

RD Sharma Solutions for Class 12 Maths Chapter 16
Tangents and Normals

EXERCISE 16.1

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1. Find the Slopes of the tangent and the normal to the following curves at the indicated points:

(i) $y = \sqrt{x^3}$ at $x = 4$

Solution:

Given $y = \sqrt{x^3}$ at $x = 4$

First, we have to find $\frac{dy}{dx}$ of given function, $f(x)$ that is to find the derivative of $f(x)$

$$y = \sqrt{x^3}$$

$$\therefore \sqrt[n]{x} = x^{\frac{1}{n}}$$

$$\Rightarrow y = (x^3)^{\frac{1}{2}}$$

$$\Rightarrow y = (x)^{\frac{3}{2}}$$

$$\therefore \frac{dy}{dx}(x^n) = n x^{n-1}$$

We know that the Slope of the tangent is $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{3}{2}(x)^{\frac{3}{2}-1}$$

$$\Rightarrow \frac{dy}{dx} = \frac{3}{2}(x)^{\frac{1}{2}}$$

Since, $x = 4$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=4} = \frac{3}{2}(4)^{\frac{1}{2}}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=4} = \frac{3}{2} \times \sqrt{4}$$

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$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=4} = \frac{3}{2} \times 2$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=4} = 3$$

The Slope of the tangent at $x = 4$ is 3

$$\Rightarrow \text{The Slope of the normal} = \frac{-1}{\text{The Slope of the tangent}}$$

$$\Rightarrow \text{The Slope of the normal} = \frac{-1}{\left(\frac{dy}{dx}\right)_{x=4}}$$

$$\Rightarrow \text{The Slope of the normal} = \frac{-1}{3}$$

(ii) $y = \sqrt{x}$ at $x = 9$

Solution:

Given $y = \sqrt{x}$ at $x = 9$

First, we have to find $\frac{dy}{dx}$ of given function, $f(x)$ that is to find the derivative of $f(x)$

$$\Rightarrow y = \sqrt{x}$$

$$\therefore \sqrt[n]{x} = x^{\frac{1}{n}}$$

$$\Rightarrow y = (x)^{\frac{1}{2}}$$

$$\therefore \frac{dy}{dx}(x^n) = n x^{n-1}$$

The Slope of the tangent is $\frac{dy}{dx}$

$$\Rightarrow y = (x)^{\frac{1}{2}}$$

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$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}(x)^{\frac{1}{2}-1}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}(x)^{-\frac{1}{2}}$$

Since, $x = 9$

$$\left(\frac{dy}{dx}\right)_{x=9} = \frac{1}{2}(9)^{-\frac{1}{2}}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=9} = \frac{1}{2} \times \frac{1}{(9)^{\frac{1}{2}}}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=9} = \frac{1}{2} \times \frac{1}{\sqrt{9}}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=9} = \frac{1}{2} \times \frac{1}{3}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=9} = \frac{1}{6}$$

The Slope of the tangent at $x = 9$ is $\frac{1}{6}$

$$\Rightarrow \text{The Slope of the normal} = \frac{-1}{\text{The Slope of the tangent}}$$

$$\Rightarrow \text{The Slope of the normal} = \frac{-1}{\left(\frac{dy}{dx}\right)_{x=9}}$$

$$\Rightarrow \text{The Slope of the normal} = \frac{-1}{\frac{1}{6}}$$

$$\Rightarrow \text{The Slope of the normal} = -6$$

(iii) $y = x^3 - x$ at $x = 2$

Solution:

First, we have to find $\frac{dy}{dx}$ of given function $f(x)$ that is to find the derivative of $f(x)$

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$$\therefore \frac{dy}{dx}(x^n) = n x^{n-1}$$

The Slope of the tangent is $\frac{dy}{dx}$

$$\Rightarrow y = x^3 - x$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy}{dx}(x^3) + 3 \times \frac{dy}{dx}(x)$$

$$\Rightarrow \frac{dy}{dx} = 3 \cdot x^{3-1} - 1 \cdot x^{1-0}$$

$$\Rightarrow \frac{dy}{dx} = 3x^2 - 1$$

Since, $x = 2$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=2} = 3 \times (2)^2 - 1$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=2} = (3 \times 4) - 1$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=2} = 12 - 1$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=2} = 11$$

\therefore The Slope of the tangent at $x = 2$ is 11

$$\Rightarrow \text{The Slope of the normal} = \frac{-1}{\text{The Slope of the tangent}}$$

$$\Rightarrow \text{The Slope of the normal} = \frac{-1}{\left(\frac{dy}{dx}\right)_{x=2}}$$

$$\Rightarrow \text{The Slope of the normal} = \frac{-1}{11}$$

(iv) $y = 2x^2 + 3 \sin x$ at $x = 0$

Solution:

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Given $y = 2x^2 + 3\sin x$ at $x = 0$

First, we have to find $\frac{dy}{dx}$ of given function $f(x)$ that is to find the derivative of $f(x)$

$$\therefore \frac{dy}{dx}(x^n) = n x^{n-1}$$

The Slope of the tangent is $\frac{dy}{dx}$

$$\Rightarrow y = 2x^2 + 3\sin x$$

$$\Rightarrow \frac{dy}{dx} = 2 \frac{dy}{dx}(x^2) + 3 \frac{dy}{dx}(\sin x)$$

$$\Rightarrow \frac{dy}{dx} = 2(2x^{2-1}) + 3(\cos x)$$

$$\therefore \frac{d}{dx}(\sin x) = \cos x$$

$$\Rightarrow \frac{dy}{dx} = 4x + 3\cos x$$

Since, $x = 0$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=0} = 4(0) + 3\cos(0)$$

We know $\cos(0) = 1$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=0} = 0 + 3(1)$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=0} = 3$$

\therefore The Slope of the tangent at $x = 0$ is 3

$$\Rightarrow \text{The Slope of the normal} = \frac{-1}{\text{The Slope of the tangent}}$$

$$\Rightarrow \text{The Slope of the normal} = \frac{-1}{\left(\frac{dy}{dx}\right)_{x=0}}$$

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$$\Rightarrow \text{The Slope of the normal} = \frac{-1}{3}$$

$$(v) x = a (\theta - \sin \theta), y = a (1 + \cos \theta) \text{ at } \theta = -\pi / 2$$

Solution:

$$\text{Given } x = a (\theta - \sin \theta), y = a (1 + \cos \theta) \text{ at } \theta = -\pi / 2$$

Here, to find $\frac{dy}{dx}$, we have to find $\frac{dy}{d\theta}$ & $\frac{dx}{d\theta}$ and divide $\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$ and we get our desired $\frac{dy}{dx}$.

$$\therefore \frac{dy}{dx}(x^n) = n x^{n-1}$$

$$\Rightarrow x = a (\theta - \sin \theta)$$

$$\Rightarrow \frac{dx}{d\theta} = a \left\{ \frac{dx}{d\theta}(\theta) - \frac{dx}{d\theta}(\sin \theta) \right\}$$

$$\Rightarrow \frac{dx}{d\theta} = a (1 - \cos \theta) \dots (1)$$

$$\therefore \frac{d}{dx}(\sin x) = \cos x$$

$$\Rightarrow y = a (1 + \cos \theta)$$

$$\Rightarrow \frac{dy}{d\theta} = a \left[\frac{dx}{d\theta}(1) + \frac{dx}{d\theta}(\cos \theta) \right]$$

$$\therefore \frac{d}{dx}(\cos x) = -\sin x$$

$$\therefore \frac{d}{dx}(\text{Constant}) = 0$$

$$\Rightarrow \frac{dy}{d\theta} = a (0 + (-\sin \theta))$$

$$\Rightarrow \frac{dy}{d\theta} = a (-\sin \theta)$$

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$$\Rightarrow \frac{dy}{d\theta} = -a \sin \theta \dots (2)$$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-a \sin \theta}{a(1 - \cos \theta)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-\sin \theta}{(1 - \cos \theta)}$$

The Slope of the tangent is $\frac{-\sin \theta}{(1 - \cos \theta)}$

Since, $\theta = \frac{-\pi}{2}$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta = \frac{-\pi}{2}} = \frac{-\sin \frac{-\pi}{2}}{(1 - \cos \frac{-\pi}{2})}$$

We know $\cos(\pi/2) = 0$ and $\sin(\pi/2) = 1$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta = \frac{-\pi}{2}} = \frac{-(-1)}{(1 - (-0))}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta = \frac{-\pi}{2}} = \frac{1}{(1 - 0)}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta = \frac{-\pi}{2}} = 1$$

∴ The Slope of the tangent at $x = \frac{-\pi}{2}$ is 1

$$\Rightarrow \text{The Slope of the normal} = \frac{-1}{\text{The Slope of the tangent}}$$

$$\Rightarrow \text{The Slope of the normal} = \frac{-1}{\left(\frac{dy}{dx}\right)_{\theta = \frac{-\pi}{2}}}$$

$$\Rightarrow \text{The Slope of the normal} = \frac{-1}{1}$$

$$\Rightarrow \text{The Slope of the normal} = -1$$

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Tangents and Normals(vi) $x = a \cos^3 \theta$, $y = a \sin^3 \theta$ at $\theta = \pi/4$ **Solution:**Given $x = a \cos^3 \theta$, $y = a \sin^3 \theta$ at $\theta = \pi/4$ Here, to find $\frac{dy}{dx}$, we have to find $\frac{dy}{d\theta}$ & $\frac{dx}{d\theta}$ and divide $\frac{dy}{d\theta}$ and we get $\frac{dy}{dx}$.

$$\therefore \frac{dy}{dx}(x^n) = n x^{n-1}$$

$$\Rightarrow x = a \cos^3 \theta$$

$$\Rightarrow \frac{dx}{d\theta} = a \left(\frac{d}{d\theta}(\cos^3 \theta) \right)$$

$$\therefore \frac{d}{dx}(\cos x) = -\sin x$$

$$\Rightarrow \frac{dx}{d\theta} = a (3\cos^{3-1} \theta \times -\sin \theta)$$

$$\Rightarrow \frac{dx}{d\theta} = a (3\cos^2 \theta \times -\sin \theta)$$

$$\Rightarrow \frac{dx}{d\theta} = -3a\cos^2 \theta \sin \theta \dots (1)$$

$$\Rightarrow y = a \sin^3 \theta$$

$$\Rightarrow \frac{dy}{d\theta} = a \left(\frac{d}{d\theta}(\sin^3 \theta) \right)$$

$$\therefore \frac{d}{dx}(\sin x) = \cos x$$

$$\Rightarrow \frac{dy}{d\theta} = a (3\sin^{3-1} \theta \cos \theta)$$

$$\Rightarrow \frac{dy}{d\theta} = a (3\sin^2 \theta \cos \theta)$$

$$\Rightarrow \frac{dy}{d\theta} = 3 a \sin^2 \theta \cos \theta \dots (2)$$

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$$\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-3a\cos^2\theta\sin\theta}{3a\sin^2\theta\cos\theta}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-\cos\theta}{\sin\theta}$$

$$\Rightarrow \frac{dy}{dx} = -\tan\theta$$

The Slope of the tangent is $-\tan\theta$

Since, $\theta = \pi/4$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{4}} = -\tan(\pi/4)$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{4}} = -1$$

We know $\tan(\pi/4) = 1$

The Slope of the tangent at $x = \pi/4$ is -1

$$\Rightarrow \text{The Slope of the normal} = \frac{-1}{\text{The Slope of the tangent}}$$

$$\Rightarrow \text{The Slope of the normal} = \frac{-1}{\left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{4}}} = \frac{-1}{-1}$$

$$\Rightarrow \text{The Slope of the normal} = \frac{-1}{-1}$$

$$\Rightarrow \text{The Slope of the normal} = 1$$

(vii) $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ at $\theta = \pi/2$

Solution:

Given $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ at $\theta = \pi/2$

Here, to find $\frac{dy}{dx}$, we have to find $\frac{dy}{d\theta}$ & $\frac{dx}{d\theta}$ and divide $\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$ and we get $\frac{dy}{dx}$.

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$$\therefore \frac{dy}{dx}(x^n) = n x^{n-1}$$

$$\Rightarrow x = a(\theta - \sin \theta)$$

$$\Rightarrow \frac{dx}{d\theta} = a \left\{ \frac{dx}{d\theta}(\theta) - \frac{dx}{d\theta}(\sin \theta) \right\}$$

$$\Rightarrow \frac{dx}{d\theta} = a(1 - \cos \theta) \dots (1)$$

$$\therefore \frac{d}{dx}(\sin x) = \cos x$$

$$\Rightarrow y = a(1 - \cos \theta)$$

$$\Rightarrow \frac{dy}{d\theta} = a \left(\frac{dy}{d\theta}(1) - \frac{dy}{d\theta}(\cos \theta) \right)$$

$$\therefore \frac{d}{dx}(\cos x) = -\sin x$$

$$\therefore \frac{d}{dx}(\text{Constant}) = 0$$

$$\Rightarrow \frac{dy}{d\theta} = a(\theta - (-\sin \theta))$$

$$\Rightarrow \frac{dy}{d\theta} = a \sin \theta \dots (2)$$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a \sin \theta}{a(1 - \cos \theta)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-\sin \theta}{(1 - \cos \theta)}$$

$$\text{The Slope of the tangent is } \frac{-\sin \theta}{(1 - \cos \theta)}$$

$$\text{Since, } \theta = \frac{\pi}{2}$$

$$\Rightarrow \left(\frac{dy}{dx} \right)_{\theta = \frac{\pi}{2}} = \frac{\sin \frac{\pi}{2}}{(1 - \cos \frac{\pi}{2})}$$

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We know $\cos(\pi/2) = 0$ and $\sin(\pi/2) = 1$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{2}} = \frac{(1)}{(1-(-0))}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{2}} = \frac{1}{(1-0)}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{2}} = 1$$

The Slope of the tangent at $x = \frac{\pi}{2}$ is 1

$$\Rightarrow \text{The Slope of the normal} = \frac{-1}{\text{The Slope of the tangent}}$$

$$\Rightarrow \text{The Slope of the normal} = \frac{-1}{\left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{2}}}$$

$$\Rightarrow \text{The Slope of the normal} = \frac{-1}{1}$$

$$\Rightarrow \text{The Slope of the normal} = -1$$

(viii) $y = (\sin 2x + \cot x + 2)^2$ at $x = \pi/2$

Solution:

Given $y = (\sin 2x + \cot x + 2)^2$ at $x = \pi/2$

First, we have to find $\frac{dy}{dx}$ of given function $f(x)$ that is to find the derivative of $f(x)$

$$\therefore \frac{dy}{dx}(x^n) = n x^{n-1}$$

The Slope of the tangent is $\frac{dy}{dx}$

$$\Rightarrow y = (\sin 2x + \cot x + 2)^2$$

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$$\frac{dy}{dx} = 2 \times (\sin 2x + \cot x + 2)^{2-1} \left\{ \frac{dy}{dx}(\sin 2x) + \frac{dy}{dx}(\cot x) + \frac{dy}{dx}(2) \right\}$$

$$\Rightarrow \frac{dy}{dx} = 2(\sin 2x + \cot x + 2) \{(\cos 2x) \times 2 + (-\operatorname{cosec}^2 x) + (0)\}$$

$$\therefore \frac{d}{dx}(\sin x) = \cos x$$

$$\therefore \frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$$

$$\Rightarrow \frac{dy}{dx} = 2(\sin 2x + \cot x + 2)(2 \cos 2x - \operatorname{cosec}^2 x)$$

Since, $x = \pi/2$

$$\left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{2}} = 2 \times (\sin 2(\pi/2) + \cot(\pi/2) + 2)(2 \cos 2(\pi/2) - \operatorname{cosec}^2(\pi/2))$$

$$\left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{2}} = 2 \times (\sin(\pi) + \cot(\pi/2) + 2) \times (2 \cos(\pi) - \operatorname{cosec}^2(\pi/2))$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{2}} = 2 \times (0 + 0 + 2) \times (2(-1) - 1)$$

We know $\sin(\pi) = 0$, $\cos(\pi) = -1$

$\cot(\pi/2) = 0$, $\operatorname{cosec}(\pi/2) = 1$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{2}} = 2(2) \times (-2 - 1)$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{2}} = 4 \times -3$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{2}} = -12$$

The Slope of the tangent at $x = \frac{\pi}{2}$ is -12

$$\Rightarrow \text{The Slope of the normal} = \frac{-1}{\text{The Slope of the tangent}}$$

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$$\Rightarrow \text{The Slope of the normal} = \frac{-1}{\left(\frac{dy}{dx}\right)_{\theta = \frac{\pi}{2}}}$$

$$\Rightarrow \text{The Slope of the normal} = \frac{-1}{-12}$$

$$\Rightarrow \text{The Slope of the normal} = \frac{1}{12}$$

$$\text{(ix) } x^2 + 3y + y^2 = 5 \text{ at } (1, 1)$$

Solution:

$$\text{Given } x^2 + 3y + y^2 = 5 \text{ at } (1, 1)$$

Here we have to differentiate the above equation with respect to x.

$$\Rightarrow \frac{d}{dx}(x^2 + 3y + y^2) = \frac{d}{dx}(5)$$

$$\Rightarrow \frac{d}{dx}(x^2) + \frac{d}{dx}(3y) + \frac{d}{dx}(y^2) = \frac{d}{dx}(5)$$

$$\therefore \frac{dy}{dx}(x^n) = n x^{n-1}$$

$$\Rightarrow 2x + 3 \times \frac{dy}{dx} + 2y \times \frac{dy}{dx} = 0$$

$$\Rightarrow 2x + \frac{dy}{dx}(3 + 2y) = 0$$

$$\Rightarrow \frac{dy}{dx}(3 + 2y) = -2x$$

$$\Rightarrow \frac{dy}{dx} = \frac{-2x}{(3 + 2y)}$$

The Slope of the tangent at (1, 1) is

$$\Rightarrow \frac{dy}{dx} = \frac{-2 \times 1}{(3 + 2 \times 1)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-2}{(3 + 2)}$$

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$$\Rightarrow \frac{dy}{dx} = \frac{-2}{5}$$

The Slope of the tangent at (1, 1) is $\frac{-2}{5}$

$$\Rightarrow \text{The Slope of the normal} = \frac{-1}{\text{The Slope of the tangent}}$$

$$\Rightarrow \text{The Slope of the normal} = \frac{-1}{\left(\frac{dy}{dx}\right)}$$

$$\Rightarrow \text{The Slope of the normal} = \frac{-1}{\frac{-2}{5}}$$

$$\Rightarrow \text{The Slope of the normal} = \frac{5}{2}$$

(x) x y = 6 at (1, 6)

Solution:

Given $xy = 6$ at (1, 6)

Here we have to use the product rule for above equation, then we get

$$\frac{d}{dx}(xy) = \frac{d}{dx}(6)$$

$$\Rightarrow x \times \frac{d}{dx}(y) + y \frac{d}{dx}(x) = \frac{d}{dx}(6)$$

$$\therefore \frac{d}{dx}(\text{Constant}) = 0$$

$$\Rightarrow x \frac{dy}{dx} + y = 0$$

$$\Rightarrow x \frac{dy}{dx} = -y$$

$$\Rightarrow \frac{dy}{dx} = \frac{-y}{x}$$

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The Slope of the tangent at (1, 6) is

$$\Rightarrow \frac{dy}{dx} = \frac{-6}{1}$$

$$\Rightarrow \frac{dy}{dx} = -6$$

The Slope of the tangent at (1, 6) is -6

$$\Rightarrow \text{The Slope of the normal} = \frac{-1}{\text{The Slope of the tangent}}$$

$$\Rightarrow \text{The Slope of the normal} = \frac{-1}{\left(\frac{dy}{dx}\right)}$$

$$\Rightarrow \text{The Slope of the normal} = \frac{-1}{-6}$$

$$\Rightarrow \text{The Slope of the normal} = \frac{1}{6}$$

2. Find the values of a and b if the Slope of the tangent to the curve $xy + ax + by = 2$ at (1, 1) is 2.

Solution:

Given the Slope of the tangent to the curve $xy + ax + by = 2$ at (1, 1) is 2

First, we will find The Slope of tangent by using product rule, we get

$$\Rightarrow xy + ax + by = 2$$

$$\Rightarrow x \frac{d}{dx}(y) + y \frac{d}{dx}(x) + a \frac{d}{dx}(x) + b \frac{d}{dx}(y) = \frac{d}{dx}(2)$$

$$\Rightarrow x \frac{dy}{dx} + y + a + b \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx}(x + b) + y + a = 0$$

$$\Rightarrow \frac{dy}{dx}(x + b) = -(a + y)$$

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$$\Rightarrow \frac{dy}{dx} = \frac{-(a+y)}{x+b}$$

Since, the Slope of the tangent to the curve $xy + ax + by = 2$ at $(1, 1)$ is 2 that is,

$$\frac{dy}{dx} = 2$$

$$\Rightarrow \left\{ \frac{-(a+y)}{x+b} \right\}_{(x=1, y=1)} = 2$$

$$\Rightarrow \frac{-(a+1)}{1+b} = 2$$

$$\Rightarrow -a - 1 = 2(1+b)$$

$$\Rightarrow -a - 1 = 2 + 2b$$

$$\Rightarrow a + 2b = -3 \dots (1)$$

Also, the point $(1, 1)$ lies on the curve $xy + ax + by = 2$, we have

$$1 \times 1 + a \times 1 + b \times 1 = 2$$

$$\Rightarrow 1 + a + b = 2$$

$$\Rightarrow a + b = 1 \dots (2)$$

From (1) & (2), we get $b = -4$

Substitute $b = -4$ in $a + b = 1$

$$a - 4 = 1$$

$$\Rightarrow a = 5$$

So the value of $a = 5$ & $b = -4$

3. If the tangent to the curve $y = x^3 + ax + b$ at $(1, -6)$ is parallel to the line $x - y + 5 = 0$, find a and b

Solution:

Given the Slope of the tangent to the curve $y = x^3 + ax + b$ at $(1, -6)$

First, we will find the slope of tangent

$$y = x^3 + ax + b$$

$$\frac{dy}{dx} = \frac{d}{dx}(x^3) + \frac{d}{dx}(ax) + \frac{d}{dx}(b)$$

$$\Rightarrow \frac{dy}{dx} = 3x^{3-1} + a \left(\frac{dx}{dx} \right) + 0$$

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$$\Rightarrow \frac{dy}{dx} = 3x^2 + a$$

The Slope of the tangent to the curve $y = x^3 + ax + b$ at $(1, -6)$ is

$$\Rightarrow \frac{dy}{dx}(x=1, y=-6) = 3(1)^2 + a$$

$$\Rightarrow \frac{dy}{dx}(x=1, y=-6) = 3 + a \dots (1)$$

The given line is $x - y + 5 = 0$

$y = x + 5$ is the form of equation of a straight line $y = mx + c$, where m is the Slope of the line.

So the slope of the line is $y = 1 \times x + 5$

So the Slope is 1. ... (2)

Also the point $(1, -6)$ lie on the tangent, so

$x = 1$ & $y = -6$ satisfies the equation, $y = x^3 + ax + b$

$$-6 = 1^3 + a \times 1 + b$$

$$\Rightarrow -6 = 1 + a + b$$

$$\Rightarrow a + b = -7 \dots (3)$$

Since, the tangent is parallel to the line, from (1) & (2)

$$\text{Hence, } 3 + a = 1$$

$$\Rightarrow a = -2$$

From (3)

$$a + b = -7$$

$$\Rightarrow -2 + b = -7$$

$$\Rightarrow b = -5$$

So the value is $a = -2$ & $b = -5$

4. Find a point on the curve $y = x^3 - 3x$ where the tangent is parallel to the chord joining $(1, -2)$ and $(2, 2)$.

Solution:

Given curve $y = x^3 - 3x$

First, we will find the Slope of the tangent

$$y = x^3 - 3x$$

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$$\frac{dy}{dx} = \frac{d}{dx}(x^3) - \frac{d}{dx}(3x)$$

$$\Rightarrow \frac{dy}{dx} = 3x^{3-1} - 3\left(\frac{dx}{dx}\right)$$

$$\Rightarrow \frac{dy}{dx} = 3x^2 - 3 \dots (1)$$

The equation of line passing through (x_0, y_0) and The Slope m is $y - y_0 = m(x - x_0)$.

So the Slope, $m = \frac{y - y_0}{x - x_0}$

The Slope of the chord joining $(1, -2)$ & $(2, 2)$

$$\Rightarrow \frac{dy}{dx} = \frac{2 - (-2)}{2 - 1}$$

$$\Rightarrow \frac{dy}{dx} = \frac{4}{1}$$

$$\Rightarrow \frac{dy}{dx} = 4 \dots (2)$$

From (1) & (2)

$$3x^2 - 3 = 4$$

$$\Rightarrow 3x^2 = 7$$

$$\Rightarrow x^2 = \frac{7}{3}$$

$$\Rightarrow x = \pm \sqrt{\frac{7}{3}}$$

$$y = x^3 - 3x$$

$$\Rightarrow y = x(x^2 - 3)$$

$$\Rightarrow y = \pm \sqrt{\frac{7}{3}} \left(\left(\pm \sqrt{\frac{7}{3}} \right)^2 - 3 \right)$$

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$$\Rightarrow y = \pm \sqrt{\frac{7}{3}} \left(\frac{7}{3} - 3 \right)$$

$$\Rightarrow y = \pm \sqrt{\frac{7}{3}} \left(\frac{-2}{3} \right)$$

$$\Rightarrow y = \mp \left(\frac{-2}{3} \right) \sqrt{\frac{7}{3}}$$

$$\text{Thus, the required point is } x = \pm \sqrt{\frac{7}{3}} \text{ \& } y = \mp \left(\frac{-2}{3} \right) \sqrt{\frac{7}{3}}$$

5. Find a point on the curve $y = x^3 - 2x^2 - 2x$ at which the tangent lines are parallel to the line $y = 2x - 3$.

Solution:

Given the curve $y = x^3 - 2x^2 - 2x$ and a line $y = 2x - 3$

First, we will find the slope of tangent

$$y = x^3 - 2x^2 - 2x$$

$$\frac{dy}{dx} = \frac{d}{dx}(x^3) - \frac{d}{dx}(2x^2) - \frac{d}{dx}(2x)$$

$$\Rightarrow \frac{dy}{dx} = 3x^{3-1} - 2 \times 2 (x^{2-1}) - 2 \times x^{1-1}$$

$$\Rightarrow \frac{dy}{dx} = 3x^2 - 4x - 2 \dots (1)$$

$y = 2x - 3$ is the form of equation of a straight line $y = mx + c$, where m is the Slope of the line.

So the slope of the line is $y = 2 \times (x) - 3$

Thus, the Slope = 2. ... (2)

From (1) & (2)

$$\Rightarrow 3x^2 - 4x - 2 = 2$$

$$\Rightarrow 3x^2 - 4x = 4$$

$$\Rightarrow 3x^2 - 4x - 4 = 0$$

We will use factorization method to solve the above Quadratic equation.

$$\Rightarrow 3x^2 - 6x + 2x - 4 = 0$$

$$\Rightarrow 3x(x-2) + 2(x-2) = 0$$

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$$\Rightarrow (x - 2)(3x + 2) = 0$$

$$\Rightarrow (x - 2) = 0 \text{ \& } (3x + 2) = 0$$

$$\Rightarrow x = 2 \text{ or}$$

$$x = -2/3$$

$$\text{Substitute } x = 2 \text{ \& } x = -2/3 \text{ in } y = x^3 - 2x^2 - 2x$$

$$\text{When } x = 2$$

$$\Rightarrow y = (2)^3 - 2 \times (2)^2 - 2 \times (2)$$

$$\Rightarrow y = 8 - (2 \times 4) - 4$$

$$\Rightarrow y = 8 - 8 - 4$$

$$\Rightarrow y = -4$$

$$\text{When } x = \frac{-2}{3}$$

$$\Rightarrow y = \left(\frac{-2}{3}\right)^3 - 2 \times \left(\frac{-2}{3}\right)^2 - 2 \times \left(\frac{-2}{3}\right)$$

$$\Rightarrow y = \left(\frac{-8}{27}\right) - 2 \times \left(\frac{4}{9}\right) + \left(\frac{4}{3}\right)$$

$$\Rightarrow y = \left(\frac{-8}{27}\right) - \left(\frac{8}{9}\right) + \left(\frac{4}{3}\right)$$

Taking LCM

$$\Rightarrow y = \frac{(-8 \times 1) - (8 \times 3) + (4 \times 9)}{27}$$

$$\Rightarrow y = \frac{-8 - 24 + 36}{27}$$

$$\Rightarrow y = \frac{4}{27}$$

$$\text{Thus, the points are } (2, -4) \text{ \& } \left(\frac{-2}{3}, \frac{4}{27}\right)$$

6. Find a point on the curve $y^2 = 2x^3$ at which the Slope of the tangent is 3

Solution:

Given the curve $y^2 = 2x^3$ and the Slope of tangent is 3

$$y^2 = 2x^3$$

Differentiating the above with respect to x

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$$\Rightarrow 2y^{2-1} \frac{dy}{dx} = 2 \times 3x^{3-1}$$

$$\Rightarrow y \frac{dy}{dx} = 3x^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{3x^2}{y}$$

Since, The Slope of tangent is 3

$$\frac{3x^2}{y} = 3$$

$$\Rightarrow \frac{x^2}{y} = 1$$

$$\Rightarrow x^2 = y$$

Substituting $x^2 = y$ in $y^2 = 2x^3$,

$$(x^2)^2 = 2x^3$$

$$x^4 - 2x^3 = 0$$

$$x^3(x - 2) = 0$$

$$x^3 = 0 \text{ or } (x - 2) = 0$$

$$x = 0 \text{ or } x = 2$$

If $x = 0$

$$\Rightarrow \frac{dy}{dx} = \frac{3(0)^2}{y}$$

$dy/dx = 0$ which is not possible.

So we take $x = 2$ and substitute it in $y^2 = 2x^3$, we get

$$y^2 = 2(2)^3$$

$$y^2 = 2 \times 8$$

$$y^2 = 16$$

$$y = 4$$

Thus, the required point is (2, 4)

7. Find a point on the curve $xy + 4 = 0$ at which the tangents are inclined at an angle of

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45° with the x-axis.

Solution:

Given the curve is $xy + 4 = 0$

If a tangent line to the curve $y = f(x)$ makes an angle θ with x – axis in the positive direction, then

$\frac{dy}{dx}$ = The Slope of the tangent = $\tan \theta$

$$xy + 4 = 0$$

Differentiating the above with respect to x

$$\Rightarrow x \frac{d}{dx}(y) + y \frac{d}{dx}(x) + \frac{d}{dx}(4) = 0$$

$$\Rightarrow x \frac{dy}{dx} + y = 0$$

$$\Rightarrow x \frac{dy}{dx} = -y$$

$$\Rightarrow \frac{dy}{dx} = \frac{-y}{x} \dots (1)$$

$$\text{Also, } \frac{dy}{dx} = \tan 45^\circ = 1 \dots (2)$$

From (1) & (2), we get,

$$\Rightarrow \frac{-y}{x} = 1$$

$$\Rightarrow x = -y$$

Substitute in $xy + 4 = 0$, we get

$$\Rightarrow x(-x) + 4 = 0$$

$$\Rightarrow -x^2 + 4 = 0$$

$$\Rightarrow x^2 = 4$$

$$\Rightarrow x = \pm 2$$

So when $x = 2$, $y = -2$

And when $x = -2$, $y = 2$

Thus, the points are $(2, -2)$ & $(-2, 2)$

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8. Find a point on the curve $y = x^2$ where the Slope of the tangent is equal to the x – coordinate of the point.

Solution:

Given the curve is $y = x^2$

$$y = x^2$$

Differentiating the above with respect to x

$$\Rightarrow \frac{dy}{dx} = 2x^{2-1}$$

$$\Rightarrow \frac{dy}{dx} = 2x \dots (1)$$

Also given the Slope of the tangent is equal to the x – coordinate,

$$\frac{dy}{dx} = x \dots (2)$$

From (1) & (2), we get,

$$2x = x$$

$$\Rightarrow x = 0.$$

Substituting this in $y = x^2$, we get,

$$y = 0^2$$

$$\Rightarrow y = 0$$

Thus, the required point is $(0, 0)$

9. At what point on the circle $x^2 + y^2 - 2x - 4y + 1 = 0$, the tangent is parallel to x – axis.

Solution:

Given the curve is $x^2 + y^2 - 2x - 4y + 1 = 0$

Differentiating the above with respect to x

$$\Rightarrow x^2 + y^2 - 2x - 4y + 1 = 0$$

$$\Rightarrow 2x^{2-1} + 2y^{2-1} \times \frac{dy}{dx} - 2 - 4 \times \frac{dy}{dx} + 0 = 0$$

$$\Rightarrow 2x + 2y \frac{dy}{dx} - 2 - 4 \frac{dy}{dx} = 0$$

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$$\Rightarrow \frac{dy}{dx}(2y - 4) = -2x + 2$$

$$\Rightarrow \frac{dy}{dx} = \frac{-2(x-1)}{2(y-2)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-(x-1)}{(y-2)} \dots (1)$$

$$\therefore \frac{dy}{dx} = \text{The Slope of the tangent} = \tan \theta$$

Since, the tangent is parallel to x – axis

$$\Rightarrow \frac{dy}{dx} = \tan (0) = 0 \dots (2)$$

Because $\tan (0) = 0$

From (1) & (2), we get,

$$\Rightarrow \frac{-(x-1)}{(y-2)} = 0$$

$$\Rightarrow -(x-1) = 0$$

$$\Rightarrow x = 1$$

Substituting $x = 1$ in $x^2 + y^2 - 2x - 4y + 1 = 0$, we get,

$$\Rightarrow 1^2 + y^2 - 2(1) - 4y + 1 = 0$$

$$\Rightarrow 1 - y^2 - 2 - 4y + 1 = 0$$

$$\Rightarrow y^2 - 4y = 0$$

$$\Rightarrow y(y-4) = 0$$

$$\Rightarrow y = 0 \text{ and } y = 4$$

Thus, the required point is (1, 0) and (1, 4)

10. At what point of the curve $y = x^2$ does the tangent make an angle of 45° with the x-axis?

Solution:

Given the curve is $y = x^2$

Differentiating the above with respect to x

$$\Rightarrow y = x^2$$

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$$\Rightarrow \frac{dy}{dx} = 2x^{2-1}$$

$$\Rightarrow \frac{dy}{dx} = 2x \dots (1)$$

$$\therefore \frac{dy}{dx} = \text{The Slope of the tangent} = \tan \theta$$

Since, the tangent make an angle of 45° with x – axis

$$\Rightarrow \frac{dy}{dx} = \tan (45^\circ) = 1 \dots (2)$$

Because $\tan (45^\circ) = 1$

From (1) & (2), we get,

$$\Rightarrow 2x = 1$$

$$\Rightarrow x = \frac{1}{2}$$

Substituting $x = \frac{1}{2}$ in $y = x^2$, we get,

$$\Rightarrow y = \left(\frac{1}{2}\right)^2$$

$$\Rightarrow y = \frac{1}{4}$$

Thus, the required point is $\left(\frac{1}{2}, \frac{1}{4}\right)$

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EXERCISE 16.2

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1. Find the equation of the tangent to the curve $\sqrt{x} + \sqrt{y} = a$, at the point $(a^2/4, a^2/4)$.**Solution:**

$$\text{Given } \sqrt{x} + \sqrt{y} = a$$

To find the slope of the tangent of the given curve we have to differentiate the given equation

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \left(\frac{dy}{dx} \right) = 0$$

$$\frac{dy}{dx} = -\frac{\sqrt{x}}{\sqrt{y}}$$

At $\left(\frac{a^2}{4}, \frac{a^2}{4}\right)$ slope m , is -1

The equation of the tangent is given by $y - y_1 = m(x - x_1)$

$$y - \frac{a^2}{4} = -1 \left(x - \frac{a^2}{4} \right)$$

$$x + y = \frac{a^2}{2}$$

2. Find the equation of the normal to $y = 2x^3 - x^2 + 3$ at $(1, 4)$.**Solution:**

$$\text{Given } y = 2x^3 - x^2 + 3$$

By differentiating the given curve, we get the slope of the tangent

$$m = \frac{dy}{dx} = 6x^2 - 2x$$

$$m = 4 \text{ at } (1, 4)$$

Normal is perpendicular to tangent so, $m_1 m_2 = -1$

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$$m(\text{normal}) = -\frac{1}{4}$$

Equation of normal is given by $y - y_1 = m(\text{normal})(x - x_1)$

$$y - 4 = \left(-\frac{1}{4}\right)(x - 1)$$

$$x + 4y = 17$$

3. Find the equation of the tangent and the normal to the following curves at the indicated points:

(i) $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at $(0, 5)$

Solution:

Given $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at $(0, 5)$

By differentiating the given curve, we get the slope of the tangent

$$\frac{dy}{dx} = 4x^3 - 18x^2 + 26x - 10$$

$$m(\text{tangent}) \text{ at } (0, 5) = -10$$

$$m(\text{normal}) \text{ at } (0, 5) = \frac{1}{10}$$

Equation of tangent is given by $y - y_1 = m(\text{tangent})(x - x_1)$

$$y - 5 = -10x$$

$$y + 10x = 5$$

Equation of normal is given by $y - y_1 = m(\text{normal})(x - x_1)$

$$y - 5 = \frac{1}{10}x$$

(ii) $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at $x = 1, y = 3$

Solution:

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Given $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at $x = 1$ $y = 3$

By differentiating the given curve, we get the slope of the tangent

$$\frac{dy}{dx} = 4x^3 - 18x^2 + 26x - 10$$

m (tangent) at $(x = 1) = 2$

Normal is perpendicular to tangent so, $m_1 m_2 = -1$

$$m(\text{normal}) \text{ at } (x = 1) = -\frac{1}{2}$$

Equation of tangent is given by $y - y_1 = m$ (tangent) $(x - x_1)$

$$y - 3 = 2(x - 1)$$

$$y = 2x + 1$$

Equation of normal is given by $y - y_1 = m$ (normal) $(x - x_1)$

$$y - 3 = -\frac{1}{2}(x - 1)$$

$$2y = 7 - x$$

(iii) $y = x^2$ at $(0, 0)$

Solution:

Given $y = x^2$ at $(0, 0)$

By differentiating the given curve, we get the slope of the tangent

$$\frac{dy}{dx} = 2x$$

m (tangent) at $(x = 0) = 0$

Normal is perpendicular to tangent so, $m_1 m_2 = -1$

$$m(\text{normal}) \text{ at } (x = 0) = \frac{1}{0}$$

We can see that the slope of normal is not defined

Equation of tangent is given by $y - y_1 = m$ (tangent) $(x - x_1)$

$$y = 0$$

Equation of normal is given by $y - y_1 = m$ (normal) $(x - x_1)$

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$$x = 0$$

$$(iv) y = 2x^2 - 3x - 1 \text{ at } (1, -2)$$

Solution:

$$\text{Given } y = 2x^2 - 3x - 1 \text{ at } (1, -2)$$

By differentiating the given curve, we get the slope of the tangent

$$\frac{dy}{dx} = 4x - 3$$

$$m(\text{tangent}) \text{ at } (1, -2) = 1$$

Normal is perpendicular to tangent so, $m_1 m_2 = -1$

$$m(\text{normal}) \text{ at } (1, -2) = -1$$

Equation of tangent is given by $y - y_1 = m(\text{tangent})(x - x_1)$

$$y + 2 = 1(x - 1)$$

$$y = x - 3$$

Equation of normal is given by $y - y_1 = m(\text{normal})(x - x_1)$

$$y + 2 = -1(x - 1)$$

$$y + x + 1 = 0$$

$$(v) y^2 = \frac{x^3}{4 - x}$$

Solution:

By differentiating the given curve, we get the slope of the tangent

$$2y \frac{dy}{dx} = \frac{(4 - x)3x^2 + x^4}{(4 - x)^2}$$

$$\frac{dy}{dx} = \frac{(4 - x)3x^2 + x^4}{2y(4 - x)^2}$$

$$m(\text{tangent}) \text{ at } (2, -2) = -2$$

$$m(\text{normal}) \text{ at } (2, -2) = \frac{1}{2}$$

Equation of tangent is given by $y - y_1 = m(\text{tangent})(x - x_1)$

$$y + 2 = -2(x - 2)$$

$$y + 2x = 2$$

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Equation of normal is given by $y - y_1 = m \text{ (normal)} (x - x_1)$

$$y + 2 = \frac{1}{2}(x - 2)$$

$$2y + 4 = x - 2$$

$$2y - x + 6 = 0$$

4. Find the equation of the tangent to the curve $x = \theta + \sin \theta$, $y = 1 + \cos \theta$ at $\theta = \pi/4$.

Solution:

Given $x = \theta + \sin \theta$, $y = 1 + \cos \theta$ at $\theta = \pi/4$

By differentiating the given equation with respect to θ , we get the slope of the tangent

$$\frac{dx}{d\theta} = 1 + \cos \theta$$

$$\frac{dy}{d\theta} = -\sin \theta$$

Dividing both the above equations

$$\frac{dy}{dx} = -\frac{\sin \theta}{1 + \cos \theta}$$

$$m \text{ at } \theta = (\pi/4) = -1 + \frac{1}{\sqrt{2}}$$

Equation of tangent is given by $y - y_1 = m \text{ (tangent)} (x - x_1)$

$$y - 1 - \frac{1}{\sqrt{2}} = \left(-1 + \frac{1}{\sqrt{2}}\right) \left(x - \frac{\pi}{4} - \frac{1}{\sqrt{2}}\right)$$

5. Find the equation of the tangent and the normal to the following curves at the indicated points:

(i) $x = \theta + \sin \theta$, $y = 1 + \cos \theta$ at $\theta = \pi/2$

Solution:

Given $x = \theta + \sin \theta$, $y = 1 + \cos \theta$ at $\theta = \pi/2$

By differentiating the given equation with respect to θ , we get the slope of the tangent

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$$\frac{dx}{d\theta} = 1 + \cos\theta$$

$$\frac{dy}{d\theta} = -\sin\theta$$

Dividing both the above equations

$$\frac{dy}{dx} = -\frac{\sin\theta}{1 + \cos\theta}$$

$$m \text{ (tangent) at } \theta = (\pi/2) = -1$$

Normal is perpendicular to tangent so, $m_1 m_2 = -1$

$$m \text{ (normal) at } \theta = (\pi/2) = 1$$

Equation of tangent is given by $y - y_1 = m \text{ (tangent)} (x - x_1)$

$$y - 1 = -1 \left(x - \frac{\pi}{2} - 1 \right)$$

Equation of normal is given by $y - y_1 = m \text{ (normal)} (x - x_1)$

$$y - 1 = 1 \left(x - \frac{\pi}{2} - 1 \right)$$

$$(ii) \ x = \frac{2at^2}{1+t^2}, \ y = \frac{2at^3}{1+t^2} \text{ at } t = \frac{1}{2}$$

Solution:

By differentiating the given equation with respect to t , we get the slope of the tangent

$$\frac{dx}{dt} = \frac{(1+t^2)4at - 2at^2(2t)}{(1+t^2)^2}$$

$$\frac{dx}{dt} = \frac{4at}{(1+t^2)^2}$$

$$\frac{dy}{dt} = \frac{(1+t^2)6at^2 - 2at^3(2t)}{(1+t^2)^2}$$

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$$\frac{dy}{dt} = \frac{6at^2 + 2at^4}{(1 + t^2)^2}$$

Now dividing $\frac{dy}{dt}$ and $\frac{dx}{dt}$ to obtain the slope of tangent

$$\frac{dy}{dx} = \frac{6at^2 + 2at^4}{4at}$$

$$m \text{ (tangent) at } t = \frac{1}{2} \text{ is } \frac{13}{16}$$

Normal is perpendicular to tangent so, $m_1 m_2 = -1$

$$m \text{ (normal) at } t = \frac{1}{2} \text{ is } -\frac{16}{13}$$

Equation of tangent is given by $y - y_1 = m \text{ (tangent)} (x - x_1)$

$$y - \frac{a}{5} = \frac{13}{16} \left(x - \frac{2a}{5} \right)$$

Equation of normal is given by $y - y_1 = m \text{ (normal)} (x - x_1)$

$$y - \frac{a}{5} = -\frac{16}{13} \left(x - \frac{2a}{5} \right)$$

(iii) $x = at^2$, $y = 2at$ at $t = 1$.

Solution:

Given $x = at^2$, $y = 2at$ at $t = 1$.

By differentiating the given equation with respect to t , we get the slope of the tangent

$$\frac{dx}{dt} = 2at$$

$$\frac{dy}{dt} = 2a$$

Now dividing $\frac{dy}{dt}$ and $\frac{dx}{dt}$ to obtain the slope of tangent

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$$\frac{dy}{dx} = \frac{1}{t}$$

m (tangent) at $t = 1$ is 1

Normal is perpendicular to tangent so, $m_1 m_2 = -1$

m (normal) at $t = 1$ is -1

Equation of tangent is given by $y - y_1 = m \text{ (tangent)} (x - x_1)$

$$y - 2a = 1(x - a)$$

Equation of normal is given by $y - y_1 = m \text{ (normal)} (x - x_1)$

$$y - 2a = -1(x - a)$$

(iv) $x = a \sec t$, $y = b \tan t$ at t .

Solution:

Given $x = a \sec t$, $y = b \tan t$ at t .

By differentiating the given equation with respect to t , we get the slope of the tangent

$$\frac{dx}{dt} = a \sec t \tan t$$

$$\frac{dy}{dt} = b \sec^2 t$$

Now dividing $\frac{dy}{dt}$ and $\frac{dx}{dt}$ to obtain the slope of tangent

$$\frac{dy}{dx} = \frac{b \operatorname{cosec} t}{a}$$

$$m \text{ (tangent) at } t = \frac{b \operatorname{cosec} t}{a}$$

Normal is perpendicular to tangent so, $m_1 m_2 = -1$

$$m \text{ (normal) at } t = -\frac{a}{b} \sin t$$

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Equation of tangent is given by $y - y_1 = m \text{ (tangent)} (x - x_1)$

$$y - b \tan t = \frac{b \operatorname{cosec} t}{a} (x - a \sec t)$$

Equation of normal is given by $y - y_1 = m \text{ (normal)} (x - x_1)$

$$y - b \tan t = -\frac{a \sin t}{b} (x - a \sec t)$$

(v) $x = a(\theta + \sin \theta)$, $y = a(1 - \cos \theta)$ at θ

Solution:

Given $x = a(\theta + \sin \theta)$, $y = a(1 - \cos \theta)$ at θ

By differentiating the given equation with respect to θ , we get the slope of the tangent

$$\frac{dx}{d\theta} = a(1 + \cos \theta)$$

$$\frac{dy}{d\theta} = a(\sin \theta)$$

Now dividing $\frac{dy}{d\theta}$ and $\frac{dx}{d\theta}$ to obtain the slope of tangent

$$\frac{dy}{dx} = \frac{\sin \theta}{1 + \cos \theta}$$

$m \text{ (tangent) at } \theta \text{ is } \frac{\sin \theta}{1 + \cos \theta}$

Normal is perpendicular to tangent so, $m_1 m_2 = -1$

$m \text{ (normal) at } \theta \text{ is } -\frac{\sin \theta}{1 + \cos \theta}$

Equation of tangent is given by $y - y_1 = m \text{ (tangent)} (x - x_1)$

$$y - a(1 - \cos \theta) = \frac{\sin \theta}{1 + \cos \theta} (x - a(\theta + \sin \theta))$$

Equation of normal is given by $y - y_1 = m \text{ (normal)} (x - x_1)$

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$$y - a(1 - \cos \theta) = \frac{1 + \cos \theta}{-\sin \theta} (x - a(\theta + \sin \theta))$$

(vi) $x = 3 \cos \theta - \cos^3 \theta$, $y = 3 \sin \theta - \sin^3 \theta$

Solution:

Given $x = 3 \cos \theta - \cos^3 \theta$, $y = 3 \sin \theta - \sin^3 \theta$

By differentiating the given equation with respect to θ , we get the slope of the tangent

$$\frac{dx}{d\theta} = -3 \sin \theta + 3 \cos^2 \theta \sin \theta$$

$$\frac{dy}{d\theta} = 3 \cos \theta - 3 \sin^2 \theta \cos \theta$$

Now dividing $\frac{dy}{d\theta}$ and $\frac{dx}{d\theta}$ to obtain the slope of tangent

$$\frac{dy}{dx} = \frac{3 \cos \theta - 3 \sin^2 \theta \cos \theta}{-3 \sin \theta + 3 \cos^2 \theta \sin \theta} = -\tan^3 \theta$$

m (tangent) at θ is $-\tan^3 \theta$

Normal is perpendicular to tangent so, $m_1 m_2 = -1$

m (normal) at θ is $\cot^3 \theta$

Equation of tangent is given by $y - y_1 = m$ (tangent) $(x - x_1)$

$$y - 3 \sin \theta + \sin^3 \theta = -\tan^3 \theta (x - 3 \cos \theta + 3 \cos^3 \theta)$$

Equation of normal is given by $y - y_1 = m$ (normal) $(x - x_1)$

$$y - 3 \sin \theta + \sin^3 \theta = \cot^3 \theta (x - 3 \cos \theta + 3 \cos^3 \theta)$$

6. Find the equation of the normal to the curve $x^2 + 2y^2 - 4x - 6y + 8 = 0$ at the point whose abscissa is 2.

Solution:

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$$\text{Given } x^2 + 2y^2 - 4x - 6y + 8 = 0$$

By differentiating the given curve, we get the slope of the tangent

$$2x + 4y \frac{dy}{dx} - 4 - 6 \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{4 - 2x}{4y - 6}$$

Finding y co-ordinate by substituting x in the given curve

$$2y^2 - 6y + 4 = 0$$

$$y^2 - 3y + 2 = 0$$

$$y = 2 \text{ or } y = 1$$

m (tangent) at $x = 2$ is 0

Normal is perpendicular to tangent so, $m_1 m_2 = -1$

m (normal) at $x = 2$ is $1/0$, which is undefined

Equation of normal is given by $y - y_1 = m \text{ (normal)} (x - x_1)$

$$x = 2$$

7. Find the equation of the normal to the curve $ay^2 = x^3$ at the point (am^2, am^3) .

Solution:

$$\text{Given } ay^2 = x^3$$

By differentiating the given curve, we get the slope of the tangent

$$2ay \frac{dy}{dx} = 3x^2$$

$$\frac{dy}{dx} = \frac{3x^2}{2ay}$$

$$m \text{ (tangent) at } (am^2, am^3) \text{ is } \frac{3m}{2}$$

Normal is perpendicular to tangent so, $m_1 m_2 = -1$

$$m \text{ (normal) at } (am^2, am^3) \text{ is } -\frac{2}{3m}$$

Equation of normal is given by $y - y_1 = m \text{ (normal)} (x - x_1)$

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$$y - am^3 = -\frac{2}{3m}(x - am^2)$$

8. The equation of the tangent at (2, 3) on the curve $y^2 = ax^3 + b$ is $y = 4x - 5$. Find the values of a and b.

Solution:

Given $y^2 = ax^3 + b$ is $y = 4x - 5$

By differentiating the given curve, we get the slope of the tangent

$$2y \frac{dy}{dx} = 3ax^2$$

$$\frac{dy}{dx} = \frac{3ax^2}{2y}$$

m (tangent) at (2, 3) = 2a

Equation of tangent is given by $y - y_1 = m \text{ (tangent)} (x - x_1)$

Now comparing the slope of a tangent with the given equation

$$2a = 4$$

$$a = 2$$

Now (2, 3) lies on the curve, these points must satisfy

$$3^2 = 2 \times 2^3 + b$$

$$b = -7$$

9. Find the equation of the tangent line to the curve $y = x^2 + 4x - 16$ which is parallel to the line $3x - y + 1 = 0$.

Solution:

Given $y = x^2 + 4x - 16$

By differentiating the given curve, we get the slope of the tangent

$$\frac{dy}{dx} = 2x + 4$$

$$m \text{ (tangent)} = 2x + 4$$

Equation of tangent is given by $y - y_1 = m \text{ (tangent)} (x - x_1)$

Now comparing the slope of a tangent with the given equation

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$$2x + 4 = 3$$

$$x = -\frac{1}{2}$$

Now substituting the value of x in the curve to find y

$$y = \frac{1}{4} - 2 - 16 = -\frac{71}{4}$$

Therefore, the equation of tangent parallel to the given line is

$$y + \frac{71}{4} = 3\left(x + \frac{1}{2}\right)$$

10. Find the equation of normal line to the curve $y = x^3 + 2x + 6$ which is parallel to the line $x + 14y + 4 = 0$.

Solution:

$$\text{Given } y = x^3 + 2x + 6$$

By differentiating the given curve, we get the slope of the tangent

$$\frac{dy}{dx} = 3x^2 + 2$$

$$m(\text{tangent}) = 3x^2 + 2$$

Normal is perpendicular to tangent so, $m_1 m_2 = -1$

$$m(\text{normal}) = \frac{-1}{3x^2 + 2}$$

Equation of normal is given by $y - y_1 = m(\text{normal})(x - x_1)$

Now comparing the slope of normal with the given equation

$$m(\text{normal}) = -\frac{1}{14}$$

$$-\frac{1}{14} = -\frac{1}{3x^2 + 2}$$

$$x = 2 \text{ or } -2$$

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Hence the corresponding value of y is 18 or -6

So, equations of normal are

$$y - 18 = -\frac{1}{14}(x - 2)$$

Or

$$y + 6 = -\frac{1}{14}(x + 2)$$

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EXERCISE 16.3

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1. Find the angle to intersection of the following curves:**(i) $y^2 = x$ and $x^2 = y$** **Solution:**Given curves $y^2 = x \dots (1)$ And $x^2 = y \dots (2)$ First curve is $y^2 = x$ Differentiating above with respect to x ,

$$\Rightarrow 2y \frac{dy}{dx} = 1$$

$$\Rightarrow m_1 = \frac{dy}{dx} = \frac{1}{2x} \dots (3)$$

The second curve is $x^2 = y$

$$\Rightarrow 2x = \frac{dy}{dx}$$

$$\Rightarrow m_2 = \frac{dy}{dx} = 2x \dots (4)$$

Substituting (1) in (2), we get

$$\Rightarrow x^2 = y$$

$$\Rightarrow (y^2)^2 = y$$

$$\Rightarrow y^4 - y = 0$$

$$\Rightarrow y(y^3 - 1) = 0$$

$$\Rightarrow y = 0 \text{ or } y = 1$$

Substituting $y = 0$ & $y = 1$ in (1) in (2),

$$x = y^2$$

When $y = 0$, $x = 0$ When $y = 1$, $x = 1$

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Substituting above values for m_1 & m_2 , we get,

When $x = 0$,

$$m_1 = \frac{dy}{dx} = \frac{1}{2 \times 0} = \infty$$

When $x = 1$,

$$m_1 = \frac{dy}{dx} = \frac{1}{2 \times 1} = \frac{1}{2}$$

Values of m_1 is ∞ & $\frac{1}{2}$

When $y = 0$,

$$m_2 = \frac{dy}{dx} = 2x = 2 \times 0 = 0$$

When $x = 1$,

$$m_2 = \frac{dy}{dx} = 3x = 2 \times 1 = 2$$

Values of m_2 is 0 & 2

When $m_1 = \infty$ & $m_2 = 0$

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$\tan \theta = \left| \frac{0 - \infty}{1 + \infty \times 0} \right|$$

$$\tan \theta = \infty$$

$$\theta = \tan^{-1}(\infty)$$

$$\therefore \tan^{-1}(\infty) = \frac{\pi}{2}$$

$$\theta = \frac{\pi}{2}$$

When $m_1 = \frac{1}{2}$ & $m_2 = 2$

Angle of intersection of two curves is given by $\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$

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$$\tan \theta = \left| \frac{2 - \frac{1}{2}}{1 + \frac{1}{2} \times 2} \right|$$

$$\tan \theta = \left| \frac{\frac{3}{2}}{2} \right|$$

$$\tan \theta = \left| \frac{3}{4} \right|$$

$$\theta = \tan^{-1}\left(\frac{3}{4}\right)$$

(ii) $y = x^2$ and $x^2 + y^2 = 20$

Solution:

Given curves $y = x^2$... (1) and $x^2 + y^2 = 20$... (2)

Now consider first curve $y = x^2$

$$\Rightarrow m_1 = \frac{dy}{dx} = 2x \dots (3)$$

Consider second curve is $x^2 + y^2 = 20$

Differentiating above with respect to x ,

$$\Rightarrow 2x + 2y \cdot \frac{dy}{dx} = 0$$

$$\Rightarrow y \cdot \frac{dy}{dx} = -x$$

$$\Rightarrow m_2 = \frac{dy}{dx} = \frac{-x}{y} \dots (4)$$

Substituting (1) in (2), we get

$$\Rightarrow y + y^2 = 20$$

$$\Rightarrow y^2 + y - 20 = 0$$

We will use factorization method to solve the above Quadratic equation

$$\Rightarrow y^2 + 5y - 4y - 20 = 0$$

$$\Rightarrow y(y + 5) - 4(y + 5) = 0$$

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$$\Rightarrow (y + 5)(y - 4) = 0$$

$$\Rightarrow y = -5 \text{ \& } y = 4$$

Substituting $y = -5$ \& $y = 4$ in (1) in (2),

$$y = x^2$$

When $y = -5$,

$$\Rightarrow -5 = x^2$$

$$\Rightarrow x = \sqrt{-5}$$

When $y = 4$,

$$\Rightarrow 4 = x^2$$

$$\Rightarrow x = \pm 2$$

Substituting above values for m_1 \& m_2 , we get,

When $x = 2$,

$$m_1 = \frac{dy}{dx} = 2 \times 2$$

$$= 4$$

When $x = 1$,

$$m_1 = \frac{dy}{dx} = 2 \times -2$$

$$= -4$$

Values of m_1 is 4 \& -4

When $y = 4$ \& $x = 2$

$$m_2 = \frac{dy}{dx} = \frac{-x}{y} = \frac{-2}{4} = \frac{-1}{2}$$

When $y = 4$ \& $x = -2$

$$m_2 = \frac{dy}{dx} = \frac{-x}{y} = \frac{2}{4} = \frac{1}{2}$$

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Values of m_2 is $\frac{-1}{2}$ & $\frac{1}{2}$

When $m_1 = \infty$ & $m_2 = 0$

Angle of intersection of two curves is given by $\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$

$$\tan \theta = \left| \frac{\frac{-1}{2} - 4}{1 + 2 \times 4} \right|$$

$$\tan \theta = \left| \frac{\frac{-9}{2}}{1 - 2} \right|$$

$$\tan \theta = \left| \frac{9}{2} \right|$$

$$\theta = \tan^{-1}\left(\frac{9}{2}\right)$$

(iii) $2y^2 = x^3$ and $y^2 = 32x$

Solution:

Given curves $2y^2 = x^3 \dots (1)$ and $y^2 = 32x \dots (2)$

First curve is $2y^2 = x^3$

Differentiating above with respect to x ,

$$\Rightarrow 4y \cdot \frac{dy}{dx} = 3x^2$$

$$\Rightarrow m_1 = \frac{dy}{dx} = \frac{3x^2}{4y} \dots (3)$$

Second curve is $y^2 = 32x$

$$\Rightarrow 2y \cdot \frac{dy}{dx} = 32$$

$$\Rightarrow y \cdot \frac{dy}{dx} = 16$$

$$\Rightarrow m_2 = \frac{dy}{dx} = \frac{16}{y} \dots (4)$$

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Substituting (2) in (1), we get

$$\Rightarrow 2y^2 = x^3$$

$$\Rightarrow 2(32x) = x^3$$

$$\Rightarrow 64x = x^3$$

$$\Rightarrow x^3 - 64x = 0$$

$$\Rightarrow x(x^2 - 64) = 0$$

$$\Rightarrow x = 0 \text{ \& } (x^2 - 64) = 0$$

$$\Rightarrow x = 0 \text{ \& } \pm 8$$

Substituting $x = 0$ & $x = \pm 8$ in (1) in (2),

$$y^2 = 32x$$

When $x = 0$, $y = 0$

When $x = 8$

$$\Rightarrow y^2 = 32 \times 8$$

$$\Rightarrow y^2 = 256$$

$$\Rightarrow y = \pm 16$$

Substituting above values for m_1 & m_2 , we get,

When $x = 0$, $y = 16$

$$m_1 = \frac{dy}{dx}$$

$$\Rightarrow \frac{3 \times 0^2}{4 \times 8}$$

$$= 0$$

When $x = 8$, $y = 16$

$$m_1 = \frac{dy}{dx}$$

$$\Rightarrow \frac{3 \times 8^2}{4 \times 16}$$

$$= 3$$

Values of m_1 is 0 & 3

When $x = 0$, $y = 0$,

$$m_2 = \frac{dy}{dx}$$

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$$\Rightarrow \frac{16}{y} = \frac{16}{0} = \infty$$

When $y = 16$,

$$m_2 = \frac{dy}{dx}$$

$$\Rightarrow \frac{16}{y} = \frac{16}{16}$$

$$= 1$$

Values of m_2 is ∞ & 1

When $m_1 = 0$ & $m_2 = \infty$

$$\Rightarrow \tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$\Rightarrow \tan \theta = \left| \frac{\infty - 0}{1 + \infty \times 0} \right|$$

$$\Rightarrow \tan \theta = \infty$$

$$\Rightarrow \theta = \tan^{-1}(\infty)$$

$$\therefore \tan^{-1}(\infty) = \frac{\pi}{2}$$

$$\Rightarrow \theta = \frac{\pi}{2}$$

When $m_1 = \frac{1}{2}$ & $m_2 = 2$

Angle of intersection of two curves is given by $\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$

$$\Rightarrow \tan \theta = \left| \frac{3-1}{1+3 \times 1} \right|$$

$$\Rightarrow \tan \theta = \left| \frac{2}{4} \right|$$

$$\Rightarrow \tan \theta = \left| \frac{1}{2} \right|$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{1}{2}\right)$$

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(iv) $x^2 + y^2 - 4x - 1 = 0$ and $x^2 + y^2 - 2y - 9 = 0$

Solution:

Given curves $x^2 + y^2 - 4x - 1 = 0 \dots (1)$ and $x^2 + y^2 - 2y - 9 = 0 \dots (2)$

First curve is $x^2 + y^2 - 4x - 1 = 0$

$$\Rightarrow x^2 - 4x + 4 + y^2 - 4 - 1 = 0$$

$$\Rightarrow (x - 2)^2 + y^2 - 5 = 0$$

Now, Subtracting (2) from (1), we get

$$\Rightarrow x^2 + y^2 - 4x - 1 - (x^2 + y^2 - 2y - 9) = 0$$

$$\Rightarrow x^2 + y^2 - 4x - 1 - x^2 - y^2 + 2y + 9 = 0$$

$$\Rightarrow -4x - 1 + 2y + 9 = 0$$

$$\Rightarrow -4x + 2y + 8 = 0$$

$$\Rightarrow 2y = 4x - 8$$

$$\Rightarrow y = 2x - 4$$

Substituting $y = 2x - 4$ in (3), we get,

$$\Rightarrow (x - 2)^2 + (2x - 4)^2 - 5 = 0$$

$$\Rightarrow (x - 2)^2 + 4(x - 2)^2 - 5 = 0$$

$$\Rightarrow (x - 2)^2(1 + 4) - 5 = 0$$

$$\Rightarrow 5(x - 2)^2 - 5 = 0$$

$$\Rightarrow (x - 2)^2 - 1 = 0$$

$$\Rightarrow (x - 2)^2 = 1$$

$$\Rightarrow (x - 2) = \pm 1$$

$$\Rightarrow x = 1 + 2 \text{ or } x = -1 + 2$$

$$\Rightarrow x = 3 \text{ or } x = 1$$

So, when $x = 3$

$$y = 2 \times 3 - 4$$

$$\Rightarrow y = 6 - 4 = 2$$

So, when $x = 1$

$$y = 2 \times 1 - 4$$

$$\Rightarrow y = 2 - 4 = -2$$

The point of intersection of two curves are $(3, 2)$ & $(1, -2)$

Now, differentiating curves (1) & (2) with respect to x , we get

$$\Rightarrow x^2 + y^2 - 4x - 1 = 0$$

$$\Rightarrow 2x + 2y \frac{dy}{dx} - 4 - 0 = 0$$

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$$\Rightarrow x + y \frac{dy}{dx} - 2 = 0$$

$$\Rightarrow y \frac{dy}{dx} = 2 - x$$

$$\Rightarrow \frac{dy}{dx} = \frac{2-x}{y} \dots (3)$$

$$\Rightarrow x^2 + y^2 - 2y - 9 = 0$$

$$\Rightarrow 2x + 2y \frac{dy}{dx} - 2 \frac{dy}{dx} - 0 = 0$$

$$\Rightarrow x + y \frac{dy}{dx} - \frac{dy}{dx} = 0$$

$$\Rightarrow x + (y-1) \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-x}{y-1} \dots (4)$$

At (3, 2) in equation (3), we get

$$\Rightarrow \frac{dy}{dx} = \frac{2-3}{2}$$

$$\Rightarrow m_1 = \frac{dy}{dx} = \frac{-1}{2}$$

At (3, 2) in equation (4), we get

$$\Rightarrow \frac{dy}{dx} = \frac{-3}{2-1}$$

$$\Rightarrow \frac{dy}{dx} = -3$$

$$\Rightarrow m_2 = \frac{dy}{dx} = -3$$

$$\text{When } m_1 = \frac{-1}{2} \text{ \& } m_2 = 0$$

Angle of intersection of two curves is given by $\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$

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$$\Rightarrow \tan \theta = \left| \frac{\frac{1}{2} + 3}{1 + \frac{3}{2}} \right| = 1$$

$$\Rightarrow \theta = \tan^{-1}(1) = \frac{\pi}{4}$$

$$(v) \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ and } x^2 + y^2 = ab$$

Solution:

Given curves $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \dots (1)$ and $x^2 + y^2 = ab \dots (2)$

Second curve is $x^2 + y^2 = ab$

$$y^2 = ab - x^2$$

Substituting this in equation (1),

$$\Rightarrow \frac{x^2}{a^2} + \frac{ab - x^2}{b^2} = 1$$

$$\Rightarrow \frac{x^2 b^2 + a^2(ab - x^2)}{a^2 b^2} = 1$$

$$\Rightarrow x^2 b^2 + a^3 b - a^2 x^2 = a^2 b^2$$

$$\Rightarrow x^2 b^2 - a^2 x^2 = a^2 b^2 - a^3 b$$

$$\Rightarrow x^2(b^2 - a^2) = a^2 b(b - a)$$

$$\Rightarrow x^2 = \frac{a^2 b(b - a)}{x^2(b^2 - a^2)}$$

$$\Rightarrow x^2 = \frac{a^2 b(b - a)}{x^2(b - a)(b + a)}$$

$$\Rightarrow x^2 = \frac{a^2 b}{(b + a)}$$

$$\therefore a^2 - b^2 = (a + b)(a - b)$$

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$$\Rightarrow x = \pm \sqrt{\frac{a^2 b}{(b+a)}} \dots (3)$$

Since, $y^2 = ab - x^2$

$$\Rightarrow y^2 = ab - \frac{a^2 b}{(b+a)}$$

$$\Rightarrow y^2 = \frac{ab^2 + a^2 b - a^2 b}{(b+a)}$$

$$\Rightarrow y^2 = \frac{ab^2}{(b+a)}$$

$$\Rightarrow y = \pm \sqrt{\frac{ab^2}{(b+a)}} \dots (4)$$

Since, curves are $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ & $x^2 + y^2 = ab$

Differentiating above with respect to x

$$\Rightarrow \frac{2x}{a^2} + \frac{2y}{b^2} \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{y}{b^2} \frac{dy}{dx} = -\frac{x}{a^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-\frac{x}{a^2}}{\frac{y}{b^2}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-b^2 x}{a^2 y}$$

$$\Rightarrow m_1 = \frac{dy}{dx} = \frac{-b^2 x}{a^2 y} \dots (5)$$

Second curve is $x^2 + y^2 = ab$

$$\Rightarrow 2x + 2y \frac{dy}{dx} = 0$$

$$\Rightarrow m_2 = \frac{dy}{dx} = \frac{-x}{y} \dots (6)$$

Substituting (3) in (4), above values for m_1 & m_2 , we get,

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At $(\sqrt{\frac{a^2b}{(b+a)}}, \sqrt{\frac{ab^2}{(b+a)}})$ in equation (5), we get

$$\Rightarrow \frac{dy}{dx} = \frac{-b^2 \times \sqrt{\frac{a^2b}{(b+a)}}}{a^2 \times \sqrt{\frac{ab^2}{(b+a)}}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-b^2 \times a \sqrt{\frac{b}{(b+a)}}}{a^2 \times b \sqrt{\frac{a}{(b+a)}}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-b^2 a \sqrt{b}}{a^2 b \sqrt{a}}$$

$$\Rightarrow m_1 = \frac{dy}{dx} = \frac{-b\sqrt{b}}{a\sqrt{a}}$$

At $(\sqrt{\frac{a^2b}{(b+a)}}, \sqrt{\frac{ab^2}{(b+a)}})$ in equation (6), we get

$$\Rightarrow \frac{dy}{dx} = \frac{-\sqrt{\frac{a^2b}{(b+a)}}}{\sqrt{\frac{ab^2}{(b+a)}}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-a \sqrt{\frac{b}{(b+a)}}}{b \sqrt{\frac{a}{(b+a)}}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-a\sqrt{b}}{b\sqrt{a}}$$

$$\Rightarrow m_2 = \frac{dy}{dx} = -\sqrt{\frac{a}{b}}$$

$$\text{When } m_1 = \frac{-b\sqrt{b}}{a\sqrt{a}} \text{ \& } m_2 = -\sqrt{\frac{a}{b}}$$

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Angle of intersection of two curves is given by $\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$

$$\Rightarrow \tan \theta = \left| \frac{\frac{-b\sqrt{b}}{a\sqrt{a}} - \sqrt{\frac{a}{b}}}{1 + \frac{-b\sqrt{b}}{a\sqrt{a}} \times -\sqrt{\frac{a}{b}}} \right|$$

$$\Rightarrow \tan \theta = \left| \frac{\frac{-b\sqrt{b}}{a\sqrt{a}} + \sqrt{\frac{a}{b}}}{1 + \frac{b}{a}} \right|$$

$$\Rightarrow \tan \theta = \left| \frac{\frac{-b\sqrt{b} \times \sqrt{b} + a\sqrt{a} \times \sqrt{a}}{a\sqrt{a} \times \sqrt{b}}}{1 + \frac{b}{a}} \right|$$

$$\Rightarrow \tan \theta = \left| \frac{\frac{-b \times b + a \times a}{a\sqrt{ab}}}{1 + \frac{b}{a}} \right|$$

$$\Rightarrow \tan \theta = \left| \frac{\frac{a^2 - b^2}{a\sqrt{ab}}}{\frac{a+b}{a}} \right|$$

$$\Rightarrow \tan \theta = \left| \frac{\frac{(a+b)(a-b)}{\sqrt{ab}}}{a+b} \right|$$

$$\Rightarrow \tan \theta = \left| \frac{(a-b)}{\sqrt{ab}} \right|$$

$$\Rightarrow \theta = \tan^{-1} \left(\frac{(a-b)}{\sqrt{ab}} \right)$$

2. Show that the following set of curves intersect orthogonally:

(i) $y = x^3$ and $6y = 7 - x^2$

Solution:

Given curves $y = x^3 \dots (1)$ and $6y = 7 - x^2 \dots (2)$

Solving (1) & (2), we get

$$\Rightarrow 6y = 7 - x^2$$

$$\Rightarrow 6(x^3) = 7 - x^2$$

$$\Rightarrow 6x^3 + x^2 - 7 = 0$$

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Since $f(x) = 6x^3 + x^2 - 7$,

We have to find $f(x) = 0$, so that x is a factor of $f(x)$.

When $x = 1$

$$f(1) = 6(1)^3 + (1)^2 - 7$$

$$f(1) = 6 + 1 - 7$$

$$f(1) = 0$$

Hence, $x = 1$ is a factor of $f(x)$.

Substituting $x = 1$ in $y = x^3$, we get

$$y = 1^3$$

$$y = 1$$

The point of intersection of two curves is $(1, 1)$

First curve $y = x^3$

Differentiating above with respect to x ,

$$\Rightarrow 6 \frac{dy}{dx} = 0 - 2x$$

$$\Rightarrow m_2 = \frac{-2x}{6}$$

$$\Rightarrow m_2 = \frac{-x}{3} \dots (4)$$

At $(1, 1)$, we have,

$$m_1 = 3x^2$$

$$\Rightarrow 3 \times (1)^2$$

$$m_1 = 3$$

At $(1, 1)$, we have,

$$\Rightarrow m_2 = \frac{-x}{3}$$

$$\Rightarrow \frac{-1}{3}$$

$$\Rightarrow m_2 = \frac{-1}{3}$$

$$\text{When } m_1 = 3 \text{ \& } m_2 = \frac{-1}{3}$$

Two curves intersect orthogonally if $m_1 m_2 = -1$

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$$\Rightarrow 3 \times \frac{-1}{3} = -1$$

\therefore Two curves $y = x^3$ & $6y = 7 - x^2$ intersect orthogonally.

(ii) $x^3 - 3xy^2 = -2$ and $3x^2y - y^3 = 2$

Solution:

Given curves $x^3 - 3xy^2 = -2 \dots (1)$ and $3x^2y - y^3 = 2 \dots (2)$

Adding (1) & (2), we get

$$\Rightarrow x^3 - 3xy^2 + 3x^2y - y^3 = -2 + 2$$

$$\Rightarrow x^3 - 3xy^2 + 3x^2y - y^3 = -0$$

$$\Rightarrow (x - y)^3 = 0$$

$$\Rightarrow (x - y) = 0$$

$$\Rightarrow x = y$$

Substituting $x = y$ on $x^3 - 3xy^2 = -2$

$$\Rightarrow x^3 - 3 \times x \times x^2 = -2$$

$$\Rightarrow x^3 - 3x^3 = -2$$

$$\Rightarrow -2x^3 = -2$$

$$\Rightarrow x^3 = 1$$

$$\Rightarrow x = 1$$

Since $x = y$

$$y = 1$$

The point of intersection of two curves is (1, 1)

First curve $x^3 - 3xy^2 = -2$

Differentiating above with respect to x ,

$$\Rightarrow 3x^2 - 3(1 \times y^2 + x \times 2y \frac{dy}{dx}) = 0$$

$$\Rightarrow 3x^2 - 3y^2 - 6xy \frac{dy}{dx} = 0$$

$$\Rightarrow 3x^2 - 3y^2 = 6xy \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{3x^2 - 3y^2}{6xy}$$

$$\Rightarrow \frac{dy}{dx} = \frac{3(x^2 - y^2)}{6xy}$$

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$$\Rightarrow m_1 = \frac{(x^2 - y^2)}{2xy} \dots (3)$$

Second curve $3x^2y - y^3 = 2$

Differentiating above with respect to x

$$\Rightarrow 3(2xxy + x^2 \times \frac{dy}{dx}) - 3y^2 \frac{dy}{dx} = 0$$

$$\Rightarrow 6xy + 3x^2 \frac{dy}{dx} - 3y^2 \frac{dy}{dx} = 0$$

$$\Rightarrow 6xy + (3x^2 - 3y^2) \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-6xy}{3x^2 - 3y^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-2xy}{x^2 - y^2}$$

$$\Rightarrow m_2 = \frac{-2xy}{x^2 - y^2} \dots (4)$$

$$\text{When } m_1 = \frac{(x^2 - y^2)}{2xy} \text{ \& } m_2 = \frac{-2xy}{x^2 - y^2}$$

Two curves intersect orthogonally if $m_1 m_2 = -1$

$$\Rightarrow \frac{(x^2 - y^2)}{2xy} \times \frac{-2xy}{x^2 - y^2} = -1$$

\therefore Two curves $x^3 - 3xy^2 = -2$ & $3x^2y - y^3 = 2$ intersect orthogonally.

(iii) $x^2 + 4y^2 = 8$ and $x^2 - 2y^2 = 4$.

Solution:

Given curves $x^2 + 4y^2 = 8 \dots (1)$ and $x^2 - 2y^2 = 4 \dots (2)$

Solving (1) & (2), we get,

From 2nd curve,

$$x^2 = 4 + 2y^2$$

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Substituting on $x^2 + 4y^2 = 8$,

$$\Rightarrow 4 + 2y^2 + 4y^2 = 8$$

$$\Rightarrow 6y^2 = 4$$

$$\Rightarrow y^2 = \frac{4}{6}$$

$$\Rightarrow y = \pm\sqrt{\frac{2}{3}}$$

Substituting on $y = \pm\sqrt{\frac{2}{3}}$, we get,

$$\Rightarrow x^2 = 4 + 2\left(\pm\sqrt{\frac{2}{3}}\right)^2$$

$$\Rightarrow x^2 = 4 + 2\left(\frac{2}{3}\right)$$

$$\Rightarrow x^2 = 4 + \frac{4}{3}$$

$$\Rightarrow x^2 = \frac{16}{3}$$

$$\Rightarrow x = \pm\sqrt{\frac{16}{3}}$$

$$\Rightarrow x = \pm\frac{4}{\sqrt{3}}$$

\therefore The point of intersection of two curves $\left(\frac{4}{\sqrt{3}}, \sqrt{\frac{2}{3}}\right)$ & $\left(-\frac{4}{\sqrt{3}}, -\sqrt{\frac{2}{3}}\right)$

Now, differentiating curves (1) & (2) with respect to x , we get

$$\Rightarrow x^2 + 4y^2 = 8$$

$$\Rightarrow 2x + 8y \cdot \frac{dy}{dx} = 0$$

$$\Rightarrow 8y \cdot \frac{dy}{dx} = -2x$$

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$$\Rightarrow \frac{dy}{dx} = \frac{-x}{4y} \dots (3)$$

$$\Rightarrow x^2 - 2y^2 = 4$$

$$\Rightarrow 2x - 4y \cdot \frac{dy}{dx} = 0$$

$$\Rightarrow x - 2y \cdot \frac{dy}{dx} = 0$$

$$\Rightarrow 4y \frac{dy}{dx} = x$$

$$\Rightarrow \frac{dy}{dx} = \frac{x}{2y} \dots (4)$$

At $(\frac{4}{\sqrt{3}}, \sqrt{\frac{2}{3}})$ in equation (3), we get

$$\Rightarrow \frac{dy}{dx} = \frac{-\frac{4}{\sqrt{3}}}{4 \times \sqrt{\frac{2}{3}}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-\frac{1}{\sqrt{3}}}{\sqrt{\frac{2}{3}}}$$

$$\Rightarrow m_1 = \frac{-1}{\sqrt{2}}$$

At $(\frac{4}{\sqrt{3}}, \sqrt{\frac{2}{3}})$ in equation (4), we get

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{4}{\sqrt{3}}}{2 \times \sqrt{\frac{2}{3}}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2}{\sqrt{2}}$$

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$$\Rightarrow \frac{dy}{dx} = \frac{2}{\sqrt{2}}$$

$$\Rightarrow \frac{dy}{dx} = \sqrt{2}$$

$$\Rightarrow m_2 = 1$$

$$\text{When } m_1 = \frac{-1}{\sqrt{2}} \text{ \& } m_2 = \sqrt{2}$$

Two curves intersect orthogonally if $m_1 m_2 = -1$

$$\Rightarrow \frac{-1}{\sqrt{2}} \times \sqrt{2} = -1$$

\therefore Two curves $x^2 + 4y^2 = 8$ \& $x^2 - 2y^2 = 4$ intersect orthogonally.

3. $x^2 = 4y$ and $4y + x^2 = 8$ at $(2, 1)$ **Solution:**

Given curves $x^2 = 4y$... (1) and $4y + x^2 = 8$... (2)

The point of intersection of two curves $(2, 1)$

Solving (1) \& (2), we get,

First curve is $x^2 = 4y$

Differentiating above with respect to x ,

$$\Rightarrow 2x = 4 \cdot \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2x}{4}$$

$$\Rightarrow m_1 = \frac{x}{2} \dots (3)$$

Second curve is $4y + x^2 = 8$

$$\Rightarrow 4 \cdot \frac{dy}{dx} + 2x = 0$$

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$$\Rightarrow \frac{dy}{dx} = \frac{-2x}{4}$$

$$\Rightarrow m_2 = \frac{-x}{2} \dots (4)$$

Substituting (2, 1) for m_1 & m_2 , we get,

$$m_1 = \frac{x}{2}$$

$$\Rightarrow \frac{2}{2}$$

$$m_1 = 1 \dots (5)$$

$$m_2 = \frac{-x}{2}$$

$$\Rightarrow \frac{-2}{2}$$

$$m_2 = -1 \dots (6)$$

When $m_1 = 1$ & $m_2 = -1$

Two curves intersect orthogonally if $m_1 m_2 = -1$

$$\Rightarrow 1 \times -1 = -1$$

\therefore Two curves $x^2 = 4y$ & $4y + x^2 = 8$ intersect orthogonally.

(ii) $x^2 = y$ and $x^3 + 6y = 7$ at (1, 1)

Solution:

Given curves $x^2 = y \dots (1)$ and $x^3 + 6y = 7 \dots (2)$

The point of intersection of two curves (1, 1)

Solving (1) & (2), we get,

First curve is $x^2 = y$

Differentiating above with respect to x ,

$$\Rightarrow 2x = \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = 2x$$

$$\Rightarrow m_1 = 2x \dots (3)$$

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Second curve is $x^3 + 6y = 7$

Differentiating above with respect to x ,

$$\Rightarrow 3x^2 + 6 \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-3x^2}{6}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-x^2}{2}$$

$$\Rightarrow m_2 = \frac{-x^2}{2} \dots (4)$$

Substituting (1, 1) for m_1 & m_2 , we get,

$$m_1 = 2x$$

$$\Rightarrow 2 \times 1$$

$$m_1 = 2 \dots (5)$$

$$m_2 = \frac{-x^2}{2}$$

$$\Rightarrow \frac{-1^2}{2}$$

$$m_2 = -\frac{1}{2} \dots (6)$$

$$\text{When } m_1 = 2 \text{ \& } m_2 = -\frac{1}{2}$$

Two curves intersect orthogonally if $m_1 m_2 = -1$

$$\Rightarrow 2 \times \frac{-1}{2} = -1$$

\therefore Two curves $x^2 = y$ & $x^3 + 6y = 7$ intersect orthogonally.

(iii) $y^2 = 8x$ and $2x^2 + y^2 = 10$ at (1, $2\sqrt{2}$)

Solution:

Given curves $y^2 = 8x \dots (1)$ and $2x^2 + y^2 = 10 \dots (2)$

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The point of intersection of two curves are $(0, 0)$ & $(1, 2\sqrt{2})$

Now, differentiating curves (1) & (2) w.r.t x , we get

$$\Rightarrow y^2 = 8x$$

$$\Rightarrow 2y \cdot \frac{dy}{dx} = 8$$

$$\Rightarrow \frac{dy}{dx} = \frac{8}{2y}$$

$$\Rightarrow \frac{dy}{dx} = \frac{4}{y} \dots (3)$$

$$\Rightarrow 2x^2 + y^2 = 10$$

Differentiating above with respect to x ,

$$\Rightarrow 4x + 2y \cdot \frac{dy}{dx} = 0$$

$$\Rightarrow 2x + y \cdot \frac{dy}{dx} = 0$$

$$\Rightarrow y \cdot \frac{dy}{dx} = -2x$$

$$\Rightarrow \frac{dy}{dx} = \frac{-2x}{y} \dots (4)$$

Substituting $(1, 2\sqrt{2})$ for m_1 & m_2 , we get,

$$m_1 = \frac{4}{y}$$

$$\Rightarrow \frac{4}{2\sqrt{2}}$$

$$m_1 = \frac{1}{\sqrt{2}} \dots (5)$$

$$m_2 = \frac{-2x}{y}$$

$$\Rightarrow \frac{-2 \times 1}{2\sqrt{2}}$$

$$m_2 = -\frac{1}{\sqrt{2}} \dots (6)$$

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$$\text{When } m_1 = \sqrt{2} \text{ \& } m_2 = \frac{-1}{\sqrt{2}}$$

Two curves intersect orthogonally if $m_1 m_2 = -1$

$$\Rightarrow \sqrt{2} \times \frac{-1}{\sqrt{2}} = -1$$

\therefore Two curves $y^2 = 8x$ \& $2x^2 + y^2 = 10$ intersect orthogonally.

4. Show that the curves $4x = y^2$ \& $4xy = k$ cut at right angles, if $k^2 = 512$.

Solution:

Given curves $4x = y^2$... (1) \& $4xy = k$... (2)

We have to prove that two curves cut at right angles if $k^2 = 512$

Now, differentiating curves (1) \& (2) w.r.t x , we get

$$\Rightarrow 4x = y^2$$

$$\Rightarrow 4 = 2y \cdot \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2}{y}$$

$$m_1 = \frac{2}{y} \text{ ... (3)}$$

$$\Rightarrow 4xy = k$$

Differentiating above with respect to x ,

$$\Rightarrow 4\left(y + x \frac{dy}{dx}\right) = 0$$

$$\Rightarrow y + x \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-y}{x}$$

$$\Rightarrow m_2 = \frac{-y}{x} \text{ ... (4)}$$

Two curves intersect orthogonally if $m_1 m_2 = -1$

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Since m_1 and m_2 cuts orthogonally,

$$\Rightarrow \frac{2}{y} \times \frac{-y}{x} = -1$$

$$\Rightarrow \frac{-2}{x} = -1$$

$$\Rightarrow x = 2$$

Now, Solving (1) & (2), we get,

$$4xy = k \text{ \& \; } 4x = y^2$$

$$\Rightarrow (y^2) y = k$$

$$\Rightarrow y^3 = k$$

$$\Rightarrow y = k^{\frac{1}{3}}$$

Substituting $y = k^{\frac{1}{3}}$ in $4x = y^2$, we get,

$$\Rightarrow 4x = (k^{\frac{1}{3}})^2$$

$$\Rightarrow 4 \times 2 = k^{\frac{2}{3}}$$

$$\Rightarrow k^{\frac{2}{3}} = 8$$

$$\Rightarrow k^2 = 8^3$$

$$\Rightarrow k^2 = 512$$

5. Show that the curves $2x = y^2$ and $2xy = k$ cut at right angles, if $k^2 = 8$.

Solution:

Given curves $2x = y^2$... (1) and $2xy = k$... (2)

We have to prove that two curves cut at right angles if $k^2 = 8$

Now, differentiating curves (1) & (2) with respect to x , we get

$$\Rightarrow 2x = y^2$$

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$$\Rightarrow 2 = 2y \cdot \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{y}$$

$$m_1 = \frac{1}{y} \dots (3)$$

$$\Rightarrow 2xy = k$$

Differentiating above with respect to x,

$$\Rightarrow 2\left(y + x \frac{dy}{dx}\right) = 0$$

$$\Rightarrow y + x \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-y}{x}$$

$$\Rightarrow m_2 = \frac{-y}{x} \dots (4)$$

Two curves intersect orthogonally if $m_1 m_2 = -1$

Since m_1 and m_2 cuts orthogonally,

$$\Rightarrow \frac{1}{y} \times \frac{-y}{x} = -1$$

$$\Rightarrow \frac{-1}{x} = -1$$

$$\Rightarrow x = 1$$

Now, solving (1) & (2), we get,

$$2xy = k \text{ \& \; } 2x = y^2$$

$$\Rightarrow (y^2) y = k$$

$$\Rightarrow y^3 = k$$

$$\Rightarrow y = k^{\frac{1}{3}}$$

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Substituting $y = k^{\frac{1}{3}}$ in $2x = y^2$, we get,

$$\Rightarrow 2x = (k^{\frac{1}{3}})^2$$

$$\Rightarrow 2 \times 1 = k^{\frac{2}{3}}$$

$$\Rightarrow k^{\frac{2}{3}} = 2$$

$$\Rightarrow k^2 = 2^3$$

$$\Rightarrow k^2 = 8$$