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Prove the following identities:

1. $\sqrt{[(1 - \cos 2x) / (1 + \cos 2x)]} = \tan x$

Solution:

Let us consider LHS:

$\sqrt{[(1 - \cos 2x) / (1 + \cos 2x)]}$

We know that $\cos 2x = 1 - 2 \sin^2 x$

$= 2 \cos^2 x - 1$

So,

$\sqrt{[(1 - \cos 2x) / (1 + \cos 2x)]} = \sqrt{[(1 - (1 - 2\sin^2 x)) / (1 + (2\cos^2 x - 1))]}$

$= \sqrt{[(1 - 1 + 2\sin^2 x) / (1 + 2\cos^2 x - 1)]}$

$= \sqrt{[2 \sin^2 x / 2 \cos^2 x]}$

$= \sin x/\cos x$

$= \tan x$

$=$ RHS

Hence proved.

2. $\sin 2x / (1 - \cos 2x) = \cot x$

Solution:

Let us consider LHS:

$\sin 2x / (1 - \cos 2x)$

We know that $\cos 2x = 1 - 2 \sin^2 x$

$\text{Sin } 2x = 2 \sin x \cos x$

So,

$\sin 2x / (1 - \cos 2x) = (2 \sin x \cos x) / (1 - (1 - 2\sin^2 x))$

$= (2 \sin x \cos x) / (1 - 1 + 2\sin^2 x)]$

$= [2 \sin x \cos x / 2 \sin^2 x]$

$= \cos x/\sin x$

$= \cot x$

$=$ RHS

Hence proved.

3. $\sin 2x / (1 + \cos 2x) = \tan x$

Solution:

Let us consider LHS:

$\sin 2x / (1 + \cos 2x)$

We know that $\cos 2x = 1 - 2 \sin^2 x$

$= 2 \cos^2 x - 1$

Sin 2x = 2 sin x cos x

So,

sin 2x / (1 + cos 2x) = [2 sin x cos x / (1 + ($2\cos^2 x$ - 1))]

= [2 sin x cos x / (1 + $2\cos^2$ x - 1)]

= [2 sin x cos x / 2 $\cos^2$ x]

= sin x/cos x

= tan x

= RHS

Hence proved.

4. $\sqrt{2+\sqrt{2+2\cos 4x}} = 2\cos x, 0 < x < \frac{\pi}{4}$

Solution:

Let us consider LHS:

$\sqrt{2+\sqrt{2+2\cos 4x}} = \sqrt{2+\sqrt{2+2(2\cos^2 2x-1)}}$

{since, cos 2x = 2 $\cos^2$ x – 1 ⇒ cos 4x = 2 $\cos^2$ 2x -1}

$= \sqrt{2+\sqrt{2+4\cos^2 2x-2}}$

$= \sqrt{2+\sqrt{4\cos^2 2x}}$

$= \sqrt{2+2\cos 2x}$

$= \sqrt{2+2(2\cos^2 x-1)}$ {since, cos 2x = 2 $\cos^2$ x – 1}

$= \sqrt{2+4\cos^2 x-2}$

$= \sqrt{4\cos^2 x}$

= 2 cos x

= RHS

Hence proved.

5. [1 – cos 2x + sin 2x] / [1 + cos 2x + sin 2x] = tan x

Solution:

Let us consider LHS:

[1 – cos 2x + sin 2x] / [1 + cos 2x + sin 2x]

We know that, cos 2x = 1 – 2 $\sin^2$ x

= 2 $\cos^2$ x – 1

Sin 2x = 2 sin x cos x
So,

$$= \frac{1-(1-2\sin^2 x)+2\sin x\cos x}{1+(2\cos^2 x-1)+2\sin x\cos x}$$

$$= \frac{1-1+2\sin^2 x+2\sin x\cos x}{1+2\cos^2 x-1+2\sin x\cos x}$$

$$= \frac{2\sin^2 x+2\sin x\cos x}{2\cos^2 x+2\sin x\cos x}$$

$$= \frac{2\sin x(\sin x+\cos x)}{2\cos x(\cos x+\sin x)}$$

$$= \frac{\sin x}{\cos x}$$

= tan x

= RHS

Hence proved.

6. [sin x + sin 2x] / [1 + cos x + cos 2x] = tan x

Solution:

Let us consider LHS:

[sin x + sin 2x] / [1 + cos x + cos 2x]

We know that, cos 2x = $\cos^2 x - \sin^2 x$

Sin 2x = 2 sin x cos x

So,

$$\frac{\sin x+\sin 2x}{1+\cos x+\cos 2x} = \frac{\sin x+2\sin x\cos x}{1+\cos x+(2\cos^2 x-1)}$$

$$= \frac{\sin x+2\sin x\cos x}{1+\cos x+2\cos^2 x-1}$$

$$= \frac{\sin x+2\sin x\cos x}{\cos x+2\cos^2 x}$$

$$= \frac{\sin x(1+2\cos x)}{\cos x(1+2\cos x)}$$

$$= \frac{\sin x}{\cos x}$$

= tan x

= RHS

Hence proved.

7. cos 2x / (1 + sin 2x) = tan (π/4 - x)

Solution:

Let us consider LHS:

cos 2x / (1 + sin 2x)

We know that, $\cos 2x = \cos^2 x - \sin^2 x$

$\sin 2x = 2 \sin x \cos x$

So,

$$\frac{\cos 2x}{1+\sin 2x} = \frac{\cos^2 x - \sin^2 x}{1 + 2\sin x\cos x}$$

$$= \frac{(\cos x - \sin x)(\cos x + \sin x)}{\sin^2 x + \cos^2 x + 2\sin x\cos x}$$

(since, $a^2 - b^2 = (a - b)(a + b)$ & $\sin^2 x + \cos^2 x = 1$)

$$= \frac{(\cos x - \sin x)(\cos x + \sin x)}{(\sin x + \cos x)^2}$$

(since, $a^2 + b^2 + 2ab = (a + b)^2$)

$$= \frac{(\cos x - \sin x)(\cos x + \sin x)}{(\sin x + \cos x)(\sin x + \cos x)}$$

$$= \frac{(\cos x - \sin x)}{(\sin x + \cos x)}$$

Multiplying numerator and denominator by $1/\sqrt{2}$

We get,

$$= \frac{\frac{1}{\sqrt{2}}(\cos x - \sin x)}{\frac{1}{\sqrt{2}}(\sin x + \cos x)}$$

$$= \frac{\left(\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x\right)}{\left(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x\right)}$$

$$= \frac{\left(\sin\frac{\pi}{4}\cos x - \cos\frac{\pi}{4}\sin x\right)}{\left(\sin\frac{\pi}{4}\sin x + \cos\frac{\pi}{4}\cos x\right)}$$ (since, $1/\sqrt{2} = \sin \pi/4$)

$$= \frac{\sin\left(\frac{\pi}{4} - x\right)}{\cos\left(\frac{\pi}{4} - x\right)}$$

By using the formulas,

sin (A – B) = sin A cos B – sin B cos A

cos (A – B) = cos A cos B + sin A sin B

= tan (π/4 - x)

= RHS

Hence proved.

8. cos x / (1 - sin x) = tan (π/4 + x/2)

Solution:

Let us consider LHS:

cos x / (1 - sin x)

We know that, cos 2x = $\cos^2 x – \sin^2 x$

Cos x = $\cos^2 x/2 – \sin^2 x/2$

Sin 2x = 2 sin x cos x

Sin x = 2 sin x/2 cos x/2

So,

$$\frac{\cos x}{1 - \sin x} = \frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{1 - 2\sin\frac{x}{2}\cos\frac{x}{2}}$$

$$= \frac{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)}{\sin^2\frac{x}{2} + \cos^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}}$$

(By using the formula, $a^2 – b^2 = (a - b)(a + b)$ & $\sin^2 x + \cos^2 x = 1$)

$$= \frac{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)}{\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2}$$

(By using the formula, $a^2 + b^2 + 2ab = (a + b)^2$)

$$= \frac{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)}{\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)}$$

$$= \frac{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)}{\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)}$$

$$= \frac{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)}{\left(\sin\frac{x}{2} - \cos\frac{x}{2}\right)}$$

Let us multiply numerator and denominator by $1/\sqrt{2}$

We get,

$$= \frac{\frac{1}{\sqrt{2}}\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)}{\frac{1}{\sqrt{2}}\left(\sin\frac{x}{2} - \cos\frac{x}{2}\right)}$$

$$= \frac{\left(\frac{1}{\sqrt{2}}\cos\frac{x}{2} + \frac{1}{\sqrt{2}}\sin\frac{x}{2}\right)}{\left(\frac{1}{\sqrt{2}}\sin\frac{x}{2} - \frac{1}{\sqrt{2}}\cos\frac{x}{2}\right)}$$

$$= \frac{\left(\sin\frac{\pi}{4}\cos\frac{x}{2} + \cos\frac{\pi}{4}\sin\frac{x}{2}\right)}{\left(\sin\frac{\pi}{4}\sin\frac{x}{2} - \cos\frac{\pi}{4}\cos\frac{x}{2}\right)}$$ (since, $1/\sqrt{2} = \sin \pi/4$)

$$= \frac{\sin\left(\frac{\pi}{4} - x\right)}{\cos\left(\frac{\pi}{4} - x\right)}$$

$= \tan (\pi/4 - x)$

= RHS

Hence proved.

9. $\cos^2\frac{\pi}{8} + \cos^2\frac{3\pi}{8} + \cos^2\frac{5\pi}{8} + \cos^2\frac{7\pi}{8} = 2$

Solution:

Let us consider LHS:

$$\cos^2\frac{\pi}{8} + \cos^2\frac{3\pi}{8} + \cos^2\frac{5\pi}{8} + \cos^2\frac{7\pi}{8}$$

We know that $\cos 2x = 2\cos^2 x - 1$

$\text{Cos } 2x + 1 = 2\cos^2 x$

$\text{Cos}^2 x = (\text{Cos } 2x + 1)/2$

So,

$$\cos^2\frac{\pi}{8} + \cos^2\frac{3\pi}{8} + \cos^2\frac{5\pi}{8} + \cos^2\frac{7\pi}{8}$$

$$= \frac{1+\cos\frac{2\pi}{8}}{2} + \frac{1+\cos\frac{6\pi}{8}}{2} + \frac{1+\cos\frac{10\pi}{8}}{2} + \frac{1+\cos\frac{14\pi}{8}}{2}$$

$$= \frac{1+\cos\frac{2\pi}{8}}{2} + \frac{1+\cos\left(\pi-\frac{2\pi}{8}\right)}{2} + \frac{1+\cos\left(\pi+\frac{2\pi}{8}\right)}{2} + \frac{1+\cos\left(2\pi-\frac{2\pi}{8}\right)}{2}$$

$$\left\{\because \pi - \frac{2\pi}{8} = \frac{6\pi}{8}; \pi + \frac{2\pi}{8} = \frac{10\pi}{8}; 2\pi - \frac{2\pi}{8} = \frac{14\pi}{8}\right\}$$

$$= \frac{1+\cos\frac{2\pi}{8}}{2} + \frac{1-\cos\frac{2\pi}{8}}{2} + \frac{1-\cos\frac{2\pi}{8}}{2} + \frac{1+\cos\frac{2\pi}{8}}{2}$$

{we know, cos (π – A) = - cos A, cos (π + A) = - cos A & cos (2π – A) = cos A}

$$= 2 \times \frac{1+\cos\frac{2\pi}{8}}{2} + 2 \times \frac{1-\cos\frac{2\pi}{8}}{2}$$

$$= 1 + \cos\frac{2\pi}{8} + 1 - \cos\frac{2\pi}{8}$$

= 2

= RHS

Hence proved.

10. $\sin^2\frac{\pi}{8} + \sin^2\frac{3\pi}{8} + \sin^2\frac{5\pi}{8} + \sin^2\frac{7\pi}{8} = 2$

Solution:

Let us consider LHS:

$$\sin^2\frac{\pi}{8} + \sin^2\frac{3\pi}{8} + \sin^2\frac{5\pi}{8} + \sin^2\frac{7\pi}{8}$$

We know that, cos 2x = 1 – 2sin² x

2sin² x = 1- cos 2x

Sin² x = (1 – cos 2x)/2

So,

$$= \frac{1-\cos\frac{2\pi}{8}}{2} + \frac{1-\cos\frac{6\pi}{8}}{2} + \frac{1-\cos\frac{10\pi}{8}}{2} + \frac{1-\cos\frac{14\pi}{8}}{2}$$

$$= \frac{1-\cos\frac{2\pi}{8}}{2} + \frac{1-\cos\left(\pi-\frac{2\pi}{8}\right)}{2} + \frac{1-\cos\left(\pi+\frac{2\pi}{8}\right)}{2} + \frac{1-\cos\left(2\pi-\frac{2\pi}{8}\right)}{2}$$

$\left\{\because \pi - \frac{2\pi}{8} = \frac{6\pi}{8}; \pi + \frac{2\pi}{8} = \frac{10\pi}{8}; 2\pi - \frac{2\pi}{8} = \frac{14\pi}{8}\right\}$

$= \frac{1-\cos\frac{2\pi}{8}}{2} + \frac{1-\left(-\cos\frac{2\pi}{8}\right)}{2} + \frac{1-\left(-\cos\frac{2\pi}{8}\right)}{2} + \frac{1-\cos\frac{2\pi}{8}}{2}$

{we know, cos (π – A) = - cos A, cos (π + A) = - cos A & cos (2π – A) = cos A}

$= \frac{1-\cos\frac{2\pi}{8}}{2} + \frac{1+\cos\frac{2\pi}{8}}{2} + \frac{1+\cos\frac{2\pi}{8}}{2} + \frac{1-\cos\frac{2\pi}{8}}{2}$

$= 2 \times \frac{1-\cos\frac{2\pi}{8}}{2} + 2 \times \frac{1+\cos\frac{2\pi}{8}}{2}$

$= 1 - \cos\frac{2\pi}{8} + 1 + \cos\frac{2\pi}{8}$

= 2

= RHS

Hence proved.

11. (cos α + cos β) ^{2} + (sin α + sin β) ^{2} = 4 cos^2 (α - β)/2

Solution:

Let us consider LHS:

(cos α + cos β)^2 + (sin α + sin β)^2

Upon expansion, we get,

(cos α + cos β)^2 + (sin α + sin β)^2 =

= cos^2 α + cos^2 β + 2 cos α cos β + sin^2 α + sin^2 β + 2 sin α sin β

= 2 + 2 cos α cos β + 2 sin α sin β

= 2 (1 + cos α cos β + sin α sin β)

= 2 (1 + cos (α - β)) [since, cos (A – B) = cos A cos B + sin A sin B]

= 2 (1 + 2 cos^2 (α - β)/2 - 1) [since, cos2x = 2cos^2 x – 1]

= 2 (2 cos^2 (α - β)/2)

= 4 cos^2 (α - β)/2

= RHS

Hence Proved.

12. sin^2 (π/8 + x/2) – sin^2 (π/8 – x/2) = 1/√2 sin x

Solution:

Let us consider LHS:

sin^2 (π/8 + x/2) – sin^2 (π/8 – x/2) we know, $\sin^2 A - \sin^2 B = \sin (A+B) \sin (A-B)$

so,

$\sin^2 (\pi/8 + x/2) - \sin^2 (\pi/8 - x/2) = \sin (\pi/8 + x/2 + \pi/8 - x/2) \sin (\pi/8 + x/2 - (\pi/8 - x/2))$

$= \sin (\pi/8 + \pi/8) \sin (\pi/8 + x/2 - \pi/8 + x/2)$

$= \sin \pi/4 \sin x$

$= 1/\sqrt{2} \sin x$ [since, since $\pi/4 = 1/\sqrt{2}$]

= RHS

Hence proved.

13. $1 + \cos^2 2x = 2 (\cos^4 x + \sin^4 x)$

Solution:

Let us consider LHS:

$1 + \cos^2 2x$

We know, $\cos 2x = \cos^2 x - \sin^2 x$

$\cos^2 x + \sin^2 x = 1$

so,

$1 + \cos^2 2x = (\cos^2 x + \sin^2 x)^2 + (\cos^2 x - \sin^2 x)^2$

$= (\cos^4 x + \sin^4 x + 2 \cos^2 x \sin^2 x) + (\cos^4 x + \sin^4 x - 2 \cos^2 x \sin^2 x)$

$= \cos^4 x + \sin^4 x + \cos^4 x + \sin^4 x$

$= 2 \cos^4 x + 2 \sin^4 x$

$= 2 (\cos^4 x + \sin^4 x)$

= RHS

Hence proved.

14. $\cos^3 2x + 3 \cos 2x = 4 (\cos^6 x - \sin^6 x)$

Solution:

Let us consider RHS:

$4 (\cos^6 x - \sin^6 x)$

Upon expansion we get,

$4 (\cos^6 x - \sin^6 x) = 4 [(\cos^2 x)^3 - (\sin^2 x)^3]$

$= 4 (\cos^2 x - \sin^2 x) (\cos^4 x + \sin^4 x + \cos^2 x \sin^2 x)$

By using the formula,

$a^3 - b^3 = (a-b) (a^2 + b^2 + ab)$

$= 4 \cos 2x (\cos^4 x + \sin^4 x + \cos^2 x \sin^2 x + \cos^2 x \sin^2 x - \cos^2 x \sin$

We know, $\cos 2x = \cos^2 x - \sin^2 x$

So,

$= 4 \cos 2x (\cos^4 x + \sin^4 x + 2 \cos^2 x \sin^2 x - \cos^2 x \sin^2 x)$

$= 4 \cos 2x [(\cos^2 x)^2 + (\sin^2 x)^2 + 2 \cos^2 x \sin^2 x - \cos^2 x \sin^2 x]$

We know, $a^2 + b^2 + 2ab = (a + b)^2$

$= 4 \cos 2x [(1)^2 – 1/4 (4 \cos^2 x \sin^2 x)]$
$= 4 \cos 2x [(1)^2 – 1/4 (2 \cos x \sin x)^2]$

We know, $\sin 2x = 2\sin x \cos x$

$= 4 \cos 2x [(1^2) – 1/4 (\sin 2x)^2]$
$= 4 \cos 2x (1 – 1/4 \sin^2 2x)$

We know, $\sin^2 x = 1 – \cos^2 x$

$= 4 \cos 2x [1 – 1/4 (1 – \cos^2 2x)]$
$= 4 \cos 2x [1 – 1/4 + 1/4 \cos^2 2x]$
$= 4 \cos 2x [3/4 + 1/4 \cos^2 2x]$
$= 4 (3/4 \cos 2x + 1/4 \cos^3 2x)$
$= 3 \cos 2x + \cos^3 2x$
$= \cos^3 2x + 3 \cos 2x$
= LHS

Hence proved.

15. $(\sin 3x + \sin x) \sin x + (\cos 3x – \cos x) \cos x = 0$

Solution:

Let us consider LHS:

$(\sin 3x + \sin x) \sin x + (\cos 3x – \cos x) \cos x$

$= (\sin 3x) (\sin x) + \sin^2 x + (\cos 3x) (\cos x) – \cos^2 x$
$= [(\sin 3x) (\sin x) + (\cos 3x) (\cos x)] + (\sin^2 x – \cos^2 x)$
$= [(\sin 3x) (\sin x) + (\cos 3x) (\cos x)] – (\cos^2 x – \sin^2 x)$
$= \cos (3x – x) – \cos 2x$

We know, $\cos 2x = \cos^2 x – \sin^2 x$

$\cos A \cos B + \sin A \sin B = \cos(A – B)$

So,

$= \cos 2x – \cos 2x$
$= 0$
= RHS

Hence Proved.

16. $\cos^2 (\pi/4 - x) – \sin^2 (\pi/4 - x) = \sin 2x$

Solution:

Let us consider LHS:

$\cos^2 (\pi/4 - x) – \sin^2 (\pi/4 - x)$

We know, $\cos^2 A – \sin^2 A = \cos 2A$

So,

$\cos^2 (\pi/4 - x) – \sin^2 (\pi/4 - x) = \cos 2 (\pi/4 - x)$
$= \cos (\pi/2 - 2x)$

= sin 2x [since, cos (π/2 - A) = sin A]
= RHS

Hence proved.

17. $\cos 4x = 1 - 8\cos^2 x + 8\cos^4 x$

Solution:

Let us consider LHS:

cos 4x

We know, $\cos 2x = 2\cos^2 x - 1$

So,

$\cos 4x = 2\cos^2 2x - 1$

$= 2(2\cos^2 2x - 1)^2 - 1$

$= 2[(2\cos^2 2x)^2 + 1^2 - 2\times 2\cos^2 x] - 1$

$= 2(4\cos^4 2x + 1 - 4\cos^2 x) - 1$

$= 8\cos^4 2x + 2 - 8\cos^2 x - 1$

$= 8\cos^4 2x + 1 - 8\cos^2 x$

= RHS

Hence Proved.

18. $\sin 4x = 4\sin x\cos^3 x - 4\cos x\sin^3 x$

Solution:

Let us consider LHS:

sin 4x

We know, $\sin 2x = 2\sin x\cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

So,

$\sin 4x = 2\sin 2x\cos 2x$

$= 2(2\sin x\cos x)(\cos^2 x - \sin^2 x)$

$= 4\sin x\cos x(\cos^2 x - \sin^2 x)$

$= 4\sin x\cos^3 x - 4\sin^3 x\cos x$

= RHS

Hence proved.

19. $3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x) = 13$

Solution:

Let us consider LHS:

$3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x)$

We know, $(a + b)^2 = a^2 + b^2 + 2ab$

$(a - b)^2 = a^2 + b^2 - 2ab$

$a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

So,

$3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x) = 3\{(\sin x - \cos x)^2\}^2 + 6\{(\sin x)^2 + (\cos x)^2 + 2\sin x\cos x)\} + 4\{(\sin^2 x)^3 + (\cos^2 x)^3\}$

$= 3\{(\sin x)^2 + (\cos x)^2 - 2\sin x\cos x)\}^2 + 6(\sin^2 x + \cos^2 x + 2\sin x\cos x) + 4\{(\sin^2 x + \cos^2 x)(\sin^4 x + \cos^4 x - \sin^2 x\cos^2 x)\}$

$= 3(1 - 2\sin x\cos x)^2 + 6(1 + 2\sin x\cos x) + 4\{(1)(\sin^4 x + \cos^4 x - \sin^2 x\cos^2 x)\}$

We know, $\sin^2 x + \cos^2 x = 1$

So,

$= 3\{1^2 + (2\sin x\cos x)^2 - 4\sin x\cos x\} + 6(1 + 2\sin x\cos x) + 4\{(\sin^2 x)^2 + (\cos^2 x)^2 + 2\sin^2 x\cos^2 x - 3\sin^2 x\cos^2 x)\}$

$= 3\{1 + 4\sin^2 x\cos^2 x - 4\sin x\cos x\} + 6(1 + 2\sin x\cos x) + 4\{(\sin^2 x + \cos^2 x)^2 - 3\sin^2 x\cos^2 x)\}$

$= 3 + 12\sin^2 x\cos^2 x - 12\sin x\cos x + 6 + 12\sin x\cos x + 4\{(1)^2 - 3\sin^2 x\cos^2 x)\}$

$= 9 + 12\sin^2 x\cos^2 x + 4(1 - 3\sin^2 x\cos^2 x)$

$= 9 + 12\sin^2 x\cos^2 x + 4 - 12\sin^2 x\cos^2 x$

$= 13$

= RHS

Hence proved.

20. $2(\sin^6 x + \cos^6 x) - 3(\sin^4 x + \cos^4 x) + 1 = 0$

Solution:

Let us consider LHS:

$2(\sin^6 x + \cos^6 x) - 3(\sin^4 x + \cos^4 x) + 1$

We know, $(a + b)^2 = a^2 + b^2 + 2ab$

$a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

So,

$2(\sin^6 x + \cos^6 x) - 3(\sin^4 x + \cos^4 x) + 1 = 2\{(\sin^2 x)^3 + (\cos^2 x)^3\} - 3\{(\sin^2 x)^2 + (\cos^2 x)^2\} + 1$

$= 2\{(\sin^2 x + \cos^2 x)(\sin^4 x + \cos^4 x - \sin^2 x\cos^2 x\} - 3\{(\sin^2 x)^2 + (\cos^2 x)^2 + 2\sin^2 x\cos^2 x - 2\sin^2 x\cos^2 x\} + 1$

$= 2\{(1)(\sin^4 x + \cos^4 x + 2\sin^2 x\cos^2 x - 3\sin^2 x\cos^2 x\} - 3\{(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x\cos^2 x\} + 1$

We know, $\sin^2 x + \cos^2 x = 1$

$= 2\{(\sin^2 x + \cos^2 x)^2 – 3 \sin^2 x \cos^2 x\} – 3\{(1)^2 – 2\sin^2 x \cos^2 x\} + 1$

$= 2\{(1)^2 – 3 \sin^2 x \cos^2 x\} – 3(1 – 2\sin^2 x \cos^2 x) + 1$

$= 2(1 – 3 \sin^2 x \cos^2 x) – 3 + 6 \sin^2 x \cos^2 x + 1$

$= 2 – 6 \sin^2 x \cos^2 x – 2 + 6 \sin^2 x \cos^2 x$

$= 0$

= RHS

Hence proved.

21. $\cos^6 x – \sin^6 x = \cos 2x (1 – 1/4 \sin^2 2x)$

Solution:

Let us consider LHS:

$\cos^6 x – \sin^6 x$

We know, $(a + b)^2 = a^2 + b^2 + 2ab$

$a^3 – b^3 = (a – b) (a^2 + b^2 + ab)$

So,

$\cos^6 x – \sin^6 x = (\cos^2 x)^3 – (\sin^2 x)^3$

$= (\cos^2 x – \sin^2 x) (\cos^4 x + \sin^4 x + \cos^2 x \sin^2 x)$

We know, $\cos 2x = \cos^2 x – \sin^2 x$

So,

$= \cos 2x [(\cos^2 x)^2 + (\sin^2 x)^2 + 2 \cos^2 x \sin^2 x – \cos^2 x \sin^2 x]$

$= \cos 2x [(\cos^2 x)^2 + (\sin^2 x)^2 – 1/4 \times 4 \cos^2 x \sin^2 x]$

We know, $\sin^2 x + \cos^2 x = 1$

So,

$= \cos 2x [(1)^2 – 1/4 \times (2 \cos x \sin x)^2]$

We know, $\sin 2x = 2 \sin x \cos x$

So,

$= \cos 2x [1 – 1/4 \times (\sin 2x)^2]$

$= \cos 2x [1 – 1/4 \times \sin^2 2x]$

= RHS

Hence proved.

22. $\tan (\pi/4 + x) + \tan (\pi/4 - x) = 2 \sec 2x$

Solution:

Let us consider LHS:

$\tan (\pi/4 + x) + \tan (\pi/4 - x)$

We know,

tan (A+B) = (tan A + tan B)/(1- tan A tan B)
tan (A-B) = (tan A - tan B)/(1+ tan A tan B)
So,

$$tan(\frac{\pi}{4}+x)+tan(\frac{\pi}{4}-x)=\frac{tan\frac{\pi}{4}+tanx}{1-tan\frac{\pi}{4}tanx}+\frac{tan\frac{\pi}{4}-tanx}{1+tan\frac{\pi}{4}tanx}$$

We know, tan π/4 = 1
So,

$$=\frac{1+\tan x}{1-\tan x}+\frac{1-\tan x}{1+\tan x}$$

$$=\frac{(1+\tan x)^2+(1-\tan x)^2}{(1-\tan x)(1+\tan x)}$$

We know, $(a-b)(a+b)=a^2-b^2$;
$(a+b)^2=a^2+b^2+2ab$ &
$(a-b)^2=a^2+b^2-2ab$
So,

$$=\frac{1^2+\tan^2 x+2\tan x+1^2+\tan^2 x-2\tan x}{1^2-\tan^2 x}$$

$$=\frac{1+\tan^2 x+1+\tan^2 x}{1-\tan^2 x}$$

$$=\frac{2(1+\tan^2 x)}{1-\tan^2 x}$$

We know, tan x = sin x/cos x
So,

$$=\frac{2\left(1+\left(\frac{\sin x}{\cos x}\right)^2\right)}{1-\left(\frac{\sin x}{\cos x}\right)^2}$$

$$=\frac{2\left(1+\frac{\sin^2 x}{\cos^2 x}\right)}{1-\frac{\sin^2 x}{\cos^2 x}}$$

$$=\frac{2\left(\frac{\cos^2 x+\sin^2 x}{\cos^2 x}\right)}{\frac{\cos^2 x-\sin^2 x}{\cos^2 x}}$$

We know, $\cos^2 x + \sin^2 x = 1$ & $\cos 2x = \cos^2 x - \sin^2 x$

So,

$$= \frac{2\left(\frac{1}{\cos^2 x}\right)}{\frac{\cos 2x}{\cos^2 x}}$$

$$= \frac{2}{\cos 2x}$$

$= 2 \sec 2x$ (since, $1/\cos 2x = \sec 2x$)

= RHS

Hence proved.

EXERCISE 9.2 PAGE NO: 9.36

Prove that:

1. $\sin 5x = 5 \sin x - 20 \sin^3 x + 16 \sin^5 x$

Solution:

Let us consider LHS:

$\sin 5x$

Now,

$\sin 5x = \sin (3x + 2x)$

But we know,

$\text{Sin} (x + y) = \sin x \cos y + \cos x \sin y$…..(i)

So,

$\sin 5x = \sin 3x \cos 2x + \cos 3x \sin 2x$

$= \sin (2x + x) \cos 2x + \cos (2x + x) \sin 2x$……..(ii)

And

$\cos (x + y) = \cos x \cos y - \sin x \sin y$……(iii)

Now substituting equation (i) and (iii) in equation (ii), we get

$\sin 5x = (\sin 2x \cos x + \cos 2x \sin x) \cos 2x + (\cos 2x \cos x - \sin 2x \sin x) \sin 2x$

$= \sin 2x \cos 2x \cos x + \cos^2 2x \sin x + (\sin 2x \cos 2x \cos x - \sin^2 2x \sin x)$

$= 2\sin 2x \cos 2x \cos x + \cos^2 2x \sin x - \sin^2 2x \sin x$ …….(iv)

Now $\sin 2x = 2\sin x \cos x$………(v)

And $\cos 2x = \cos^2 x - \sin^2 x$……..(vi)

Substituting equation (v) and (vi) in equation (iv), we get

$\sin 5x = 2(2\sin x \cos x) (\cos^2 x - \sin^2 x) \cos x + (\cos^2 x - \sin^2 x)^2 \sin x - (2\sin x \cos x)^2 \sin x$

$= 4(\sin x \cos^2 x) ([1 - \sin^2 x] - \sin^2 x) + ([1 - \sin^2 x] - \sin^2 x)^2 \sin x - (4\sin^2 x \cos^2 x)\sin x$

(as $\cos^2 x + \sin^2 x = 1 \Rightarrow \cos^2 x = 1 - \sin^2 x$)

$\sin 5x = 4(\sin x [1 - \sin^2 x]) (1 - 2\sin^2 x) + (1 - 2\sin^2 x)^2 \sin x - 4\sin^3 x [1 - \sin^2 x]$

$= 4\sin x (1 - \sin^2 x) (1 - 2\sin^2 x) + (1 - 4\sin^2 x + 4\sin^4 x) \sin x - 4\sin^3 x + 4\sin^5 x$

$= (4\sin x - 4\sin^3 x) (1 - 2\sin^2 x) + \sin x - 4\sin^3 x + 4\sin^5 x - 4\sin^3 x + 4\sin^5 x$

$= 4\sin x - 8\sin^3 x - 4\sin^3 x + 8\sin^5 x + \sin x - 8\sin^3 x + 8\sin^5 x$

$= 5\sin x - 20\sin^3 x + 16\sin^5 x$

= RHS

Hence proved.

2. $4 (\cos^3 10^\circ + \sin^3 20^\circ) = 3 (\cos 10^\circ + \sin 20^\circ)$

Solution:

Let us consider LHS:

$4(\cos^3 10^\circ + \sin^3 20^\circ)$
We know that, $\sin 60^\circ = \sqrt{3}/2 = \cos 30^\circ$
$\sin 30^\circ = \cos 60^\circ = 1/2$
So,
$\sin(3\times20^\circ) = \cos(3\times10^\circ)$
$3\sin 20^\circ - 4\sin^3 20^\circ = 4\cos^3 10^\circ - 3\cos 10^\circ$
(we know, $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$ and $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$)
So,
$4(\cos^3 10^\circ+\sin^3 20^\circ) = 3(\sin 20^\circ+\cos 10^\circ)$
= RHS
Hence proved.

3. $\cos^3 x \sin 3x + \sin^3 x \cos 3x = 3/4 \sin 4x$
Solution:
We know that,
$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$
So, $4\cos^3\theta = \cos 3\theta + 3\cos\theta$
$\cos^3\theta = [\cos 3\theta + 3\cos\theta]/4$ …… (i)
Similarly,
$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$
$4\sin^3\theta = 3\sin\theta - \sin 3\theta$
$\sin^3\theta = [3\sin\theta - \sin 3\theta]/4$ …….. (ii)
Now,
Let us consider LHS:
$\cos^3 x \sin 3x + \sin^3 x \cos 3x$
Substituting the values from equation (i) and (ii), we get
$\cos^3 x \sin 3x + \sin^3 x \cos 3x = (\cos 3x + 3\cos x)/4 \sin 3x + (3\sin x - \sin 3x)/4 \cos 3x$
$= 1/4 (\sin 3x \cos 3x + 3 \sin 3x \text{ cox } x + 3\sin x \cos 3x - \sin 3x \cos 3x)$
$= 1/4 (3(\sin 3x \cos x + \sin x \cos 3x) + 0)$
$= 1/4 (3 \sin(3x + x))$
(We know, $\sin(x + y) = \sin x \cos y + \cos x \sin y$)
$= 3/4 \sin 4x$
= RHS
Hence proved.

4. $\sin 5x = 5\cos^4 x \sin x - 10\cos^2 x \sin^3 x + \sin^5 x$
Solution:
Let us consider LHS:
$\sin 5x$

Now,
sin 5x = sin (3x + 2x)
But we know,
Sin (x + y) = sin x cos y + cos x sin y…..(i)
So,
sin 5x = sin 3x cos 2x + cos 3x sin 2x
= sin (2x + x) cos 2x + cos (2x + x) sin 2x……..(ii)
And
cos (x + y) = cos x cos y – sin x sin y……(iii)
Now substituting equation (i) and (iii) in equation (ii), we get
sin 5x = (sin 2x cos x + cos 2x sin x) cos 2x + (cos 2x cos x – sin 2x sin x) sin 2x … (iv)

Now sin 2x = 2sin x cos x………(v)
And cos 2x = $\cos^2 x - \sin^2 x$………(vi)
Substituting equation (v) and (vi) in equation (iv), we get
sin 5x = $[(2 \sin x \cos x) \cos x + (\cos^2 x - \sin^2 x) \sin x] (\cos^2 x - \sin^2 x) + [(\cos^2 x - \sin^2 x) \cos x - (2 \sin x \cos x) \sin x)] (2 \sin x \cos x)$
= $[2 \sin x \cos^2 x + \sin x \cos^2 x - \sin^3 x] (\cos^2 x - \sin^2 x) + [\cos^3 x - \sin^2 x \cos x - 2 \sin^2 x \cos x] (2 \sin x \cos x)$
= $\cos^2 x [3 \sin x \cos^2 x - \sin^3 x] - \sin^2 x [3 \sin x \cos^2 x - \sin^3 x] + 2 \sin x \cos^4 x - 2 \sin^3 x \cos^2 x - 4 \sin^3 x \cos^2 x$
= $3 \sin x \cos^4 x - \sin^3 x \cos^2 x - 3 \sin^3 x \cos^2 x - \sin^5 x + 2 \sin x \cos^4 x - 2 \sin^3 x \cos^2 x - 4 \sin^3 x \cos^2 x$
= $5 \sin x \cos^4 x - 10 \sin^3 x \cos^2 x + \sin^5 x$
= RHS
Hence proved.

5. sin 5x = 5 sin x – 20 $\sin^3$ x + 16 $\sin^5$ x
Solution:
Let us consider LHS:
sin 5x
Now,
sin 5x = sin (3x + 2x)
But we know,
Sin (x + y) = sin x cos y + cos x sin y…..(i)
So,
sin 5x = sin 3x cos 2x + cos 3x sin 2x
= sin (2x + x) cos 2x + cos (2x + x) sin 2x……..(ii)
And

cos (x + y) = cos x cos y – sin x sin y……(iii)

Now substituting equation (i) and (iii) in equation (ii), we get

$\sin 5x = (\sin 2x \cos x + \cos 2x \sin x) \cos 2x + (\cos 2x \cos x - \sin 2x \sin x) \sin 2x$

$= \sin 2x \cos 2x \cos x + \cos^2 2x \sin x + (\sin 2x \cos 2x \cos x - \sin^2 2x \sin x)$

$= 2\sin 2x \cos 2x \cos x + \cos^2 2x \sin x - \sin^2 2x \sin x$ …….(iv)

Now sin 2x = 2sin x cos x………(v)

And $\cos 2x = \cos^2 x - \sin^2 x$………(vi)

Substituting equation (v) and (vi) in equation (iv), we get

$\sin 5x = 2(2\sin x \cos x)(\cos^2 x - \sin^2 x) \cos x + (\cos^2 x - \sin^2 x)^2 \sin x - (2\sin x \cos x)^2 \sin x$

$= 4(\sin x \cos^2 x)([1 - \sin^2 x] - \sin^2 x) + ([1 - \sin^2 x] - \sin^2 x)^2 \sin x - (4\sin^2 x \cos^2 x)\sin x$

(as $\cos^2 x + \sin^2 x = 1 \Rightarrow \cos^2 x = 1 - \sin^2 x$)

$\sin 5x = 4(\sin x [1 - \sin^2 x])(1 - 2\sin^2 x) + (1 - 2\sin^2 x)^2 \sin x - 4\sin^3 x [1 - \sin^2 x]$

$= 4\sin x (1 - \sin^2 x)(1 - 2\sin^2 x) + (1 - 4\sin^2 x + 4\sin^4 x) \sin x - 4\sin^3 x + 4\sin^5 x$

$= (4\sin x - 4\sin^3 x)(1 - 2\sin^2 x) + \sin x - 4\sin^3 x + 4\sin^5 x - 4\sin^3 x + 4\sin^5 x$

$= 4\sin x - 8\sin^3 x - 4\sin^3 x + 8\sin^5 x + \sin x - 8\sin^3 x + 8\sin^5 x$

$= 5\sin x - 20\sin^3 x + 16\sin^5 x$

= RHS

Hence proved.

7. $\tan x + \tan\left(\frac{\pi}{3} + x\right) - \tan\left(\frac{\pi}{3} - x\right) = 3\tan 3x$

Solution:

Let us consider LHS:

$\tan x + \tan\left(\frac{\pi}{3} + x\right) - \tan\left(\frac{\pi}{3} - x\right)$

$$= \tan x + \left(\frac{\tan\frac{\pi}{3} + \tan x}{1 - \tan x \tan\frac{\pi}{3}}\right) - \left(\frac{\tan\frac{\pi}{3} - \tan x}{1 + \tan x \tan\frac{\pi}{3}}\right)$$

We know that,

$$\tan(A + B) = \left(\frac{\tan A + \tan B}{1 - \tan A \tan B}\right) \text{ and } \tan(A - B) = \left(\frac{\tan A - \tan B}{1 + \tan A \tan B}\right)$$

So,

$$= \tan x + \left(\frac{\sqrt{3} + \tan x}{1 - \sqrt{3}\tan x}\right) - \left(\frac{\sqrt{3} - \tan x}{1 + \sqrt{3}\tan x}\right)$$

$$= \tan x + \left(\frac{(1 + \sqrt{3}\tan x)(\sqrt{3} + \tan x) - (1 - \sqrt{3}\tan x)(\sqrt{3} - \tan x)}{\left(1 - \tan x(\sqrt{3})\right)\left(1 + \tan x(\sqrt{3})\right)}\right)$$

Simplify and cancel the similar terms of different sign in the above expression we get,

$$= \tan x + \left(\frac{(0 + 6\tan x + 2\tan x + 0)}{(1 - 3\tan^2 x)}\right)$$

$$= \tan x + \left(\frac{8\tan x}{(1 - 3\tan^2 x)}\right)$$

$$= \left(\frac{\tan x\,(1 - 3\tan^2 x) + 8\tan x}{(1 - 3\tan^2 x)}\right)$$

$$= \left(\frac{(\tan x - 3\tan^3 x) + 8\tan x}{(1 - 3\tan^2 x)}\right)$$

$$= \left(\frac{9\tan x - 3\tan^3 x}{(1 - 3\tan^2 x)}\right)$$

$$= 3\left(\frac{3\tan x - \tan^3 x}{(1 - 3\tan^2 x)}\right)$$

= 3 tan 3x (since, $\tan 3x = (3\tan x - \tan^3 x)/(1-3\tan^2 x)$)

= RHS

Hence proved.

EXERCISE 9.3 PAGE NO: 9.42

Prove that:

1. $\sin^2 2\pi/5 - \sin^2 \pi/3 = (\sqrt{5} - 1)/8$

Solution:

Let us consider LHS:

$\sin^2 2\pi/5 - \sin^2 \pi/3 = \sin^2 (\pi/2 - \pi/10) - \sin^2 \pi/3$

we know, $\sin (90° - A) = \cos A$

So, $\sin^2 (\pi/2 - \pi/10) = \cos^2 \pi/10$

$\sin \pi/3 = \sqrt{3}/2$

Then the above equation becomes,

$= \cos^2 \pi/10 - (\sqrt{3}/2)^2$

We know, $\cos \pi/10 = \sqrt{(10+2\sqrt{5})}/4$

the above equation becomes,

$= [\sqrt{(10+2\sqrt{5})}/4]^2 - 3/4$

$= [10 + 2\sqrt{5}]/16 - 3/4$

$= [10 + 2\sqrt{5} - 12]/16$

$= [2\sqrt{5} - 2]/16$

$= [\sqrt{5} - 1]/8$

= RHS

Hence proved.

2. $\sin^2 24° - \sin^2 6° = (\sqrt{5} - 1)/8$

Solution:

Let us consider LHS:

$\sin^2 24° - \sin^2 6°$

we know, $\sin (A + B) \sin (A - B) = \sin^2 A - \sin^2 B$

Then the above equation becomes,

$\sin^2 24° - \sin^2 6° = \sin (24° + 6°) - \sin (24° - 6°)$

$= \sin 30° - \sin 18°$

$= \sin 30° - (\sqrt{5} - 1)/4$ [since, $\sin 18° = (\sqrt{5} - 1)/4$]

$= 1/2 \times (\sqrt{5} - 1)/4$

$= (\sqrt{5} - 1)/8$

= RHS

Hence proved.

3. $\sin^2 42° - \cos^2 78° = (\sqrt{5} + 1)/8$

Solution:

Let us consider LHS:

$\sin^2 42^\circ - \cos^2 78^\circ = \sin^2 (90^\circ - 48^\circ) - \cos^2 (90^\circ - 12^\circ)$

$= \cos^2 48^\circ - \sin^2 12^\circ$ [since, sin (90 - A) = cos A and cos (90 - A) = sin A]

We know, $\cos (A + B) \cos (A - B) = \cos^2 A - \sin^2 B$

Then the above equation becomes,

$= \cos^2 (48^\circ + 12^\circ) \cos (48^\circ - 12^\circ)$

$= \cos 60^\circ \cos 36^\circ$ [since, $\cos 36^\circ = (\sqrt{5} + 1)/4$]

$= 1/2 \times (\sqrt{5} + 1)/4$

$= (\sqrt{5} + 1)/8$

= RHS

Hence proved.

4. cos 78° cos 42° cos 36° = 1/8

Solution:

Let us consider LHS:

$\cos 78^\circ \cos 42^\circ \cos 36^\circ$

Let us multiply and divide by 2 we get,

$\cos 78^\circ \cos 42^\circ \cos 36^\circ = 1/2 (2 \cos 78^\circ \cos 42^\circ \cos 36^\circ)$

We know, $2 \cos A \cos B = \cos (A + B) + \cos (A - B)$

Then the above equation becomes,

$= 1/2 (\cos (78^\circ + 42^\circ) + \cos (78^\circ - 42^\circ)) \times \cos 36^\circ$

$= 1/2 (\cos 120^\circ + \cos 36^\circ) \times \cos 36^\circ$

$= 1/2 (\cos (180^\circ - 60^\circ) + \cos 36^\circ) \times \cos 36^\circ$

$= 1/2 (-\cos (60^\circ) + \cos 36^\circ) \times \cos 36^\circ$ [since, $\cos(180^\circ - A) = - A$]

$= 1/2 (-1/2 + (\sqrt{5} + 1)/4) ((\sqrt{5} + 1)/4)$ [since, $\cos 36^\circ = (\sqrt{5} + 1)/4$]

$= 1/2 (\sqrt{5} + 1 - 2)/4 ((\sqrt{5} + 1)/4)$

$= 1/2 (\sqrt{5} - 1)/4) ((\sqrt{5} + 1)/4)$

$= 1/2 ((\sqrt{5})^2 - 1^2)/16$

$= 1/2 (5-1)/16$

$= 1/2 (4/16)$

$= 1/8$

= RHS

Hence proved.

5. cos π/15 cos 2π/15 cos 4π/15 cos 7π/15 = 1/16

Solution:

Let us consider LHS:

$\cos \pi/15 \cos 2\pi/15 \cos 4\pi/15 \cos 7\pi/15$

Let us multiply and divide by $2 \sin \pi/15$, we get,

$= [2 \sin \pi/15 \cos \pi/15] \cos 2\pi/15 \cos 4\pi/15 \cos 7\pi/15] / 2 \sin \pi/15$

We know, 2sin A cos A = sin 2A

Then the above equation becomes,

$= [(\sin 2\pi/15) \cos 2\pi/15 \cos 4\pi/15 \cos 7\pi/15] / 2 \sin \pi/15$

Now, multiply and divide by 2 we get,

$= [(2 \sin 2\pi/15 \cos 2\pi/15) \cos 4\pi/15 \cos 7\pi/15] / 2 \times 2 \sin \pi/15$

We know, 2sin A cos A = sin 2A

Then the above equation becomes,

$= [(\sin 4\pi/15) \cos 4\pi/15 \cos 7\pi/15] / 4 \sin \pi/15$

Now, multiply and divide by 2 we get,

$= [(2 \sin 4\pi/15 \cos 4\pi/15) \cos 7\pi/15] / 2 \times 4 \sin \pi/15$

We know, 2sin A cos A = sin 2A

Then the above equation becomes,

$= [(\sin 8\pi/15) \cos 7\pi/15] / 8 \sin \pi/15$

Now, multiply and divide by 2 we get,

$= [2 \sin 8\pi/15 \cos 7\pi/15] / 2 \times 8 \sin \pi/15$

We know, 2sin A cos B = sin (A+B) + sin (A–B)

Then the above equation becomes,

$= [\sin (8\pi/15 + 7\pi/15) + \sin (8\pi/15 - 7\pi/15)] / 16 \sin \pi/15$

$= [\sin (\pi) + \sin (\pi/15)] / 16 \sin \pi/15$

$= [0 + \sin (\pi/15)] / 16 \sin \pi/15$

$= \sin (\pi/15) / 16 \sin \pi/15$

$= 1/16$

= RHS

Hence proved.