

RD Sharma Solutions for Class 12 Maths Chapter 14
Differentials, Errors and Approximations

EXERCISE 14.1

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1. If $y = \sin x$ and x changes from $\pi/2$ to $22/14$, what is the approximate change in y ?**Solution:**Given $y = \sin x$ and x changes from $\frac{\pi}{2}$ to $\frac{22}{14}$.Let $x = \frac{\pi}{2}$ so that $x + \Delta x = \frac{22}{14}$

$$\Rightarrow \frac{\pi}{2} + \Delta x = \frac{22}{14}$$

$$\therefore \Delta x = \frac{22}{14} - \frac{\pi}{2}$$

On differentiating y with respect to x , we get

$$\frac{dy}{dx} = \frac{d}{dx}(\sin x)$$

We know that $\frac{d}{dx}(\sin x) = \cos x$

$$\therefore \frac{dy}{dx} = \cos x$$

When $x = \frac{\pi}{2}$, we have $\frac{dy}{dx} = \cos\left(\frac{\pi}{2}\right)$.

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=\frac{\pi}{2}} = 0$$

We know that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right) \Delta x$$

$$\text{Here, } \frac{dy}{dx} = 0 \text{ and } \Delta x = \frac{22}{14} - \frac{\pi}{2}$$

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$$\Rightarrow \Delta y = (0) \left(\frac{22}{14} - \frac{\pi}{2} \right)$$

$$\therefore \Delta y = 0$$

Thus, there is approximately no change in y .

2. The radius of a sphere shrinks from 10 to 9.8 cm. Find approximately the decrease in its volume.

Solution:

Given the radius of a sphere changes from 10 cm to 9.8 cm.

Let x be the radius of the sphere and Δx be the change in the value of x .

Hence, we have $x = 10$ and $x + \Delta x = 9.8$

$$\Rightarrow 10 + \Delta x = 9.8$$

$$\Rightarrow \Delta x = 9.8 - 10$$

$$\therefore \Delta x = -0.2$$

The volume of a sphere of radius x is given by

$$V = \frac{4}{3} \pi x^3$$

On differentiating V with respect to x , we get

$$\frac{dV}{dx} = \frac{d}{dx} \left(\frac{4}{3} \pi x^3 \right)$$

$$\Rightarrow \frac{dV}{dx} = \frac{4\pi}{3} \frac{d}{dx} (x^3)$$

$$\text{We know } \frac{d}{dx} (x^n) = nx^{n-1}$$

$$\Rightarrow \frac{dV}{dx} = \frac{4\pi}{3} (3x^2)$$

$$\therefore \frac{dV}{dx} = 4\pi x^2$$

$$\text{When } x = 10, \text{ we have } \frac{dV}{dx} = 4\pi(10)^2.$$

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$$\Rightarrow \left(\frac{dV}{dx}\right)_{x=10} = 4\pi \times 100$$

$$\Rightarrow \left(\frac{dV}{dx}\right)_{x=10} = 400\pi$$

We know that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right) \Delta x$$

$$\text{Here, } \frac{dV}{dx} = 400\pi \text{ and } \Delta x = -0.2$$

$$\Rightarrow \Delta V = (400\pi)(-0.2)$$

$$\therefore \Delta V = -80\pi$$

Thus, the approximate decrease in the volume of the sphere is $80\pi \text{ cm}^3$.

3. A circular metal plate expands under heating so that its radius increases by $k\%$. Find the approximate increase in the area of the plate, if the radius of the plate before heating is 10 cm.

Solution:

Given the radius of a circular plate initially is 10 cm and it increases by $k\%$.

Let x be the radius of the circular plate, and Δx is the change in the value of x .

$$\text{Hence, we have } x = 10 \text{ and } \Delta x = \frac{k}{100} \times 10$$

$$\therefore \Delta x = 0.1k$$

The area of a circular plate of radius x is given by

$$A = \pi x^2$$

On differentiating A with respect to x , we get

$$\frac{dA}{dx} = \frac{d}{dx}(\pi x^2)$$

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$$\Rightarrow \frac{dA}{dx} = \pi \frac{d}{dx}(x^2)$$

$$\text{We know } \frac{d}{dx}(x^n) = nx^{n-1}$$

$$\Rightarrow \frac{dA}{dx} = \pi(2x)$$

$$\therefore \frac{dA}{dx} = 2\pi x$$

$$\text{When } x = 10, \text{ we have } \frac{dA}{dx} = 2\pi(10)$$

$$\Rightarrow \left(\frac{dA}{dx}\right)_{x=10} = 20\pi$$

We know that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right) \Delta x$$

$$\text{Here, } \frac{dA}{dx} = 20\pi \text{ and } \Delta x = 0.1\text{k}$$

$$\Rightarrow \Delta A = (20\pi)(0.1\text{k})$$

$$\therefore \Delta A = 2\text{k}\pi$$

Thus, the approximate increase in the area of the circular plate is $2\text{k}\pi \text{ cm}^2$.

4. Find the percentage error in calculating the surface area of a cubical box if an error of 1% is made in measuring the lengths of the edges of the cube.

Solution:

Given the error in the measurement of the edge of a cubical box is 1%.

Let x be the edge of the cubical box, and Δx is the error in the value of x .

$$\text{Hence, we have } \Delta x = \frac{1}{100} \times x$$

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$$\therefore \Delta x = 0.01x$$

The surface area of a cubical box of radius x is given by

$$S = 6x^2$$

On differentiating A with respect to x , we get

$$\frac{dS}{dx} = \frac{d}{dx}(6x^2)$$

$$\Rightarrow \frac{dS}{dx} = 6 \frac{d}{dx}(x^2)$$

$$\text{We know } \frac{d}{dx}(x^n) = nx^{n-1}$$

$$\Rightarrow \frac{dS}{dx} = 6(2x)$$

$$\therefore \frac{dS}{dx} = 12x$$

We know that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right) \Delta x$$

$$\text{Here, } \frac{dS}{dx} = 12x \text{ and } \Delta x = 0.01x$$

$$\Rightarrow \Delta S = (12x)(0.01x)$$

$$\therefore \Delta S = 0.12x^2$$

The percentage error is,

$$\text{Error} = \frac{0.12x^2}{6x^2} \times 100\%$$

$$\Rightarrow \text{Error} = 0.02 \times 100\%$$

$$\therefore \text{Error} = 2\%$$

Thus, the error in calculating the surface area of the cubical box is 2%.

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5. If there is an error of 0.1% in the measurement of the radius of a sphere, find approximately the percentage error in the calculation of the volume of the sphere.

Solution:

Given the error in the measurement of the radius of a sphere is 0.1%. Let x be the radius of the sphere and Δx be the error in the value of x .

Hence, we have $\Delta x = \frac{0.1}{100} \times x$

$$\therefore \Delta x = 0.001x$$

The volume of a sphere of radius x is given by

$$V = \frac{4}{3} \pi x^3$$

On differentiating V with respect to x , we get

$$\frac{dV}{dx} = \frac{d}{dx} \left(\frac{4}{3} \pi x^3 \right)$$

$$\Rightarrow \frac{dV}{dx} = \frac{4\pi}{3} \frac{d}{dx} (x^3)$$

We know $\frac{d}{dx} (x^n) = nx^{n-1}$

$$\Rightarrow \frac{dV}{dx} = \frac{4\pi}{3} (3x^2)$$

$$\therefore \frac{dV}{dx} = 4\pi x^2$$

We know that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx} \right) \Delta x$$

$$\text{Here, } \frac{dV}{dx} = 4\pi x^2 \text{ and } \Delta x = 0.001x$$

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$$\therefore \Delta V = 0.004\pi x^3$$

The percentage error is,

$$\text{Error} = \frac{0.004\pi x^3}{\frac{4}{3}\pi x^3} \times 100\%$$

$$\Rightarrow \text{Error} = \frac{0.004 \times 3}{4} \times 100\%$$

$$\Rightarrow \text{Error} = 0.003 \times 100\%$$

$$\therefore \text{Error} = 0.3\%$$

Thus, the error in calculating the volume of the sphere is 0.3%.

**6. The pressure p and the volume v of a gas are connected by the relation $pv^{1.4} = \text{const.}$
Find the percentage error in p corresponding to a decrease of $\frac{1}{2}\%$ in v .**

Solution:

Given $pv^{1.4} = \text{constant}$ and the decrease in v is $\frac{1}{2}\%$.

$$\text{Hence, we have } \Delta v = -\frac{\frac{1}{2}}{100} \times v$$

$$\therefore \Delta v = -0.005v$$

We have $pv^{1.4} = \text{constant}$

Taking log on both sides, we get

$$\log(pv^{1.4}) = \log(\text{constant})$$

$$\Rightarrow \log p + \log v^{1.4} = 0 \quad [\because \log(ab) = \log a + \log b]$$

$$\Rightarrow \log p + 1.4 \log v = 0 \quad [\because \log(a^m) = m \log a]$$

On differentiating both sides with respect to v , we get

$$\frac{d}{dp}(\log p) \times \frac{dp}{dv} + \frac{d}{dv}(1.4 \log v) = 0$$

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$$\Rightarrow \frac{d}{dp}(\log p) \times \frac{dp}{dv} + 1.4 \frac{d}{dv}(\log v) = 0$$

$$\text{We know } \frac{d}{dx}(\log x) = \frac{1}{x}$$

$$\Rightarrow \frac{1}{p} \times \frac{dp}{dv} + 1.4 \times \frac{1}{v} = 0$$

$$\Rightarrow \frac{1}{p} \frac{dp}{dv} + \frac{1.4}{v} = 0$$

$$\Rightarrow \frac{1}{p} \frac{dp}{dv} = -\frac{1.4}{v}$$

$$\therefore \frac{dp}{dv} = -\frac{1.4}{v} p$$

We know that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right) \Delta x$$

$$\text{Here, } \frac{dp}{dv} = -\frac{1.4}{v} p \text{ and } \Delta v = -0.005v$$

$$\Rightarrow \Delta p = \left(-\frac{1.4}{v} p\right) (-0.005v)$$

$$\Rightarrow \Delta p = (-1.4p) (-0.005)$$

$$\therefore \Delta p = 0.007p$$

The percentage error is,

$$\text{Error} = \frac{0.007p}{p} \times 100\%$$

$$\Rightarrow \text{Error} = 0.007 \times 100\%$$

$$\therefore \text{Error} = 0.7\%$$

Thus, the error in p corresponding to the decrease in v is 0.7%.

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7. The height of a cone increases by $k\%$, its semi-vertical angle remaining the same.
What is the approximate percentage increase in (i) in total surface area, and (ii) in the volume, assuming that k is small.

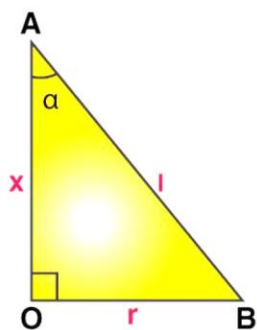
Solution:

Given the height of a cone increases by $k\%$. Let x be the height of the cone and Δx be the change in the value of x .

Hence, we have $\Delta x = \frac{k}{100} \times x$

$\therefore \Delta x = 0.01kx$

Let us assume the radius, the slant height and the semi-vertical angle of the cone to be r , l and α respectively as shown in the figure below.



From the above figure, using trigonometry, we have

$$\tan \alpha = \frac{OB}{OA}$$

$$\Rightarrow \tan \alpha = \frac{r}{x}$$

$$\therefore r = x \tan (\alpha)$$

We also have

$$\cos \alpha = \frac{OA}{AB}$$

$$\Rightarrow \cos \alpha = \frac{x}{l}$$

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$$\Rightarrow l = \frac{x}{\cos \alpha}$$

$$\therefore l = x \sec (\alpha)$$

(i) The total surface area of the cone is given by

$$S = \pi r^2 + \pi r l$$

From above, we have $r = x \tan (\alpha)$ and $l = x \sec (\alpha)$.

$$\Rightarrow S = \pi (x \tan (\alpha))^2 + \pi (x \tan (\alpha)) (x \sec (\alpha))$$

$$\Rightarrow S = \pi x^2 \tan^2 \alpha + \pi x^2 \tan (\alpha) \sec (\alpha)$$

$$\Rightarrow S = \pi x^2 \tan (\alpha) [\tan (\alpha) + \sec (\alpha)]$$

On differentiating S with respect to x , we get

$$\frac{dS}{dx} = \frac{d}{dx} [\pi x^2 \tan \alpha (\tan \alpha + \sec \alpha)]$$

$$\Rightarrow \frac{dS}{dx} = \pi \tan \alpha (\tan \alpha + \sec \alpha) \frac{d}{dx} (x^2)$$

$$\text{We know } \frac{d}{dx} (x^n) = nx^{n-1}$$

$$\Rightarrow \frac{dS}{dx} = \pi \tan \alpha (\tan \alpha + \sec \alpha) (2x)$$

$$\therefore \frac{dS}{dx} = 2\pi x \tan \alpha (\tan \alpha + \sec \alpha)$$

We know that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx} \right) \Delta x$$

$$\text{Here, } \frac{dS}{dx} = 2\pi x \tan \alpha (\tan \alpha + \sec \alpha) \text{ and } \Delta x = 0.01kx$$

$$\Rightarrow \Delta S = (2\pi x \tan (\alpha) [\tan (\alpha) + \sec (\alpha)]) (0.01kx)$$

$$\therefore \Delta S = 0.02 k \pi x^2 \tan (\alpha) [\tan (\alpha) + \sec (\alpha)]$$

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The percentage increase in S is,

$$\text{Increase} = \frac{\Delta S}{S} \times 100\%$$

$$\Rightarrow \text{Increase} = \frac{0.02k\pi x^2 \tan \alpha (\tan \alpha + \sec \alpha)}{\pi x^2 \tan \alpha (\tan \alpha + \sec \alpha)} \times 100\%$$

$$\Rightarrow \text{Increase} = 0.02k \times 100\%$$

$$\therefore \text{Increase} = 2k\%$$

Thus, the approximate increase in the total surface area of the cone is $2k\%$.

(ii) The volume of the cone is given by

$$V = \frac{1}{3} \pi r^2 x$$

From above, we have $r = x \tan \alpha$.

$$\Rightarrow V = \frac{1}{3} \pi (x \tan \alpha)^2 x$$

$$\Rightarrow V = \frac{1}{3} \pi (x^2 \tan^2 \alpha) x$$

$$\Rightarrow V = \frac{1}{3} \pi x^3 \tan^2 \alpha$$

On differentiating V with respect to x , we get

$$\frac{dV}{dx} = \frac{d}{dx} \left(\frac{1}{3} \pi x^3 \tan^2 \alpha \right)$$

$$\Rightarrow \frac{dV}{dx} = \frac{1}{3} \pi \tan^2 \alpha \frac{d}{dx} (x^3)$$

$$\text{We know } \frac{d}{dx} (x^n) = nx^{n-1}$$

$$\Rightarrow \frac{dV}{dx} = \frac{1}{3} \pi \tan^2 \alpha (3x^2)$$

$$\therefore \frac{dV}{dx} = \pi x^2 \tan^2 \alpha$$

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We know that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right) \Delta x$$

Here, $\frac{dV}{dx} = \pi x^2 \tan^2 \alpha$ and $\Delta x = 0.01kx$

$$\Rightarrow \Delta V = (\pi x^2 \tan^2 \alpha) (0.01kx)$$

$$\therefore \Delta V = 0.01k\pi x^3 \tan^2 \alpha$$

The percentage increase in V is,

$$\text{Increase} = \frac{\Delta V}{V} \times 100\%$$

$$\Rightarrow \text{Increase} = \frac{0.01k\pi x^3 \tan^2 \alpha}{\frac{1}{3}\pi x^3 \tan^2 \alpha} \times 100\%$$

$$\Rightarrow \text{Increase} = \frac{0.01k}{\frac{1}{3}} \times 100\%$$

$$\Rightarrow \text{Increase} = 0.03k \times 100\%$$

$$\therefore \text{Increase} = 3k\%$$

Thus, the approximate increase in the volume of the cone is $3k\%$.

8. Show that the relative error in computing the volume of a sphere, due to an error in measuring the radius, is approximately equal to three times the relative error in the radius.

Solution:

Let the error in measuring the radius of a sphere be $k\%$.

Let x be the radius of the sphere and Δx be the error in the value of x

Hence, we have $\Delta x = \frac{k}{100} \times x$

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$$\therefore \Delta x = 0.01kx$$

The volume of a sphere of radius x is given by

$$V = \frac{4}{3} \pi x^3$$

On differentiating V with respect to x , we get

$$\frac{dV}{dx} = \frac{d}{dx} \left(\frac{4}{3} \pi x^3 \right)$$

$$\Rightarrow \frac{dV}{dx} = \frac{4\pi}{3} \frac{d}{dx} (x^3)$$

$$\text{We know } \frac{d}{dx} (x^n) = nx^{n-1}$$

$$\Rightarrow \frac{dV}{dx} = \frac{4\pi}{3} (3x^2)$$

$$\therefore \frac{dV}{dx} = 4\pi x^2$$

We know that if $y = f(x)$ and Δx is a small increment in x , then the corresponding increment in y , $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx} \right) \Delta x$$

$$\text{Here, } \frac{dV}{dx} = 4\pi x^2 \text{ and } \Delta x = 0.01kx$$

$$\Rightarrow \Delta V = (4\pi x^2) (0.01kx)$$

$$\therefore \Delta V = 0.04k\pi x^3$$

The percentage error is,

$$\text{Error} = \frac{0.04k\pi x^3}{\frac{4}{3} \pi x^3} \times 100\%$$

$$\Rightarrow \text{Error} = \frac{0.04k \times 3}{4} \times 100\%$$

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$$\Rightarrow \text{Error} = 0.03k \times 100\%$$

$$\therefore \text{Error} = 3k\%$$