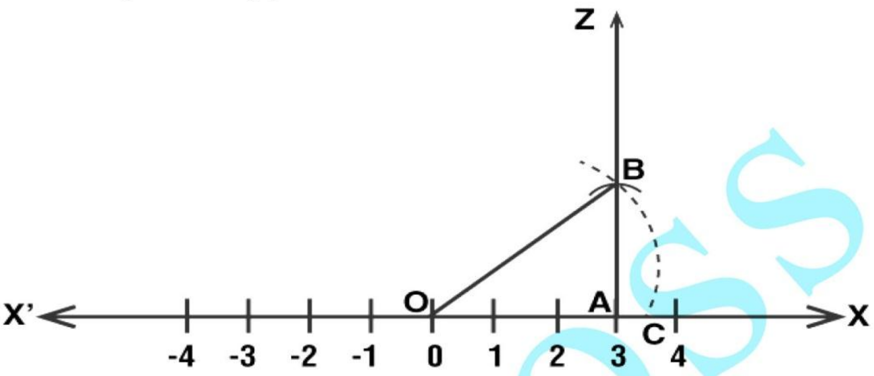
Draw a number line.
Take a point O which is corresponding to zero.
Consider a point A so that OA = 3 units.

Construct a perpendicular at A and name it as AZ
Draw a cut off arc AB = 1unit

On the basis of Pythagoras Theorem,
We know that
$OB^2 = OA^2 + AB^2$
$= 3^2 + 1^2$
$= 9 + 1$
$= 10$
OB = $\sqrt{10}$

Consider O as the centre and OB = $\sqrt{10}$ as the radius construct an arc which cuts the line at the point C.
Thus, OB = OC = $\sqrt{10}$

$\sqrt{10}$ is represented by point C on the number line.



4. Locate $\sqrt{8}$ on the number line.
Solution:

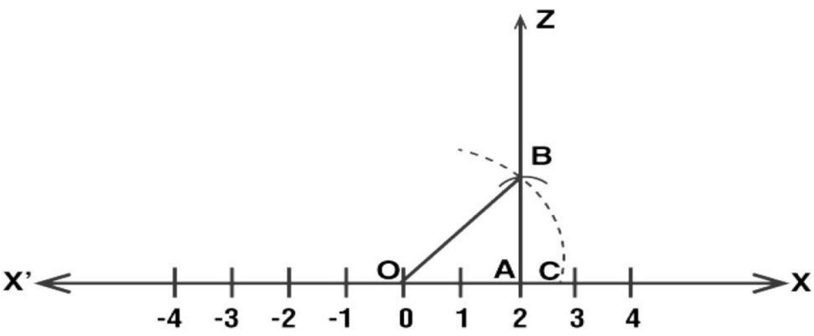
Draw a number line.
Take a point O which is corresponding to zero.
Consider a point A so that OA = 2 units.

Construct a perpendicular at A and name it as AZ
Draw a cut off arc AB = 2 units

On the basis of Pythagoras Theorem,
We know that
$OB^2 = OA^2 + AB^2$
$= 2^2 + 2^2$
$= 4 + 4$
$= 8$
OB = $\sqrt{8}$

Consider O as the centre and OB = $\sqrt{8}$ as the radius construct an arc which cuts the line at the point C.
Thus, OB = OC = $\sqrt{8}$

$\sqrt{8}$ is represented by point C on the number line.

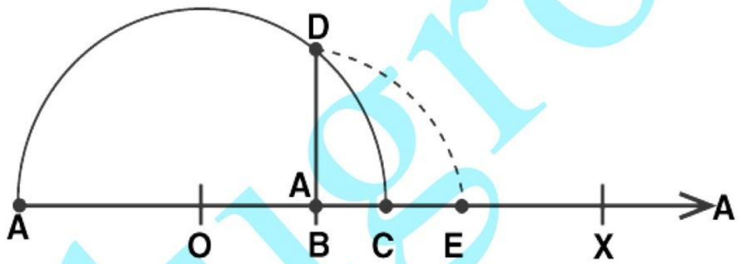


5. Represent $\sqrt{4.7}$ geometrically on the number line.
Solution:

Construct a line segment of AB = 4.7 units
Extend it from B to C where BC = 1 unit

Let O be the midpoint of AC
Let O be the centre and OA be the radius, construct a semicircle.
Draw a perpendicular at B and name it as BD intersecting the semicircle at D.
Thus, BD = $\sqrt{4.7}$ units

Considering B as the centre and BD as the radius, construct an arc meeting the point E on AC.
Thus, BE = BD = $\sqrt{4.7}$ units

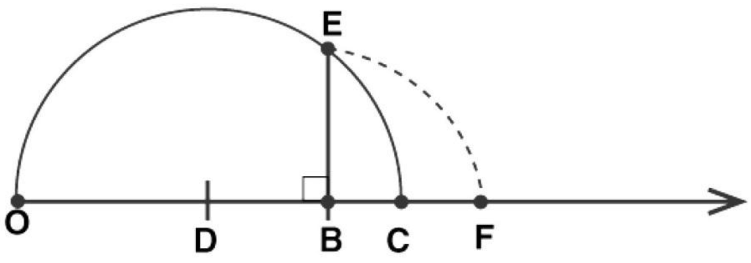


6. Represent $\sqrt{10.5}$ on the number line.
Solution:

Construct a line segment of OB = 10.5 units
Extend it from B to C where BC = 1 unit

Let D be the midpoint of OC
Let D be the centre and DO be the radius, construct a semicircle.
Draw a perpendicular at B and name it as BE intersecting the semicircle at E.
Thus, BE = $\sqrt{10.5}$ units

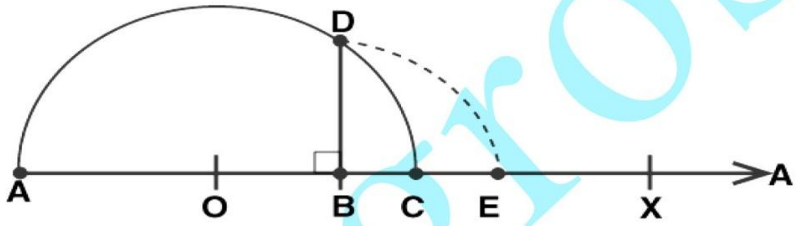
Considering B as the centre and BE as the radius, construct an arc meeting the point F on AC.
Thus, BF = BE = $\sqrt{10.5}$ units



7. Represent $\sqrt{7.28}$ geometrically on the number line.
Solution:

Construct a line segment of AB = 7.28 units
Extend it from B to C where BC = 1 unit

Let O be the midpoint of AC
Let O be the centre and OA be the radius, construct a semicircle.
Draw a perpendicular to AC at B and name it as BD intersecting the semicircle at D.
Thus, BD = $\sqrt{7.28}$ units
Considering D as the centre and BD as the radius, construct an arc meeting the point E on AC.
Thus, BE = BD = $\sqrt{7.28}$ units



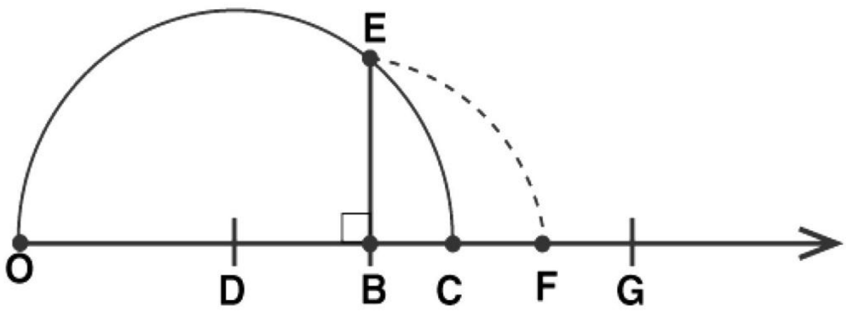
8. Represent $(1 + \sqrt{9.5})$ on the number line.
Solution:

Construct a line segment of OB = 9.5 units
Extend it from B to C where BC = 1 unit

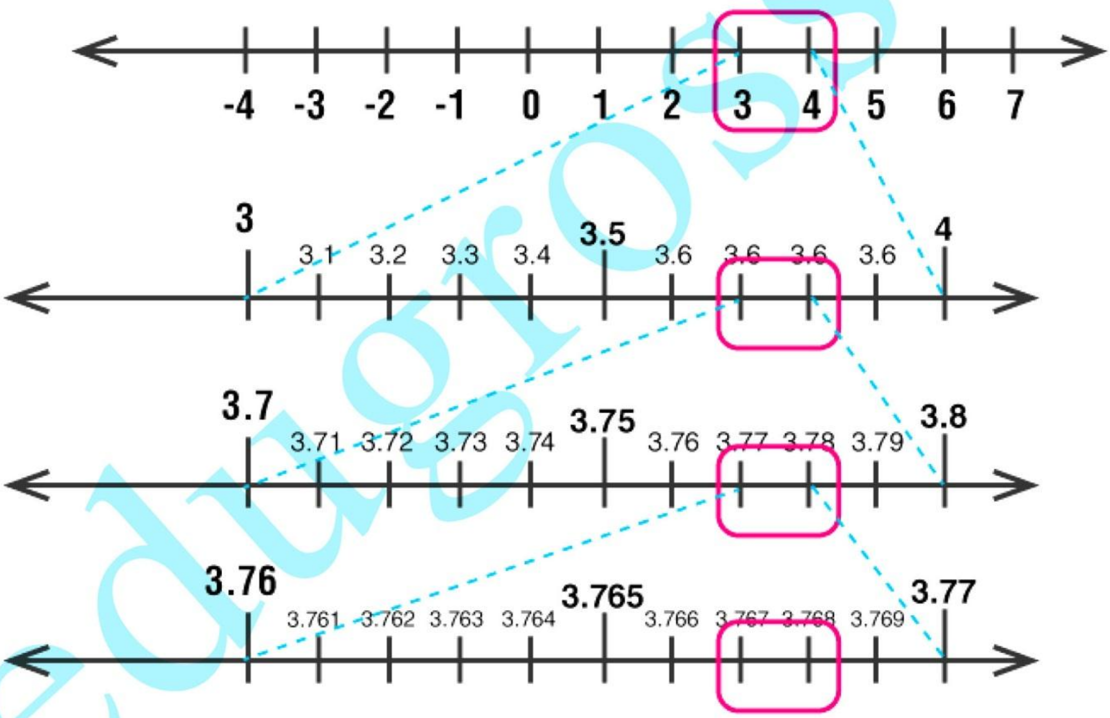
Let D be the midpoint of OC
Let D be the centre and DO be the radius, construct a semicircle.
Draw a perpendicular to AC at B and name it as BE intersecting the semicircle at E.
Thus, BE = $\sqrt{9.5}$ units

Considering B as the centre and BE as the radius, construct an arc meeting the point F on AC.
Thus, BF = BE = $\sqrt{10.5}$ units

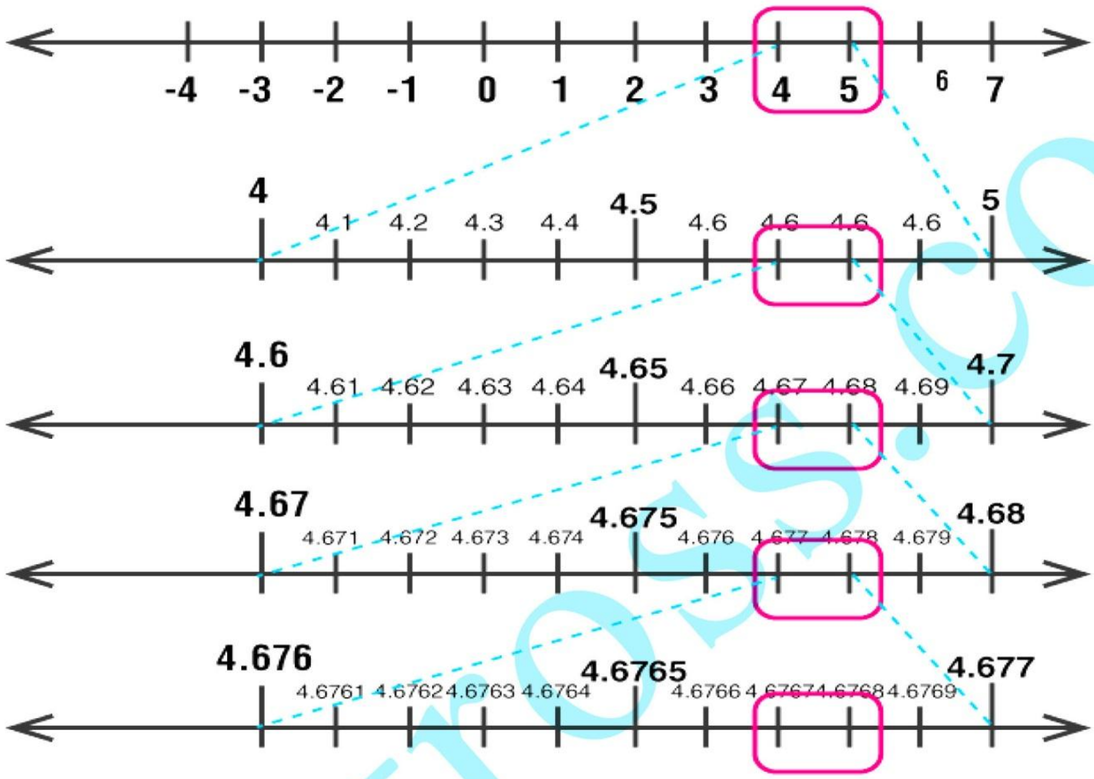
Now extend BF to G in such a way FG = 1 unit
Thus, BG = $(1 + \sqrt{9.5})$



9. Visualize the representation of 3.765 on the number line using successive magnification.
Solution:



10. Visualize the representation of $4.\overline{67}$ on the number line up to 4 decimal places.
Solution:


-4 -3 -2 -1 0 1 2 3 4 5 6 7
4
4.1 4.2 4.3 4.4
4.5
4.6 4.6 4.6 4.6
5
4.6
4.61 4.62 4.63 4.64
4.65
4.66 4.67 4.68 4.69
4.7
4.67
4.671 4.672 4.673 4.674
4.675
4.676 4.677 4.678 4.679
4.68
4.676
4.6761 4.6762 4.6763 4.6764
4.6765
4.6766 4.6767 4.6768 4.6769
4.677

EXERCISE 1(F) PAGE: 43

1. Write the rationalising factor of the denominator in $\frac{1}{\sqrt{2}+\sqrt{3}}$.

Solution:

The rationalising factor of the denominator in $\frac{1}{\sqrt{2}+\sqrt{3}}$ is $(\sqrt{2}-\sqrt{3})$.

2. Rationalise the denominator of each of the following.

(i) $\frac{1}{\sqrt{7}}$

(ii) $\frac{\sqrt{5}}{2\sqrt{3}}$

(iii) $\frac{1}{2+\sqrt{3}}$

(iv) $\frac{1}{\sqrt{5}-2}$

(v) $\frac{1}{5+3\sqrt{2}}$

(vi) $\frac{1}{\sqrt{7}-\sqrt{6}}$

(vii) $\frac{4}{\sqrt{11}-\sqrt{7}}$

(viii) $\frac{1+\sqrt{2}}{2-\sqrt{2}}$

(ix) $\frac{3-2\sqrt{2}}{3+2\sqrt{2}}$

Solution:

(i) Multiply both numerator and denominator by $\sqrt{7}$

So we get,

$\frac{1}{\sqrt{7}}$

On further calculation

$= \frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$

Removing the similar terms

$= \frac{\sqrt{7}}{7}$

(ii) Multiply both numerator and denominator by $\sqrt{3}$

So we get,

$\frac{\sqrt{5}}{2\sqrt{3}}$

On further calculation

$= \frac{\sqrt{5}}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

We get

$= \frac{\sqrt{15}}{2\times3}$

$= \frac{\sqrt{15}}{6}$

(iii) Consider a and b as integers

The rationalising factors are (a + $\sqrt{b}$) and (a - $\sqrt{b}$)
It can be written as (a + $\sqrt{b}$) (a - $\sqrt{b}$) = (a^2 – b)
So we get,

$\frac{1}{2+\sqrt{3}}$

It can written as

$= \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$

According to the formula $a^2 - b^2 = (a+b)(a-b)$

$= \frac{2-\sqrt{3}}{2^2-\sqrt{3}^2}$

On further calculation

$= \frac{2-\sqrt{3}}{4-3}$

So we get

$= \frac{2-\sqrt{3}}{1}$

$= 2 - \sqrt{3}$

(iv) Consider a and b as integers

The rationalising factors are (a + $\sqrt{b}$) and (a - $\sqrt{b}$)
It can be written as (a + $\sqrt{b}$) (a - $\sqrt{b}$) = (a^2 – b)
So we get,

$$\frac{1}{\sqrt{5}-2}$$

$$= \frac{1}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2}$$

According to the formula $a^2 - b^2 = (a+b)(a-b)$

$$= \frac{\sqrt{5}+2}{\sqrt{5}^2 - 2^2}$$

On further calculation

$$= \frac{\sqrt{5}+2}{5-4}$$

So we get

$$= \frac{\sqrt{5}+2}{1}$$

$$= \sqrt{5} + 2$$

(v) Consider a and b as integers as x as natural number

The rationalising factor are (a + b$\sqrt{x}$) and (a - b$\sqrt{x}$)
We can write it as (a + b$\sqrt{x}$) (a – b$\sqrt{x}$) = ($a^2 - b^2x$)
So we get,

$$\frac{1}{5+3\sqrt{2}}$$

$$= \frac{1}{5+3\sqrt{2}} \times \frac{5-3\sqrt{2}}{5-3\sqrt{2}}$$

According to the formula $a^2 - b^2 = (a+b)(a-b)$

$$= \frac{5-3\sqrt{2}}{5^2 - 3\sqrt{2}^2}$$

On further calculation

$= \frac{5-3\sqrt{2}}{25-18}$

So we get

$= \frac{5-3\sqrt{2}}{7}$

(vi) $\frac{1}{\sqrt{7}-\sqrt{6}}$

$= \frac{1}{\sqrt{7}-\sqrt{6}} \times \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}}$

According to the formula $a^2 - b^2 = (a+b)(a-b)$

$= \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}^2 - \sqrt{6}^2}$

On further calculation

$= \frac{\sqrt{7}+\sqrt{6}}{7-6}$

So we get

$= \sqrt{7} + \sqrt{6}$

(vii) $\frac{4}{\sqrt{11}-\sqrt{7}}$

$= \frac{4}{\sqrt{11}-\sqrt{7}} \times \frac{\sqrt{11}+\sqrt{7}}{\sqrt{11}+\sqrt{7}}$

According to the formula $a^2 - b^2 = (a+b)(a-b)$

$= \frac{4\ (\sqrt{11}+\sqrt{7})}{\sqrt{11}^2 - \sqrt{7}^2}$

On further calculation

$= \frac{4\ (\sqrt{11}+\sqrt{7})}{11-7}$

So we get

$= \frac{4\,(\sqrt{11}+\sqrt{7})}{4}$

$= (\sqrt{11} + \sqrt{7})$

(viii) $\frac{1+\sqrt{2}}{2-\sqrt{2}}$

$= \frac{1+\sqrt{2}}{2-\sqrt{2}} \times \frac{2+\sqrt{2}}{2+\sqrt{2}}$

According to the formula $a^2 - b^2 = (a+b)(a-b)$

$= \frac{(1+\sqrt{2})(2+\sqrt{2})}{2^2-\sqrt{2}^2}$

On further calculation

$= \frac{1\times 2+\sqrt{2}+2\sqrt{2}+\sqrt{2}^2}{4-2}$

So we get

$= \frac{2+3\sqrt{2}+2}{2}$

$= \frac{4+3\sqrt{2}}{2}$

(ix) $\frac{3-2\sqrt{2}}{3+2\sqrt{2}}$

$= \frac{3-2\sqrt{2}}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}}$

According to the formula $a^2 - b^2 = (a+b)(a-b)$

$= \frac{(3-2\sqrt{2})^2}{(3)^2-(2\sqrt{2})^2}$

On further calculation

$= \frac{3^2-2\times 3\times 2\sqrt{2}+2\sqrt{2}^2}{9-4\times 2}$

$= \frac{9-12\sqrt{2}+8}{9-8}$

So we get

$= \frac{17-12\sqrt{2}}{1}$

$= 17 - 12\sqrt{2}$

3. It being given that $\sqrt{2}$ = 1.414, $\sqrt{3}$ = 1.732, $\sqrt{5}$ = 2.236 and $\sqrt{10}$ = 3.162, find the value to three places of decimals, of each of the following.

(i) $\frac{2}{\sqrt{5}}$

(ii) $\frac{2-\sqrt{3}}{\sqrt{3}}$

(iii) $\frac{\sqrt{10}-\sqrt{5}}{\sqrt{2}}$

Solution:

(i) $\frac{2}{\sqrt{5}}$

It can be written as

$= \frac{2}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$

By multiplication

$= \frac{2\sqrt{5}}{\sqrt{5}^2}$

So we get

$= \frac{2\sqrt{5}}{5}$

Substituting the value of $\sqrt{5}$

$= \frac{2 \times 2.236}{5}$

So we get

$= \frac{4.472}{5}$

= 0.8944

= 0.894

(ii) $\frac{2-\sqrt{3}}{\sqrt{3}}$

It can be written as

$= \frac{2-\sqrt{3}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

By multiplication

$= \frac{\sqrt{3}\,(2-\sqrt{3})}{\sqrt{3}^2}$

So we get

$= \frac{2\sqrt{3}-3}{3}$

Substituting the value of $\sqrt{3}$

$= \frac{3.464-3}{3}$

We get

$= \frac{0.464}{3}$

By division

= 0.1546

= 0.155

(iii) $\frac{\sqrt{10}-\sqrt{5}}{\sqrt{2}}$

It can be written as

$= \frac{\sqrt{5 \times 2}-\sqrt{5}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

By multiplication

$= \frac{2\sqrt{5} - \sqrt{10}}{\sqrt{2}^2}$

So we get

$= \frac{2\sqrt{5} - \sqrt{10}}{2}$

Substituting the value of $\sqrt{5}$ and $\sqrt{10}$

$= \frac{2 \times 2.236 - 3.162}{2}$

On further calculation

$= \frac{4.472 - 3.162}{2}$

$= \frac{1.31}{2}$

By division

$= 0.655$

4. Find rational numbers a and b such that

(i) $\frac{\sqrt{2}-1}{\sqrt{2}+1} = a + b\sqrt{2}$

(ii) $\frac{2-\sqrt{5}}{2+\sqrt{5}} = a\sqrt{5} + b$

(iii) $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} = a + b\sqrt{6}$

(iv) $\frac{5+2\sqrt{3}}{7+4\sqrt{3}} = a + b\sqrt{3}$

Solution:

(i) $\frac{\sqrt{2}-1}{\sqrt{2}+1}$

It can be written as

$= \frac{\sqrt{2}-1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}$

According to the formula $a^2 - b^2 = (a+b)(a-b)$ and $(a-b)^2 = a^2 + b^2 - 2ab$

$= \frac{(\sqrt{2}-1)^2}{\sqrt{2}^2 - 1^2}$

On further calculation

$= \frac{\sqrt{2}^2 - 2\sqrt{2}+1}{2-1}$

So we get

$= \frac{2-2\sqrt{2}+1}{1}$

$= 3 - 2\sqrt{2}$

Based on the question,

$\frac{\sqrt{2}-1}{\sqrt{2}+1} = a + b\sqrt{2}$

$3 - 2\sqrt{2} = a + b\sqrt{2}$

So we get,

a = 3 and b = -2

(ii) $\frac{2-\sqrt{5}}{2+\sqrt{5}}$

It can be written as

$= \frac{2-\sqrt{5}}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}}$

According to the formula $a^2 - b^2 = (a+b)(a-b)$ and $(a-b)^2 = a^2 + b^2 - 2ab$

$= \frac{(2-\sqrt{5})^2}{2^2 - \sqrt{5}^2}$

$= \frac{2^2 - 2 \times 2\sqrt{5} + \sqrt{5}^2}{4-5}$

On further calculation

$= \frac{4-4\sqrt{5}+5}{-1}$

$= -(-4\sqrt{5} + 9)$

So we get

$= 4\sqrt{5} - 9$

As per the question,

$\frac{2-\sqrt{5}}{2+\sqrt{5}} = a\sqrt{5} + b$

$4\sqrt{5} - 9 = a\sqrt{5} + b$

Therefore, a = 4 and b = -9

(iii) $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

It can be written as

$= \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

According to the formula $a^2 - b^2 = (a+b)(a-b)$ and $(a+b)^2 = a^2 + b^2 + 2ab$

$= \frac{(\sqrt{3}+\sqrt{2})^2}{\sqrt{3}^2 - \sqrt{2}^2}$

On further calculation

$= \frac{\sqrt{3}^2 + 2 \times \sqrt{3} \times \sqrt{2} + \sqrt{2}^2}{3-2}$

So we get

$= \frac{3+2\sqrt{6}+2}{1}$

$= 5 + 2\sqrt{6}$

So we get,

$\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} = a + b\sqrt{6}$

a = 5 and b = 2

(iv) $\frac{5+2\sqrt{3}}{7+4\sqrt{3}}$

It can be written as

$= \frac{5+2\sqrt{3}}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}}$

According to the formula $a^2 - b^2 = (a+b)(a-b)$

$= \frac{5 \times 7 - 5 \times 4\sqrt{3} + 2\sqrt{3} \times 7 - 2\sqrt{3} \times 4\sqrt{3}}{7^2 - 4\sqrt{3}^2}$

On further calculation

$= \frac{35 - 20\sqrt{3} + 14\sqrt{3} - 8 \times 3}{49 - 16 \times 3}$

So we get

$= \frac{35 - 6\sqrt{3} - 24}{49 - 48}$

By subtraction we get

$= \frac{11 - 6\sqrt{3}}{1}$

$= 11 - 6\sqrt{3}$

We know,

$\frac{5+2\sqrt{3}}{7+4\sqrt{3}} = a + b\sqrt{3}$

Therefore, a = 11 and b = -6

5. It being given that $\sqrt{3} = 1.732$, $\sqrt{5} = 2.236$, $\sqrt{6} = 2.449$ and $\sqrt{10} = 3.162$, find to three places of decimal, the value of each of the following.

(i) $\frac{1}{\sqrt{6}+\sqrt{5}}$

(ii) $\frac{6}{\sqrt{5}+\sqrt{3}}$

(iii) $\frac{1}{4\sqrt{3}-3\sqrt{5}}$

(iv) $\frac{3+\sqrt{5}}{3-\sqrt{5}}$

(v) $\frac{1+2\sqrt{3}}{2-\sqrt{3}}$

(vi) $\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$

Solution:

(i) $\frac{1}{\sqrt{6}+\sqrt{5}}$

It can be written as

$= \frac{1}{\sqrt{6}+\sqrt{5}} \times \frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}-\sqrt{5}}$

According to the formula $a^2 - b^2 = (a+b)(a-b)$

$= \frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}^2-\sqrt{5}^2}$

On further calculation

$= \frac{\sqrt{6}-\sqrt{5}}{6-5}$

$= \frac{\sqrt{6}-\sqrt{5}}{1}$

So we get

$= \sqrt{6} - \sqrt{5}$

Substituting the values

= 2.449 – 2.236

= 0.213

(ii) $\frac{6}{\sqrt{5}+\sqrt{3}}$

It can be written as

$= \frac{6}{\sqrt{5}+\sqrt{3}} \times \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

According to the formula $a^2 - b^2 = (a+b)(a-b)$

$= \frac{6(\sqrt{5}-\sqrt{3})}{\sqrt{5}^2-\sqrt{3}^2}$

On further calculation

$= \frac{6\,(\sqrt{5} - \sqrt{3})}{5-3}$

$= \frac{6\,(\sqrt{5} - \sqrt{3})}{2}$

So we get

$= 3(\sqrt{5} - \sqrt{3})$

Substituting the values

= 3 (2.236 – 1.732)

By multiplication

= 3 × 0.504

= 1.512

(iii) $\frac{1}{4\sqrt{3} - 3\sqrt{5}}$

It can be written as

$= \frac{1}{4\sqrt{3} - 3\sqrt{5}} \times \frac{4\sqrt{3}+3\sqrt{5}}{4\sqrt{3}+3\sqrt{5}}$

According to the formula $a^2 - b^2 = (a + b)(a - b)$

$= \frac{4\sqrt{3}+3\sqrt{5}}{4\sqrt{3}^2 - 3\sqrt{5}^2}$

On further calculation

$= \frac{4 \times 1.732 + 3 \times 2.236}{16 \times 3 - 9 \times 5}$

So we get

$= \frac{6.928+6.708}{48-45}$

By division

$= \frac{13.836}{3}$

= 4.545

(iv) $\frac{3+\sqrt{5}}{3-\sqrt{5}}$

It can be written as

$= \frac{3+\sqrt{5}}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}}$

According to the formula $a^2 - b^2 = (a+b)(a-b)$

$= \frac{(3+\sqrt{5})^2}{3^3-\sqrt{5}^2}$

On further calculation

$= \frac{3^2+2\times3\sqrt{5}+\sqrt{5}^2}{9-5}$

$= \frac{9+6\sqrt{5}+5}{4}$

So we get

$= \frac{14+6\sqrt{5}}{4}$

$= \frac{7+3\sqrt{5}}{2}$

Substituting the value

$= \frac{7+3\times2.236}{2}$

$= \frac{7+6.708}{2}$

By division

$= \frac{13.708}{2}$

= 6.854

(v) $\frac{1+2\sqrt{3}}{2-\sqrt{3}}$

It can be written as

$= \frac{1+2\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$

According to the formula $a^2 - b^2 = (a+b)(a-b)$

$= \frac{2+\sqrt{3}+2\sqrt{3}\times 2+2\sqrt{3}\times\sqrt{3}}{2^2-\sqrt{3}^2}$

On further calculation

$= \frac{2+\sqrt{3}+4\sqrt{3}+6}{4-3}$

So we get

$= \frac{8+5\sqrt{3}}{1}$

Substituting the value

= 8 + 5 × 1.732

We get

= 8 + 8.660

= 16.660

(vi) $\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$

It can be written as

$= \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}} \times \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}}$

According to the formula $a^2 - b^2 = (a+b)(a-b)$

$= \frac{(\sqrt{5}+\sqrt{2})^2}{\sqrt{5}^2-\sqrt{2}^2}$

On further calculation

$= \frac{\sqrt{5}^2+2\times\sqrt{5}\times\sqrt{2}+\sqrt{2}^2}{5-2}$

So we get

$= \frac{5+2\sqrt{10}+2}{3}$

$= \frac{7+2\sqrt{10}}{3}$

Substituting the value

$= \frac{7+2\times 3.162}{3}$

We get

$= \frac{7+6.324}{3}$

By division

$= \frac{13.324}{3}$

= 4.441

6. Simplify by rationalising the denominator.

(i) $\frac{7\sqrt{3}-5\sqrt{2}}{\sqrt{48}+\sqrt{18}}$

(ii) $\frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}}$

Solution:

(i) $\frac{7\sqrt{3}-5\sqrt{2}}{\sqrt{48}+\sqrt{18}}$

It can be written as

$= \frac{7\sqrt{3}-5\sqrt{2}}{\sqrt{16\times 3}+\sqrt{9\times 2}}$

On further calculation

$= \frac{7\sqrt{3}-5\sqrt{2}}{4\sqrt{3}+3\sqrt{2}}$

We can write it as

$= \frac{7\sqrt{3}-5\sqrt{2}}{4\sqrt{3}+3\sqrt{2}} \times \frac{4\sqrt{3}-3\sqrt{2}}{4\sqrt{3}-3\sqrt{2}}$

According to the formula $a^2 - b^2 = (a + b)(a - b)$

$$= \frac{7\sqrt{3} \times 4\sqrt{3} - 7\sqrt{3} \times 3\sqrt{2} - 5\sqrt{2} \times 4\sqrt{3} + 5\sqrt{2} \times 3\sqrt{2}}{4\sqrt{3}^2 - 3\sqrt{2}^2}$$

On further calculation

$$= \frac{28 \times 3 - 21\sqrt{6} - 20\sqrt{6} + 15 \times 2}{16 \times 3 - 9 \times 2}$$

$$= \frac{84 - 41\sqrt{6} + 30}{48 - 18}$$

We get

$$= \frac{114 - 41\sqrt{6}}{30}$$

(ii) $\frac{2\sqrt{6} - \sqrt{5}}{3\sqrt{5} - 2\sqrt{6}}$

It can be written as

$$= \frac{2\sqrt{6} - \sqrt{5}}{3\sqrt{5} - 2\sqrt{6}} \times \frac{3\sqrt{5} + 2\sqrt{6}}{3\sqrt{5} + 2\sqrt{6}}$$

According to the formula $a^2 - b^2 = (a + b)(a - b)$

$$= \frac{2\sqrt{6} \times 3\sqrt{5} + 2\sqrt{6}^2 - \sqrt{5} \times 3\sqrt{5} - \sqrt{5} \times 2\sqrt{6}}{3\sqrt{5}^2 - 2\sqrt{6}^2}$$

On further calculation

$$= \frac{6\sqrt{30} + 24 - 15 - 2\sqrt{30}}{45 - 24}$$

So we get

$$= \frac{4\sqrt{30} + 9}{21}$$

7. Simplify

(i) $\frac{4 + \sqrt{5}}{4 - \sqrt{5}} + \frac{4 - \sqrt{5}}{4 + \sqrt{5}}$

(ii) $\frac{1}{\sqrt{3}-\sqrt{2}} - \frac{2}{\sqrt{5}-\sqrt{3}} - \frac{3}{\sqrt{2}-\sqrt{5}}$

(iii) $\frac{2+\sqrt{3}}{2-\sqrt{3}} + \frac{2-\sqrt{3}}{2+\sqrt{3}} + \frac{\sqrt{3}-1}{\sqrt{3}+1}$

(iv) $\frac{2\sqrt{6}}{\sqrt{2}+\sqrt{3}} + \frac{6\sqrt{2}}{\sqrt{6}+\sqrt{3}} - \frac{8\sqrt{3}}{\sqrt{6}+\sqrt{2}}$

Solution:

(i) In order to rationalise the denominator we should multiply the numerator and denominator with rationalising factor.

Consider a and b are integers

The rationalising factors are (a +$\sqrt{b}$) and (a -$\sqrt{b}$)

We can write (a +$\sqrt{b}$) (a -$\sqrt{b}$) = (a^2 – b)

Consider the first term

So we get,

$\frac{4+\sqrt{5}}{4-\sqrt{5}}$

It can be written as

$= \frac{4+\sqrt{5}}{4-\sqrt{5}} \times \frac{4+\sqrt{5}}{4+\sqrt{5}}$

According to the formula $a^2 - b^2 = (a+b)(a-b)$

$= \frac{(4+\sqrt{5})^2}{4^2-\sqrt{5}^2}$

So we get

$= \frac{4^2+2\times4\times\sqrt{5}+\sqrt{5}^2}{16-5}$

On further calculation

$= \frac{16+8\sqrt{5}+5}{11}$

$= \frac{21+8\sqrt{5}}{11}$ ………. (1)

Consider the second term

So we get,

$$\frac{4-\sqrt{5}}{4+\sqrt{5}}$$

It can be written as

$$=\frac{4-\sqrt{5}}{4+\sqrt{5}} \times \frac{4-\sqrt{5}}{4-\sqrt{5}}$$

According to the formula $a^2 - b^2 = (a+b)(a-b)$

$$=\frac{(4-\sqrt{5})^2}{4^2-\sqrt{5}^2}$$

So we get

$$=\frac{4^2-2\times 4\times\sqrt{5}+\sqrt{5}^2}{16-5}$$

On further calculation

$$=\frac{16-8\sqrt{5}+5}{11}$$

$$=\frac{21-8\sqrt{5}}{11} \text{ ……….. } (2)$$

Add both equations (1) and (2)

$$\frac{4+\sqrt{5}}{4-\sqrt{5}}+\frac{4-\sqrt{5}}{4+\sqrt{5}}$$

By addition we get

$$=\frac{21+8\sqrt{5}}{11}+\frac{21-8\sqrt{5}}{11}$$

By taking LCM

$$=\frac{21+8\sqrt{5}+21-8\sqrt{5}}{11}$$

So we get

$$=\frac{42}{11}$$

(ii) $\frac{1}{\sqrt{3}-\sqrt{2}} - \frac{2}{\sqrt{5}-\sqrt{3}} - \frac{3}{\sqrt{2}-\sqrt{5}}$

It can be written as

$= \frac{1}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} - \frac{2}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} - \frac{3}{\sqrt{2}-\sqrt{5}} \times \frac{\sqrt{2}+\sqrt{5}}{\sqrt{2}+\sqrt{5}}$

According to the formula $a^2 - b^2 = (a+b)(a-b)$

$= \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}^2-\sqrt{2}^2} - \frac{2(\sqrt{5}+\sqrt{3})}{\sqrt{5}^2-\sqrt{3}^2} - \frac{3(\sqrt{2}+\sqrt{5})}{\sqrt{2}^2-\sqrt{5}^2}$

On further calculation

$= \frac{\sqrt{3}+\sqrt{2}}{3-2} - \frac{2(\sqrt{5}+\sqrt{3})}{5-3} - \frac{3(\sqrt{2}+\sqrt{5})}{2-5}$

So we get

$= \frac{\sqrt{3}+\sqrt{2}}{1} - \frac{2(\sqrt{5}+\sqrt{3})}{2} - \frac{3(\sqrt{2}+\sqrt{5})}{-3}$

$= (\sqrt{3}+\sqrt{2}) - (\sqrt{5}+\sqrt{3}) + (\sqrt{2}+\sqrt{5})$

On further simplification

$= \sqrt{3}+\sqrt{2} - \sqrt{5} - \sqrt{3} + \sqrt{2} + \sqrt{5}$

$= 2\sqrt{2}$

(iii) $\frac{2+\sqrt{3}}{2-\sqrt{3}} + \frac{2-\sqrt{3}}{2+\sqrt{3}} + \frac{\sqrt{3}-1}{\sqrt{3}+1}$

It can be written as

$= \frac{2+\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2-\sqrt{3}}$

According to the formula $a^2 - b^2 = (a+b)(a-b)$

$= \frac{(2+\sqrt{3})^2}{2^2-\sqrt{3}^2} + \frac{(2-\sqrt{3})^2}{2^2-\sqrt{3}^2} + \frac{(\sqrt{3}-1)^2}{\sqrt{3}^2-1}$

On further calculation

$= \frac{2^2 + 2 \times 2\sqrt{3} + \sqrt{3}^2}{4-3} + \frac{2^2 - 2 \times 2\sqrt{3} + \sqrt{3}^2}{4-3} + \frac{\sqrt{3}^2 - 2\sqrt{3} + 1}{3-1}$

So we get

$= \frac{4+4\sqrt{3}+3}{1} + \frac{4-4\sqrt{3}+3}{1} + \frac{3-2\sqrt{3}+1}{2}$

$= 7 + 4\sqrt{3} + 7 - 4\sqrt{3} + \frac{4-2\sqrt{3}}{2}$

On further simplification

$= 14 + 2 - \sqrt{3}$

$= 16 - \sqrt{3}$

(iv) $\frac{2\sqrt{6}}{\sqrt{2}+\sqrt{3}} + \frac{6\sqrt{2}}{\sqrt{6}+\sqrt{3}} - \frac{8\sqrt{3}}{\sqrt{6}+\sqrt{2}}$

It can be written as

$= \frac{2\sqrt{6}}{\sqrt{2}+\sqrt{3}} \times \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}} + \frac{6\sqrt{2}}{\sqrt{6}+\sqrt{3}} \times \frac{\sqrt{6}-\sqrt{3}}{\sqrt{6}-\sqrt{3}} - \frac{8\sqrt{3}}{\sqrt{6}+\sqrt{2}} \times \frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}$

According to the formula $a^2 - b^2 = (a+b)(a-b)$

$= \frac{2\sqrt{6}\,(\sqrt{2}-\sqrt{3})}{\sqrt{2}^2 - \sqrt{3}^2} + \frac{6\sqrt{2}\,(\sqrt{6}-\sqrt{3})}{\sqrt{6}^2 - \sqrt{3}^2} - \frac{8\sqrt{3}\,(\sqrt{6}-\sqrt{2})}{\sqrt{6}^2 - \sqrt{2}^2}$

On further calculation

$= \frac{2\sqrt{12}-2\sqrt{18}}{2-3} + \frac{6\sqrt{12}-6\sqrt{6}}{6-3} - \frac{8\sqrt{18}-8\sqrt{6}}{6-2}$

So we get

$= \frac{2\sqrt{4\times3}-2\sqrt{9\times2}}{-1} + \frac{6\sqrt{4\times3}-6\sqrt{6}}{3} - \frac{8\sqrt{9\times2}-8\sqrt{6}}{4}$

$= \frac{4\sqrt{3}-6\sqrt{2}}{-1} + \frac{12\sqrt{3}-6\sqrt{6}}{3} - \frac{24\sqrt{2}-8\sqrt{6}}{4}$

On further simplification

$= \frac{4\sqrt{3}-6\sqrt{2}}{-1} + \frac{4\sqrt{3}-2\sqrt{6}}{1} - \frac{6\sqrt{2}-2\sqrt{6}}{1}$

$= -4\sqrt{3} + 6\sqrt{2} + 4\sqrt{3} - 2\sqrt{6} - 6\sqrt{2} + 2\sqrt{6}$

$= 0$

8. Prove that

(i) $\frac{1}{3+\sqrt{7}} + \frac{1}{\sqrt{7}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{3}} + \frac{1}{\sqrt{3}+1} = 1$

(ii) $\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \frac{1}{\sqrt{4}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{6}} + \frac{1}{\sqrt{6}+\sqrt{7}} + \frac{1}{\sqrt{7}+\sqrt{8}} + \frac{1}{\sqrt{8}+\sqrt{9}} = 2$

Solution:

(i) Let us prove LHS = RHS

LHS

$= \frac{1}{3+\sqrt{7}} + \frac{1}{\sqrt{7}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{3}} + \frac{1}{\sqrt{3}+1}$

It can be written as

$= \frac{1}{3+\sqrt{7}} \times \frac{3-\sqrt{7}}{3-\sqrt{7}} + \frac{1}{\sqrt{7}+\sqrt{5}} \times \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{3}} \times \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}} + \frac{1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$

According to the formula $a^2 - b^2 = (a+b)(a-b)$

$= \frac{3-\sqrt{7}}{3^2-\sqrt{7}^2} + \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}^2-\sqrt{5}^2} + \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}^2-\sqrt{3}^2} + \frac{\sqrt{3}-1}{\sqrt{3}^2-1^2}$

On further calculation

$= \frac{3-\sqrt{7}}{9-7} + \frac{\sqrt{7}-\sqrt{5}}{7-5} + \frac{\sqrt{5}-\sqrt{3}}{5-3} + \frac{\sqrt{3}-1}{3-1}$

So we get

$= \frac{3-\sqrt{7}}{2} + \frac{\sqrt{7}-\sqrt{5}}{2} + \frac{\sqrt{5}-\sqrt{3}}{2} + \frac{\sqrt{3}-1}{2}$

$= \frac{3-\sqrt{7}+\sqrt{7}-\sqrt{5}+\sqrt{5}-\sqrt{3}+\sqrt{3}-1}{2}$

By dividing we get

$= \frac{2}{2}$

= 1

= RHS

(ii) Prove LHS = RHS

LHS

$$=\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\frac{1}{\sqrt{4}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{6}}+\frac{1}{\sqrt{6}+\sqrt{7}}+\frac{1}{\sqrt{7}+\sqrt{8}}+\frac{1}{\sqrt{8}+\sqrt{9}}$$

It can be written as

$$=\frac{1}{1+\sqrt{2}}\times\frac{1-\sqrt{2}}{1-\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}\times\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}\times\frac{\sqrt{3}-\sqrt{4}}{\sqrt{3}-\sqrt{4}}+\frac{1}{\sqrt{4}+\sqrt{5}}\times\frac{\sqrt{4}-\sqrt{5}}{\sqrt{4}-\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{6}}\times\frac{\sqrt{5}-\sqrt{6}}{\sqrt{5}-\sqrt{6}}$$

$$+\frac{1}{\sqrt{6}+\sqrt{7}}\times\frac{\sqrt{6}-\sqrt{7}}{\sqrt{6}-\sqrt{7}}+\frac{1}{\sqrt{7}+\sqrt{8}}\times\frac{\sqrt{7}-\sqrt{8}}{\sqrt{7}-\sqrt{8}}+\frac{1}{\sqrt{8}+\sqrt{9}}\times\frac{\sqrt{8}-\sqrt{9}}{\sqrt{8}-\sqrt{9}}$$

According to the formula $a^2-b^2=(a+b)(a-b)$

$$=\frac{1-\sqrt{2}}{1-\sqrt{2}^2}+\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}^2-\sqrt{3}^2}+\frac{\sqrt{3}-\sqrt{4}}{\sqrt{3}^2-\sqrt{4}^2}+\frac{\sqrt{4}-\sqrt{5}}{\sqrt{4}^2-\sqrt{5}^2}+\frac{\sqrt{5}-\sqrt{6}}{\sqrt{5}^2-\sqrt{6}^2}+\frac{\sqrt{6}-\sqrt{7}}{\sqrt{6}^2-\sqrt{7}^2}+\frac{\sqrt{7}-\sqrt{8}}{\sqrt{7}^2-\sqrt{8}^2}+\frac{\sqrt{8}-\sqrt{9}}{\sqrt{8}^2-\sqrt{9}^2}$$

On further calculation

$$=\frac{1-\sqrt{2}}{1-2}+\frac{\sqrt{2}-\sqrt{3}}{2-3}+\frac{\sqrt{3}-\sqrt{4}}{3-4}+\frac{\sqrt{4}-\sqrt{5}}{4-5}+\frac{\sqrt{5}-\sqrt{6}}{5-6}+\frac{\sqrt{6}-\sqrt{7}}{6-7}+\frac{\sqrt{7}-\sqrt{8}}{7-8}+\frac{\sqrt{8}-\sqrt{9}}{8-9}$$

So we get

$$=\frac{1-\sqrt{2}}{-1}+\frac{\sqrt{2}-\sqrt{3}}{-1}+\frac{\sqrt{3}-\sqrt{4}}{-1}+\frac{\sqrt{4}-\sqrt{5}}{-1}+\frac{\sqrt{5}-\sqrt{6}}{-1}+\frac{\sqrt{6}-\sqrt{7}}{-1}+\frac{\sqrt{7}-\sqrt{8}}{-1}+\frac{\sqrt{8}-\sqrt{9}}{-1}$$

On further simplification

$$=-1+\sqrt{2}-\sqrt{2}+\sqrt{3}-\sqrt{3}+\sqrt{4}-\sqrt{4}+\sqrt{5}-\sqrt{5}+\sqrt{6}-\sqrt{6}+\sqrt{7}-\sqrt{7}+\sqrt{8}-\sqrt{8}+\sqrt{9}$$

We get

= -1+3

= 2

= RHS

9. Find the values of a and b if

$$\frac{7+3\sqrt{5}}{3+\sqrt{5}} - \frac{7-3\sqrt{5}}{3-\sqrt{5}} = a + b\sqrt{5}$$

Solution:

LHS

$$= \frac{7+3\sqrt{5}}{3+\sqrt{5}} - \frac{7-3\sqrt{5}}{3-\sqrt{5}}$$

It can be written as

$$= \frac{7+3\sqrt{5}}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}} - \frac{7-3\sqrt{5}}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}}$$

According to the formula $a^2 - b^2 = (a+b)(a-b)$

$$= \frac{7 \times 3 - 7\sqrt{5} + 3\sqrt{5} \times 3 - 3\sqrt{5} \times \sqrt{5}}{3^2 - \sqrt{5}^2} - \frac{7 \times 3 + 7\sqrt{5} - 3\sqrt{5} \times 3 - 3\sqrt{5} \times \sqrt{5}}{3^2 - \sqrt{5}^2}$$

On further calculation

$$= \frac{21 - 7\sqrt{5} + 9\sqrt{5} - 15}{9-5} - \frac{21 + 7\sqrt{5} - 9\sqrt{5} - 15}{9-5}$$

$$= \frac{6+2\sqrt{5}}{4} - \frac{6-2\sqrt{5}}{4}$$

So we get

$$= \frac{6+2\sqrt{5}-6+2\sqrt{5}}{4}$$

By dividing

$$= \frac{0+4\sqrt{5}}{4}$$

$= 0 + \sqrt{5}$

We know that,

$$\frac{7+3\sqrt{5}}{3+\sqrt{5}} - \frac{7-3\sqrt{5}}{3-\sqrt{5}} = a + b\sqrt{5}$$

$0 + \sqrt{5} = a + b\sqrt{5}$

a = 0 and b = 1

10. Simplify $\frac{\sqrt{13}-\sqrt{11}}{\sqrt{13}+\sqrt{11}}+\frac{\sqrt{13}+\sqrt{11}}{\sqrt{13}-\sqrt{11}}$.

Solution:

$$\frac{\sqrt{13}-\sqrt{11}}{\sqrt{13}+\sqrt{11}}+\frac{\sqrt{13}+\sqrt{11}}{\sqrt{13}-\sqrt{11}}$$

It can be written as

$$=\frac{(\sqrt{13}-\sqrt{11})^2+(\sqrt{13}+\sqrt{11})^2}{(\sqrt{13}+\sqrt{11})(\sqrt{13}-\sqrt{11})}$$

According to the formula $a^2-b^2=(a+b)(a-b)$

$$=\frac{\left(\sqrt{13}^2-2\sqrt{13}\times\sqrt{11}+\sqrt{11}^2\right)+(\sqrt{13}^2+2\sqrt{13}\times\sqrt{11}+\sqrt{11}^2)}{\sqrt{13}^2-\sqrt{11}^2}$$

On further calculation

$$=\frac{(\sqrt{13}-2\sqrt{143}+11)+(13+2\sqrt{143}+11)}{13-11}$$

So we get

$$=\frac{24-2\sqrt{143}+24+2\sqrt{143}}{2}$$

$$=\frac{48}{2}$$

By division we get

$= 24$

11. If x=3+2$\sqrt{2}$, check whether x+$\frac{1}{x}$ is rational or irrational.

Solution:

We know that,

$x = 3 + 2\sqrt{2}$

To find,

$x+\frac{1}{x}$

$= 3 + 2\sqrt{2} + \frac{1}{3 + 2\sqrt{2}}$

It can be written as

$= 3 + 2\sqrt{2} + \frac{1}{3 + 2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}}$

According to the formula $a^2 - b^2 = (a + b)(a - b)$

$= 3 + 2\sqrt{2} + \frac{3-2\sqrt{2}}{3^2-2\sqrt{2}^2}$

On further calculation

$= 3 + 2\sqrt{2} + \frac{3-2\sqrt{2}}{9-8}$

$= 3 + 2\sqrt{2} + \frac{3-2\sqrt{2}}{1}$

So we get

$= 3 + 2\sqrt{2} + 3 - 2\sqrt{2}$

$= 6$

Therefore, $x+\frac{1}{x}$ is rational.

12. If x = 2 - $\sqrt{3}$, find the value of $(x - \frac{1}{x})^3$.

Solution:

We know that,

$x = 2 - \sqrt{3}$

To find,

$x - \frac{1}{x}$

$= 2 - \sqrt{3} - \frac{1}{2 - \sqrt{3}}$

It can be written as

$= 2 - \sqrt{3} - \frac{1}{2 - \sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$

According to the formula $a^2 - b^2 = (a + b)(a - b)$

$= 2 - \sqrt{3} - \frac{2+\sqrt{3}}{2^2 - \sqrt{3}^2}$

On further calculation

$= 2 - \sqrt{3} - \frac{2+\sqrt{3}}{4-3}$

So we get

$= 2 - \sqrt{3} - 2 - \sqrt{3}$

$= -2\sqrt{3}$

$(x - \frac{1}{x})^3 = -2\sqrt{3} = -24\sqrt{3}$

13. If x = 9 - 4$\sqrt{5}$, find the value of $x^2 + \frac{1}{x^2}$.

Solution:

We know that,

$x = 9 - 4\sqrt{5}$

So $\frac{1}{x}$ can be written as

$= \frac{1}{9 - 4\sqrt{5}}$

It can be written as

$= \frac{1}{9 - 4\sqrt{5}} \times \frac{9+ 4\sqrt{5}}{9+ 4\sqrt{5}}$

According to the formula $a^2 - b^2 = (a + b)(a - b)$

$= \frac{9+ 4\sqrt{5}}{9^2 - 4\sqrt{5}^2}$

On further calculation

$= \frac{9+4\sqrt{5}}{81-90}$

So we get

$= 9 + 4\sqrt{5}$

$x + \frac{1}{x}$

By substituting the values we get

$= 9 - 4\sqrt{5} + 9 + 4\sqrt{5}$

$= 18$

$$(x + \frac{1}{x})^2 = 18^2 = 324$$

According to the formula $(a+b)^2 = a^2 + b^2 + 2ab$

$$(x^2 + \frac{1}{x^2}) + 2 \times x \times \frac{1}{x} = 324$$

On further calculation

$$(x^2 + \frac{1}{x^2}) + 2 = 324$$

Therefore, $(x^2 + \frac{1}{x^2}) = 322$

14. If $x = \frac{5-\sqrt{21}}{2}$, find the value of $x + \frac{1}{x}$.

Solution:

Given,

$x = \frac{5-\sqrt{21}}{2}$

We can write $\frac{1}{x}$ as,

$\frac{1}{x}$

$= \frac{2}{5-\sqrt{21}}$

It can be written as

$= \frac{2}{5-\sqrt{21}} \times \frac{5+\sqrt{21}}{5+\sqrt{21}}$

According to the formula $a^2 - b^2 = (a+b)(a-b)$

$= \frac{2(5+\sqrt{21})}{5^2-\sqrt{21}^2}$

So we get

$= \frac{2(5+\sqrt{21})}{25-21}$

On further simplification

$= \frac{2(5+\sqrt{21})}{4}$

$= \frac{5+\sqrt{21}}{2}$

$x + \frac{1}{x}$

Substituting the values

$= \frac{5-\sqrt{21}}{2} + \frac{5+\sqrt{21}}{2}$

By addition

$= \frac{5-\sqrt{21}+5+\sqrt{21}}{2}$

By dividing

$= \frac{10}{2}$

$=5$

15. If a = 3 - 2$\sqrt{2}$, find the value of $a^2 - \frac{1}{a^2}$.

Solution:

Given,

a = 3 - $2\sqrt{2}$

We can write a^2 as

= $(3 - 2\sqrt{2})^2$

According to the formula $(a-b)^2 = a^2 + b^2 - 2ab$

= 3^2- 2× 3 × $2\sqrt{2}$ + $(2\sqrt{2})^2$

On further calculation

= 9 - $12\sqrt{2}$ + 8

= 17 - $12\sqrt{2}$

We can write $\frac{1}{a^2}$ as

$= \frac{1}{17-12\sqrt{2}} \times \frac{17+12\sqrt{2}}{17+12\sqrt{2}}$

According to the formula $a^2 - b^2 = (a+b)(a-b)$

$= \frac{17+12\sqrt{2}}{17^2-(12\sqrt{2})^2}$

So we get

$= \frac{17+12\sqrt{2}}{289-288}$

= $17 + 12\sqrt{2}$

So a^2 - $\frac{1}{a^2}$ can be written as

= $(17 - 12\sqrt{2})$ - $(17 + 12\sqrt{2})$

We get

= 17 - $12\sqrt{2}$ - 17 − $12\sqrt{2}$

= - $24\sqrt{2}$

16. If x = $\sqrt{13}$ + $2\sqrt{3}$, find the value of x - $\frac{1}{x}$.

Solution:

We know that,

x = $\sqrt{13} + 2\sqrt{3}$

$\frac{1}{x}$

$= \frac{1}{\sqrt{13} + 2\sqrt{3}}$

It can be written as

$= \frac{1}{\sqrt{13} + 2\sqrt{3}} \times \frac{\sqrt{13} - 2\sqrt{3}}{\sqrt{13} - 2\sqrt{3}}$

According to the formula $a^2 - b^2 = (a + b)(a - b)$

$= \frac{\sqrt{13} - 2\sqrt{3}}{\sqrt{13}^2 - (2\sqrt{3})^2}$

On further calculation

$= \frac{\sqrt{13} - 2\sqrt{3}}{13 - 12}$

$= \sqrt{13} - 2\sqrt{3}$

So x - $\frac{1}{x}$ can be written as,

$= \sqrt{13} + 2\sqrt{3} - \sqrt{13} + 2\sqrt{3}$

$= 4\sqrt{3}$

17. If x = 2 +$\sqrt{3}$, find the value of $x^3 + \frac{1}{x^3}$.

Solution:

x = 2 +$\sqrt{3}$

$\frac{1}{x}$

$= \frac{1}{2 + \sqrt{3}}$

It can be written as

$= \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$

According to the formula $a^2 - b^2 = (a+b)(a-b)$

$= \frac{2-\sqrt{3}}{2^2 - \sqrt{3}^2}$

On further calculation

$= \frac{2-\sqrt{3}}{4-3}$

$= 2 - \sqrt{3}$

$x + \frac{1}{x}$

Substituting the values

$= 2 + \sqrt{3} + 2 - \sqrt{3}$

$= 4$

$(x + \frac{1}{x})^3 = 4^3$

According to the formula $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$(x^3 + \frac{1}{x^3}) + 3 \times x \times \frac{1}{x} (x + \frac{1}{x}) = 64$

On further calculation

$(x^3 + \frac{1}{x^3}) + 3 \times 4 = 64$

So we get

$(x^3 + \frac{1}{x^3}) + 12 = 64$

$(x^3 + \frac{1}{x^3}) = 52$

18. If $x = \frac{5-\sqrt{3}}{5+\sqrt{3}}$ and $y = \frac{5+\sqrt{3}}{5-\sqrt{3}}$, show that $x - y = -\frac{10\sqrt{3}}{11}$.

Solution:

We know that, x = $\frac{5-\sqrt{3}}{5+\sqrt{3}}$

So we get,

$= \frac{5-\sqrt{3}}{5+\sqrt{3}} \times \frac{5-\sqrt{3}}{5-\sqrt{3}}$

According to the formula $a^2 - b^2 = (a+b)(a-b)$

$= \frac{(5-\sqrt{3})^2}{5^2-\sqrt{3}^2}$

On further calculation

$= \frac{25-10\sqrt{3}+3}{25-3}$

So we get

$= \frac{28-10\sqrt{3}}{22}$

$= \frac{14-5\sqrt{3}}{11}$

We know that, y = $\frac{5+\sqrt{3}}{5-\sqrt{3}}$

So we get,

$= \frac{5+\sqrt{3}}{5-\sqrt{3}} \times \frac{5+\sqrt{3}}{5+\sqrt{3}}$

According to the formula $a^2 - b^2 = (a+b)(a-b)$

$= \frac{(5+\sqrt{3})^2}{5^2-\sqrt{3}^2}$

On further calculation

$= \frac{25+10\sqrt{3}+3}{25-3}$

So we get

$= \frac{28+10\sqrt{3}}{22}$

$= \frac{14+5\sqrt{3}}{11}$

So we can write x – y as,

$= \frac{14-5\sqrt{3}}{11} - \frac{14+5\sqrt{3}}{11}$

On further calculation

$= \frac{14-5\sqrt{3}-14-5\sqrt{3}}{11}$

We get

$= \frac{-10\sqrt{3}}{11}$

19. If a = $\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$ and b = $\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$, show that $3a^2$+ 4ab - $3b^2$ = 4 + $\frac{56}{3}\sqrt{10}$.

Solution:

We know that a = $\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$

So we get,

$= \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}} \times \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}}$

According to the formula $a^2 - b^2 = (a+b)(a-b)$

$= \frac{(\sqrt{5}+\sqrt{2})^2}{\sqrt{5}^2-\sqrt{2}^2}$

On further calculation

$= \frac{5+2\sqrt{10}+2}{5-2}$

So we get

$= \frac{7+2\sqrt{10}}{3}$

We know that b = $\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$

So we get,

$= \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$

According to the formula $a^2 - b^2 = (a+b)(a-b)$

$= \frac{(\sqrt{5}-\sqrt{2})^2}{\sqrt{5}^2 - \sqrt{2}^2}$

On further calculation

$= \frac{5-2\sqrt{10}+2}{5-2}$

So we get

$= \frac{7-2\sqrt{10}}{3}$

$a^2 = (\frac{7+2\sqrt{10}}{3})^2 = \frac{49+28\sqrt{10}+40}{9} = \frac{89+28\sqrt{10}}{9}$

$b^2 = (\frac{7-2\sqrt{10}}{3})^2 = \frac{49-28\sqrt{10}+40}{9} = \frac{89-28\sqrt{10}}{9}$

We know that LHS

$= 3a^2 + 4ab - 3b^2$

Substituting the values

$= 3 \times \frac{89+28\sqrt{10}}{9} + 4 \times \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}} - 3 \times \frac{89-28\sqrt{10}}{9}$

So we get

$= \frac{89+28\sqrt{10}}{3} + 4 - \frac{89-28\sqrt{10}}{3}$

On further calculation

$= 4 + \frac{89+28\sqrt{10}-89+28\sqrt{10}}{3}$

$= 4 + \frac{56}{3}\sqrt{10}$

= RHS

20. If a = $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ and b = $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$, find the value of $a^2 + b^2$ – 5ab.

Solution:

a = $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

So we get,

$= \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

According to the formula $a^2 - b^2 = (a+b)(a-b)$

$= \frac{(\sqrt{3}-\sqrt{2})^2}{\sqrt{3}^2-\sqrt{2}^2}$

On further calculation

$= \frac{3+2-2\sqrt{6}}{3-2}$

So we get

$= 5 - 2\sqrt{6}$

b = $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

So we get,

$= \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

According to the formula $a^2 - b^2 = (a+b)(a-b)$

$= \frac{(\sqrt{3}+\sqrt{2})^2}{\sqrt{3}^2-\sqrt{2}^2}$

On further calculation

$= \frac{3+2+2\sqrt{6}}{3-2}$

So we get

$= 5 + 2\sqrt{6}$

$a^2 = (5 - 2\sqrt{6})^2 = 25 - 20\sqrt{6} + 24 = 49 - 20\sqrt{6}$

$b^2 = (5 + 2\sqrt{6})^2 = 25 + 20\sqrt{6} + 24 = 49 + 20\sqrt{6}$

So $a^2 + b^2 - 5ab$

Substituting the values we get

$= (49 - 20\sqrt{6}) + (49 + 20\sqrt{6}) - 5 \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

On further calculation

$= 98 - 5$

$= 93$

21. If $p = \frac{3-\sqrt{5}}{3+\sqrt{5}}$ and $q = \frac{3+\sqrt{5}}{3-\sqrt{5}}$, find the value of p^2+q^2.

Solution:

$p = \frac{3-\sqrt{5}}{3+\sqrt{5}}$

So we get,

$= \frac{3-\sqrt{5}}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}}$

According to the formula $a^2 - b^2 = (a+b)(a-b)$

$= \frac{(3-\sqrt{5})^2}{3^2-\sqrt{5}^2}$

On further calculation

$= \frac{9+5-6\sqrt{5}}{9-5}$

$= \frac{14-6\sqrt{5}}{4}$

So we get,

$= \frac{7-3\sqrt{5}}{2}$

$q = \frac{3+\sqrt{5}}{3-\sqrt{5}}$

So we get,

$= \frac{3+\sqrt{5}}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}}$

According to the formula $a^2 - b^2 = (a+b)(a-b)$

$= \frac{(3+\sqrt{5})^2}{3^2-\sqrt{5}^2}$

On further calculation

$= \frac{9+5+6\sqrt{5}}{9-5}$

$= \frac{14+6\sqrt{5}}{4}$

So we get,

$= \frac{7+3\sqrt{5}}{2}$

We can write p^2+q^2 as

$= (\frac{7-3\sqrt{5}}{2})^2 + (\frac{7+3\sqrt{5}}{2})^2$

$= \frac{49+45-42\sqrt{5}}{4} + \frac{49+45+42\sqrt{5}}{4}$

On further calculation

$= \frac{94-42\sqrt{5}}{4} + \frac{94+42\sqrt{5}}{4}$

By taking 2 as common

$= \frac{47-21\sqrt{5}}{2} + \frac{47+21\sqrt{5}}{2}$

So we get

$= \frac{47-21\sqrt{5}+47+21\sqrt{5}}{2}$

$= \frac{94}{2}$

$= 47$

22. Rationalise the denominator of each of the following.

(i) $\frac{1}{\sqrt{7}+\sqrt{6}-\sqrt{13}}$

(ii) $\frac{3}{\sqrt{3}+\sqrt{5}-\sqrt{2}}$

(iii) $\frac{4}{2+\sqrt{3}+\sqrt{7}}$

Solution:

(i) $\frac{1}{\sqrt{7}+\sqrt{6}-\sqrt{13}}$

It can be written as

$= \frac{1}{(\sqrt{7}+\sqrt{6})-\sqrt{13}} \times \frac{(\sqrt{7}+\sqrt{6})+\sqrt{13}}{(\sqrt{7}+\sqrt{6})+\sqrt{13}}$

According to the formula $a^2 - b^2 = (a+b)(a-b)$

$= \frac{(\sqrt{7}+\sqrt{6})+\sqrt{13}}{(\sqrt{7}+\sqrt{6})^2-\sqrt{13}^2}$

On further calculation

$= \frac{\sqrt{7}+\sqrt{6}+\sqrt{13}}{(7+6+2\sqrt{42})-13}$

$= \frac{\sqrt{7}+\sqrt{6}+\sqrt{13}}{13+2\sqrt{42}-13}$

So we get

$= \frac{\sqrt{7}+\sqrt{6}+\sqrt{13}}{2\sqrt{42}}$

We can also write it as

$= \frac{\sqrt{7}+\sqrt{6}+\sqrt{13}}{2\sqrt{42}} \times \frac{\sqrt{42}}{\sqrt{42}}$

$= \frac{\sqrt{7\times42}+\sqrt{6\times42}+\sqrt{13\times42}}{2(\sqrt{42})^2}$

On further calculation

$= \frac{\sqrt{7\times7\times6}+\sqrt{6\times6\times7}+\sqrt{546}}{2\times42}$

So we get

$= \frac{7\sqrt{6}+6\sqrt{7}+\sqrt{546}}{84}$

(ii) $\frac{3}{\sqrt{3}+\sqrt{5}-\sqrt{2}}$

It can be written as

$= \frac{3}{(\sqrt{3}+\sqrt{5})-\sqrt{2}} \times \frac{(\sqrt{3}+\sqrt{5})+\sqrt{2}}{(\sqrt{3}+\sqrt{5})+\sqrt{2}}$

According to the formula $a^2 - b^2 = (a+b)(a-b)$

$= \frac{3(\sqrt{3}+\sqrt{5}+\sqrt{2})}{(\sqrt{3}+\sqrt{5})^2-\sqrt{2}^2}$

On further calculation

$= \frac{3\sqrt{3}+3\sqrt{5}+3\sqrt{2}}{(3+5+2\sqrt{15})-2}$

$= \frac{3\sqrt{3}+3\sqrt{5}+3\sqrt{2}}{8+2\sqrt{15}-2}$

So we get

$= \frac{3\sqrt{3}+3\sqrt{5}+3\sqrt{2}}{6+2\sqrt{15}}$

We can also write it as

$= \frac{3\sqrt{3}+3\sqrt{5}+3\sqrt{2}}{6+2\sqrt{15}} \times \frac{6-2\sqrt{15}}{6-2\sqrt{15}}$

According to the formula $a^2 - b^2 = (a+b)(a-b)$

$= \frac{18\sqrt{3}-6\sqrt{45}+18\sqrt{5}-6\sqrt{75}+18\sqrt{2}-6\sqrt{30}}{6^2-(2\sqrt{15})^2}$

On further calculation

$= \frac{18\sqrt{3}-6\sqrt{9\times5}+18\sqrt{5}-6\sqrt{25\times3}+18\sqrt{2}-6\sqrt{30}}{36-60}$

$= \frac{18\sqrt{3}-18\sqrt{5}+18\sqrt{5}-30\sqrt{3}+18\sqrt{2}-6\sqrt{30}}{-24}$

So we get

$= \frac{-12\sqrt{3}+18\sqrt{2}-6\sqrt{30}}{-24}$

Taking 6 as common in the numerator we get

$= \frac{-6(2\sqrt{3}-3\sqrt{2}+\sqrt{30})}{-24}$

$= \frac{2\sqrt{3}-3\sqrt{2}+\sqrt{30}}{4}$

(iii) $\frac{4}{2+\sqrt{3}+\sqrt{7}}$

It can be written as

$= \frac{4}{(2+\sqrt{3})+\sqrt{7}} \times \frac{(2+\sqrt{3})-\sqrt{7}}{(2+\sqrt{3})-\sqrt{7}}$

According to the formula $a^2 - b^2 = (a+b)(a-b)$

$= \frac{4(2+\sqrt{3}-\sqrt{7})}{(2+\sqrt{3})^2-\sqrt{7}^2}$

On further calculation

$= \frac{8+4\sqrt{3}-4\sqrt{7}}{4+4\sqrt{3}+3-7}$

So we get

$= \frac{8+4\sqrt{3}-4\sqrt{7}}{4\sqrt{3}}$

We can write it as

$= \frac{8+4\sqrt{3}-4\sqrt{7}}{4\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

On further calculation

$= \frac{8\sqrt{3}+12-4\sqrt{21}}{12}$

$= \frac{2\sqrt{3}+3-\sqrt{21}}{3}$

23. Given, $\sqrt{2}$ = 1.414 and $\sqrt{6}$ = 2.449, find the value of $\frac{1}{\sqrt{3}-\sqrt{2}-1}$ correct to 3 places of decimal.

Solution:

$\sqrt{2}$ = 1.414 and $\sqrt{6}$ = 2.449

$\frac{1}{\sqrt{3}-\sqrt{2}-1}$

It can be written as

$= \frac{1}{(\sqrt{3}-\sqrt{2})-1} \times \frac{(\sqrt{3}-\sqrt{2})+1}{(\sqrt{3}-\sqrt{2})+1}$

According to the formula $a^2 - b^2 = (a+b)(a-b)$

$= \frac{\sqrt{3}-\sqrt{2}+1}{(\sqrt{3}-\sqrt{2})^2 - 1^2}$

On further calculation

$= \frac{\sqrt{3}-\sqrt{2}+1}{3+2-2\sqrt{6}-1}$

So we get

$= \frac{\sqrt{3}-\sqrt{2}+1}{4-2\sqrt{6}}$

We can also write it as

$= \frac{\sqrt{3}-\sqrt{2}+1}{4-2\sqrt{6}} \times \frac{4+2\sqrt{6}}{4+2\sqrt{6}}$

According to the formula $a^2 - b^2 = (a+b)(a-b)$

$= \frac{4\sqrt{3}+2\sqrt{18} - 4\sqrt{2}-2\sqrt{12}+4+2\sqrt{6}}{4^2 - (2\sqrt{6})^2}$

On further calculation

$= \frac{4\sqrt{3}+6\sqrt{2} - 4\sqrt{2}-4\sqrt{3}+4+2\sqrt{6}}{16-24}$

So we get

$= \frac{2\sqrt{2}+4+2\sqrt{6}}{-8}$

Substituting the values

$= \frac{2.828+4+4.898}{-8}$

$= - \frac{11.726}{8}$

By division

= -1.466

24. If x = $\frac{1}{2-\sqrt{3}}$, find the value of x^3- 2x^2- 7x +5.

Solution:

We know that x can be written as

$= \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$

According to the formula $a^2 - b^2 = (a+b)(a-b)$

$= \frac{2+\sqrt{3}}{2^2 - \sqrt{3}^2}$

On further calculation

$= \frac{2+\sqrt{3}}{4-3}$

$= 2 + \sqrt{3}$

So we get,

$x^3 - 2x^2 - 7x + 5$

$= (2 + \sqrt{3})^3 - 2(2 + \sqrt{3})^2 - 7(2 + \sqrt{3}) + 5$

According to the formula $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$ and $(a + b)^2 = a^2 + b^2 + 2ab$

$= [2^3 + \sqrt{3}^3 + 3 \times 2\sqrt{3}(2 + \sqrt{3})] - 2[2^2 + 2 \times 2\sqrt{3} + \sqrt{3}^2] - 7 \times 2 - 7\sqrt{3} + 5$

On further calculation

$= [8 + 3\sqrt{3} + 6\sqrt{3}(2 + \sqrt{3})] - 2[4 + 4\sqrt{3} + 3] - 14 - 7\sqrt{3} + 5$

$= [8 + 3\sqrt{3} + 12\sqrt{3} + 18] - 2[7 + 4\sqrt{3}] - 9 - 7\sqrt{3}$

We get

$= 26 + 15\sqrt{3} - 14 - 8\sqrt{3} - 9 - 7\sqrt{3}$

$= 3$

25. Evaluate $\frac{15}{\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}}$, it being given that $\sqrt{5} = 2.236$ and $\sqrt{10} = 3.162$.

Solution:

$$\frac{15}{\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}}$$

It can be written as

$$= \frac{15}{\sqrt{10}+\sqrt{4\times5}+\sqrt{4\times10}-\sqrt{5}-\sqrt{16\times5}}$$

On further calculation

$$= \frac{15}{\sqrt{10}+2\sqrt{5}+2\sqrt{10}-\sqrt{5}-4\sqrt{5}}$$

$$= \frac{15}{3\sqrt{10}-3\sqrt{5}}$$

By taking 3 as common in the denominator

$$= \frac{5}{\sqrt{10}-\sqrt{5}}$$

It can be written as

$$= \frac{5}{\sqrt{10}-\sqrt{5}} \times \frac{\sqrt{10}+\sqrt{5}}{\sqrt{10}+\sqrt{5}}$$

According to the formula $a^2 - b^2 = (a+b)(a-b)$

$= \frac{5(\sqrt{10}+\sqrt{5})}{\sqrt{10}^2 - \sqrt{5}^2}$

On further calculation

$= \frac{5(\sqrt{10}+\sqrt{5})}{10-5}$

Taking 5 as common

$= \frac{5(\sqrt{10}+\sqrt{5})}{5}$

So we get

$= \sqrt{10} + \sqrt{5}$

Substituting the values

= 3.162+2.236

= 5.398

EXERCISE 1(G) PAGE: 53

1. Simplify

(i) $2^{\frac{2}{3}} \times 2^{\frac{1}{3}}$

(ii) $2^{\frac{2}{3}} \times 2^{\frac{1}{5}}$

(iii) $7^{\frac{5}{6}} \times 7^{\frac{2}{3}}$

(iv) $1296^{\frac{1}{4}} \times 1296^{\frac{1}{2}}$

Solution:

(i) $2^{\frac{2}{3}} \times 2^{\frac{1}{3}}$

It can be written as

$= 2^{\frac{2}{3}+\frac{1}{3}}$

So we get

$= 2^{\frac{3}{3}}$

$=2^1$

$= 2$

(ii) $2^{\frac{2}{3}} \times 2^{\frac{1}{5}}$

It can be written as

$= 2^{\frac{2}{3}+\frac{1}{5}}$

So we get

$= 2^{\frac{10+3}{15}}$

$= 2^{\frac{13}{15}}$

(iii) $7^{\frac{5}{6}} \times 7^{\frac{2}{3}}$

It can be written as

$= 7^{\frac{5}{6}+\frac{2}{3}}$

$= 7^{\frac{5+4}{6}}$

So we get

$= 7^{\frac{9}{6}}$

$= 7^{\frac{3}{2}}$

(iv) $1296^{\frac{1}{4}} \times 1296^{\frac{1}{2}}$

It can be written as

$= (6^4)^{\frac{1}{4}} \times (6^4)^{\frac{1}{2}}$

$= 6^{4 \times \frac{1}{4}} \times 6^{4 \times \frac{1}{2}}$

So we get

$= 6 \times 6^2$

$= 6 \times 36$

$= 216$

2. Simplify

(i) $\frac{6^{\frac{1}{4}}}{6^{\frac{1}{5}}}$

(ii) $\frac{8^{\frac{1}{2}}}{8^{\frac{2}{3}}}$

(iii) $\frac{5^{\frac{6}{7}}}{5^{\frac{2}{3}}}$

Solution:

(i) $\frac{6^{\frac{1}{4}}}{6^{\frac{1}{5}}}$

It can be written as

$= 6^{\left(\frac{1}{4}-\frac{1}{5}\right)}$

So we get

$= 6^{\frac{5-4}{20}}$

$= 6^{\frac{1}{20}}$

(ii) $\frac{8^{\frac{1}{2}}}{8^{\frac{2}{3}}}$

It can be written as

$= 8^{\left(\frac{1}{2}-\frac{2}{3}\right)}$

So we get

$= 8^{\frac{3-4}{6}}$

$= 8^{-\frac{1}{6}}$

(iii) $\frac{5^{\frac{6}{7}}}{5^{\frac{2}{3}}}$

It can be written as

$= 5^{\left(\frac{6}{7}-\frac{2}{3}\right)}$

So we get

$= 5^{\frac{18-14}{21}}$

$= 5^{\frac{4}{21}}$

3. Simplify

(i) $3^{\frac{1}{4}} \times 5^{\frac{1}{4}}$

(ii) $2^{\frac{5}{8}} \times 3^{\frac{5}{8}}$

(iii) $6^{\frac{1}{2}} \times 7^{\frac{1}{2}}$

Solution:

(i) $3^{\frac{1}{4}} \times 5^{\frac{1}{4}}$

It can be written as

$= (3 \times 5)^{\frac{1}{4}}$

So we get

$= (15)^{\frac{1}{4}}$

(ii) $2^{\frac{5}{8}} \times 3^{\frac{5}{8}}$

It can be written as

$= (2 \times 3)^{\frac{5}{8}}$

So we get

$= (6)^{\frac{5}{8}}$

(iii) $6^{\frac{1}{2}} \times 7^{\frac{1}{2}}$

It can be written as

$= (6 \times 7)^{\frac{1}{2}}$

So we get

$= (42)^{\frac{1}{2}}$

4. Simplify

(i) $(3^4)^{\frac{1}{4}}$

(ii) $(3^{\frac{1}{3}})^4$

(iii) $(\frac{1}{3^4})^{\frac{1}{2}}$

Solution:

(i) $(3^4)^{\frac{1}{4}}$

It can be written as

$= 3^{4\times\frac{1}{4}}$

So we get

$= 3^1$

=3

(ii) $(3^{\frac{1}{3}})^4$

It can be written as

$= 3^{\frac{1}{3}\times 4}$

So we get

$= 3^{\frac{4}{3}}$

(iii) $(\frac{1}{3^4})^{\frac{1}{2}}$

It can be written as

$= (3^{-4})^{\frac{1}{2}}$

So we get

$= 3^{-4\times\frac{1}{2}}$

= 3 -

5. Evaluate

(i) $125^{\frac{1}{3}}$

(ii) $64^{\frac{1}{6}}$

(iii) $25^{\frac{3}{2}}$

(iv) $81^{\frac{3}{4}}$

(v) $64^{-\frac{1}{2}}$

(vi) $8^{-\frac{1}{3}}$

Solution:

(i) $125^{\frac{1}{3}}$

It can be written as

$= (5^3)^{\frac{1}{3}}$

$= 5^{3\times\frac{1}{3}}$

So we get

$= 5^1$

= 5

(ii) $64^{\frac{1}{6}}$

It can be written as

$= (2^6)^{\frac{1}{6}}$

$= 2^{6\times\frac{1}{6}}$

So we get

$= 2^1$

= 2

(iii) $25^{\frac{3}{2}}$

It can be written as

$= (5^2)^{\frac{3}{2}}$

So we get

$= 5^{2\times\frac{3}{2}}$

On further calculation

$= 5^3$

= 125

(iv) $81^{\frac{3}{4}}$

It can be written as

$= (3^4)^{\frac{3}{4}}$

So we get

$= 3^{4\times\frac{3}{4}}$

On further calculation

$= 3^3$

= 27

(v) $64^{-\frac{1}{2}}$

It can be written as

$= \frac{1}{64^{\frac{1}{2}}}$

So we get

$$= \frac{1}{(8^2)^{\frac{1}{2}}}$$

$$= \frac{1}{8^{2\times\frac{1}{2}}}$$

On further calculation

$$= \frac{1}{8^1}$$

$$= \frac{1}{8}$$

(vi) $8^{-\frac{1}{3}}$

It can be written as

$$= \frac{1}{8^{\frac{1}{3}}}$$

$$= \frac{1}{(2^3)^{\frac{1}{3}}}$$

So we get

$$= \frac{1}{2^{3\times\frac{1}{3}}}$$

On further calculation

$$= \frac{1}{2^1}$$

$$= \frac{1}{2}$$

6. If a = 2, b = 3 find the values of

(i) $(a^b + b^a)^{-1}$

(ii) $(a^a + b^b)^{-1}$

Solution:

(i) Given,

a = 2, b = 3

$(a^b + b^a)^{-1}$

$= \frac{1}{a^b+b^a}$

Substituting the values of a and b

$= \frac{1}{2^3+3^2}$

So we get

$= \frac{1}{8+9}$

$= \frac{1}{17}$

(ii) Given,

a = 2, b = 3

$(a^a + b^b)^{-1}$

$= \frac{1}{a^a+b^b}$

Substituting the values of a and b

$= \frac{1}{2^2+3^3}$

So we get

$= \frac{1}{4+27}$

$= \frac{1}{31}$

7. Simplify

(i) $\left(\frac{81}{49}\right)^{-\frac{3}{2}}$

(ii) $\mathbf{14641^{0.25}}$

(iii) $\left(\frac{32}{243}\right)^{-\frac{4}{5}}$

(iv) $\left(\frac{7776}{243}\right)^{-\frac{3}{5}}$

Solution:

(i) $\left(\frac{81}{49}\right)^{-\frac{3}{2}}$

It can be written as

$= \left(\frac{49}{81}\right)^{\frac{3}{2}}$

$= \left(\frac{7^2}{9^2}\right)^{\frac{3}{2}}$

On further calculation

$= \frac{7^{2\times\frac{3}{2}}}{9^{2\times\frac{3}{2}}}$

$= \frac{7^3}{9^3}$

So we get

$= \frac{343}{729}$

(ii) $14641^{0.25}$

It can be written as

$= (14641)^{\frac{1}{4}}$

$= (11^4)^{\frac{1}{4}}$

So we get

$= 11^{4\times\frac{1}{4}}$

$= 11$

(iii) $(\frac{32}{243})^{-\frac{4}{5}}$

It can be written as

$= (\frac{243}{32})^{\frac{4}{5}}$

Multiples of 3 and 2

$= (\frac{3^5}{2^5})^{\frac{4}{5}}$

$= \frac{3^{5\times\frac{4}{5}}}{2^{5\times\frac{4}{5}}}$

So we get

$= \frac{3^4}{2^4}$

$= \frac{81}{16}$

(iv) $(\frac{7776}{243})^{-\frac{3}{5}}$

It can be written as

$= (\frac{243}{7776})^{\frac{3}{5}}$

$= (\frac{3^5}{6^5})^{\frac{3}{5}}$

So we get

$= \frac{3^{5\times\frac{3}{5}}}{6^{5\times\frac{3}{5}}}$

On further calculation

$= \frac{3^3}{6^3}$

$= \frac{1}{8}$

8. Evaluate

(i) $\frac{4}{216^{-\frac{2}{3}}}+\frac{1}{256^{-\frac{3}{4}}}+\frac{2}{243^{-\frac{1}{5}}}$

(ii) $(\frac{64}{125})^{-\frac{2}{3}}+(\frac{256}{625})^{-\frac{1}{4}}+(\frac{3}{7})^{0}$

(iii) $(\frac{81}{16})^{-\frac{3}{4}}[(\frac{25}{9})^{-\frac{3}{2}}\div(\frac{5}{2})^{-3}]$

(iv) $\frac{25^{\frac{5}{2}}\times729^{\frac{1}{3}}}{125^{\frac{2}{3}}\times27^{\frac{2}{3}}\times8^{\frac{4}{3}}}$

Solution:

(i) $\frac{4}{216^{-\frac{2}{3}}}+\frac{1}{256^{-\frac{3}{4}}}+\frac{2}{243^{-\frac{1}{5}}}$

It can be written as

$=\frac{4}{(6^3)^{-\frac{2}{3}}}+\frac{1}{(4^4)^{-\frac{3}{4}}}+\frac{2}{(3^5)^{-\frac{1}{5}}}$

$=\frac{4}{6^{3\times(-\frac{2}{3})}}+\frac{1}{4^{4\times(-\frac{3}{4})}}+\frac{2}{3^{5\times(-\frac{1}{5})}}$

On further calculation

$=\frac{4}{6^{-2}}+\frac{1}{4^{-3}}+\frac{2}{3^{-1}}$

So we get

$= 4\times6^2+1\times\ 4^3+2\ \times3$

By addition

$= 4\ \times36+64+6$

= 144 + 70

= 214

(ii) $(\frac{64}{125})^{-\frac{2}{3}}+(\frac{256}{625})^{-\frac{1}{4}}+(\frac{3}{7})^{0}$

It can be written as

$= (\frac{125}{64})^{\frac{2}{3}} + (\frac{625}{256})^{\frac{1}{4}} + 1$

$= (\frac{5^3}{4^3})^{\frac{2}{3}} + (\frac{5^4}{4^4})^{\frac{1}{4}} + 1$

On further calculation

$= \frac{5^{3\times\frac{2}{3}}}{4^{3\times\frac{2}{3}}} + \frac{5^{4\times\frac{1}{4}}}{4^{4\times\frac{1}{4}}} + 1$

So we get

$= \frac{5^2}{4^2} + \frac{5}{4} + 1$

$= \frac{25}{16} + \frac{5}{4} + 1$

By addition

$= \frac{25+20+16}{16}$

$= \frac{61}{16}$

(iii) $(\frac{81}{16})^{-\frac{3}{4}}[(\frac{25}{9})^{-\frac{3}{2}} \div (\frac{5}{2})^{-3}]$

It can be written as

$= \frac{16}{81}^{\frac{3}{4}} [(\frac{9}{25})^{\frac{3}{2}} \div (\frac{2}{5})^3]$

$= (\frac{2^4}{3^4})^{\frac{3}{4}} [(\frac{3^2}{5^2})^{\frac{3}{2}} \div \frac{2^3}{5^3}]$

On further calculation

$= \frac{2^{4\times\frac{3}{4}}}{3^{4\times\frac{3}{4}}} [\frac{3^{2\times\frac{3}{2}}}{5^{2\times\frac{3}{2}}} \div \frac{8}{125}]$

$= \frac{2^3}{3^3} [\frac{3^3}{5^3} \times \frac{125}{8}]$

So we get

$= \frac{8}{27}[\frac{27}{125} \times \frac{125}{8}]$

By multiplication

$= \frac{8}{27} \times \frac{27}{8}$

$= 1$

(iv) $\frac{25^{\frac{5}{2}} \times 729^{\frac{1}{3}}}{125^{\frac{2}{3}} \times 27^{\frac{2}{3}} \times 8^{\frac{4}{3}}}$

It can be written as

$= \frac{(5^2)^{\frac{5}{2}} \times (9^3)^{\frac{1}{3}}}{(5^3)^{\frac{2}{3}} \times (3^3)^{\frac{2}{3}} \times (2^3)^{\frac{4}{3}}}$

$= \frac{5^{2\times\frac{5}{2}} \times 9^{3\times\frac{1}{3}}}{5^{3\times\frac{2}{3}} \times 3^{3\times\frac{2}{3}} \times 2^{3\times\frac{4}{3}}}$

On further calculation

$= \frac{5^5 \times 9^1}{5^2 \times 3^2 \times 2^4}$

So we get

$= \frac{5^3}{2^4}$

$= \frac{125}{16}$

9. Evaluate

(i) $(1^3 + 2^3 + 3^3)^{\frac{1}{2}}$

(ii) $[5(8^{\frac{1}{3}} + 27^{\frac{1}{3}})^3]^{\frac{1}{4}}$

(iii) $\frac{2^0 + 7^0}{5^0}$

(iv) $[(16)^{\frac{1}{2}}]^{\frac{1}{2}}$

Solution:

(i) $(1^3 + 2^3 + 3^3)^{\frac{1}{2}}$

It can be written as

$= (1 + 8 + 27)^{\frac{1}{2}}$

On further calculation

$= 36^{\frac{1}{2}}$

So we get

$= (6^2)^{\frac{1}{2}}$

$= 6$

(ii) $[5(8^{\frac{1}{3}} + 27^{\frac{1}{3}})^3]^{\frac{1}{4}}$

It can be written as

$=[5\left(2^{3\times\frac{1}{3}} + 3^{3\times\frac{1}{3}}\right)^3]^{\frac{1}{4}}$

On further calculation

$= [5\ (2 + 3)^3]^{\frac{1}{4}}$

$= [5\ (5)^3]^{\frac{1}{4}}$

So we get

$= [5^4]^{\frac{1}{4}}$

$= 5$

(iii) $\frac{2^0 + 7^0}{5^0}$

It can be written as

$=\frac{1+1}{1}$

On further calculation

$=\frac{2}{1}$

So we get

=2

(iv) $[(16)^{\frac{1}{2}}]^{\frac{1}{2}}$

It can be written as

$=[(4^2)^{\frac{1}{2}}]^{\frac{1}{2}}$

On further calculation

$=4^{\frac{1}{2}}$

So we get

$=2^{2\times\frac{1}{2}}$

$=2$

10. Prove that

(i) $[8^{-\frac{2}{3}}\times 2^{\frac{1}{2}}\times 25^{-\frac{5}{4}}]\div[32^{-\frac{2}{5}}\times 125^{-\frac{5}{6}}]=\sqrt{2}$

(ii) $(\frac{64}{125})^{-\frac{2}{3}}+\frac{1}{(\frac{256}{625})^{\frac{1}{4}}}+\frac{\sqrt{25}}{\sqrt[3]{64}}=\frac{65}{16}$

(iii) $[7\{(81)^{\frac{1}{4}}+(256)^{\frac{1}{4}}\}^{\frac{1}{4}}]^4=16807$

Solution:

(i) We know that the LHS

$=[8^{-\frac{2}{3}}\times 2^{\frac{1}{2}}\times 25^{-\frac{5}{4}}]\div[32^{-\frac{2}{5}}\times 125^{-\frac{5}{6}}]$

It can be written as

$= [2^{3\times(-\frac{2}{3})} \times \sqrt{2} \times 5^{2\times(-\frac{5}{4})}] \div [2^{5\times(-\frac{2}{5})} \times 5^{3\times(-\frac{5}{6})}]$

On further calculation

$= [2^{-2} \times \sqrt{2} \times 5^{(-\frac{5}{2})}] \div [2^{-2} \times 5^{(-\frac{5}{2})}]$

So we get

$= \frac{2^{-2} \times \sqrt{2} \times 5^{(-\frac{5}{2})}}{2^{-2} \times 5^{(-\frac{5}{2})}}$

$= \sqrt{2}$

= RHS

(ii) We know that the LHS

$= (\frac{64}{125})^{-\frac{2}{3}} + \frac{1}{(\frac{256}{625})^{\frac{1}{4}}} + \frac{\sqrt{25}}{\sqrt[3]{64}}$

It can be written as

$= (\frac{125}{64})^{\frac{2}{3}} + (\frac{625}{256})^{\frac{1}{4}} + \frac{\sqrt{5^3}}{\sqrt[3]{4^3}}$

On further calculation

$= \frac{5^{3\times\frac{2}{3}}}{4^{3\times\frac{2}{3}}} + \frac{5^{4\times\frac{1}{4}}}{4^{4\times\frac{1}{4}}} + \frac{5}{4}$

So we get

$= \frac{25}{16} + \frac{5}{4} + \frac{5}{4}$

By taking LCM

$= \frac{25+20+20}{16}$

We get

$= \frac{65}{16}$

= RHS

(iii) We know that the LHS

$= [7\{(81)^{\frac{1}{4}} + (256)^{\frac{1}{4}}\}^{\frac{1}{4}}]^4$

It can be written as

$= [7\{3^{4\times\frac{1}{4}} + 4^{4\times\frac{1}{4}}\}^{\frac{1}{4}}]^4$

On further calculation

$= [7\{3 + 4\}^{\frac{1}{4}}]^4$

$= [7\{7\}^{\frac{1}{4}}]^4$

So we get

$= 7^4 \times 7^{\frac{1}{4}\times 4}$

By multiplication

$= 7^4 \times 7$

$= 7^5$

= 16807

=RHS

11. Simplify $\sqrt[4]{\sqrt[3]{x^2}}$ and express the result in the exponential form of x.

Solution:

$\sqrt[4]{\sqrt[3]{x^2}}$

It can be written as

$= (\sqrt[3]{x^2})^{\frac{1}{4}}$

On further calculation

$= (x^2)^{\frac{1}{3} \times \frac{1}{4}}$

So we get

$= x^{2 \times \frac{1}{12}}$

$= x^{\frac{1}{6}}$

12. Simplify the product $\sqrt[3]{2} \times \sqrt[4]{2} \times \sqrt[12]{32}$.

Solution:

$\sqrt[3]{2} \times \sqrt[4]{2} \times \sqrt[12]{32}$

It can be written as

$= 2^{\frac{1}{3}} \times 2^{\frac{1}{4}} \times 32^{\frac{1}{12}}$

$= 2^{\frac{1}{3}} \times 2^{\frac{1}{4}} \times 2^{5 \times \frac{1}{12}}$

On further calculation

$= 2^{\frac{1}{3}} \times 2^{\frac{1}{4}} \times 2^{\frac{5}{12}}$

$= 2^{\frac{1}{3} + \frac{1}{4} + \frac{5}{12}}$

By taking LCM as 12

$= 2^{\frac{4+3+5}{12}}$

So we get

$= 2^{\frac{12}{12}}$

$= 2$

13. Simplify

(i) $\left(\frac{15^{\frac{1}{3}}}{9^{\frac{1}{4}}}\right)^{-6}$

(ii) $\left(\frac{12^{\frac{1}{5}}}{27^{\frac{1}{5}}}\right)^{\frac{5}{2}}$

(iii) $\left(\frac{15^{\frac{1}{4}}}{3^{\frac{1}{2}}}\right)^{-2}$

Solution:

(i) $\left(\frac{15^{\frac{1}{3}}}{9^{\frac{1}{4}}}\right)^{-6}$

It can be written as

$= \left(\frac{9^{\frac{1}{4}}}{15^{\frac{1}{3}}}\right)^{6}$

On further calculation

$= \left(\frac{3^{2\times\frac{1}{4}}}{15^{\frac{1}{3}}}\right)^{6}$

$= \left(\frac{3^{\frac{1}{2}}}{15^{\frac{1}{3}}}\right)^{6}$

So we get

$= \frac{3^{\frac{1}{2}\times 6}}{15^{\frac{1}{3}\times 6}}$

By division

$= \frac{3^3}{15^2}$

We get

$= \frac{27}{225}$

(ii) $\left(\frac{12^{\frac{1}{5}}}{27^{\frac{1}{5}}}\right)^{\frac{5}{2}}$

It can be written as

$= \frac{12^{\frac{1}{5}\times\frac{5}{2}}}{27^{\frac{1}{5}\times\frac{5}{2}}}$

$= \frac{12^{\frac{1}{2}}}{27^{\frac{1}{2}}}$

On further calculation

$= \frac{\sqrt{12}}{\sqrt{27}}$

We can write it as

$= \frac{\sqrt{4\times3}}{\sqrt{9\times3}}$

So we get

$= \frac{2\sqrt{3}}{3\sqrt{3}}$

By cancelling the similar terms

$= \frac{2}{3}$

(iii) $\left(\frac{15^{\frac{1}{4}}}{3^{\frac{1}{2}}}\right)^{-2}$

It can be written as

$= \left(\frac{3^{\frac{1}{2}}}{15^{\frac{1}{4}}}\right)^{2}$

On further calculation

$= \frac{3^{\frac{1}{2}\times2}}{15^{\frac{1}{4}\times2}}$

So we get

$= \frac{3}{15^{\frac{1}{2}}}$

14. Find the value of x in each of the following.

(i) $\sqrt[3]{5x+2} = 2$

(ii) $\sqrt[3]{3x-2} = 4$

(iii) $(\frac{3}{4})^3 \left(\frac{4}{3}\right)^{-7} = (\frac{3}{4})^{2x}$

(iv) $5^{x-3} \times 3^{2x-8} = 225$

(v) $\frac{3^{3x} . 3^{2x}}{3^x} = \sqrt[4]{3^{20}}$

Solution:

(i) To find x,

$\sqrt[3]{5x+2} = 2$

It can be written as

$(5x+2)^{\frac{1}{5}} = 2$

On further calculation

$[(5x+2)^{\frac{1}{5}}]^5 = 2^5$

$5x + 2 = 32$

So we get

$5x = 30$

By division

$x = 6$

(ii) To find x,

$\sqrt[3]{3x-2} = 4$

It can be written as

$(3x-2)^{\frac{1}{3}} = 4$

On further calculation

$[(3x-2)^{\frac{1}{3}}]^{3} = 4^{3}$

So we get

$3x - 2 = 64$

Adding 64+2 we get

$3x = 66$

By division

$x = 22$

(iii) In order to find x,

$(\frac{3}{4})^{3}\left(\frac{4}{3}\right)^{-7} = (\frac{3}{4})^{2x}$

It can be written as

$(\frac{3}{4})^{3}\left(\frac{3}{4}\right)^{7} = (\frac{3}{4})^{2x}$

On further calculation

$(\frac{3}{4})^{3+7} = (\frac{3}{4})^{2x}$

So we get

$(\frac{3}{4})^{10} = (\frac{3}{4})^{2x}$

Consider the degrees to find x

$2x = 10$

Dividing 10 by 2 we get

$x = 5$

(iv) It is given that,

$5^{x-3} \times 3^{2x-8} = 225$

$5^{x-3} \times 3^{2x-8} = 5^2 \times 3^2$

So we get,

x – 3 = 2 and 2x-8 = 2

x = 2+3 and 2x = 2+8

Where,

x = 5

(v) It is given that,

$\frac{3^{3x} . 3^{2x}}{3^x} = \sqrt[4]{3^{20}}$

It can be written as

$\frac{3^{3x+2x}}{3^x} = 3^{20\times\frac{1}{4}}$

On further calculation

$\frac{3^{5x}}{3^x} = 3^5$

So we get

$3^{4x} = 3^5$

Consider the degrees to find x

4x = 5

By division

x = $\frac{5}{4}$

15. Prove that

(i) $\sqrt{x^{-1}y}\,.\sqrt{y^{-1}x}\,.\sqrt{z^{-1}x} = 1$

(ii) $(\frac{1}{x^{a-b}})^{\frac{1}{a-c}} \cdot (\frac{1}{x^{b-c}})^{\frac{1}{b-a}} \cdot (\frac{1}{x^{c-a}})^{\frac{1}{c-b}} = 1$

(iii) $\frac{x^{a(b-c)}}{x^{b(a-c)}} \div (\frac{x^b}{x^a})^c = 1$

(iv) $\frac{(x^{a+b})^2\,(x^{b+c})^2\,(x^{c+a})^2}{(x^a\,x^b\,x^c)^4} = 1$

Solution:

(i) We know that, LHS

$= \sqrt{x^{-1}y}\,.\sqrt{y^{-1}x}\,.\sqrt{z^{-1}x}$

It can be written as

$= \sqrt{\frac{y}{x}} \times \sqrt{\frac{z}{y}} \times \sqrt{\frac{x}{z}}$

On further calculation

$= \sqrt{\frac{y \times z \times x}{x \times y \times z}}$

So we get

$= \sqrt{1}$

$= 1$

= RHS

(ii) Given LHS

$= (\frac{1}{x^{a-b}})^{\frac{1}{a-c}} \cdot (\frac{1}{x^{b-c}})^{\frac{1}{b-a}} \cdot (\frac{1}{x^{c-a}})^{\frac{1}{c-b}}$

It can be written as

$= (x)^{\frac{1}{(a-b)} \times \frac{1}{(a-c)}} \times (x)^{\frac{1}{(b-c)} \times \frac{1}{(b-a)}} \times (x)^{\frac{1}{(c-a)} \times \frac{1}{(c-b)}}$

Addition of degrees

$= (x)^{\frac{1}{(a-b)(a-c)}+\frac{1}{(b-c)(b-a)}+\frac{1}{(c-a)(c-b)}}$

Taking the negative sign out we get

$= (x)^{-\frac{1}{(a-b)(c-a)}-\frac{1}{(b-c)(a-b)}-\frac{1}{(c-a)(b-c)}}$

Taking the LCM

$= (x)^{\frac{-(b-c)-(c-a)-(a-b)}{(a-b)(b-c)(c-a)}}$

So we get

$= (x)^{\frac{-b+c-c+a-a+b}{(a-b)(b-c)(c-a)}}$

$= x^0$

$= 1$

= RHS

(iii) To prove that LHS is equal to RHS

$= \frac{x^{a(b-c)}}{x^{b(a-c)}} \div (\frac{x^b}{x^a})^c$

It can be written as

$= \frac{x^{ab-ac}}{x^{ab-bc}} \div \frac{x^{bc}}{x^{ac}}$

On further calculation

$= x^{ab-ac-ab+bc} \div x^{bc-ac}$

So we get

$= x^{bc-ac} \div x^{bc-ac}$

$= 1$

=RHS

(iv) We know that LHS

$$= \frac{(x^{a+b})^2\,(x^{b+c})^2\,(x^{c+a})^2}{(x^a\,x^b\,x^c)^4}$$

It can be written as

$$= \frac{(x^{2a+2b})(x^{2b+2c})(x^{2c+2a})}{x^{4a}\,x^{4b}\,x^{4c}}$$

On further calculation

$$= \frac{x^{2a+2b+2b+2c+2c+2a}}{x^{4a+4b+4c}}$$

So we get

$$= \frac{x^{4a+4b+4c}}{x^{4a+4b+4c}}$$

We get

= 1

= RHS

16. If x is a positive real number and exponents are rational numbers, simplify

$$\left(\frac{x^b}{x^c}\right)^{b+c-a} \cdot \left(\frac{x^c}{x^a}\right)^{c+a-b} \cdot \left(\frac{x^a}{x^b}\right)^{a+b-c}$$

Solution:

Given,

$$\left(\frac{x^b}{x^c}\right)^{b+c-a} \cdot \left(\frac{x^c}{x^a}\right)^{c+a-b} \cdot \left(\frac{x^a}{x^b}\right)^{a+b-c}$$

It can be written as

$$= \left(\frac{x^{b^2+bc-ab}}{x^{bc+c^2-ac}}\right) \cdot \left(\frac{x^{c^2+ac-bc}}{x^{ac+a^2-ab}}\right) \cdot \left(\frac{x^{a^2+ab-ac}}{x^{ab+b^2-bc}}\right)$$

On further calculation

$$= (x^{b^2+bc-ab-bc-c^2+ac})\,(x^{c^2+ac-bc-ac-a^2+ab})(x^{a^2+ab-ac-ab-b^2+bc})$$

$= (x^{b^2-ab-c^2+ac})\ (x^{c^2-bc-a^2+ab})(x^{a^2-ac-b^2+bc})$

By grouping the terms

$= x^{b^2-ab-c^2+ac+c^2-bc-a^2+ab+a^2-ac-b^2+bc}$

We get

$= x^0$

$= 1$

17. If $\frac{9^n \times 3^2 \times (3^{-\frac{n}{2}})^{-2} - (27)^n}{3^{3m} \times 2^3} = \frac{1}{27}$, prove that m-n = 1.

Solution:

We know that

$$\frac{9^n \times 3^2 \times (3^{-\frac{n}{2}})^{-2} - (27)^n}{3^{3m} \times 2^3} = \frac{1}{27}$$

It can be written as

$$\frac{3^{2n} \times 3^2 \times (\frac{1}{3^{\frac{n}{2}}})^{-2} - (3^3)^n}{3^{3m} \times 8} = \frac{1}{3^3}$$

$$\frac{3^{2n+2}(3^{\frac{n}{2}})^2 - 3^{3n}}{3^{3m} \times 8} = \frac{1}{3^3}$$

On further calculation

$$\frac{3^{2n+2} \times 3^n - 3^{3n}}{3^{3m} \times 8} = \frac{1}{3^3}$$

Considering the degrees in the numerator we get

$$\frac{3^{2n+2+n} - 3^{3n}}{3^{3m} \times 8} = \frac{1}{3^3}$$

$$\frac{3^{3n+2} - 3^{3n}}{3^{3m} \times 8} = \frac{1}{3^3}$$

So we get

$$\frac{3^{3n}(3^2-1)}{3^{3m}\times 8} = \frac{1}{3^3}$$

$$\frac{3^{3n}\times 8}{3^{3m}\times 8} = \frac{1}{3^3}$$

It can be written as

$$\frac{1}{3^{3m+3n}} = \frac{1}{3^3}$$

So we get

3m – 3n = 3

Taking 3 as common

m – n = 1

18. Write the following in ascending order of magnitude.

$\sqrt[6]{6}, \sqrt[3]{7}, \sqrt[4]{8}$.

Solution:

We can write

$$\sqrt[6]{6} = 6^{\frac{1}{6}}$$

$$\sqrt[3]{7} = 7^{\frac{1}{3}}$$

$$\sqrt[4]{8} = 8^{\frac{1}{4}}$$

We know that the LCM of

6, 3 and 4 is 12

So we get,

$$\sqrt[6]{6} = 6^{\frac{1}{6}} = 6^{\frac{1}{6}\times\frac{2}{2}} = 6^{\frac{2}{12}} = (6^2)^{\frac{1}{12}} = (36)^{\frac{1}{12}}$$

$$\sqrt[3]{7} = 7^{\frac{1}{3}} = 7^{\frac{1}{3}\times\frac{4}{4}} = 7^{\frac{4}{12}} = (7^4)^{\frac{1}{12}} = (2401)^{\frac{1}{12}}$$

$$\sqrt[4]{8} = 8^{\frac{1}{4}} = 8^{\frac{1}{4}\times\frac{3}{3}} = 8^{\frac{3}{12}} = (8^3)^{\frac{1}{12}} = (512)^{\frac{1}{12}}$$

We know that $36 < 512 < 2401$

$(36)^{\frac{1}{12}} < (512)^{\frac{1}{12}} < (2401)^{\frac{1}{12}}$

Therefore, $\sqrt[6]{6} < \sqrt[4]{8} < \sqrt[3]{7}$