

EXERCISE 5.1

PAGE NO: 5.6

1. If a matrix has 8 elements, what are the possible orders it can have? What if it has 5 elements?

Solution:

If a matrix is of order $m \times n$ elements, it has $m \times n$ elements. So, if the matrix has 8 elements, we will find the ordered pairs m and n .

$$m \times n = 8$$

Then, ordered pairs m and n will be

$$m \times n \text{ be } (8 \times 1), (1 \times 8), (4 \times 2), (2 \times 4)$$

Now, if it has 5 elements

Possible orders are $(5 \times 1), (1 \times 5)$.

$$2. \text{ If } A = [a_{ij}] = \begin{bmatrix} 2 & 3 & -5 \\ 1 & 4 & 9 \\ 0 & 7 & -2 \end{bmatrix} \text{ and } B = [b_{ij}] = \begin{bmatrix} 2 & -1 \\ -3 & 4 \\ 1 & 2 \end{bmatrix} \text{ then find}$$

$$(i) a_{22} + b_{21}$$

$$(ii) a_{11}b_{11} + a_{22}b_{22}$$

Solution:

(i)

We know that

$$A = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \dots (i)$$

$$\text{And } B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix} \dots (ii)$$

Also given that

$$A = [a_{ij}] = \begin{bmatrix} 2 & 3 & -5 \\ 1 & 4 & 9 \\ 0 & 7 & -2 \end{bmatrix} \text{ and } B = [b_{ij}] = \begin{bmatrix} 2 & -1 \\ -3 & 4 \\ 1 & 2 \end{bmatrix}$$

Now, Comparing with equation (1) and (2)

$$a_{22} = 4 \text{ and } b_{21} = -3$$

$$a_{22} + b_{21} = 4 + (-3) = 1$$

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Algebra of Matrices

(ii)

We know that

$$A = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \dots (i)$$

$$\text{And } B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix} \dots (ii)$$

Also given that

$$A = [a_{ij}] = \begin{bmatrix} 2 & 3 & -5 \\ 1 & 4 & 9 \\ 0 & 7 & -2 \end{bmatrix} \text{ and } B = [b_{ij}] = \begin{bmatrix} 2 & -1 \\ -3 & 4 \\ 1 & 2 \end{bmatrix}$$

Now, Comparing with equation (1) and (2)

$$a_{11} = 2, a_{22} = 4, b_{11} = 2, b_{22} = 4$$

$$a_{11} b_{11} + a_{22} b_{22} = 2 \times 2 + 4 \times 4 = 4 + 16 = 20$$

3. Let A be a matrix of order 3×4 . If R_1 denotes the first row of A and C_2 denotes its second column, then determine the orders of matrices R_1 and C_2 .

Solution:Given A be a matrix of order 3×4 .

$$\text{So, } A = [a_{ij}]_{3 \times 4}$$

$$R_1 = \text{first row of } A = [a_{11}, a_{12}, a_{13}, a_{14}]$$

$$\text{So, order of matrix } R_1 = 1 \times 4$$

 C_2 = second column of

$$A = \begin{bmatrix} a_{12} \\ a_{22} \\ a_{32} \end{bmatrix}$$

Therefore order of $C_2 = 3 \times 1$

4. Construct a 2×3 matrix $A = [a_{ij}]$ whose elements a_{ij} are given by:

$$(i) a_{ij} = i \times j$$

$$(ii) a_{ij} = 2i - j$$

$$(iii) a_{ij} = i + j$$

$$(iv) a_{ij} = (i + j)^2 / 2$$

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices**Solution:**(i) Given $a_{ij} = i \times j$ Let $A = [a_{ij}]_{2 \times 3}$ So, the elements in a 2×3 matrix are $[a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}]$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$$a_{11} = 1 \times 1 = 1$$

$$a_{12} = 1 \times 2 = 2$$

$$a_{13} = 1 \times 3 = 3$$

$$a_{21} = 2 \times 1 = 2$$

$$a_{22} = 2 \times 2 = 4$$

$$a_{23} = 2 \times 3 = 6$$

Substituting these values in matrix A we get,

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix}$$

(ii) Given $a_{ij} = 2i - j$ Let $A = [a_{ij}]_{2 \times 3}$ So, the elements in a 2×3 matrix are $a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$$a_{11} = 2 \times 1 - 1 = 2 - 1 = 1$$

$$a_{12} = 2 \times 1 - 2 = 2 - 2 = 0$$

$$a_{13} = 2 \times 1 - 3 = 2 - 3 = -1$$

$$a_{21} = 2 \times 2 - 1 = 4 - 1 = 3$$

$$a_{22} = 2 \times 2 - 2 = 4 - 2 = 2$$

$$a_{23} = 2 \times 2 - 3 = 4 - 3 = 1$$

Substituting these values in matrix A we get,

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 2 & 1 \end{bmatrix}$$

(iii) Given $a_{ij} = i + j$ Let $A = [a_{ij}]_{2 \times 3}$ So, the elements in a 2×3 matrix are

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Algebra of Matrices $a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$$a_{11} = 1 + 1 = 2$$

$$a_{12} = 1 + 2 = 3$$

$$a_{13} = 1 + 3 = 4$$

$$a_{21} = 2 + 1 = 3$$

$$a_{22} = 2 + 2 = 4$$

$$a_{23} = 2 + 3 = 5$$

Substituting these values in matrix A we get,

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$$

(iv) Given $a_{ij} = (i + j)^2/2$ Let $A = [a_{ij}]_{2 \times 3}$ So, the elements in a 2×3 matrix are $a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

Let $A = [a_{ij}]_{2 \times 3}$ So, the elements in a 2×3 matrix are $a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$$a_{11} = \frac{(1+1)^2}{2} = \frac{2^2}{2} = \frac{4}{2} = 2$$

$$a_{12} = \frac{(1+2)^2}{2} = \frac{3^2}{2} = \frac{9}{2} = 4.5$$

$$a_{13} = \frac{(1+3)^2}{2} = \frac{4^2}{2} = \frac{16}{2} = 8$$

$$a_{21} = \frac{(2+1)^2}{2} = \frac{3^2}{2} = \frac{9}{2} = 4.5$$

$$a_{22} = \frac{(2+2)^2}{2} = \frac{4^2}{2} = \frac{16}{2} = 8$$

$$a_{23} = \frac{(2+3)^2}{2} = \frac{5^2}{2} = \frac{25}{2} = 12.5$$

Substituting these values in matrix A we get,

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$$A = \begin{bmatrix} 2 & 4.5 & 8 \\ 4.5 & 8 & 12.5 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & \frac{9}{2} & 8 \\ \frac{9}{2} & 8 & \frac{25}{2} \end{bmatrix}$$

5. Construct a 2×2 matrix $A = [a_{ij}]$ whose elements a_{ij} are given by:

(i) $(i + j)^2 / 2$

(ii) $a_{ij} = (i - j)^2 / 2$

(iii) $a_{ij} = (i - 2j)^2 / 2$

(iv) $a_{ij} = (2i + j)^2 / 2$

(v) $a_{ij} = |2i - 3j| / 2$

(vi) $a_{ij} = |-3i + j| / 2$

(vii) $a_{ij} = e^{2ix} \sin x j$

Solution:

(i) Given $(i + j)^2 / 2$

Let $A = [a_{ij}]_{2 \times 2}$

So, the elements in a 2×2 matrix are

$a_{11}, a_{12}, a_{21}, a_{22}$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$$a_{11} = \frac{(1+1)^2}{2} = \frac{2^2}{2} = \frac{4}{2} = 2$$

$$a_{12} = \frac{(1+2)^2}{2} = \frac{3^2}{2} = \frac{9}{2} = 4.5$$

$$a_{21} = \frac{(2+1)^2}{2} = \frac{3^2}{2} = \frac{9}{2} = 4.5$$

$$a_{22} = \frac{(2+2)^2}{2} = \frac{4^2}{2} = \frac{16}{2} = 8$$

Substituting these values in matrix A we get,

$$A = \begin{bmatrix} 2 & 4.5 \\ 4.5 & 8 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{bmatrix}$$

(ii) Given $a_{ij} = (i - j)^2 / 2$

Let $A = [a_{ij}]_{2 \times 2}$

So, the elements in a 2×2 matrix are

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Algebra of Matrices $a_{11}, a_{12}, a_{21}, a_{22}$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$$a_{11} = \frac{(1-1)^2}{2} = \frac{0^2}{2} = 0$$

$$a_{12} = \frac{(1-2)^2}{2} = \frac{1^2}{2} = \frac{1}{2} = 0.5$$

$$a_{21} = \frac{(2-1)^2}{2} = \frac{1^2}{2} = \frac{1}{2} = 0.5$$

$$a_{22} = \frac{(2-2)^2}{2} = \frac{0^2}{2} = 0$$

Substituting these values in matrix A we get,

$$A = \begin{bmatrix} 0 & 0.5 \\ 0.5 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{bmatrix}$$

(iii) Given $a_{ij} = (i - 2j)^2 / 2$ Let $A = [a_{ij}]_{2 \times 2}$ So, the elements in a 2×2 matrix are $a_{11}, a_{12}, a_{21}, a_{22}$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$$a_{11} = \frac{(1-2 \times 1)^2}{2} = \frac{1^2}{2} = 0.5$$

$$a_{12} = \frac{(1-2 \times 2)^2}{2} = \frac{3^2}{2} = \frac{9}{2} = 4.5$$

$$a_{21} = \frac{(2-2 \times 1)^2}{2} = \frac{0^2}{2} = 0$$

$$a_{22} = \frac{(2-2 \times 2)^2}{2} = \frac{2^2}{2} = \frac{4}{2} = 2$$

Substituting these values in matrix A we get,

$$A = \begin{bmatrix} 0.5 & 4.5 \\ 0 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} \frac{1}{2} & \frac{9}{2} \\ 0 & 2 \end{bmatrix}$$

(iv) Given $a_{ij} = (2i + j)^2 / 2$

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Algebra of Matrices

Let $A = [a_{ij}]_{2 \times 2}$

So, the elements in a 2×2 matrix are

$a_{11}, a_{12}, a_{21}, a_{22}$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$$a_{11} = \frac{(2 \times 1 + 1)^2}{2} = \frac{3^2}{2} = \frac{9}{2} = 4.5$$

$$a_{12} = \frac{(2 \times 1 + 2)^2}{2} = \frac{4^2}{2} = \frac{16}{2} = 8$$

$$a_{21} = \frac{(2 \times 2 + 1)^2}{2} = \frac{5^2}{2} = \frac{25}{2} = 12.5$$

$$a_{22} = \frac{(2 \times 2 + 2)^2}{2} = \frac{6^2}{2} = \frac{36}{2} = 18$$

Substituting these values in matrix A we get,

$$A = \begin{bmatrix} 4.5 & 8 \\ 12.5 & 18 \end{bmatrix}$$

$$A = \begin{bmatrix} \frac{9}{2} & 8 \\ \frac{25}{2} & 18 \end{bmatrix}$$

(v) Given $a_{ij} = |2i - 3j|/2$

Let $A = [a_{ij}]_{2 \times 2}$

So, the elements in a 2×2 matrix are

$a_{11}, a_{12}, a_{21}, a_{22}$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$$a_{11} = \frac{|2 \times 1 - 3 \times 1|}{2} = \frac{1}{2} = 0.5$$

$$a_{12} = \frac{|2 \times 1 - 3 \times 2|}{2} = \frac{4}{2} = 2$$

$$a_{21} = \frac{|2 \times 2 - 3 \times 1|}{2} = \frac{4 - 3}{2} = \frac{1}{2} = 0.5$$

$$a_{22} = \frac{|2 \times 2 - 3 \times 2|}{2} = \frac{2}{2} = 1$$

Substituting these values in matrix A we get,

$$A = \begin{bmatrix} 0.5 & 2 \\ 0.5 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} \frac{1}{2} & 2 \\ \frac{1}{2} & 1 \end{bmatrix}$$

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Algebra of Matrices(vi) Given $a_{ij} = |-3i + j|/2$ Let $A = [a_{ij}]_{2 \times 2}$ So, the elements in a 2×2 matrix are $a_{11}, a_{12}, a_{21}, a_{22}$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$$a_{11} = \frac{|-3 \times 1 + 1|}{2} = \frac{2}{2} = 1$$

$$a_{12} = \frac{|-3 \times 1 + 2|}{2} = \frac{1}{2} = 0.5$$

$$a_{21} = \frac{|-3 \times 2 + 1|}{2} = \frac{5}{2} = 2.5$$

$$a_{22} = \frac{|-3 \times 2 + 2|}{2} = \frac{4}{2} = 2$$

Substituting these values in matrix A we get,

$$A = \begin{bmatrix} 1 & 0.5 \\ 2.5 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{5}{2} & 2 \end{bmatrix}$$

(vii) Given $a_{ij} = e^{2ix} \sin x \cdot j$ Let $A = [a_{ij}]_{2 \times 2}$ So, the elements in a 2×2 matrix are $a_{11}, a_{12}, a_{21}, a_{22},$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$$a_{11} = e^{2 \times 1x} \sin x \times 1 = e^{2x} \sin x$$

$$a_{12} = e^{2 \times 1x} \sin x \times 2 = e^{2x} \sin 2x$$

$$a_{21} = e^{2 \times 2x} \sin x \times 1 = e^{4x} \sin x$$

$$a_{22} = e^{2 \times 2x} \sin x \times 2 = e^{4x} \sin 2x$$

Substituting these values in matrix A we get,

$$A = \begin{bmatrix} e^{2x} \sin x & e^{2x} \sin 2x \\ e^{4x} \sin x & e^{4x} \sin 2x \end{bmatrix}$$

6. Construct a 3×4 matrix $A = [a_{ij}]$ whose elements a_{ij} are given by:

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Algebra of Matrices

(i) $a_{ij} = i + j$

(ii) $a_{ij} = i - j$

(iii) $a_{ij} = 2i$

(iv) $a_{ij} = j$

(v) $a_{ij} = \frac{1}{2} |-3i + j|$

Solution:

(i) Given $a_{ij} = i + j$

Let $A = [a_{ij}]_{2 \times 3}$

So, the elements in a 3×4 matrix are

$a_{11}, a_{12}, a_{13}, a_{14}, a_{21}, a_{22}, a_{23}, a_{24}, a_{31}, a_{32}, a_{33}, a_{34}$

$$A = \begin{bmatrix} a_{11} & \cdots & a_{14} \\ \vdots & \ddots & \vdots \\ a_{31} & \cdots & a_{34} \end{bmatrix}$$

$a_{11} = 1 + 1 = 2$

$a_{12} = 1 + 2 = 3$

$a_{13} = 1 + 3 = 4$

$a_{14} = 1 + 4 = 5$

$a_{21} = 2 + 1 = 3$

$a_{22} = 2 + 2 = 4$

$a_{23} = 2 + 3 = 5$

$a_{24} = 2 + 4 = 6$

$a_{31} = 3 + 1 = 4$

$a_{32} = 3 + 2 = 5$

$a_{33} = 3 + 3 = 6$

$a_{34} = 3 + 4 = 7$

Substituting these values in matrix A we get,

$$A = \begin{bmatrix} 2 & \cdots & 5 \\ \vdots & \ddots & \vdots \\ 4 & \cdots & 7 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \end{bmatrix}$$

(ii) Given $a_{ij} = i - j$

Let $A = [a_{ij}]_{2 \times 3}$

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Algebra of Matrices

So, the elements in a 3×4 matrix are

$a_{11}, a_{12}, a_{13}, a_{14}, a_{21}, a_{22}, a_{23}, a_{24}, a_{31}, a_{32}, a_{33}, a_{34}$

$$A = \begin{bmatrix} a_{11} & \cdots & a_{14} \\ \vdots & \ddots & \vdots \\ a_{31} & \cdots & a_{34} \end{bmatrix}$$

$$a_{11} = 1 - 1 = 0$$

$$a_{12} = 1 - 2 = -1$$

$$a_{13} = 1 - 3 = -2$$

$$a_{14} = 1 - 4 = -3$$

$$a_{21} = 2 - 1 = 1$$

$$a_{22} = 2 - 2 = 0$$

$$a_{23} = 2 - 3 = -1$$

$$a_{24} = 2 - 4 = -2$$

$$a_{31} = 3 - 1 = 2$$

$$a_{32} = 3 - 2 = 1$$

$$a_{33} = 3 - 3 = 0$$

$$a_{34} = 3 - 4 = -1$$

Substituting these values in matrix A we get,

$$A = \begin{bmatrix} 0 & \cdots & -3 \\ \vdots & \ddots & \vdots \\ 2 & \cdots & -1 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & -1 & -2 & -3 \\ 1 & 0 & -1 & -2 \\ 2 & 1 & 0 & -1 \end{bmatrix}$$

(iii) Given $a_{ij} = 2i$

Let $A = [a_{ij}]_{2 \times 3}$

So, the elements in a 3×4 matrix are

$a_{11}, a_{12}, a_{13}, a_{14}, a_{21}, a_{22}, a_{23}, a_{24}, a_{31}, a_{32}, a_{33}, a_{34}$

$$A = \begin{bmatrix} a_{11} & \cdots & a_{14} \\ \vdots & \ddots & \vdots \\ a_{31} & \cdots & a_{34} \end{bmatrix}$$

$$a_{11} = 2 \times 1 = 2$$

$$a_{12} = 2 \times 1 = 2$$

$$a_{13} = 2 \times 1 = 2$$

$$a_{14} = 2 \times 1 = 2$$

$$a_{21} = 2 \times 2 = 4$$

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Algebra of Matrices

$$a_{22} = 2 \times 2 = 4$$

$$a_{23} = 2 \times 2 = 4$$

$$a_{24} = 2 \times 2 = 4$$

$$a_{31} = 2 \times 3 = 6$$

$$a_{32} = 2 \times 3 = 6$$

$$a_{33} = 2 \times 3 = 6$$

$$a_{34} = 2 \times 3 = 6$$

Substituting these values in matrix A we get,

$$A = \begin{bmatrix} 2 & \cdots & 2 \\ \vdots & \ddots & \vdots \\ 6 & \cdots & 6 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 2 & 2 & 2 \\ 4 & 4 & 4 & 4 \\ 6 & 6 & 6 & 6 \end{bmatrix}$$

(iv) Given $a_{ij} = j$

Let $A = [a_{ij}]_{2 \times 3}$

So, the elements in a 3×4 matrix are

$a_{11}, a_{12}, a_{13}, a_{14}, a_{21}, a_{22}, a_{23}, a_{24}, a_{31}, a_{32}, a_{33}, a_{34}$

$$A = \begin{bmatrix} a_{11} & \cdots & a_{14} \\ \vdots & \ddots & \vdots \\ a_{31} & \cdots & a_{34} \end{bmatrix}$$

$$a_{11} = 1$$

$$a_{12} = 2$$

$$a_{13} = 3$$

$$a_{14} = 4$$

$$a_{21} = 1$$

$$a_{22} = 2$$

$$a_{23} = 3$$

$$a_{24} = 4$$

$$a_{31} = 1$$

$$a_{32} = 2$$

$$a_{33} = 3$$

$$a_{34} = 4$$

Substituting these values in matrix A we get,

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Algebra of Matrices

$$A = \begin{bmatrix} 1 & \dots & 4 \\ \vdots & \ddots & \vdots \\ 1 & \dots & 4 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{bmatrix}$$

(vi) Given $a_{ij} = \frac{1}{2} |-3i + j|$

Let $A = [a_{ij}]_{2 \times 3}$

So, the elements in a 3×4 matrix are

$a_{11}, a_{12}, a_{13}, a_{14}, a_{21}, a_{22}, a_{23}, a_{24}, a_{31}, a_{32}, a_{33}, a_{34}$

$$A = \begin{bmatrix} a_{11} & \dots & a_{14} \\ \vdots & \ddots & \vdots \\ a_{31} & \dots & a_{34} \end{bmatrix}$$

$$a_{11} = \frac{1}{2} (-3 \times 1 + 1) = \frac{1}{2} (-3 + 1) = \frac{1}{2} (-2) = -1$$

$$a_{12} = \frac{1}{2} (-3 \times 1 + 2) = \frac{1}{2} (-3 + 2) = \frac{1}{2} (-1) = -\frac{1}{2}$$

$$a_{13} = \frac{1}{2} (-3 \times 1 + 3) = \frac{1}{2} (-3 + 3) = \frac{1}{2} (0) = 0$$

$$a_{14} = \frac{1}{2} (-3 \times 1 + 4) = \frac{1}{2} (-3 + 4) = \frac{1}{2} (1) = \frac{1}{2}$$

$$a_{21} = \frac{1}{2} (-3 \times 2 + 1) = \frac{1}{2} (-6 + 1) = \frac{1}{2} (-5) = -\frac{5}{2}$$

$$a_{22} = \frac{1}{2} (-3 \times 2 + 2) = \frac{1}{2} (-6 + 2) = \frac{1}{2} (-4) = -2$$

$$a_{23} = \frac{1}{2} (-3 \times 2 + 3) = \frac{1}{2} (-6 + 3) = \frac{1}{2} (-3) = -\frac{3}{2}$$

$$a_{24} = \frac{1}{2} (-3 \times 2 + 4) = \frac{1}{2} (-6 + 4) = \frac{1}{2} (-2) = -1$$

$$a_{31} = \frac{1}{2} (-3 \times 3 + 1) = \frac{1}{2} (-9 + 1) = \frac{1}{2} (-8) = -4$$

$$a_{32} = \frac{1}{2} (-3 \times 3 + 2) = \frac{1}{2} (-9 + 2) = \frac{1}{2} (-7) = -\frac{7}{2}$$

$$a_{33} = \frac{1}{2} (-3 \times 3 + 3) = \frac{1}{2} (-9 + 3) = \frac{1}{2} (-6) = -3$$

$$a_{34} = \frac{1}{2} (-3 \times 3 + 4) = \frac{1}{2} (-9 + 4) = \frac{1}{2} (-5) = -\frac{5}{2}$$

Substituting these values in matrix A we get,

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Algebra of Matrices

$$A = \begin{bmatrix} -1 & \cdots & \frac{1}{2} \\ \vdots & \ddots & \vdots \\ -4 & \cdots & -\frac{5}{2} \end{bmatrix}$$

Multiplying by negative sign we get,

7. Construct a 4×3 matrix $A = [a_{ij}]$ whose elements a_{ij} are given by:

(i) $a_{ij} = 2i + i/j$

(ii) $a_{ij} = (i - j)/(i + j)$

(iii) $a_{ij} = i$

Solution:

(i) Given $a_{ij} = 2i + i/j$

Let $A = [a_{ij}]_{4 \times 3}$

So, the elements in a 4×3 matrix are

$a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}, a_{31}, a_{32}, a_{33}, a_{41}, a_{42}, a_{43}$

$$A = \begin{bmatrix} a_{11} & \cdots & a_{13} \\ \vdots & \ddots & \vdots \\ a_{41} & \cdots & a_{43} \end{bmatrix}$$

$$a_{11} = 2 \times 1 + \frac{1}{1} = 2 + 1 = 3$$

$$a_{12} = 2 \times 1 + \frac{1}{2} = 2 + \frac{1}{2} = \frac{5}{2}$$

$$a_{13} = 2 \times 1 + \frac{1}{3} = 2 + \frac{1}{3} = \frac{7}{3}$$

$$a_{21} = 2 \times 2 + \frac{2}{1} = 4 + 2 = 6$$

$$a_{22} = 2 \times 2 + \frac{2}{2} = 4 + 1 = 5$$

$$a_{23} = 2 \times 2 + \frac{2}{3} = 4 + \frac{2}{3} = \frac{14}{3}$$

$$a_{31} = 2 \times 3 + \frac{3}{1} = 6 + 3 = 9$$

$$a_{32} = 2 \times 3 + \frac{3}{2} = 6 + \frac{3}{2} = \frac{15}{2}$$

$$a_{33} = 2 \times 3 + \frac{3}{3} = 6 + 1 = 7$$

$$a_{41} = 2 \times 4 + \frac{4}{1} = 8 + 4 = 12$$

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

$$a_{42} = 2 \times 4 + \frac{4}{2} = 8 + 2 = 10$$

$$a_{43} = 2 \times 4 + \frac{4}{3} = 8 + \frac{4}{3} = \frac{28}{3}$$

Substituting these values in matrix A we get,

$$A = \begin{bmatrix} 3 & \dots & \frac{7}{3} \\ \vdots & \ddots & \vdots \\ 12 & \dots & \frac{28}{3} \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & \frac{5}{2} & \frac{7}{3} \\ 6 & 5 & \frac{14}{3} \\ 9 & \frac{15}{2} & 7 \\ 12 & 10 & \frac{28}{3} \end{bmatrix}$$

(ii) Given $a_{ij} = (i - j) / (i + j)$

Let $A = [a_{ij}]_{4 \times 3}$

So, the elements in a 4×3 matrix are

$a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}, a_{31}, a_{32}, a_{33}, a_{41}, a_{42}, a_{43}$

$$A = \begin{bmatrix} a_{11} & \dots & a_{13} \\ \vdots & \ddots & \vdots \\ a_{41} & \dots & a_{43} \end{bmatrix}$$

$$a_{11} = \frac{1-1}{1+1} = \frac{0}{2} = 0$$

$$a_{12} = \frac{1-2}{1+2} = \frac{-1}{3}$$

$$a_{13} = \frac{1-3}{1+3} = \frac{-2}{4} = -\frac{1}{2}$$

$$a_{21} = \frac{2-1}{2+1} = \frac{1}{3}$$

$$a_{22} = \frac{2-2}{2+2} = \frac{0}{4} = 0$$

$$a_{23} = \frac{2-3}{2+3} = \frac{-1}{5}$$

$$a_{31} = \frac{3-1}{3+1} = \frac{2}{4} = \frac{1}{2}$$

$$a_{32} = \frac{3-2}{3+2} = \frac{1}{5}$$

$$a_{33} = \frac{3-3}{3+3} = \frac{0}{6} = 0$$

$$a_{41} = \frac{4-1}{4+1} = \frac{3}{5}$$

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

$$a_{42} = \frac{4-2}{4+2} = \frac{2}{6} = \frac{1}{3}$$

$$a_{43} = \frac{4-3}{4+3} = \frac{1}{7}$$

Substituting these values in matrix A we get,

$$A = \begin{bmatrix} 0 & \cdots & -\frac{1}{2} \\ \vdots & \ddots & \vdots \\ \frac{3}{5} & \cdots & \frac{1}{7} \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & -\frac{1}{3} & -\frac{1}{2} \\ \frac{1}{3} & 0 & \frac{2}{5} \\ \frac{1}{2} & \frac{1}{5} & 0 \\ \frac{3}{5} & \frac{1}{3} & \frac{1}{7} \end{bmatrix}$$

(iii) Given $a_{ij} = i$

Let $A = [a_{ij}]_{4 \times 3}$

So, the elements in a 4×3 matrix are

$a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}, a_{31}, a_{32}, a_{33}, a_{41}, a_{42}, a_{43}$

$$A = \begin{bmatrix} a_{11} & \cdots & a_{13} \\ \vdots & \ddots & \vdots \\ a_{41} & \cdots & a_{43} \end{bmatrix}$$

$$a_{11} = 1$$

$$a_{12} = 1$$

$$a_{13} = 1$$

$$a_{21} = 2$$

$$a_{22} = 2$$

$$a_{23} = 2$$

$$a_{31} = 3$$

$$a_{32} = 3$$

$$a_{33} = 3$$

$$a_{41} = 4$$

$$a_{42} = 4$$

$$a_{43} = 4$$

Substituting these values in matrix A we get,

$$A = \begin{bmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 4 & \cdots & 4 \end{bmatrix}$$

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \\ 4 & 4 & 4 \end{bmatrix}$$

8. Find x , y , a and b if

$$\begin{bmatrix} 3x + 4y & 2 & x - 2y \\ a + b & 2a - b & -1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 4 \\ 5 & -5 & -1 \end{bmatrix}$$

Solution:

Given

$$\begin{bmatrix} 3x + 4y & 2 & x - 2y \\ a + b & 2a - b & -1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 4 \\ 5 & -5 & -1 \end{bmatrix}$$

Given that two matrices are equal.

We know that if two matrices are equal then the elements of each matrices are also equal.

Therefore by equating them we get,

$$3x + 4y = 2 \dots\dots (1)$$

$$x - 2y = 4 \dots\dots (2)$$

$$a + b = 5 \dots\dots (3)$$

$$2a - b = -5 \dots\dots (4)$$

Multiplying equation (2) by 2 and adding to equation (1), we get

$$3x + 4y + 2x - 4y = 2 + 8$$

$$\Rightarrow 5x = 10$$

$$\Rightarrow x = 2$$

Now, substituting the value of x in equation (1)

$$3 \times 2 + 4y = 2$$

$$\Rightarrow 6 + 4y = 2$$

$$\Rightarrow 4y = 2 - 6$$

$$\Rightarrow 4y = -4$$

$$\Rightarrow y = -1$$

Now by adding equation (3) and (4)

$$a + b + 2a - b = 5 + (-5)$$

$$\Rightarrow 3a = 5 - 5 = 0$$

$$\Rightarrow a = 0$$

Now, again by substituting the value of a in equation (3), we get

$$0 + b = 5$$

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

$$\Rightarrow b = 5$$

$$\therefore a = 0, b = 5, x = 2 \text{ and } y = -1$$

9. Find x, y, a and b if

$$\begin{bmatrix} 2x - 3y & a - b & 3 \\ 1 & x + 4y & 3a + 4b \end{bmatrix} = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 6 & 29 \end{bmatrix}$$

Solution:

$$\begin{bmatrix} 2x - 3y & a - b & 3 \\ 1 & x + 4y & 3a + 4b \end{bmatrix} = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 6 & 29 \end{bmatrix}$$

We know that if two matrices are equal then the elements of each matrices are also equal.

Given that two matrices are equal.

Therefore by equating them we get,

$$2a + b = 4 \dots\dots (1)$$

$$\text{And } a - 2b = -3 \dots\dots (2)$$

$$\text{And } 5c - d = 11 \dots\dots (3)$$

$$4c + 3d = 24 \dots\dots (4)$$

Multiplying equation (1) by 2 and adding to equation (2)

$$4a + 2b + a - 2b = 8 - 3$$

$$\Rightarrow 5a = 5$$

$$\Rightarrow a = 1$$

Now, substituting the value of a in equation (1)

$$2 \times 1 + b = 4$$

$$\Rightarrow 2 + b = 4$$

$$\Rightarrow b = 4 - 2$$

$$\Rightarrow b = 2$$

Multiplying equation (3) by 3 and adding to equation (4)

$$15c - 3d + 4c + 3d = 33 + 24$$

$$\Rightarrow 19c = 57$$

$$\Rightarrow c = 3$$

Now, substituting the value of c in equation (4)

$$4 \times 3 + 3d = 24$$

$$\Rightarrow 12 + 3d = 24$$

$$\Rightarrow 3d = 24 - 12$$

$$\Rightarrow 3d = 12$$

$$\Rightarrow d = 4$$

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

$\therefore a = 1, b = 2, c = 3$ and $d = 4$

10. Find the values of a, b, c and d from the following equations:

$$\begin{bmatrix} 2a + b & a - 2b \\ 5c - d & 4c + 3d \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$$

Solution:

Given

$$\begin{bmatrix} 2a + b & a - 2b \\ 5c - d & 4c + 3d \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$$

We know that if two matrices are equal then the elements of each matrices are also equal.

Given that two matrices are equal.

Therefore by equating them we get,

$$2a + b = 4 \dots\dots (1)$$

$$\text{And } a - 2b = -3 \dots\dots (2)$$

$$\text{And } 5c - d = 11 \dots\dots (3)$$

$$4c + 3d = 24 \dots\dots (4)$$

Multiplying equation (1) by 2 and adding to equation (2)

$$4a + 2b + a - 2b = 8 - 3$$

$$\Rightarrow 5a = 5$$

$$\Rightarrow a = 1$$

Now, substituting the value of a in equation (1)

$$2 \times 1 + b = 4$$

$$\Rightarrow 2 + b = 4$$

$$\Rightarrow b = 4 - 2$$

$$\Rightarrow b = 2$$

Multiplying equation (3) by 3 and adding to equation (4)

$$15c - 3d + 4c + 3d = 33 + 24$$

$$\Rightarrow 19c = 57$$

$$\Rightarrow c = 3$$

Now, substituting the value of c in equation (4)

$$4 \times 3 + 3d = 24$$

$$\Rightarrow 12 + 3d = 24$$

$$\Rightarrow 3d = 24 - 12$$

$$\Rightarrow 3d = 12$$

$$\Rightarrow d = 4$$

**RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices**

$\therefore a = 1, b = 2, c = 3$ and $d = 4$

EXERCISE 5.2

PAGE NO: 5.18

1. Compute the following sums:

$$(i) \begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} -2 & 4 \\ 1 & 3 \end{bmatrix}$$

$$(ii) \begin{bmatrix} 2 & 1 & 3 \\ 0 & 3 & 5 \\ -1 & 2 & 5 \end{bmatrix} + \begin{bmatrix} 1 & -2 & 3 \\ 2 & 6 & 1 \\ 0 & -3 & 1 \end{bmatrix}$$

Solution:

(i) Given

$$\begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} -2 & 4 \\ 1 & 3 \end{bmatrix}$$

Corresponding elements of two matrices should be added

Therefore, we get

$$= \begin{bmatrix} 3-2 & -2+4 \\ 1+1 & 4+3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ 2 & 7 \end{bmatrix}$$

$$\text{Therefore, } \begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} -2 & 4 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 7 \end{bmatrix}$$

(ii) Given

$$\begin{bmatrix} 2 & 1 & 3 \\ 0 & 3 & 5 \\ -1 & 2 & 5 \end{bmatrix} + \begin{bmatrix} 1 & -2 & 3 \\ 2 & 6 & 1 \\ 0 & -3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2+1 & 1-2 & 3+3 \\ 0+2 & 3+6 & 5+1 \\ -1+0 & 2-3 & 5+1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -1 & 6 \\ 2 & 9 & 6 \\ -1 & -1 & 6 \end{bmatrix}$$

Therefore,

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Algebra of Matrices

$$\begin{bmatrix} 2 & 1 & 3 \\ 0 & 3 & 5 \\ -1 & 2 & 5 \end{bmatrix} + \begin{bmatrix} 1 & -2 & 3 \\ 2 & 6 & 1 \\ 0 & -3 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 6 \\ 2 & 9 & 6 \\ -1 & -1 & 6 \end{bmatrix}$$

$$2. \text{ Let } A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} \quad \text{and } C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}.$$

Find each of the following:

(i) $2A - 3B$

(ii) $B - 4C$

(iii) $3A - C$

(iv) $3A - 2B + 3C$

Solution:

(i) Given

$$A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} \quad \text{and } C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}.$$

First we have to compute $2A$

$$2A = 2 \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 8 \\ 6 & 4 \end{bmatrix}$$

Now by computing $3B$ we get,

$$= 3B = 3 \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 9 \\ -6 & 15 \end{bmatrix}$$

Now by we have to compute $2A - 3B$ we get

$$= 2A - 3B = \begin{bmatrix} 4 & 8 \\ 6 & 4 \end{bmatrix} - \begin{bmatrix} 3 & 9 \\ -6 & 15 \end{bmatrix} = \begin{bmatrix} 4-3 & 8-9 \\ 6+6 & 4-15 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 \\ 12 & -11 \end{bmatrix}$$

Therefore

$$2A - 3B = \begin{bmatrix} 1 & -1 \\ 12 & -11 \end{bmatrix}$$

(ii) Given $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} \quad \text{and } C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}.$

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Algebra of Matrices

First we have to compute $4C$,

$$4C = 4 \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} -8 & 20 \\ 12 & 16 \end{bmatrix}$$

Now,

$$B - 4C = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} - \begin{bmatrix} -8 & 20 \\ 12 & 16 \end{bmatrix}$$

$$= \begin{bmatrix} 1 + 8 & 3 - 20 \\ -2 - 12 & 5 - 16 \end{bmatrix} = \begin{bmatrix} 9 & -17 \\ -14 & -11 \end{bmatrix}$$

Therefore we get,

$$B - 4C = \begin{bmatrix} 9 & -17 \\ -14 & -11 \end{bmatrix}$$

(iii) Given

$$A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}.$$

First we have to compute $3A$,

$$3A = 3 \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 12 \\ 9 & 6 \end{bmatrix}$$

Now,

$$= 3A - C = \begin{bmatrix} 6 & 12 \\ 9 & 6 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 6 + 2 & 12 - 5 \\ 9 - 3 & 6 - 4 \end{bmatrix} = \begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$$

Therefore,

$$3A - C = \begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$$

(iv) Given

$$A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}.$$

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

First we have to compute $3A$

$$3A = 3 \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 12 \\ 9 & 6 \end{bmatrix}$$

Now we have to compute $2B$

$$= 3C = 3 \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} -6 & 15 \\ 9 & 12 \end{bmatrix}$$

By computing $3C$ we get,

$$= 3C = 3 \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} -6 & 15 \\ 9 & 12 \end{bmatrix}$$

$$= 3A - 2B + 3C = \begin{bmatrix} 6 & 12 \\ 9 & 6 \end{bmatrix} - \begin{bmatrix} 2 & 6 \\ -4 & 10 \end{bmatrix} + \begin{bmatrix} -6 & 15 \\ 9 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 6 - 2 - 6 & 12 - 6 + 15 \\ 9 + 4 + 9 & 6 - 10 + 12 \end{bmatrix} = \begin{bmatrix} -2 & 21 \\ 22 & 8 \end{bmatrix}$$

Therefore,

$$3A - 2B + 3C = \begin{bmatrix} -2 & 21 \\ 22 & 8 \end{bmatrix}$$

3. If $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 0 & 2 \\ 3 & 4 & 1 \end{bmatrix}$, $C = \begin{bmatrix} -1 & 2 & 3 \\ 2 & 1 & 0 \end{bmatrix}$, find

(i) $A + B$ and $B + C$

(ii) $2B + 3A$ and $3C - 4B$

Solution:

(i) Consider $A + B$,

$A + B$ is not possible because matrix A is an order of 2×2 and Matrix B is an order of 2×3 , so the Sum of the matrix is only possible when their order is same.

Now consider $B + C$

$$\Rightarrow B + C = \begin{bmatrix} -1 & 0 & 2 \\ 3 & 4 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 2 & 3 \\ 2 & 1 & 0 \end{bmatrix}$$

$$\Rightarrow B + C = \begin{bmatrix} -1 - 1 & 0 + 2 & 2 + 3 \\ 3 + 2 & 4 + 1 & 1 + 0 \end{bmatrix}$$

$$\Rightarrow B + C = \begin{bmatrix} -2 & 2 & 5 \\ 5 & 5 & 1 \end{bmatrix}$$

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

(ii) Consider $2B + 3A$

$2B + 3A$ also does not exist because the order of matrix B and matrix A is different, so we cannot find the sum of these matrix.

Now consider $3C - 4B$,

$$\Rightarrow 3C - 4B = 3 \begin{bmatrix} -1 & 2 & 3 \\ 2 & 1 & 0 \end{bmatrix} - 4 \begin{bmatrix} -1 & 0 & 2 \\ 3 & 4 & 1 \end{bmatrix}$$

$$\Rightarrow 3C - 4B = \begin{bmatrix} -3 & 6 & 9 \\ 6 & 3 & 0 \end{bmatrix} - \begin{bmatrix} -4 & 0 & 8 \\ 12 & 16 & 4 \end{bmatrix}$$

$$\Rightarrow 3C - 4B = \begin{bmatrix} -3+4 & 6-0 & 9-8 \\ 6-12 & 3-16 & 0-4 \end{bmatrix}$$

$$\Rightarrow 3C - 4B = \begin{bmatrix} 1 & 6 & 1 \\ -6 & -13 & -4 \end{bmatrix}$$

4. Let $A = \begin{bmatrix} -1 & 0 & 2 \\ 3 & 1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -2 & 5 \\ 1 & -3 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & -5 & 2 \\ 6 & 0 & -4 \end{bmatrix}$. Compute $2A - 3B + 4C$

Solution:

Given

$$A = \begin{bmatrix} -1 & 0 & 2 \\ 3 & 1 & 4 \end{bmatrix}, B = \begin{bmatrix} 0 & -2 & 5 \\ 1 & -3 & 1 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & -5 & 2 \\ 6 & 0 & -4 \end{bmatrix}$$

Now we have to compute $2A - 3B + 4C$

$$2A - 3B + 4C = 2 \begin{bmatrix} -1 & 0 & 2 \\ 3 & 1 & 4 \end{bmatrix} - 3 \begin{bmatrix} 0 & -2 & 5 \\ 1 & -3 & 1 \end{bmatrix} + 4 \begin{bmatrix} 1 & -5 & 2 \\ 6 & 0 & -4 \end{bmatrix}$$

$$\Rightarrow 2A - 3B + 4C = \begin{bmatrix} -2 & 0 & 4 \\ 6 & 2 & 8 \end{bmatrix} - \begin{bmatrix} 0 & -6 & 15 \\ 3 & -9 & 3 \end{bmatrix} + \begin{bmatrix} 4 & -20 & 8 \\ 24 & 0 & -16 \end{bmatrix}$$

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

$$\Rightarrow 2A - 3B + 4C = \begin{bmatrix} -2 - 0 + 4 & 0 + 6 - 20 & 4 - 15 + 8 \\ 6 - 3 + 24 & 2 + 9 + 0 & 8 - 3 - 16 \end{bmatrix}$$

$$\Rightarrow 2A - 3B + 4C = \begin{bmatrix} 2 & -14 & -3 \\ 27 & 11 & -11 \end{bmatrix}$$

5. If $A = \text{diag} (2 \ -5 \ 9)$, $B = \text{diag} (1 \ 1 \ -4)$ and $C = \text{diag} (-6 \ 3 \ 4)$, find

(i) $A - 2B$

(ii) $B + C - 2A$

(iii) $2A + 3B - 5C$

Solution:

(i) Given $A = \text{diag} (2 \ -5 \ 9)$, $B = \text{diag} (1 \ 1 \ -4)$ and $C = \text{diag} (-6 \ 3 \ 4)$

Here,

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -4 \end{bmatrix}$$

$$A - 2B$$

$$\Rightarrow A - 2B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 9 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -4 \end{bmatrix}$$

$$\Rightarrow A - 2B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 9 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -8 \end{bmatrix}$$

$$\Rightarrow A - 2B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -7 & 0 \\ 0 & 0 & 17 \end{bmatrix} = \text{diag} (0 \ -7 \ 17)$$

(ii) Given $A = \text{diag} (2 \ -5 \ 9)$, $B = \text{diag} (1 \ 1 \ -4)$ and $C = \text{diag} (-6 \ 3 \ 4)$

We have to find $B + C - 2A$

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Algebra of Matrices

Here,

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 9 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -4 \end{bmatrix}$$

Now we have to compute $B + C - 2A$

$$\Rightarrow B + C - 2A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -4 \end{bmatrix} + \begin{bmatrix} -6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix} - 2 \begin{bmatrix} 2 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$\Rightarrow B + C - 2A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -4 \end{bmatrix} + \begin{bmatrix} -6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & -10 & 0 \\ 0 & 0 & 18 \end{bmatrix}$$

$$\Rightarrow B + C - 2A = \begin{bmatrix} 1-6-4 & 0+0-0 & 0+0-0 \\ 0+0-0 & 1+3+10 & 0+0-0 \\ 0+0-0 & 0+0-0 & -4+4-18 \end{bmatrix}$$

$$\Rightarrow B + C - 2A = \begin{bmatrix} -9 & 0 & 0 \\ 0 & 14 & 0 \\ 0 & 0 & -18 \end{bmatrix} = \text{diag}(-9, 14, -18)$$

(iii) Given $A = \text{diag}(2, -5, 9)$, $B = \text{diag}(1, 1, -4)$ and $C = \text{diag}(-6, 3, 4)$

Now we have to find $2A + 3B - 5C$

Here,

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -4 \end{bmatrix}$$

$$\text{and } C = \begin{bmatrix} -6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

Now consider $2A + 3B - 5C$

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Algebra of Matrices

$$\begin{aligned}\Rightarrow 2A + 3B - 5C &= 2 \begin{bmatrix} 2 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 9 \end{bmatrix} + 3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -4 \end{bmatrix} - 5 \begin{bmatrix} -6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix} \\ \Rightarrow 2A + 3B - 5C &= \begin{bmatrix} 4 & 0 & 0 \\ 0 & -10 & 0 \\ 0 & 0 & 18 \end{bmatrix} + \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -12 \end{bmatrix} - \begin{bmatrix} -30 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 20 \end{bmatrix} \\ \Rightarrow 2A + 3B - 5C &= \begin{bmatrix} 4+3+30 & 0+0-0 & 0+0-0 \\ 0+0-0 & -10+3-15 & 0+0-0 \\ 0+0-0 & 0+0-0 & 18-12-20 \end{bmatrix} \\ \Rightarrow 2A + 3B - 5C &= \begin{bmatrix} 37 & 0 & 0 \\ 0 & -22 & 0 \\ 0 & 0 & -14 \end{bmatrix} \\ &= \text{diag}(37 \ -22 \ -14)\end{aligned}$$

6. Given the matrices

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & -1 & 0 \\ 0 & 2 & 4 \end{bmatrix}, B = \begin{bmatrix} 9 & 7 & -1 \\ 3 & 5 & 4 \\ 2 & 1 & 6 \end{bmatrix} \text{ and } C = \begin{bmatrix} 2 & -4 & 3 \\ 1 & -1 & 0 \\ 9 & 4 & 5 \end{bmatrix}$$

Verify that $(A + B) + C = A + (B + C)$

Solution:

Given

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & -1 & 0 \\ 0 & 2 & 4 \end{bmatrix}, B = \begin{bmatrix} 9 & 7 & -1 \\ 3 & 5 & 4 \\ 2 & 1 & 6 \end{bmatrix} \text{ and } C = \begin{bmatrix} 2 & -4 & 3 \\ 1 & -1 & 0 \\ 9 & 4 & 5 \end{bmatrix}$$

Now we have to verify $(A + B) + C = A + (B + C)$

First consider LHS, $(A + B) + C$,

$$\begin{aligned}&= \left(\begin{bmatrix} 2 & 1 & 1 \\ 3 & -1 & 0 \\ 0 & 2 & 4 \end{bmatrix} + \begin{bmatrix} 9 & 7 & -1 \\ 3 & 5 & 4 \\ 2 & 1 & 6 \end{bmatrix} \right) + \begin{bmatrix} 2 & -4 & 3 \\ 1 & -1 & 0 \\ 9 & 4 & 5 \end{bmatrix} \\ &= \left(\begin{bmatrix} 2+9 & 1+7 & 1-1 \\ 3+3 & -1+5 & 0+4 \\ 0+2 & 2+1 & 4+6 \end{bmatrix} \right) + \begin{bmatrix} 2 & -4 & 3 \\ 1 & -1 & 0 \\ 9 & 4 & 5 \end{bmatrix}\end{aligned}$$

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

$$= \begin{bmatrix} 11 & 8 & 0 \\ 6 & 4 & 4 \\ 2 & 3 & 10 \end{bmatrix} + \begin{bmatrix} 2 & -4 & 3 \\ 1 & -1 & 0 \\ 9 & 4 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 11+2 & 8-4 & 0+3 \\ 6+1 & 4-1 & 4+0 \\ 2+9 & 3+4 & 10+5 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & 4 & 3 \\ 7 & 3 & 4 \\ 11 & 7 & 15 \end{bmatrix}$$

Now consider RHS, that is $A + (B + C)$

$$= \begin{bmatrix} 2 & 1 & 1 \\ 3 & -1 & 0 \\ 0 & 2 & 4 \end{bmatrix} + \left(\begin{bmatrix} 9 & 7 & -1 \\ 3 & 5 & 4 \\ 2 & 1 & 6 \end{bmatrix} + \begin{bmatrix} 2 & -4 & 3 \\ 1 & -1 & 0 \\ 9 & 4 & 5 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 2 & 1 & 1 \\ 3 & -1 & 0 \\ 0 & 2 & 4 \end{bmatrix} + \left(\begin{bmatrix} 9+2 & 7-4 & -1+3 \\ 3+1 & 5-1 & 4+0 \\ 2+9 & 1+4 & 6+5 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 2 & 1 & 1 \\ 3 & -1 & 0 \\ 0 & 2 & 4 \end{bmatrix} + \begin{bmatrix} 11 & 3 & 2 \\ 4 & 4 & 4 \\ 11 & 5 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} 2+11 & 1+3 & 1+2 \\ 3+4 & -1+4 & 0+4 \\ 0+11 & 2+5 & 4+11 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & 4 & 3 \\ 7 & 3 & 4 \\ 11 & 7 & 15 \end{bmatrix}$$

Therefore LHS = RHS

Hence $(A + B) + C = A + (B + C)$

7. Find the matrices X and Y,

$$\text{if } X + Y = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} \text{ and } X - Y = \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$$

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices**Solution:**

Consider,

$$(X + Y) + (X - Y) = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$$

Now by simplifying we get,

$$\Rightarrow 2X = \begin{bmatrix} 5+3 & 2+6 \\ 0+0 & 9-1 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} 8 & 8 \\ 0 & 8 \end{bmatrix}$$

$$\Rightarrow X = \frac{1}{2} \begin{bmatrix} 8 & 8 \\ 0 & 8 \end{bmatrix}$$

Therefore,

$$\Rightarrow X = \begin{bmatrix} 4 & 4 \\ 0 & 4 \end{bmatrix}$$

Again consider,

$$(X + Y) - (X - Y) = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$$

$$\Rightarrow X + Y - X + Y = \begin{bmatrix} 5-3 & 2-6 \\ 0-0 & 9+1 \end{bmatrix}$$

Now by simplifying we get,

$$\Rightarrow 2Y = \begin{bmatrix} 2 & -4 \\ 0 & 10 \end{bmatrix}$$

$$\Rightarrow Y = \frac{1}{2} \begin{bmatrix} 2 & -4 \\ 0 & 10 \end{bmatrix}$$

$$\Rightarrow Y = \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix}$$

Therefore,

$$X = \begin{bmatrix} 4 & 4 \\ 0 & 4 \end{bmatrix} \text{ and } Y = \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix}$$

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

8. Find X , if $X = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and $2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$.

Solution:

Given

$$2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$$

Now by transposing, we get

$$\Rightarrow 2X + \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} 1-3 & 0-2 \\ -3-1 & 2-4 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix}$$

$$\Rightarrow X = \frac{1}{2} \begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix}$$

Therefore,

$$\Rightarrow X = \begin{bmatrix} -1 & -1 \\ -2 & -1 \end{bmatrix}$$

9. Find matrices X and Y , if $2X - Y = \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix}$ and $X + 2Y = \begin{bmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{bmatrix}$.

Solution:

Given

$$(2X - Y) = \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix} \dots (1)$$

$$(X + 2Y) = \begin{bmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{bmatrix} \dots (2)$$

Now by multiplying equation (1) and (2) we get,

$$2(2X - Y) = 2 \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix}$$

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

$$\Rightarrow 4X - 2Y = \begin{bmatrix} 12 & -12 & 0 \\ -8 & 4 & 2 \end{bmatrix} \dots (3)$$

Now by adding equation (2) and (3) we get,

$$(4X - 2Y) + (X + 2Y) = \begin{bmatrix} 12 & -12 & 0 \\ -8 & 4 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{bmatrix}$$

$$\Rightarrow 5X = \begin{bmatrix} 12+3 & -12+2 & 0+5 \\ -8-2 & 4+1 & 2-7 \end{bmatrix}$$

$$\Rightarrow 5X = \begin{bmatrix} 15 & -10 & 5 \\ -10 & 5 & -5 \end{bmatrix}$$

$$\Rightarrow X = \frac{1}{5} \begin{bmatrix} 15 & -10 & 5 \\ -10 & 5 & -5 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} 3 & -2 & 1 \\ -2 & 1 & -1 \end{bmatrix}$$

Now by substituting X in equation (2) we get,

$$(X + 2Y) = \begin{bmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 & -2 & 1 \\ -2 & 1 & -1 \end{bmatrix} + 2Y = \begin{bmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{bmatrix}$$

$$\Rightarrow 2Y = \begin{bmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{bmatrix} - \begin{bmatrix} 3 & -2 & 1 \\ -2 & 1 & -1 \end{bmatrix}$$

$$\Rightarrow 2Y = \begin{bmatrix} 3-3 & 2+2 & 5-1 \\ -2+2 & 1-1 & -7+1 \end{bmatrix}$$

$$\Rightarrow Y = \begin{bmatrix} 0 & 2 & 2 \\ 0 & 0 & -3 \end{bmatrix}$$

10. If $X - Y = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ and $X + Y = \begin{bmatrix} 3 & 5 & 1 \\ -1 & 1 & 1 \\ 11 & 8 & 0 \end{bmatrix}$ find X and Y.

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Algebra of Matrices**Solution:**

Consider

$$X - Y + X + Y = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 5 & 1 \\ -1 & 1 & 4 \\ 11 & 8 & 0 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} 1+3 & 1+5 & 1+1 \\ 1-1 & 1+1 & 0+4 \\ 1+11 & 0+8 & 0+0 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} 4 & 6 & 2 \\ 0 & 2 & 4 \\ 12 & 8 & 0 \end{bmatrix}$$

$$\Rightarrow X = \frac{1}{2} \begin{bmatrix} 4 & 6 & 2 \\ 0 & 2 & 4 \\ 12 & 8 & 0 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 1 & 2 \\ 6 & 4 & 0 \end{bmatrix}$$

Now, again consider

$$(X - Y) - (X + Y) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 3 & 5 & 1 \\ -1 & 1 & 4 \\ 11 & 8 & 0 \end{bmatrix}$$

$$\Rightarrow X - Y - X - Y = \begin{bmatrix} 1-3 & 1-5 & 1-1 \\ 1-1 & 1-1 & 0-4 \\ 1-11 & 0-8 & 0-0 \end{bmatrix}$$

$$\Rightarrow -2Y = \begin{bmatrix} -2 & -4 & 0 \\ 0 & 0 & -4 \\ -10 & -8 & 0 \end{bmatrix}$$

$$\Rightarrow Y = -\frac{1}{2} \begin{bmatrix} -2 & -4 & 0 \\ 0 & 0 & -4 \\ -10 & -8 & 0 \end{bmatrix}$$

$$\Rightarrow Y = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 2 \\ 5 & 4 & 0 \end{bmatrix}$$

Therefore,

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

$$X = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 1 & 2 \\ 6 & 4 & 0 \end{bmatrix}$$

And

$$Y = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 2 \\ 5 & 4 & 0 \end{bmatrix}$$

EXERCISE 5.3

PAGE NO: 5.41

1. Compute the indicated products:

$$(i) \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$(ii) \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ -3 & 2 & 1 \end{bmatrix}$$

$$(iii) \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$$

Solution:

(i) Consider

$$\begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a \times a + b \times b & a \times (-b) + b \times a \\ (-b) \times a + a \times b & (-b) \times (-b) + a \times a \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a^2 + b^2 & -ab + ab \\ -ab + ab & b^2 + a^2 \end{bmatrix}$$

On simplification we get,

$$\Rightarrow \begin{bmatrix} a^2 + b^2 & 0 \\ 0 & a^2 + b^2 \end{bmatrix}$$

(ii) Consider

$$\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ -3 & 2 & -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 \times 1 + (-2) \times (-3) & 1 \times 2 + (-2) \times 2 & 1 \times 3 + (-2) \times (-1) \\ 2 \times 1 + 3 \times (-3) & 2 \times 2 + 3 \times 2 & 2 \times 3 + 3 \times (-1) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 + 6 & 2 - 4 & 3 + 2 \\ 2 - 9 & 4 + 6 & 6 - 3 \end{bmatrix}$$

On simplification we get,

$$\Rightarrow \begin{bmatrix} 7 & -2 & 5 \\ -7 & 10 & 3 \end{bmatrix}$$

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Algebra of Matrices

(iii) Consider

$$\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 \times 1 + 3 \times 0 + 4 \times 3 & 2 \times (-3) + 3 \times 2 + 4 \times 0 & 2 \times 5 + 3 \times 4 + 4 \times 5 \\ 3 \times 1 + 4 \times 0 + 5 \times 3 & 3 \times (-3) + 4 \times 2 + 5 \times 0 & 3 \times 5 + 4 \times 4 + 5 \times 5 \\ 4 \times 1 + 5 \times 0 + 6 \times 3 & 4 \times (-3) + 5 \times 2 + 6 \times 0 & 4 \times 5 + 5 \times 4 + 6 \times 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 + 0 + 12 & -6 + 6 + 0 & 10 + 12 + 20 \\ 3 + 0 + 15 & -9 + 8 + 0 & 15 + 16 + 25 \\ 4 + 0 + 18 & -12 + 10 + 0 & 20 + 20 + 30 \end{bmatrix}$$

On simplification we get,

$$\Rightarrow \begin{bmatrix} 14 & 0 & 42 \\ 18 & -1 & 56 \\ 22 & -2 & 70 \end{bmatrix}$$

2. Show that $AB \neq BA$ in each of the following cases:

$$(i) A = \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$$

$$(ii) A = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$(iii) A = \begin{bmatrix} 1 & 3 & 0 \\ 1 & 1 & 0 \\ 4 & 1 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 5 & 1 \end{bmatrix}$$

Solution:

(i) Consider,

$$AB = \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 10 - 3 & 5 - 4 \\ 12 + 21 & 6 + 28 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix} \dots\dots\dots(1)$$

Again consider,

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Algebra of Matrices

$$BA = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} 10 + 6 & -2 + 7 \\ 15 + 24 & -3 + 28 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix} \dots\dots\dots(2)$$

From equation (1) and (2), it is clear that
 $AB \neq BA$

(ii) Consider,

$$AB = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} -1 + 0 + 0 & -2 + 1 + 0 & -3 + 0 + 0 \\ 0 + 0 + 1 & 0 - 1 + 1 & 0 + 0 + 0 \\ 2 + 0 + 4 & 4 + 3 + 4 & 6 + 0 + 0 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} -1 & -1 & -3 \\ 1 & 0 & 0 \\ 6 & 11 & 6 \end{bmatrix} \dots\dots\dots(1)$$

Now again consider,

$$BA = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} -1 + 0 + 6 & 1 - 2 + 9 & 0 + 2 + 12 \\ 0 + 0 + 0 & 0 - 1 + 0 & 0 + 1 + 0 \\ -1 + 0 + 0 & 1 - 1 + 0 & 0 + 1 + 0 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} 5 & 8 & 14 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix} \dots\dots\dots(2)$$

From equation (1) and (2), it is clear that
 $AB \neq BA$

(iii) Consider,

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Algebra of Matrices

$$AB = \begin{bmatrix} 1 & 3 & 0 \\ 1 & 1 & 0 \\ 4 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 5 & 1 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 0+3+0 & 1+0+0 & 0+0+0 \\ 0+1+0 & 1+0+0 & 0+0+0 \\ 0+1+0 & 4+0+0 & 0+0+0 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 3 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 4 & 0 \end{bmatrix} \dots\dots\dots(1)$$

Now again consider,

$$BA = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 5 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 0 \\ 1 & 1 & 0 \\ 4 & 1 & 0 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} 0+1+0 & 0+1+1 & 0+0+0 \\ 1+0+0 & 3+0+0 & 0+0+0 \\ 0+5+4 & 0+5+1 & 0+0+0 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 3 & 0 \\ 9 & 6 & 0 \end{bmatrix} \dots\dots\dots(2)$$

From equation (1) and (2), it is clear that
 $AB \neq BA$

3. Compute the products AB and BA whichever exists in each of the following cases:

(i) $A = \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$

(ii) $A = \begin{bmatrix} 3 & 2 \\ -1 & 0 \\ -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 5 & 6 \\ 0 & 1 & 2 \end{bmatrix}$

(iii) $A = \begin{bmatrix} 1 & -1 & 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 0 \\ 1 \\ 3 \\ 2 \end{bmatrix}$

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Algebra of Matrices

$$(iv) \begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix} + \begin{bmatrix} a & b & c & d \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

Solution:

(i) Consider,

$$AB = \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 1-4 & 2-6 & 3-2 \\ 2+6 & 4+9 & 6+3 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} -3 & -4 & 1 \\ 8 & 13 & 9 \end{bmatrix}$$

BA does not exist

Because the number of columns in B is greater than the rows in A

(ii) Consider,

$$AB = \begin{bmatrix} 3 & 2 \\ -1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 4 & 5 & 6 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 12+0 & 15+2 & 18+4 \\ -4+0 & -5+0 & -6+0 \\ -4+0 & -5+1 & -6+2 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 12 & 17 & 22 \\ -4 & -5 & -6 \\ -4 & -4 & -4 \end{bmatrix}$$

Again consider,

$$BA = \begin{bmatrix} 4 & 5 & 6 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ -1 & 0 \\ -1 & 1 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} 12-5-6 & 8+0+6 \\ 0-1-2 & 0+0+2 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} 1 & 14 \\ -3 & 2 \end{bmatrix}$$

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Algebra of Matrices

(iii) Consider,

$$AB = \begin{bmatrix} 1 & -1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 3 \\ 2 \end{bmatrix}$$

$$AB = [0 + (-1) + 6 + 6]$$

$$AB = 11$$

Again consider,

$$BA = \begin{bmatrix} 0 \\ 1 \\ 3 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 & 3 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & -1 & 2 & 3 \\ 3 & -3 & 6 & 9 \\ 2 & -2 & 4 & 6 \end{bmatrix}$$

(iv) Consider,

$$\begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix} + \begin{bmatrix} a & b & c & d \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$$\Rightarrow [ac + bd] + [a^2 + b^2 + c^2 + d^2]$$

$$[a^2 + b^2 + c^2 + d^2 + ac + bd]$$

4. Show that $AB \neq BA$ in each of the following cases:

$$(i) A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & -1 & -1 \\ 3 & 0 & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix}$$

$$(ii) A = \begin{bmatrix} 10 & -4 & -1 \\ -11 & 5 & 0 \\ 9 & -5 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 1 & 3 & 2 \end{bmatrix}$$

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices**Solution:**

(i) Consider,

$$AB = \begin{bmatrix} 1 & 3 & -1 \\ 2 & -1 & -1 \\ 3 & 0 & -1 \end{bmatrix} \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} -2-3+6 & 3+6-9 & -1-3+4 \\ -4+1+6 & 6-2-9 & -2+1+4 \\ -6-0+6 & 9+0-9 & -3-0+4 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 1 & 0 & 0 \\ 3 & -5 & 3 \\ 0 & 0 & 1 \end{bmatrix} \dots\dots\dots(1)$$

Again consider,

$$BA = \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix} \begin{bmatrix} 1 & 3 & -1 \\ 2 & -1 & -1 \\ 3 & 0 & -1 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} -2+6-3 & -6-3+0 & 2-3+1 \\ -1+4-3 & -3-2+0 & 1-2+1 \\ -6+18-12 & -18-9+0 & 6-9+4 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} 1 & -9 & 0 \\ 0 & -5 & 0 \\ 0 & -27 & 1 \end{bmatrix} \dots\dots\dots(2)$$

From equation (1) and (2), it is clear that
 $AB \neq BA$

(ii) Consider,

$$AB = \begin{bmatrix} 10 & -4 & -1 \\ -11 & 5 & 0 \\ 9 & -5 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 1 & 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 10-12-1 & 20-16-3 & 10-8-2 \\ -11+15+0 & -22+20+0 & -11+10+0 \\ 9-15+1 & 18-20+3 & 9-10+2 \end{bmatrix}$$

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

$$AB = \begin{bmatrix} -3 & 1 & 0 \\ 4 & -2 & -1 \\ -5 & 1 & 1 \end{bmatrix} \dots\dots(1)$$

Again consider,

$$BA = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 1 & 3 & 2 \end{bmatrix} \begin{bmatrix} 10 & -4 & -1 \\ -11 & 5 & 0 \\ 9 & -5 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 10 - 22 + 9 & -4 + 10 - 5 & -9 + 0 + 1 \\ 30 - 44 + 10 & -12 + 20 - 10 & -3 + 0 + 2 \\ 10 - 33 + 18 & -4 + 15 - 10 & -1 + 0 + 2 \end{bmatrix}$$

$$BA = \begin{bmatrix} -3 & 1 & 0 \\ 4 & -2 & -1 \\ -5 & 1 & 1 \end{bmatrix} \dots\dots(2)$$

From equation (1) and (2) it is clear that,
 $AB \neq BA$

5. Evaluate the following:

$$(i) \left(\begin{bmatrix} 1 & 3 \\ -1 & -4 \end{bmatrix} + \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix} \right) \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

$$(ii) \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

$$(iii) \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 2 & 3 \end{bmatrix} \left(\begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 2 \end{bmatrix} \right)$$

Solution:

(i) Given

$$\left(\begin{bmatrix} 1 & 3 \\ -1 & -4 \end{bmatrix} + \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix} \right) \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

First we have to add first two matrix,

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Algebra of Matrices

$$\Rightarrow \left(\begin{bmatrix} 1+3 & 3-2 \\ -1-1 & -4+1 \end{bmatrix} \right) \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4+2 & 12+4 & 20+6 \\ -2-6 & -6-12 & -10-18 \end{bmatrix}$$

On simplifying, we get

$$\Rightarrow \begin{bmatrix} 6 & 16 & 26 \\ -8 & -18 & -28 \end{bmatrix}$$

(ii) Given,

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

First we have to multiply first two given matrix,

$$\Rightarrow [1+4+0 \quad 0+0+3 \quad 2+2+6] \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

$$\Rightarrow [5 \quad 3 \quad 10] \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

$$\Rightarrow [10+12+60]$$

$$= 82$$

(iii) Given

$$\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 2 & 3 \end{bmatrix} \left(\begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 2 \end{bmatrix} \right)$$

First we have subtract the matrix which is inside the bracket,

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1-0 & 0-1 & 2-2 \\ 2-1 & 0-0 & 1-2 \end{bmatrix}$$

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Algebra of Matrices

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1-1 & -1+0 & 0+1 \\ 0+2 & 0+0 & 0-2 \\ 2+3 & -2+0 & 0-3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & -1 & 1 \\ 2 & 0 & -2 \\ 5 & -2 & -3 \end{bmatrix}$$

6. If $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, show that $A^2 = B^2 = C^2 = I_2$

Solution:

Given

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \text{ and } C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

We know that,

$$A^2 = AA$$

$$\Rightarrow A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1+0 & 0+1 \\ 0+0 & 0+1 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \dots\dots\dots(1)$$

Again we know that,

$$B^2 = BB$$

$$\Rightarrow B^2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\Rightarrow B^2 = \begin{bmatrix} 1+0 & 0-0 \\ 0-0 & 0+1 \end{bmatrix}$$

$$\Rightarrow B^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \dots\dots\dots(2)$$

Now, consider,

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Algebra of Matrices

$$C^2 = C C$$

$$\Rightarrow B^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\Rightarrow B^2 = \begin{bmatrix} 0+1 & 0+0 \\ 0+0 & 1+0 \end{bmatrix}$$

$$\Rightarrow B^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \dots\dots\dots(3)$$

We have,

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \dots\dots\dots(4)$$

Now, from equation (1), (2), (3) and (4), it is clear that $A^2 = B^2 = C^2 = I_2$

7.If $A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$, find $3A^2 - 2B + I$

Solution:

Given

$$A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$$

Consider,

$$A^2 = A A$$

$$\Rightarrow A^2 = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 4-3 & -2-2 \\ 6+6 & -3+4 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1 & -4 \\ 12 & 1 \end{bmatrix}$$

Now we have to find,

$$3A^2 - 2B + I$$

$$\Rightarrow 3A^2 - 2B + I = 3 \begin{bmatrix} 1 & -4 \\ 12 & 1 \end{bmatrix} - 2 \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow 3A^2 - 2B + I = \begin{bmatrix} 3 & -12 \\ 36 & 3 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

$$\Rightarrow 3A^2 - 2B + I = \begin{bmatrix} 3 - 0 + 1 & -12 - 8 + 0 \\ 36 + 2 + 0 & 3 - 14 + 1 \end{bmatrix}$$

$$\Rightarrow 3A^2 - 2B + I = \begin{bmatrix} 4 & -20 \\ 38 & -10 \end{bmatrix}$$

8. If $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$, prove that $(A - 2I)(A - 3I) = 0$.

Solution:

Given

$$A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$$

Consider,

$$\Rightarrow (A - 2I)(A - 3I) = \left(\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \left(\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} - 3 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right)$$

$$\Rightarrow (A - 2I)(A - 3I) = \left(\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \right) \left(\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \right)$$

$$\Rightarrow (A - 2I)(A - 3I) = \begin{bmatrix} 4 - 2 & 2 - 0 \\ -1 - 0 & 1 - 2 \end{bmatrix} \begin{bmatrix} 4 - 3 & 2 - 0 \\ -1 - 0 & 1 - 3 \end{bmatrix}$$

$$\Rightarrow (A - 2I)(A - 3I) = \begin{bmatrix} 2 & 2 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$$

$$\Rightarrow (A - 2I)(A - 3I) = \begin{bmatrix} 2 - 2 & 4 - 4 \\ -1 + 1 & -2 + 2 \end{bmatrix}$$

$$\Rightarrow (A - 2I)(A - 3I) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow (A - 2I)(A - 3I) = 0$$

Hence the proof.

9. If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, show that $A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ and $A^3 = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

Solution:

Given,

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

Consider,

$$A^2 = A A$$

$$A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1+0 & 1+1 \\ 0+0 & 0+1 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

Again consider,

$$A^3 = A^2 A$$

$$A^3 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^3 = \begin{bmatrix} 1+0 & 1+2 \\ 0+0 & 0+1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

Hence the proof.

10. If $A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$, show that $A^2 = 0$

Solution:

Given,

$$A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$$

Consider,

$$A^2 = A A$$

$$\Rightarrow A^2 = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix} \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} a^2b^2 - a^2b^2 & ab^3 - ab^3 \\ -a^3b + a^3b & -a^2b^2 + a^2b^2 \end{bmatrix}$$

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

$$\Rightarrow A^2 \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow A^2 = 0$$

Hence the proof.

11. If $A = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$, find A^2

Solution:

Given,

$$A = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$$

Consider,

$$A^2 = A A$$

$$\Rightarrow A^2 = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix} \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} \cos^2(2\theta) - \sin^2(2\theta) & \cos(2\theta)\sin 2\theta + \cos(2\theta)\sin 2\theta \\ -\cos(2\theta)\sin 2\theta - \sin 2\theta\cos 2\theta & -\sin^2(2\theta) + \cos^2(2\theta) \end{bmatrix}$$

We know that,

$$\cos^2\theta - \sin^2\theta = \cos^2(2\theta)$$

$$\Rightarrow A^2 = \begin{bmatrix} \cos(2 \times 2\theta) & 2 \sin 2\theta \cos 2\theta \\ -2 \sin 2\theta \cos(2\theta) & \cos(2 \times 2\theta) \end{bmatrix}$$

Again we have,

$$\sin 2\theta = 2 \sin\theta \cos\theta$$

$$\Rightarrow A^2 = \begin{bmatrix} \cos 4\theta & \sin(2 \times 2\theta) \\ -\sin(2 \times 2\theta) & \cos 4\theta \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} \cos 4\theta & \sin 4\theta \\ -\sin 4\theta & \cos 4\theta \end{bmatrix}$$

12. If $A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$ show that $AB = BA = 0_{3 \times 3}$

Solution:

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

Given,

$$A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$$

Consider,

$$AB = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix} \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -2 - 3 + 5 & 6 + 9 - 15 & 5 + 15 - 20 \\ 1 + 4 - 5 & -3 - 12 + 15 & -5 - 15 + 20 \\ -1 - 3 + 4 & 3 + 9 - 12 & 5 + 15 - 20 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$AB = 0_{3 \times 3} \dots\dots(1)$$

Again consider,

$$BA = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix} \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} -2 - 3 + 5 & 3 + 12 - 15 & 5 + 15 - 20 \\ 2 + 3 - 5 & -3 - 12 + 15 & -5 - 15 + 20 \\ -2 - 3 + 5 & 3 + 9 - 12 & 5 + 15 - 20 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$BA = 0_{3 \times 3} \dots\dots(2)$$

From equation (1) and (2) $AB = BA = 0_{3 \times 3}$

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

13. If $A = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}$ and $B = \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$ show that $AB = BA = O_{3 \times 3}$

Solution:

Given

$$A = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$$

Consider,

$$AB = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix} \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 0 + abc - abc & 0 + b^2c - b^2c & 0 + bc^2 - bc^2 \\ -a^2c + 0 + a^2c & -abc + 0 + abc & -ac^2 + 0 + ac^2 \\ a^2b - a^2b + 0 & ab^2 - ab^2 + 0 & abc - abc + 0 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow AB = O_{3 \times 3} \dots (1)$$

Again consider,

$$BA = \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix} \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} 0 - abc + abc & a^2c + 0 - a^2c & -a^2b + a^2b + 0 \\ 0 - b^2c + b^2c & abc + 0 - abc & -ab^2 + ab^2 + 0 \\ 0 - bc^2 + bc^2 & ac^2 + 0 - ac^2 & -abc + abc + 0 \end{bmatrix}$$

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

$$\Rightarrow BA = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow BA = O_{3 \times 3} \dots (2)$$

From equation (1) and (2) $AB = BA = O_{3 \times 3}$

14. If $A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ show that $AB = A$ and $BA = B$.

Solution:

Given

$$A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

Now consider,

$$AB = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix} \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 + 3 - 5 & -4 - 9 + 10 & -8 - 12 + 15 \\ -2 - 4 + 5 & 2 + 12 - 10 & 4 + 16 - 15 \\ 2 + 3 - 4 & -2 - 9 + 18 & -4 - 12 + 12 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$$

Therefore $AB = A$

Again consider, BA we get,

$$BA = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$$

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

$$= \begin{bmatrix} 4 + 2 - 4 & -6 - 8 + 12 & -10 - 10 + 16 \\ -2 - 3 + 4 & 3 + 12 - 12 & 5 + 15 - 16 \\ 2 + 2 - 3 & -3 - 8 + 9 & -5 - 10 + 12 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

Hence $BA = B$

Hence the proof.

15. Let $A = \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & 5 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}$, compute $A^2 - B^2$.

Solution:

Given,

$$A = \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & 5 & 5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}$$

Consider,

$$A^2 = \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & 5 & 5 \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & 5 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 + 3 + 5 & -1 - 3 - 5 & 1 + 3 - 5 \\ -3 - 9 + 15 & 3 + 9 + 15 & -3 - 9 + 15 \\ -5 + 15 + 25 & 5 - 15 + 25 & -5 + 15 + 25 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} -1 & 9 & -1 \\ 3 & 27 & 3 \\ 35 & 15 & 35 \end{bmatrix} \dots\dots(1)$$

Now again consider, B^2

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

$$\begin{aligned}
 B^2 &= \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix} \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix} \\
 &= \begin{bmatrix} 0+4+3 & 0-12+12 & 0-12+12 \\ 0-3+3 & 4+9-12 & 3+9-12 \\ 0+4-4 & -4-12+16 & -3-12+16 \end{bmatrix} \\
 &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \dots\dots(2)
 \end{aligned}$$

Now by subtracting equation (2) from equation (1) we get,

$$\begin{aligned}
 A^2 - B^2 &= \begin{bmatrix} -1 & 9 & -1 \\ 3 & 27 & 3 \\ 35 & 15 & 35 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} -2 & 9 & -1 \\ 3 & 26 & 3 \\ 35 & 15 & 34 \end{bmatrix}
 \end{aligned}$$

16. For the following matrices verify the associativity of matrix multiplication i.e. (AB)C = A(BC)

$$(i) A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ -1 & 2 \\ 0 & 3 \end{bmatrix}, \text{ and } C = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$(ii) A = \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}, \text{ and } C = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution:

(i) Given

$$A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ -1 & 2 \\ 0 & 3 \end{bmatrix}, \text{ and } C = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Consider,

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

$$(AB)C = \left(\begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 2 \\ 0 & 3 \end{bmatrix} \right) \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 - 2 + 0 & 0 + 4 + 0 \\ -1 + 0 + 0 & 0 + 0 + 3 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 4 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 - 4 \\ -1 - 3 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} -5 \\ -4 \end{bmatrix} \dots\dots(1)$$

Now consider RHS,

$$A(BC) = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} 1 & 0 \\ -1 & 2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 + 0 \\ -1 - 2 \\ 0 - 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 - 6 + 0 \\ -1 + 0 - 3 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} -5 \\ -4 \end{bmatrix} \dots\dots(2)$$

From equation (1) and (2), it is clear that $(AB)C = A(BC)$

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

(ii) Given,

$$A = \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}, \text{ and } C = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Consider the LHS,

$$(AB)C = \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 + 0 + 6 & -4 + 2 - 3 & 4 + 4 + 3 \\ 1 + 0 + 4 & -1 + 1 - 2 & 1 + 2 + 2 \\ 3 + 0 + 2 & -3 + 0 - 1 & 3 + 0 + 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & -5 & 11 \\ 5 & -2 & 5 \\ 5 & -4 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 10 - 15 + 0 & 20 + 0 + 0 & -10 + 5 + 11 \\ 5 - 6 + 0 & 10 + 0 + 0 & -5 - 2 + 5 \\ 5 - 12 + 0 & 10 + 0 + 0 & -5 - 4 + 4 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} -5 & 20 & -4 \\ -1 & 10 & -2 \\ -7 & 10 & -5 \end{bmatrix} \dots (1)$$

Now consider RHS,

$$A(BC) = \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 - 3 + 0 & 2 + 0 + 0 & -1 - 1 + 1 \\ 0 + 3 + 0 & 0 + 0 + 0 & 0 + 1 + 2 \\ 2 - 3 + 0 & 4 + 0 + 0 & -2 - 1 + 1 \end{bmatrix}$$

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

$$= \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 & 2 & 1 \\ 3 & 0 & 3 \\ -1 & 4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -8 + 6 - 3 & 8 + 0 + 12 & -4 + 6 - 6 \\ -2 + 3 - 2 & 2 + 0 + 8 & -1 + 3 - 4 \\ -6 + 0 - 1 & 6 + 0 + 4 & -3 + 0 - 2 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} -5 & 20 & -4 \\ -1 & 10 & -2 \\ -7 & 10 & -5 \end{bmatrix} \dots\dots(2)$$

From equation (1) and (2), it is clear that $(AB)C = A(BC)$

17. For the following matrices verify the distributivity of matrix multiplication over matrix addition i.e. $A(B + C) = AB + AC$.

$$(i) A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}, B = \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}, \text{ and } C = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$$

$$(ii) A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, \text{ and } C = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

Solution:

(i) Given

$$A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}, B = \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}, \text{ and } C = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$$

Consider LHS,

$$A(B + C) = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \left(\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -1 + 0 & 0 + 1 \\ 2 + 1 & 1 - 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 3 & 0 \end{bmatrix}$$

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

$$= \begin{bmatrix} -1-3 & 1+0 \\ 0+6 & 0+0 \end{bmatrix}$$

$$A(B + C) = \begin{bmatrix} -4 & 1 \\ 6 & 0 \end{bmatrix} \dots\dots(1)$$

Now consider RHS,

$$AB + AC = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1-2 & 0-1 \\ 0+4 & 0+2 \end{bmatrix} + \begin{bmatrix} 0+-1 & 1+1 \\ 0+2 & 0-2 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -1 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 2 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -3-1 & -1+2 \\ 4+2 & 2-2 \end{bmatrix}$$

$$AB + AC = \begin{bmatrix} -4 & 1 \\ 6 & 0 \end{bmatrix} \dots\dots(2)$$

From equation (1) and (2), it is clear that $A(B + C) = AB + AC$

(ii) Given,

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, \text{ and } C = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

Consider the LHS

$$A(B + C) = \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 0+1 & 1-1 \\ 1+0 & 1+1 \end{bmatrix}$$

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

$$= \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2-1 & 0+2 \\ 1+1 & 0+2 \\ -1+2 & 0+4 \end{bmatrix}$$

$$A(B + C) = \begin{bmatrix} 1 & -2 \\ 2 & 2 \\ 1 & 4 \end{bmatrix} \dots\dots(1)$$

Now consider RHS,

$$AB + AC = \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0+1 & 2-1 \\ 0+1 & 1+1 \\ 0+2 & -1+2 \end{bmatrix} + \begin{bmatrix} 2+0 & -2-1 \\ 1+0 & -1+1 \\ -1+0 & 1+2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 1 \\ 1 & 2 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 2 & -3 \\ 1 & 0 \\ -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1+2 & 1-3 \\ 1+1 & 2+0 \\ 2-1 & 1+3 \end{bmatrix}$$

$$AB + AC = \begin{bmatrix} 1 & -2 \\ 2 & 2 \\ 1 & 4 \end{bmatrix} \dots\dots(2)$$

$$18. \text{ If } A = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix}, \text{ and } C = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix},$$

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

verify that $A(B - C) = AB - AC$.

Solution:

Given,

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

Consider the LHS,

$$A(B - C) = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix} \left(\begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 6 \\ -1 & 0 & 3 \\ -1 & 1 & 1 \end{bmatrix}$$

$$A(B - C) = \begin{bmatrix} 1 & -2 & -8 \\ -2 & 0 & -21 \\ 0 & 1 & 16 \end{bmatrix}$$

Now consider RHS

$$AB - AC = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 5 & -8 \\ 2 & 14 & -15 \\ -3 & -9 & 13 \end{bmatrix} - \begin{bmatrix} 1 & 7 & 0 \\ 4 & 14 & 6 \\ -3 & -10 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -2 & -8 \\ -2 & 0 & -21 \\ 0 & 1 & 16 \end{bmatrix}$$

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

From the above equations LHS = RHS

Therefore, $A(B - C) = AB - AC$.

19. Compute the elements a_{43} and a_{22} of the matrix:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 2 \\ 0 & 3 & 2 \\ 4 & 0 & 4 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -3 & 2 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 & 2 & -2 \\ 3 & -3 & 4 & -4 & 0 \end{bmatrix}$$

Solution:

Given

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 2 \\ 0 & 3 & 2 \\ 4 & 0 & 4 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -3 & 2 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 & 2 & -2 \\ 3 & -3 & 4 & -4 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} -3 & 2 \\ 12 & 4 \\ -1 & 12 \\ 24 & 8 \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 & 2 & -2 \\ 3 & -3 & 4 & -4 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 6 & -9 & 11 & -14 & 6 \\ 12 & 0 & 4 & 8 & -24 \\ 36 & -37 & 49 & -50 & 2 \\ 24 & 0 & 8 & 16 & -48 \end{bmatrix}$$

From the above matrix, $a_{43} = 8$ and $a_{22} = 0$

20. If $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix}$ and I is the identity matrix of order 3, that $A^3 =$
 $pI + qA + rA^2$

Solution:

Given

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix}$$

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

Consider,

$$A^2 = A \cdot A$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix}$$

$$= \begin{bmatrix} 0+0+0 & 0+0+0 & 0+1+0 \\ 0+0+p & 0+0+q & 0+0+r \\ 0+0+pr & p+0+qr & 0+q+r^2 \end{bmatrix}$$

Again consider,

$$A^3 = A^2 \cdot A$$

$$= \begin{bmatrix} 0+0+0 & 0+0+0 & 0+1+0 \\ 0+0+p & 0+0+q & 0+0+r \\ 0+0+pr & p+0+qr & 0+q+r^2 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix}$$

$$= \begin{bmatrix} 0+0+p & 0+0+q & 0+0+r \\ 0+0+pr & p+0+qr & 0+q+r^2 \\ 0+0+pq+pr^2 & pr+0+q^2+qr^2 & 0+p+qr+qr+r^2 \end{bmatrix}$$

$$= \begin{bmatrix} p & q & r \\ pr & p+qr & q+r^2 \\ pq+pr^2 & pr+q^2+qr^2 & p+2qr+r^2 \end{bmatrix}$$

Now, consider the RHS

$$pI + qA + rA^2$$

$$= p \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + q \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix} + r \begin{bmatrix} 0 & 0 & 1 \\ p & q & r \\ pr & p+qr & q+r^2 \end{bmatrix}$$

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

$$= \begin{bmatrix} p & q & r \\ pr & p + qr & q + r^2 \\ pq + pr^2 & pr + q^2 + qr^2 & p + 2qr + r^2 \end{bmatrix}$$

Therefore, $A^3 = pI + qA + rA^2$

Hence the proof.

21. If ω is a complex cube root of unity, show that

$$\left(\begin{bmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{bmatrix} + \begin{bmatrix} \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \\ \omega & \omega^2 & 1 \end{bmatrix} \right) \begin{bmatrix} 1 \\ \omega \\ \omega^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Solution:

Given

$$\left(\begin{bmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{bmatrix} + \begin{bmatrix} \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \\ \omega & \omega^2 & 1 \end{bmatrix} \right) \begin{bmatrix} 1 \\ \omega \\ \omega^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

It is also given that ω is a complex cube root of unity,

Consider the LHS,

$$= \begin{bmatrix} 1 + \omega & \omega + \omega^2 & \omega^2 + 1 \\ \omega + \omega^2 & \omega^2 + 1 & 1 + \omega \\ \omega^2 + \omega & 1 + \omega^2 & \omega + 1 \end{bmatrix} \begin{bmatrix} 1 \\ \omega \\ \omega^2 \end{bmatrix}$$

We know that $1 + \omega + \omega^2 = 0$ and $\omega^3 = 1$

$$= \begin{bmatrix} -\omega^2 & -1 & -\omega \\ -1 & -\omega & -\omega^2 \\ -1 & -\omega & -\omega^2 \end{bmatrix} \begin{bmatrix} 1 \\ \omega \\ \omega^2 \end{bmatrix}$$

Now by simplifying we get,

$$= \begin{bmatrix} -\omega^2 & -\omega & -\omega^3 \\ -1 & -\omega^2 & -\omega^4 \\ -1 & -\omega^2 & -\omega^4 \end{bmatrix}$$

Again by substituting $1 + \omega + \omega^2 = 0$ and $\omega^3 = 1$ in above matrix we get,

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Therefore LHS = RHS

Hence the proof.

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

22. If $A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$, show that $A^2 = A$

Solution:

Given,

$$A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$$

Consider A^2

$$A^2 = A.A$$

$$= \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix} \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 4 + 3 - 5 & -6 - 12 + 15 & -10 - 15 + 20 \\ -2 - 4 + 5 & 3 + 16 - 15 & 5 + 20 - 20 \\ 2 + 3 - 4 & -3 - 12 + 12 & -5 - 15 + 16 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix} = A$$

Therefore $A^2 = A$

23. If $A = \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}$, show that $A^2 = I_3$

Solution:

Given

$$A = \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}$$

Consider A^2 ,

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

$$A^2 = A.A$$

$$= \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix} \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 16 - 3 - 12 & -4 + 0 + 4 & 16 + 4 + 12 \\ 12 + 0 - 12 & -3 + 0 + 4 & -12 + 0 + 12 \\ 12 - 3 - 9 & -3 + 0 + 3 & -12 + 4 + 9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3$$

Hence $A^2 = I_3$

24. (i) If $\begin{bmatrix} 1 & 1 & x \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 0$, find x .

(ii) If $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$, find x .

Solution:

(i) Given

$$\begin{bmatrix} 1 & 1 & x \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\Rightarrow [1 + 2x + 0 \quad x + 0 + 2 \quad 2 + 1 + 0] \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow [2x + 4 \quad x + 2 \quad 2x + 4] \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 0$$

$$= [2x + 1 + 2 + x + 3] = 0$$

$$= [3x + 6] = 0$$

$$= 3x = -6$$

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

$$x = -6/3$$

$$x = -2$$

(ii) Given,

$$\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2-6 & -6+12 \\ 5-14 & -15+28 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -4 & 6 \\ -9 & 13 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$$

On comparing the above matrix we get,

$$x = 13$$

25. If $\begin{bmatrix} x & 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 2 \\ 1 & 0 & 2 \\ 0 & 2 & -4 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = 0$, find x .

Solution:

Given

$$\begin{bmatrix} x & 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 2 \\ 1 & 0 & 2 \\ 0 & 2 & -4 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix}$$

$$\Rightarrow [2x + 4 + 0 \quad x + 0 + 2 \quad 2x + 8 - 4] \begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = 0$$

$$\Rightarrow [2x + 4 \quad x + 2 \quad 2x + 4] \begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = 0$$

$$\Rightarrow [(2x + 4)x + 4(x + 2) - 1(2x + 4)] = 0$$

$$\Rightarrow 2x^2 + 4x + 4x + 8 - 2x - 4 = 0$$

$$\Rightarrow 2x^2 + 6x + 4 = 0$$

$$\Rightarrow 2x^2 + 2x + 4x + 4 = 0$$

$$\Rightarrow 2x(x + 1) + 4(x + 1) = 0$$

$$\Rightarrow (x + 1)(2x + 4) = 0$$

$$\Rightarrow x = -1 \text{ or } x = -2$$

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

Hence, $x = -1$ or $x = -2$

26. If $\begin{bmatrix} 1 & -1 & x \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 \\ 2 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = 0$, find x .

Solution:

Given

$$\begin{bmatrix} 1 & -1 & x \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 \\ 2 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = 0$$

By multiplying we get,

$$\Rightarrow [0 - 2 + x \quad x \quad (-1) - 3 + x] \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow [x - 2 \quad x \quad x - 4] \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = 0$$

$$[(x - 2) \times 0 + x \times 1 + (x - 4) \times 1] = 0$$

$$\Rightarrow x + x - 4 = 0$$

$$\Rightarrow 2x = 4 \Rightarrow x = 2$$

27. If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then prove that $A^2 - A + 2I = 0$.

Solution:

Given

$$A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \text{ and } I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Now we have to prove $A^2 - A + 2I = 0$

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

Now, we will find the matrix for A^2 , we get

$$A^2 = A \times A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 3 \times 3 + (-2 \times 4) & 3 \times (-2) + (-2 \times -2) \\ 4 \times 3 + (-2 \times 4) & 4 \times (-2) + (-2 \times -2) \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 9 - 8 & -6 + 4 \\ 12 - 8 & -8 + 4 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} \dots \dots \dots (i)$$

Now, we will find the matrix for $2I$, we get

$$2I = 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow 2I = \begin{bmatrix} 2 \times 1 & 2 \times 0 \\ 2 \times 0 & 2 \times 1 \end{bmatrix}$$

$$\Rightarrow 2I = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \dots \dots \dots (ii)$$

$$A^2 - A + 2I$$

Substitute corresponding values from eqn (i) and eqn (ii), we get

$$\Rightarrow = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} - \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\Rightarrow = \begin{bmatrix} 1 - 3 + 2 & -2 - (-2) + 0 \\ 4 - 4 + 0 & -4 - (-2) + 2 \end{bmatrix}$$

$$\Rightarrow = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

Therefore,

$$A^2 - A + 2I = 0$$

Hence proved

28. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then prove that $A^2 = 5A + \lambda I$.

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices**Solution:**

Given

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \text{ and } I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Now, we have to find A^2 ,

$$A^2 = A \times A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 3 \times 3 + (1 \times -1) & 3 \times 1 + 1 \times 2 \\ (-1 \times 3) + 2 \times (-1) & (-1 \times 1) + 2 \times 2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} \dots \dots \dots (i)$$

Now, we will find the matrix for $5A$, we get

$$5A = 5 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$\Rightarrow 5A = \begin{bmatrix} 5 \times 3 & 5 \times 1 \\ 5 \times (-1) & 5 \times 2 \end{bmatrix}$$

$$\Rightarrow 5A = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} \dots \dots \dots (ii)$$

So,

$$A^2 = 5A + \lambda I$$

Substitute corresponding values from eqn (i) and eqn (ii), we get

$$\Rightarrow \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

$$\begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} 15 + \lambda & 5 + 0 \\ -5 + 0 & 10 + \lambda \end{bmatrix}$$

And to satisfy the above condition of equality, the corresponding entries of the matrices should be equal,

Hence,

$$8 = 15 + \lambda \Rightarrow \lambda = -7$$

$$3 = 10 + \lambda \Rightarrow \lambda = -7$$

29. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ show that $A^2 - 5A + 7I_2 = 0$.

Solution:

Given

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

I_2 is an identity matrix of size 2, so

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

To show that

$$A^2 - 5A + 7I_2 = 0$$

Now, we will find the matrix for A^2 , we get

$$A^2 = A \times A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 3 \times 3 + (1 \times -1) & 3 \times 1 + 1 \times 2 \\ (-1 \times 3) + 2 \times (-1) & (-1 \times 1) + 2 \times 2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 9 - 1 & 3 + 2 \\ -3 - 2 & -1 + 4 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} \dots \dots \dots (i)$$

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

Now, we will find the matrix for $5A$, we get

$$5A = 5 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$\Rightarrow 5A = \begin{bmatrix} 5 \times 3 & 5 \times 1 \\ 5 \times (-1) & 5 \times 2 \end{bmatrix}$$

$$\Rightarrow 5A = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} \dots\dots\dots (ii)$$

Now,

$$7I_2 = 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} \dots\dots\dots (iii)$$

So,

$$A^2 - 5A + 7I_2$$

Substitute corresponding values from eqn (i), (ii) and (iii), we get

$$\Rightarrow = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$\Rightarrow = \begin{bmatrix} 8 - 15 + 7 & 5 - 5 + 0 \\ -5 - (-5) + 0 & 3 - 10 + 7 \end{bmatrix}$$

$$\Rightarrow = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

Hence the proof.

30. If $A = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$ show that $A^2 - 2A + 3I_2 = 0$.

Solution:

Given

$$A = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$$

I_2 is an identity matrix of size 2, so

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

Now we have to show,

$$A^2 - 2A + 3I_2 = 0$$

Now, we will find the matrix for A^2 , we get

$$\begin{aligned} A^2 &= A \times A = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} \\ \Rightarrow A^2 &= \begin{bmatrix} 2 \times 2 + (3 \times -1) & 2 \times 3 + 3 \times 0 \\ (-1 \times 2) + 0 \times (-1) & (-1 \times 3) + 0 \times 0 \end{bmatrix} \\ \Rightarrow A^2 &= \begin{bmatrix} 4 - 3 & 6 + 0 \\ -2 + 0 & -3 + 0 \end{bmatrix} \\ \Rightarrow A^2 &= \begin{bmatrix} 1 & 6 \\ -2 & -3 \end{bmatrix} \dots \dots \dots (i) \end{aligned}$$

Now, we will find the matrix for $2A$, we get

$$\begin{aligned} 2A &= 2 \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} \\ \Rightarrow 2A &= \begin{bmatrix} 2 \times 2 & 2 \times 3 \\ 2 \times (-1) & 2 \times 0 \end{bmatrix} \\ \Rightarrow 2A &= \begin{bmatrix} 4 & 6 \\ -2 & 0 \end{bmatrix} \dots \dots \dots (ii) \end{aligned}$$

Now,

$$3I_2 = 3 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \dots \dots \dots (iii)$$

So,

$$A^2 - 2A + 3I_2$$

Substitute corresponding values from eqn (i), (ii) and (iii), we get

$$\Rightarrow = \begin{bmatrix} 1 & 6 \\ -2 & -3 \end{bmatrix} - \begin{bmatrix} 4 & 6 \\ -2 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$\Rightarrow = \begin{bmatrix} 1 - 4 + 3 & 6 - 6 + 0 \\ -2 - (-2) + 0 & -3 - 0 + 3 \end{bmatrix}$$

$$\Rightarrow = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

Hence the proof.

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

31. Show that the matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ satisfies the equation $A^3 - 4A^2 + A = 0$.

Solution:

Given

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

To show that $A^3 - 4A^2 + A = 0$

Now, we will find the matrix for A^2 , we get

$$A^2 = (A \times A) = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 2 \times 2 + (3 \times 1) & 2 \times 3 + 3 \times 2 \\ 1 \times 2 + 2 \times 1 & 1 \times 3 + 2 \times 2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 4 + 3 & 6 + 6 \\ 2 + 2 & 3 + 4 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} \dots \dots \dots (i)$$

Now, we will find the matrix for A^3 , we get

$$A^3 = A^2 \times A = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$\Rightarrow A^3 = \begin{bmatrix} 7 \times 2 + 12 \times 1 & 7 \times 3 + 12 \times 2 \\ 4 \times 2 + 7 \times 1 & 4 \times 3 + 7 \times 2 \end{bmatrix}$$

$$\Rightarrow A^3 = \begin{bmatrix} 14 + 12 & 21 + 24 \\ 8 + 7 & 12 + 14 \end{bmatrix}$$

$$\Rightarrow A^3 = \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix} \dots \dots \dots (ii)$$

So,

$$A^3 - 4A^2 + A$$

Substitute corresponding values from eqn (i) and (ii), we get

$$\Rightarrow = \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix} - 4 \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

$$\begin{aligned}
 &\Rightarrow = \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix} - \begin{bmatrix} 4 \times 7 & 4 \times 12 \\ 4 \times 4 & 4 \times 7 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \\
 &\Rightarrow = \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix} - \begin{bmatrix} 28 & 48 \\ 16 & 28 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \\
 &\Rightarrow = \begin{bmatrix} 26 - 28 + 2 & 45 - 48 + 3 \\ 15 - 16 + 1 & 26 - 28 + 2 \end{bmatrix} \\
 &\Rightarrow = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0
 \end{aligned}$$

Therefore,

$$A^3 - 4A^2 + A = 0$$

Hence matrix A satisfies the given equation.

32. Show that the matrix $A = \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}$ satisfies the equation $A^2 - 12A - I = 0$.

Solution:

Given

$$A = \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}$$

I is an identity matrix so $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

To show that $A^2 - 12A - I = 0$

Now, we will find the matrix for A^2 , we get

$$A^2 = A \times A = \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix} \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 5 \times 5 + 3 \times 12 & 5 \times 3 + 3 \times 7 \\ 12 \times 5 + 7 \times 12 & 12 \times 3 + 7 \times 7 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 25 + 36 & 15 + 21 \\ 60 + 84 & 36 + 49 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 61 & 36 \\ 144 & 85 \end{bmatrix} \dots \dots \dots (i)$$

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

Now, we will find the matrix for $12A$, we get

$$12A = 12 \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}$$

$$\Rightarrow 12A = \begin{bmatrix} 12 \times 5 & 12 \times 3 \\ 12 \times 12 & 12 \times 7 \end{bmatrix}$$

$$\Rightarrow 12A = \begin{bmatrix} 60 & 36 \\ 144 & 84 \end{bmatrix} \dots \dots \dots (ii)$$

So,

$$A^2 - 12A - I$$

Substitute corresponding values from eqn (i) and (ii), we get

$$\Rightarrow = \begin{bmatrix} 61 & 36 \\ 144 & 85 \end{bmatrix} - \begin{bmatrix} 60 & 36 \\ 144 & 84 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow = \begin{bmatrix} 61 - 60 - 1 & 36 - 36 - 0 \\ 144 - 144 - 0 & 85 - 84 - 1 \end{bmatrix}$$

$$\Rightarrow = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

Therefore,

$$A^2 - 12A - I = 0$$

Hence matrix A is the root of the given equation.

33. If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$ find $A^2 - 5A - 14I$.

Solution:

Given

$$A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$$

I is identity matrix so

$$14I = 14 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$$

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

To find $A^2 - 5A - 14I$

Now, we will find the matrix for A^2 , we get

$$A^2 = A \times A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 3 \times 3 + (-5 \times -4) & 3 \times (-5) + (-5 \times 2) \\ (-4 \times 3) + (2 \times -4) & (-4 \times -5) + 2 \times 2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 9 + 20 & -15 - 10 \\ -12 - 8 & 20 + 4 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} \dots \dots \dots (i)$$

Now, we will find the matrix for $5A$, we get

$$5A = 5 \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$$

$$\Rightarrow 5A = \begin{bmatrix} 5 \times 3 & 5 \times (-5) \\ 5 \times (-4) & 5 \times 2 \end{bmatrix}$$

$$\Rightarrow 5A = \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix} \dots \dots \dots (ii)$$

So,

$$A^2 - 5A - 14I$$

Substitute corresponding values from eqn (i) and (ii), we get

$$\Rightarrow = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} - \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$$

$$\Rightarrow = \begin{bmatrix} 29 - 15 - 14 & -25 + 25 - 0 \\ -20 + 20 - 0 & 24 - 10 - 14 \end{bmatrix}$$

$$\Rightarrow = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

34. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ show that $A^2 - 5A + 7I = 0$. Use this to find A^4 .

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices**Solution:**

Given

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

I is identity matrix so

$$7I = 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

To show that $A^2 - 5A + 7I = 0$ Now, we will find the matrix for A^2 , we get

$$A^2 = A \times A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 3 \times 3 + (1 \times -1) & 3 \times 1 + 1 \times 2 \\ (-1 \times 3) + (2 \times -1) & (-1 \times 1) + 2 \times 2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 9 - 1 & 3 + 2 \\ -3 - 2 & -1 + 4 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} \dots \dots \dots (i)$$

Now, we will find the matrix for $5A$, we get

$$5A = 5 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$\Rightarrow 5A = \begin{bmatrix} 5 \times 3 & 5 \times 1 \\ 5 \times (-1) & 5 \times 2 \end{bmatrix}$$

$$\Rightarrow 5A = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} \dots \dots \dots (ii)$$

So,

$$A^2 - 5A + 7I$$

Substitute corresponding values from eqn (i) and (ii), we get

$$\Rightarrow = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} - \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

$$\Rightarrow = \begin{bmatrix} 8 - 15 - 7 & 5 - 5 - 0 \\ -5 + 5 - 0 & 3 - 10 - 7 \end{bmatrix}$$

$$\Rightarrow = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

Therefore

$$A^2 - 5A + 7I = 0$$

Hence proved

We will find A^4

$$A^2 - 5A + 7I = 0$$

Multiply both sides by A^2 , we get

$$A^2(A^2 - 5A + 7I) = A^2(0)$$

$$\Rightarrow A^4 - 5A^2.A + 7I.A^2$$

$$\Rightarrow A^4 = 5A^2.A - 7I.A^2$$

$$\Rightarrow A^4 = 5A^2A - 7A^2$$

As multiplying by the identity matrix, I don't change anything. Now will substitute the corresponding values we get

$$\Rightarrow A^4 = 5 \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - 7 \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$\Rightarrow A^4 = 5 \begin{bmatrix} 24 - 5 & 8 + 10 \\ -15 - 3 & -5 + 6 \end{bmatrix} - 7 \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$\Rightarrow A^4 = 5 \begin{bmatrix} 19 & 18 \\ -18 & 1 \end{bmatrix} - 7 \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$\Rightarrow A^4 = \begin{bmatrix} 5 \times 19 & 5 \times 18 \\ 5 \times (-18) & 5 \times 1 \end{bmatrix} - \begin{bmatrix} 7 \times 8 & 7 \times 5 \\ 7 \times (-5) & 7 \times 3 \end{bmatrix}$$

$$\Rightarrow A^4 = \begin{bmatrix} 95 & 90 \\ -90 & 5 \end{bmatrix} - \begin{bmatrix} 56 & 35 \\ -35 & 21 \end{bmatrix}$$

$$\Rightarrow A^4 = \begin{bmatrix} 95 - 56 & 90 - 35 \\ -90 + 35 & 5 - 21 \end{bmatrix}$$

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

$$\Rightarrow A^4 = \begin{bmatrix} 39 & 55 \\ -55 & -16 \end{bmatrix}$$

35. If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ find k such that $A^2 = kA - 2I_2$.

Solution:

Given

$$A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

I_2 is an identity matrix of size 2, so

$$2I_2 = 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

Also given,

$$A^2 = kA - 2I_2$$

Now, we will find the matrix for A^2 , we get

$$A^2 = A \times A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 3 \times 3 + (-2 \times 4) & 3 \times (-2) + (-2 \times -2) \\ (4 \times 3) + (-2 \times 4) & (4 \times -2) + (-2 \times -2) \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 9 - 8 & -6 + 4 \\ 12 - 8 & -8 + 4 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} \dots \dots \dots (i)$$

Now, we will find the matrix for kA , we get

$$kA = k \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$\Rightarrow kA = \begin{bmatrix} k \times 3 & k \times (-2) \\ k \times 4 & k \times (-2) \end{bmatrix}$$

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

So,

$$A^2 = kA - 2I_2$$

Substitute corresponding values from eqn (i) and (ii), we get

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k-0 \\ 4k-0 & -2k-2 \end{bmatrix}$$

And to satisfy the above condition of equality, the corresponding entries of the matrices should be equal

$$\text{Hence, } 3k - 2 = 1 \Rightarrow k = 1$$

Therefore, the value of k is 1

36. If $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$ find k such that $A^2 - 8A + kI = 0$.

Solution:

Given

$$A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$$

I is identity matrix, so

$$kI = k \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$$

$$\text{Also given, } A^2 - 8A + kI = 0$$

Now, we have to find A^2 , we get

$$A^2 = A \times A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1 \times 1 + 0 & 0 + 0 \\ (-1 \times 1) + 7 \times (-1) & 0 + 7 \times 7 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} \dots \dots \dots (i)$$

Now, we will find the matrix for $8A$, we get

$$8A = 8 \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$$

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

So,

$$A^2 = kA - 2I_2$$

Substitute corresponding values from eqn (i) and (ii), we get

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k-0 \\ 4k-0 & -2k-2 \end{bmatrix}$$

And to satisfy the above condition of equality, the corresponding entries of the matrices should be equal

$$\text{Hence, } 3k - 2 = 1 \Rightarrow k = 1$$

Therefore, the value of k is 1

36. If $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$ find k such that $A^2 - 8A + kI = 0$.

Solution:

Given

$$A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$$

I is identity matrix, so

$$kI = k \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$$

$$\text{Also given, } A^2 - 8A + kI = 0$$

Now, we have to find A^2 , we get

$$A^2 = A \times A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1 \times 1 + 0 & 0 + 0 \\ (-1 \times 1) + 7 \times (-1) & 0 + 7 \times 7 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} \dots \dots \dots (i)$$

Now, we will find the matrix for $8A$, we get

$$8A = 8 \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$$

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

$$\Rightarrow 8A = \begin{bmatrix} 8 \times 1 & 8 \times 0 \\ 8 \times (-1) & 8 \times 7 \end{bmatrix}$$

$$\Rightarrow 8A = \begin{bmatrix} 8 & 0 \\ -8 & 56 \end{bmatrix} \dots \dots \dots (ii)$$

So,
 $A^2 - 8A + kI = 0$

Substitute corresponding values from eqn (i) and (ii), we get

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ -8 & 56 \end{bmatrix} + \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 1 - 8 + k & 0 - 0 + 0 \\ -8 + 8 + 0 & 49 - 56 + k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

And to satisfy the above condition of equality, the corresponding entries of the matrices should be equal

Hence,

$$1 - 8 + k = 0 \Rightarrow k = 7$$

Therefore, the value of k is 7

37. If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ and $f(x) = x^2 - 2x - 3$ show that $f(A) = 0$

Solution:

Given

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

To show that $f(A) = 0$

Substitute $x = A$ in $f(x)$, we get

$$f(A) = A^2 - 2A - 3I \dots \dots \dots (i)$$

I is identity matrix, so

$$3I = 3 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

Now, we will find the matrix for A^2 , we get

$$A^2 = A \times A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1 \times 1 + 2 \times 2 & 1 \times 2 + 2 \times 1 \\ 2 \times 1 + 1 \times 2 & 2 \times 2 + 1 \times 1 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1 + 4 & 2 + 2 \\ 2 + 2 & 4 + 1 \end{bmatrix}$$

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

$$\Rightarrow A^2 = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} \dots \dots \dots (ii)$$

Now, we will find the matrix for $2A$, we get

$$2A = 2 \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

$$\Rightarrow 2A = \begin{bmatrix} 2 \times 1 & 2 \times 2 \\ 2 \times 2 & 2 \times 1 \end{bmatrix}$$

$$\Rightarrow 2A = \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} \dots \dots \dots (iii)$$

Substitute corresponding values from eqn (ii) and (iii) in eqn (i), we get

$$f(A) = A^2 - 2A - 3I$$

$$\Rightarrow f(A) = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$\Rightarrow f(A) = \begin{bmatrix} 5-2-3 & 4-4-0 \\ 4-4-0 & 5-2-3 \end{bmatrix}$$

$$\Rightarrow f(A) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

So,

$$\Rightarrow f(A) = 0$$

Hence Proved

38. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ then find λ, μ so that $A^2 = \lambda A + \mu I$

Solution:

Given

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \text{ and } I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

So

$$\mu I = \mu \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \mu & 0 \\ 0 & \mu \end{bmatrix}$$

Now, we will find the matrix for A^2 , we get

$$A^2 = A \times A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 2 \times 2 + 3 \times 1 & 2 \times 3 + 3 \times 2 \\ 1 \times 2 + 2 \times 1 & 1 \times 3 + 2 \times 2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} \dots \dots \dots (i)$$

Now, we will find the matrix for λA , we get

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

$$\lambda A = \lambda \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$\Rightarrow \lambda A = \begin{bmatrix} \lambda \times 2 & \lambda \times 3 \\ \lambda \times 1 & \lambda \times 2 \end{bmatrix}$$

$$\Rightarrow \lambda A = \begin{bmatrix} 2\lambda & 3\lambda \\ \lambda & 2\lambda \end{bmatrix} \dots \dots \dots (ii)$$

But given, $A^2 = \lambda A + \mu I$

Substitute corresponding values from equation (i) and (ii), we get

$$\Rightarrow \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} = \begin{bmatrix} 2\lambda & 3\lambda \\ \lambda & 2\lambda \end{bmatrix} + \begin{bmatrix} \mu & 0 \\ 0 & \mu \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} = \begin{bmatrix} 2\lambda + \mu & 3\lambda + 0 \\ \lambda + 0 & 2\lambda + \mu \end{bmatrix}$$

And to satisfy the above condition of equality, the corresponding entries of the matrices should be equal

$$\text{Hence, } \lambda + 0 = 4 \Rightarrow \lambda = 4$$

$$\text{And also, } 2\lambda + \mu = 7$$

Substituting the obtained value of λ in the above equation, we get

$$2(4) + \mu = 7 \Rightarrow 8 + \mu = 7 \Rightarrow \mu = -1$$

Therefore, the value of λ and μ are 4 and -1 respectively

39. Find the value of x for which the matrix product

$$\begin{bmatrix} 2 & 0 & 7 \\ 0 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} -x & 14x & 7x \\ 0 & 1 & 0 \\ x & -4x & -2x \end{bmatrix} \text{ equal to an identity matrix.}$$

Solution:

We know,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

is identity matrix of size 3.

So according to the given criteria

$$\begin{bmatrix} 2 & 0 & 7 \\ 0 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} -x & 14x & 7x \\ 0 & 1 & 0 \\ x & -4x & -2x \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now we will multiply the two matrices on LHS using the formula $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \dots + a_{in}b_{nj}$, we get

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

$$\begin{aligned}
 & \begin{bmatrix} 2 \times (-x) + 0 + 7 \times x & 2 \times 14x + 0 + 7 \times (-4x) & 2 \times 7x + 0 + 7 \times (-2x) \\ 0 + 0 + 0 & 0 + 1 \times 1 + 0 & 0 + 0 + 0 \\ 1 \times (-x) + 0 + 1 \times x & 1 \times 14x + (-2 \times 1) + (1 \times -4x) & 1 \times 7x + 0 + 1 \times (-2x) \end{bmatrix} \\
 &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
 &\Rightarrow \begin{bmatrix} 5x & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 10x - 2 & 5x \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
 \end{aligned}$$

And to satisfy the above condition of equality, the corresponding entries of the matrices should be equal

So we get

$$5x = 1 \Rightarrow x = \frac{1}{5}$$

So the value of x is $\frac{1}{5}$

EXERCISE 5.4

PAGE NO: 5.54

1. Let $A = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$ verify that

(i) $(2A)^T = 2A^T$

(ii) $(A + B)^T = A^T + B^T$

(iii) $(A - B)^T = A^T - B^T$

(iv) $(AB)^T = B^T A^T$

Solution:

(i) Given

$$A = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$$

Consider,

$$(2A)^T = 2A^T$$

Put the value of A

$$\Rightarrow \left(2 \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix} \right)^T = 2 \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 4 & -6 \\ -14 & 10 \end{bmatrix}^T = 2 \begin{bmatrix} 2 & -7 \\ -3 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & -14 \\ -6 & 10 \end{bmatrix} = \begin{bmatrix} 4 & -14 \\ -6 & 10 \end{bmatrix}$$

L.H.S = R.H.S

(ii) Given

$$A = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$$

Consider,

$$(A + B)^T = A^T + B^T$$

$$\Rightarrow \left(\begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix} \right)^T = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}^T + \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 2+1 & -3+0 \\ -7+2 & 5-4 \end{bmatrix}^T = \begin{bmatrix} 2 & -7 \\ -3 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 0 & -4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 & -3 \\ -5 & 1 \end{bmatrix}^T = \begin{bmatrix} 3 & -5 \\ -3 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 & -5 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -3 & 1 \end{bmatrix}$$

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

$$\text{L.H.S} = \text{R.H.S}$$

Hence proved.

(iii) Given

$$A = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$$

Consider,

$$(A - B)^T = A^T - B^T$$

$$\Rightarrow \left(\begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix} \right)^T = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}^T - \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 2-1 & -3-0 \\ -7-2 & 5+4 \end{bmatrix}^T = \begin{bmatrix} 2 & -7 \\ -3 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 0 & -4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -3 \\ -9 & 9 \end{bmatrix}^T = \begin{bmatrix} 1 & -9 \\ -3 & 9 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -9 \\ -3 & 9 \end{bmatrix} = \begin{bmatrix} 1 & -9 \\ -3 & 9 \end{bmatrix}$$

$$\text{L.H.S} = \text{R.H.S}$$

(iv) Given

$$A = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$$

$$(AB)^T = B^T A^T$$

$$\Rightarrow \left(\begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix} \right)^T = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}^T \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}^T$$

$$\begin{bmatrix} 2-6 & 0+12 \\ -7+10 & 0-20 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 \\ 0 & -4 \end{bmatrix} \begin{bmatrix} 2 & -7 \\ -3 & 5 \end{bmatrix}$$

$$\begin{bmatrix} -4 & 12 \\ 3 & -20 \end{bmatrix}^T = \begin{bmatrix} 2-6 & -7+10 \\ 0+12 & 0-20 \end{bmatrix}$$

$$\begin{bmatrix} -4 & 3 \\ 12 & -20 \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ 12 & -20 \end{bmatrix}$$

So,

$$(AB)^T = B^T A^T$$

$$2. A = \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 & 4 \end{bmatrix} \text{ verify that } (AB)^T = B^T A^T$$

Solution:

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

Given

$$A = \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} \text{ and } B = [1 \ 0 \ 4]$$

$$(AB)^T = B^T A^T$$

$$\Rightarrow \left(\begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} [1 \ 0 \ 4] \right)^T = [1 \ 0 \ 4]^T \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 3 & 0 & 12 \\ 5 & 0 & 20 \\ 2 & 0 & 8 \end{bmatrix}^T = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix} [3 \ 5 \ 2]$$

$$\Rightarrow \begin{bmatrix} 3 & 5 & 2 \\ 0 & 0 & 0 \\ 12 & 20 & 8 \end{bmatrix} = \begin{bmatrix} 3 & 5 & 2 \\ 0 & 0 & 0 \\ 12 & 20 & 8 \end{bmatrix}$$

L.H.S = R.H.S

So, $(AB)^T = B^T A^T$

3. Let $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$ find A^T, B^T and verify that

(i) $(A + B)^T = A^T + B^T$

(ii) $(AB)^T = B^T A^T$

(iii) $(2A)^T = 2 A^T$

Solution:

(i) Given

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

Consider,

$$(A + B)^T = A^T + B^T$$

$$\left(\begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} \right)^T = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}^T + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}^T$$

$$\left(\begin{bmatrix} 1+1 & -1+2 & 0+3 \\ 2+2 & 1+1 & 3+3 \\ 1+0 & 2+1 & 1+1 \end{bmatrix} \right)^T = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 0 & 3 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \\ 3 & 3 & 1 \end{bmatrix}$$

RD Sharma Solutions for Class 12 Maths Chapter 5
Algebra of Matrices

$$\begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & 6 \\ 1 & 3 & 2 \end{bmatrix}^T = \begin{bmatrix} 1+1 & 2+2 & 1+0 \\ -1+2 & 1+1 & 2+1 \\ 0+3 & 3+3 & 1+1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 & 1 \\ 1 & 2 & 3 \\ 3 & 6 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 1 \\ 1 & 2 & 3 \\ 3 & 6 & 2 \end{bmatrix}$$

L.H.S = R.H.S

So, $(A+B)^T = A^T + B^T$

(ii) Given

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

Consider,

$$(AB)^T = B^T A^T$$

$$\left(\begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} \right)^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}^T \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}^T$$

$$\begin{bmatrix} 1-2+0 & 2-1+0 & 3-3+0 \\ 2+2+0 & 4+1+3 & 6+3+3 \\ 1+4+0 & 2+2+1 & 3+6+1 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \\ 3 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 0 & 3 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & 0 \\ 4 & 8 & 12 \\ 5 & 5 & 10 \end{bmatrix}^T = \begin{bmatrix} 1-2+0 & 2+2+0 & 1+4+0 \\ 2-1+0 & 4+1+3 & 2+2+1 \\ 3-3+0 & 6+3+3 & 3+6+1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 4 & 5 \\ 1 & 8 & 5 \\ 0 & 12 & 10 \end{bmatrix} = \begin{bmatrix} -1 & 4 & 5 \\ 1 & 8 & 5 \\ 0 & 12 & 10 \end{bmatrix}$$

L.H.S = R.H.S

So, $(AB)^T = B^T A^T$

(iii) Given

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

Consider,

$$(2A)^T = 2A^T$$

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Algebra of Matrices

$$\Rightarrow \left(2 \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix} \right)^T = 2 \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 2 & -2 & 0 \\ 4 & 2 & 6 \\ 2 & 4 & 2 \end{bmatrix}^T = 2 \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 0 & 3 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 4 & 2 \\ -2 & 2 & 4 \\ 0 & 6 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 2 \\ -2 & 2 & 4 \\ 0 & 6 & 2 \end{bmatrix}$$

L.H.S = R.H.S

So,

$$(2A)^T = 2A^T$$

4. If $A = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}$, $B = [1 \ 3 \ -6]$, verify that $(AB)^T = B^T A^T$

Solution:

Given

$$A = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}, B = [1 \ 3 \ -6]$$

Consider,

$$(AB)^T = B^T A^T$$

$$\Rightarrow \left(\begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix} [1 \ 3 \ -6] \right)^T = [1 \ 3 \ -6]^T \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} -2 & -6 & -12 \\ 4 & 12 & -24 \\ 5 & 15 & -30 \end{bmatrix}^T = \begin{bmatrix} 1 \\ 3 \\ -6 \end{bmatrix}^T [-2 \ 4 \ 5]$$

$$\Rightarrow \begin{bmatrix} -2 & 4 & 5 \\ -6 & 12 & 15 \\ -12 & -24 & -30 \end{bmatrix} = \begin{bmatrix} -2 & 4 & 5 \\ -6 & 12 & 15 \\ -12 & -24 & -30 \end{bmatrix}$$

L.H.S = R.H.S

So,

$$(AB)^T = B^T A^T$$

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Algebra of Matrices

5. If $A = \begin{bmatrix} 2 & 4 & -1 \\ -1 & 0 & 2 \end{bmatrix}$, $\begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 2 & 1 \end{bmatrix}$, find $(AB)^T$

Solution:

Given

$$A = \begin{bmatrix} 2 & 4 & -1 \\ -1 & 0 & 2 \end{bmatrix}, \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 2 & 1 \end{bmatrix}$$

Now we have to find $(AB)^T$

$$\begin{aligned} &\Rightarrow \left(\begin{bmatrix} 2 & 4 & -1 \\ -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 2 & 1 \end{bmatrix} \right)^T \\ &\Rightarrow \begin{bmatrix} 6 - 4 - 2 & 8 + 8 - 1 \\ -3 - 0 + 4 & -4 + 0 + 2 \end{bmatrix}^T \\ &\Rightarrow \begin{bmatrix} 0 & 15 \\ 1 & -2 \end{bmatrix}^T \\ &\Rightarrow \begin{bmatrix} 0 & 1 \\ 15 & -2 \end{bmatrix} \end{aligned}$$

So,

$$(AB)^T = \begin{bmatrix} 0 & 1 \\ 15 & -2 \end{bmatrix}$$

EXERCISE 5.5

PAGE NO: 5.60

1. If $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, prove that $A - A^T$ is a skew-symmetric matrix.

Solution:

Given

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$$

Consider,

$$(A - A^T) = \left(\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}^T \right)$$

$$= \left(\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 2-2 & 3-4 \\ 4-3 & 5-5 \end{bmatrix}$$

$$(A - A^T) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \dots (i)$$

$$-(A - A^T)^T = -\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}^T$$

$$= -\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$-(A - A^T) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \dots (ii)$$

From (i) and (ii) we can see that

A skew-symmetric matrix is a square matrix whose transpose equal to its negative, that is,

$$X = -X^T$$

So, $A - A^T$ is a skew-symmetric.

2. If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, show that $A - A^T$ is a skew-symmetric matrix.

Solution:

Given

$$A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

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Algebra of Matrices

Consider,

$$(A - A^T) = \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix} \dots (i)$$

$$-(A - A^T)^T = -\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}^T$$

$$= -\begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix}$$

$$-(A - A^T) = \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix} \dots (ii)$$

From (i) and (ii) we can see that

A skew-symmetric matrix is a square matrix whose transpose equals its negative, that is,

$$X = -X^T$$

So, $A - A^T$ is a skew-symmetric matrix.

3. If the matrix $A = \begin{bmatrix} 5 & 2 & x \\ y & z & -3 \\ 4 & t & -7 \end{bmatrix}$, is a symmetric matrix find x, y, z and t

Solution:

Given,

$$A = \begin{bmatrix} 5 & 2 & x \\ y & z & -3 \\ 4 & t & -7 \end{bmatrix} \text{ is a symmetric matrix.}$$

We know that $A = [a_{ij}]_{m \times n}$ is a symmetric matrix if $a_{ij} = a_{ji}$

So,

$$x = a_{13} = a_{31} = 4$$

$$y = a_{21} = a_{12} = 2$$

$$z = a_{22} = a_{22} = z$$

$$t = a_{32} = a_{23} = -3$$

Hence, $x = 4, y = 2, t = -3$ and z can have any value.

$$A = \begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 3 \\ -2 & 5 & 8 \end{bmatrix}$$

4. Let $A = \begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 3 \\ -2 & 5 & 8 \end{bmatrix}$. Find matrices X and Y such that $X + Y = A$, where X is a symmetric and Y is a skew-symmetric matrix.

Solution:

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Algebra of Matrices

Given, $A = \begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 3 \\ -2 & 5 & 8 \end{bmatrix}$ Then $A^T = \begin{bmatrix} 3 & 1 & -2 \\ 2 & 4 & 5 \\ 7 & 3 & 8 \end{bmatrix}$

$$\begin{aligned} X &= \frac{1}{2}(A + A^T) \\ &= \frac{1}{2} \left(\begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 3 \\ -2 & 5 & 8 \end{bmatrix} + \begin{bmatrix} 3 & 1 & -2 \\ 2 & 4 & 5 \\ 7 & 3 & 8 \end{bmatrix} \right) \\ &= \frac{1}{2} \begin{bmatrix} 3+3 & 2+1 & 7-2 \\ 1+2 & 4+4 & 3+5 \\ -2+7 & 5+3 & 8+8 \end{bmatrix} \\ &= \frac{1}{2} \begin{bmatrix} 6 & 3 & 5 \\ 3 & 8 & 8 \\ 5 & 8 & 16 \end{bmatrix} \\ X &= \begin{bmatrix} 3 & \frac{3}{2} & \frac{5}{2} \\ \frac{3}{2} & 4 & 4 \\ \frac{5}{2} & 4 & 8 \end{bmatrix} \end{aligned}$$

Now,

$$\begin{aligned} Y &= \frac{1}{2}(A - A^T) \\ &= \frac{1}{2} \left(\begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 3 \\ -2 & 5 & 8 \end{bmatrix} - \begin{bmatrix} 3 & 1 & -2 \\ 2 & 4 & 5 \\ 7 & 3 & 8 \end{bmatrix} \right) \\ &= \frac{1}{2} \begin{bmatrix} 3-3 & 2-1 & 7+2 \\ 1-2 & 4-4 & 3-5 \\ -2-7 & 5-3 & 8-8 \end{bmatrix} \\ &= \frac{1}{2} \begin{bmatrix} 0 & 1 & 9 \\ -1 & 0 & -2 \\ -9 & 2 & 0 \end{bmatrix} \\ Y &= \begin{bmatrix} 0 & \frac{1}{2} & \frac{9}{2} \\ -\frac{1}{2} & 0 & -1 \\ -\frac{9}{2} & 1 & 0 \end{bmatrix} \end{aligned}$$

Now,

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Algebra of Matrices

$$X^T = \begin{bmatrix} 3 & \frac{3}{2} & \frac{5}{2} \\ \frac{3}{2} & 4 & 4 \\ \frac{5}{2} & 4 & 8 \end{bmatrix}^T = \begin{bmatrix} 3 & \frac{3}{2} & \frac{5}{2} \\ \frac{3}{2} & 4 & 4 \\ \frac{5}{2} & 4 & 8 \end{bmatrix} = X$$

$\Rightarrow X$ is a symmetric matrix.

Now,

$$-Y^T = -\begin{bmatrix} 0 & \frac{1}{2} & \frac{9}{2} \\ -\frac{1}{2} & 0 & -1 \\ \frac{9}{2} & 1 & 0 \end{bmatrix}^T = -\begin{bmatrix} 0 & -\frac{1}{2} & -\frac{9}{2} \\ \frac{1}{2} & 0 & 1 \\ \frac{9}{2} & -1 & 0 \end{bmatrix}$$

$$-Y^T = \begin{bmatrix} 0 & \frac{1}{2} & \frac{9}{2} \\ -\frac{1}{2} & 0 & -1 \\ -\frac{9}{2} & 1 & 0 \end{bmatrix}$$

$$-Y^T = Y$$

Y is a skew symmetric matrix.

And,

$$X + Y = \begin{bmatrix} 3 & \frac{3}{2} & \frac{5}{2} \\ \frac{3}{2} & 4 & 4 \\ \frac{5}{2} & 4 & 8 \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{2} & \frac{9}{2} \\ -\frac{1}{2} & 0 & -1 \\ -\frac{9}{2} & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3+0 & \frac{3}{2}+\frac{1}{2} & \frac{5}{2}+\frac{9}{2} \\ \frac{3}{2}-\frac{1}{2} & 4+0 & 4-1 \\ \frac{5}{2}-\frac{9}{2} & 4+1 & 8+0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 3 \\ -2 & 5 & 8 \end{bmatrix} = A$$

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Hence, $X + Y = A$

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