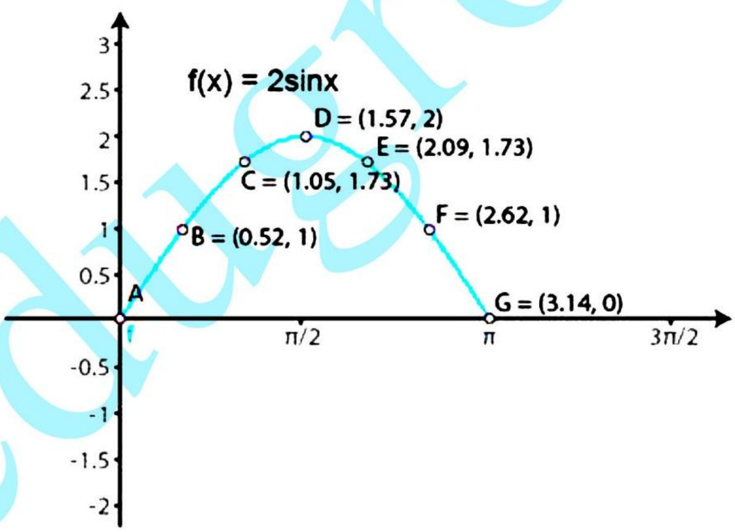
EXERCISE 6.1 PAGE NO: 6.5

1. Sketch the graphs of the following functions:
(i) f (x) = 2 sin x, $0 \le x \le \pi$
(ii) g (x) = 3 sin (x – π/4), $0 \le x \le 5\pi/4$
(iii) h (x) = 2 sin 3x, $0 \le x \le 2\pi/3$
(iv) ϕ (x) = 2 sin (2x - π/3), $0 \le x \le 7\pi/3$
(v) Ψ (x) = 4 sin 3 (x – π/4), $0 \le x \le 2\pi$
(vi) θ (x) = sin (x/2 - π/4), $0 \le x \le 4\pi$
(vii) u (x) = $\sin^2$ x, $0 \le x \le 2\pi$ υ (x) = |sin x|, $0 \le x \le 2\pi$
(viii) f (x) = 2 sin πx, $0 \le x \le 2$
Solution:
(i) f (x) = 2 sin x, $0 \le x \le \pi$
We know that g (x) = sin x is a periodic function with period π.
So, f (x) = 2 sin x is a periodic function with period π. So, we will draw the graph of f (x) = 2 sin x in the interval [0, π]. The values of f (x) = 2 sin x at various points in [0, π] are listed in the following table:

x	0(A)	π/6 (B)	π/3 (C)	π/2 (D)	2π/3 (E)	5π/6 (F)	Π (G)
f (x) = 2 sin x	0	1	√3 = 1.73	2	√3 = 1.73	1	0

The required curve is:

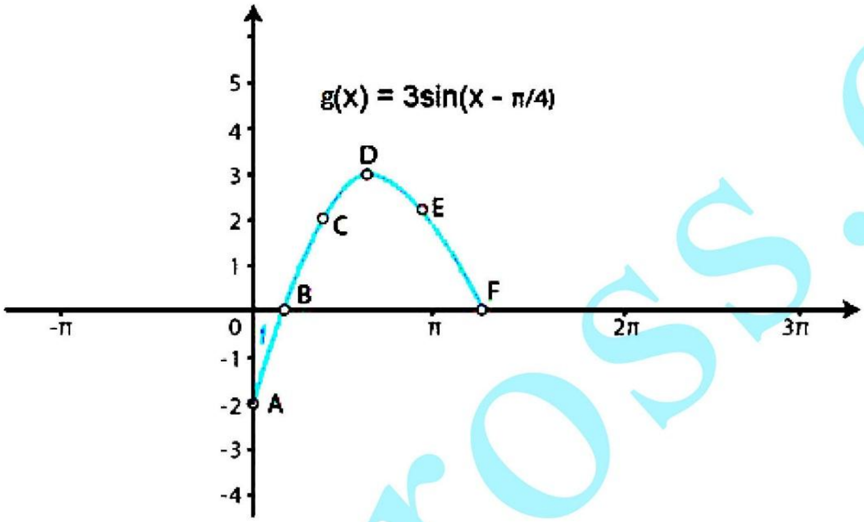


(ii) g (x) = 3 sin (x – π/4), $0 \le x \le 5\pi/4$
We know that if f (x) is a periodic function with period T, then f (ax + b) is periodic with

period T/|a|.
So, g (x) = 3 sin (x - π/4) is a periodic function with period π. So, we will draw the graph of g (x) = 3 sin (x - π/4) in the interval [0, 5π/4]. The values of g (x) = 3 sin (x - π/4) at various points in [0, 5π/4] are listed in the following table:

x	0(A)	π/4 (B)	π/2 (C)	3π/4 (D)	π (E)	5π/4 (F)
g (x) = 3 sin (x - π/4)	-3/√2 = -2.1	0	3/√2 = 2.12	3	3/√2 = 2.12	0

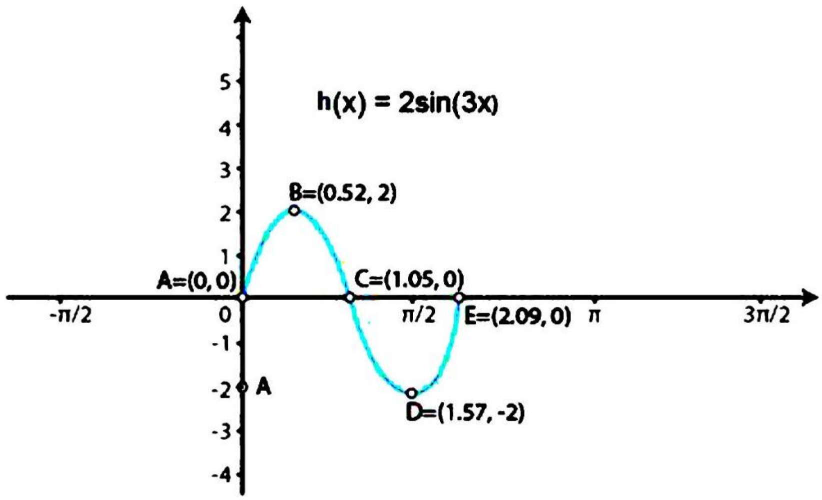
The required curve is:



(iii) h (x) = 2 sin 3x, 0 ≤ x ≤ 2π/3
We know that g (x) = sin x is a periodic function with period 2π.
So, h (x) = 2 sin 3x is a periodic function with period 2π/3. So, we will draw the graph of h (x) = 2 sin 3x in the interval [0, 2π/3]. The values of h (x) = 2 sin 3x at various points in [0, 2π/3] are listed in the following table:

x	0 (A)	π/6 (B)	π/3 (C)	π/2 (D)	2π/3 (E)
h (x) = 2 sin 3x	0	2	0	-2	0

The required curve is:



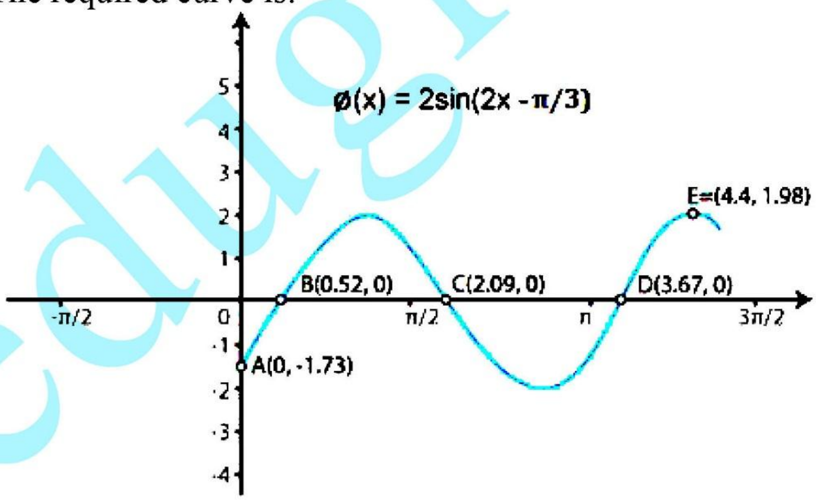
(iv) $\phi(x) = 2\sin(2x - \pi/3)$, $0 \le x \le 7\pi/3$

We know that if f(x) is a periodic function with period T, then f (ax + b) is periodic with period T/|a|.

So, $\phi(x) = 2\sin(2x - \pi/3)$ is a periodic function with period π. So, we will draw the graph of $\phi(x) = 2\sin(2x - \pi/3)$, in the interval [0, 7π/5]. The values of $\phi(x) = 2\sin(2x - \pi/3)$, at various points in [0, 7π/5] are listed in the following table:

x	0 (A)	π/6 (B)	2π/3 (C)	7π/6 (D)	7π/5 (E)
$\phi(x) = 2\sin(2x - \pi/3)$	$-\sqrt{3} = -1.73$	0	0	0	1.98

The required curve is:

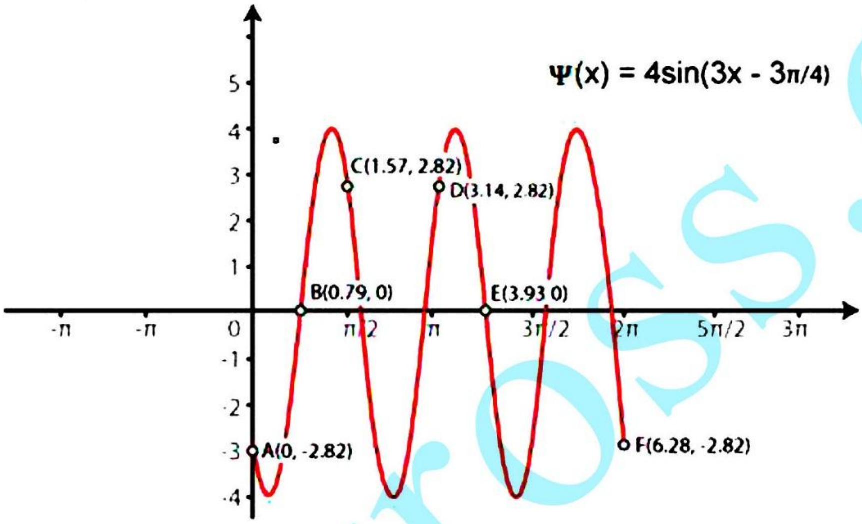


(v) $\Psi(x) = 4\sin 3(x - \pi/4)$, $0 \le x \le 2\pi$

We know that if f(x) is a periodic function with period T, then f (ax + b) is periodic with period T/|a|.
So, Ψ (x) = 4 sin 3 (x – π/4) is a periodic function with period 2π. So, we will draw the graph of Ψ (x) = 4 sin 3 (x – π/4) in the interval [0, 2π]. The values of Ψ (x) = 4 sin 3 (x – π/4) at various points in [0, 2π] are listed in the following table:

x	0 (A)	π/4 (B)	π/2 (C)	π (D)	5π/4 (E)	2π (F)
Ψ (x) = 4 sin 3 (x – π/4)	-2√2 = -2.82	0	2√2 = 2.82	0	1.98	-2√2 = -2.82

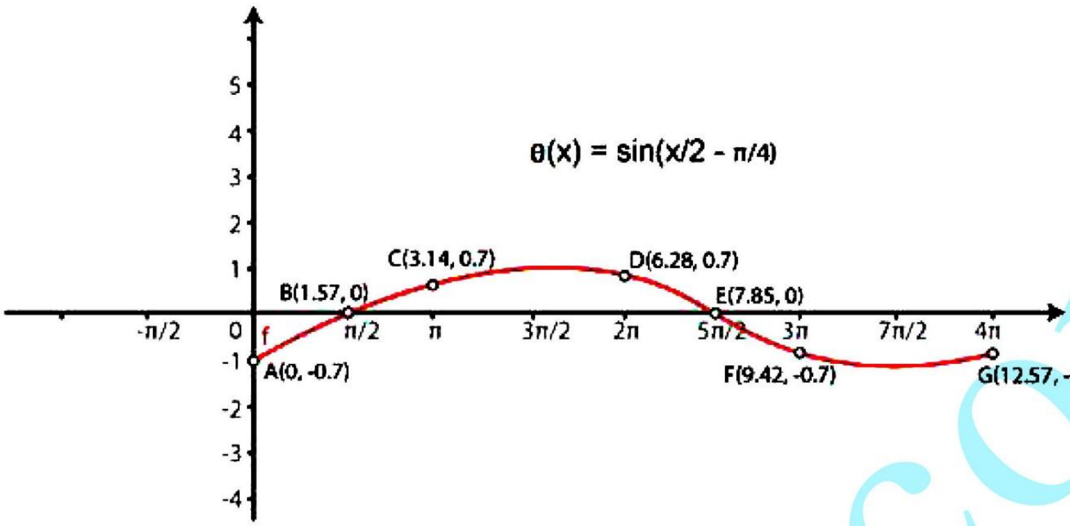
The required curve is:



(vi) θ (x) = sin (x/2 - π/4), 0 ≤ x ≤ 4π
We know that if f(x) is a periodic function with period T, then f (ax + b) is periodic with period T/|a|.
So, θ (x) = sin (x/2 - π/4) is a periodic function with period 4π. So, we will draw the graph of θ (x) = sin (x/2 - π/4) in the interval [0, 4π]. The values of θ (x) = sin (x/2 - π/4) at various points in [0, 4π] are listed in the following table:

x	0 (A)	π/2 (B)	π (C)	2π (D)	5π/2 (E)	3π (F)	4π (G)
θ (x) = sin (x/2 - π/4)	-1/√2 = -0.7	0	1/√2 = 0.7	1/√2 = 0.7	0	-1/√2 = -0.7	-1/√2 = -0.7

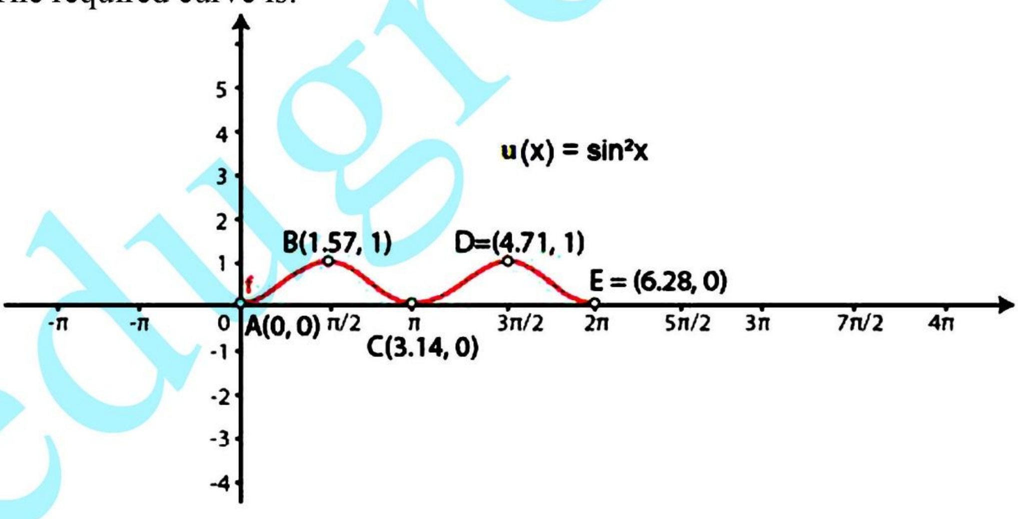
The required curve is:



(vii) u (x) = $\sin^2$ x, 0 ≤ x ≤ 2π υ (x) = |sin x|, 0 ≤ x ≤ 2π
We know that g (x) = sin x is a periodic function with period π.
So, u (x) = $\sin^2$ x is a periodic function with period 2π. So, we will draw the graph of u (x) = $\sin^2$ x in the interval [0, 2π]. The values of u (x) = $\sin^2$ x at various points in [0, 2π] are listed in the following table:

x	0 (A)	π/2 (B)	Π (C)	3π/2 (D)	2π (E)
u (x) = $\sin^2$ x	0	1	0	1	0

The required curve is:

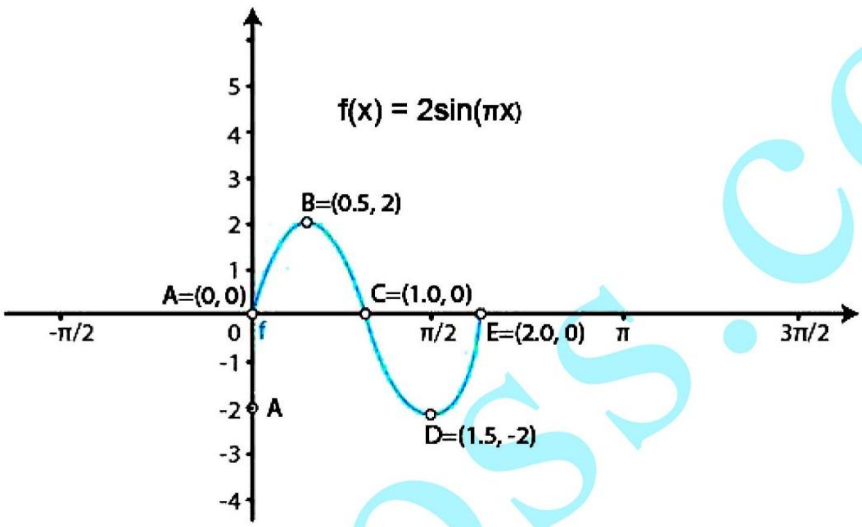


(viii) f (x) = 2 sin πx, 0 ≤ x ≤ 2
We know that g (x) = sin x is a periodic function with period 2π.
So, f (x) = 2 sin πx is a periodic function with period 2. So, we will draw the graph of f

(x) = 2 sin πx in the interval [0, 2]. The values of f (x) = 2 sin πx at various points in [0, 2] are listed in the following table:

x	0 (A)	1/2 (B)	1 (C)	3/2 (D)	2 (E)
f (x) = 2 sin πx	0	2	0	-2	0

The required curve is:



2. Sketch the graphs of the following pairs of functions on the same axes:
(i) f (x) = sin x, g (x) = sin (x + π/4)
(ii) f (x) = sin x, g (x) = sin 2x
(iii) f (x) = sin 2x, g (x) = 2 sin x
(iv) f (x) = sin x/2, g (x) = sin x
Solution:
(i) f (x) = sin x, g (x) = sin (x + π/4)
We know that the functions f (x) = sin x and g (x) = sin (x + π/4) are periodic functions with periods 2π and 7π/4.
The values of these functions are tabulated below:

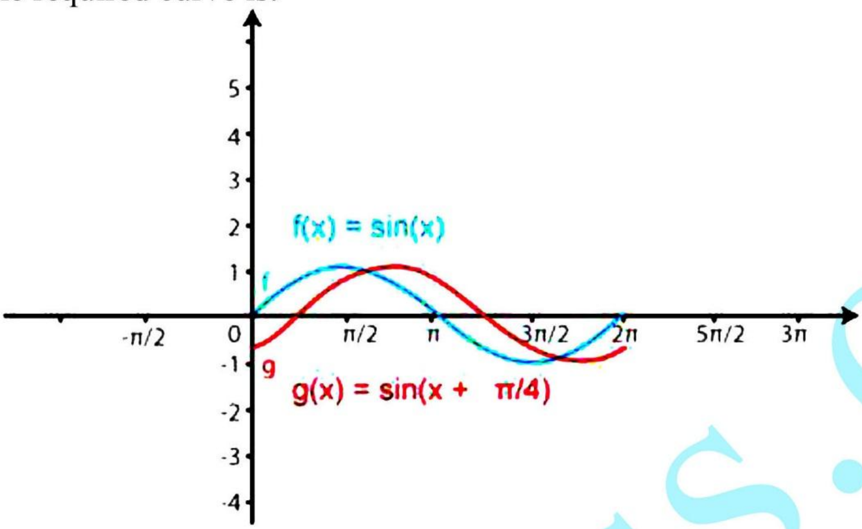
Values of f (x) = sin x in [0, 2π]

x	0	π/2	π	3π/2	2π
f (x) = sin x	0	1	0	-1	0

Values of g (x) = sin (x + π/4) in [0, 7π/4]

x	0	π/4	3π/4	5π/4	7π/4
g (x) = sin (x + π/4)	1/√2 = 0.7	1	0	-1	0

The required curve is:



(ii) f (x) = sin x, g (x) = sin 2x
We know that the functions f(x) = sin x and g (x) = sin 2x are periodic functions with periods 2π and π.
The values of these functions are tabulated below:

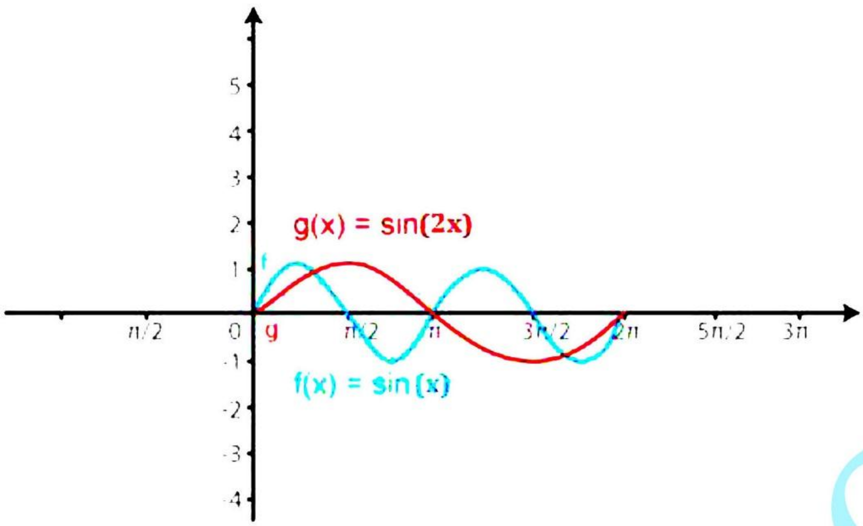
Values of f (x) = sin x in [0, 2π]

x	0	π/2	π	3π/2	2π
f (x) = sin x	0	1	0	-1	0

Values of g (x) = sin (2x) in [0, π]

x	0	π/4	π/2	3π/4	π	5π/4	3π/2	7π/4	2π
g (x) = sin (2x)	0	1	0	-1	0	1	0	-1	0

The required curve is:



(iii) f (x) = sin 2x, g (x) = 2 sin x

We know that the functions f(x) = sin 2x and g (x) = 2 sin x are periodic functions with periods π and π.

The values of these functions are tabulated below:

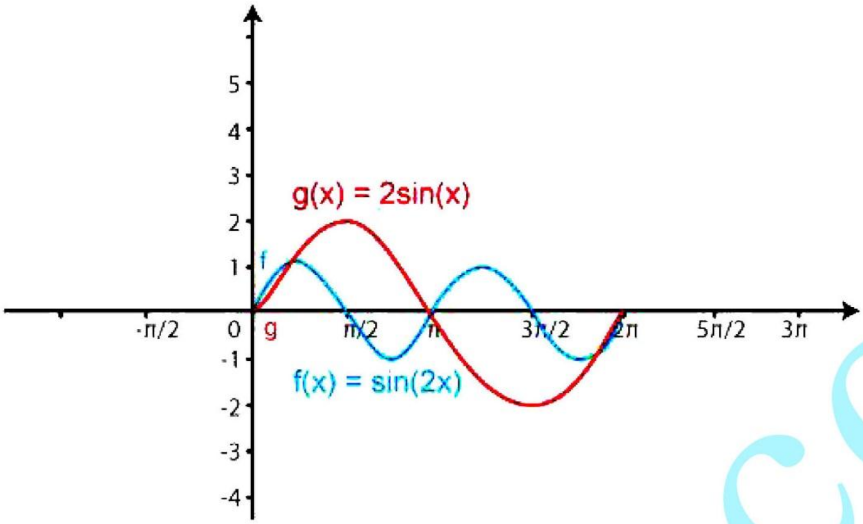
Values of f (x) = sin (2x) in [0, π]

x	0	π/4	π/2	3π/4	π	5π/4	3π/2	7π/4	2π
f (x) = sin (2x)	0	1	0	-1	0	1	0	-1	0

Values of g (x) = 2 sin x in [0, π]

x	0	π/2	π	3π/2	2π
g (x) = 2 sin x	0	1	0	-1	0

The required curve is:



(iv) f (x) = sin x/2, g (x) = sin x
We know that the functions f(x) = sin x/2 and g (x) = sin x are periodic functions with periods π and 2π.
The values of these functions are tabulated below:

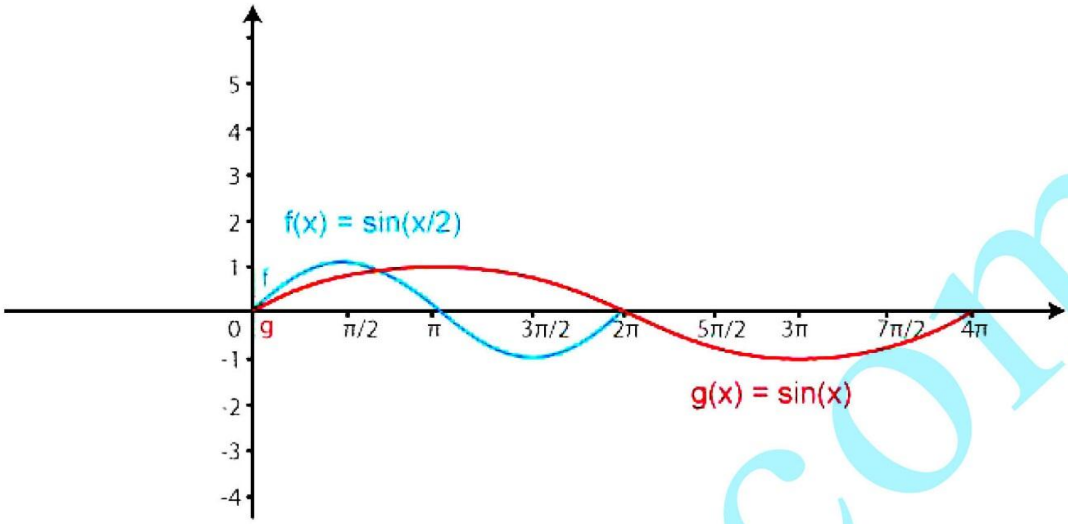
Values of f (x) = sin x/2 in [0, π]

x	0	π	2π	3π	4π
f (x) = sin x/2	0	1	0	-1	0

Values of g (x) = sin (x) in [0, 2π]

x	0	π/2	π	3π/2	2π	5π/2	3π	7π/2	4π
g (x) = sin (x)	0	1	0	-1	0	1	0	-1	0

The required curve is:

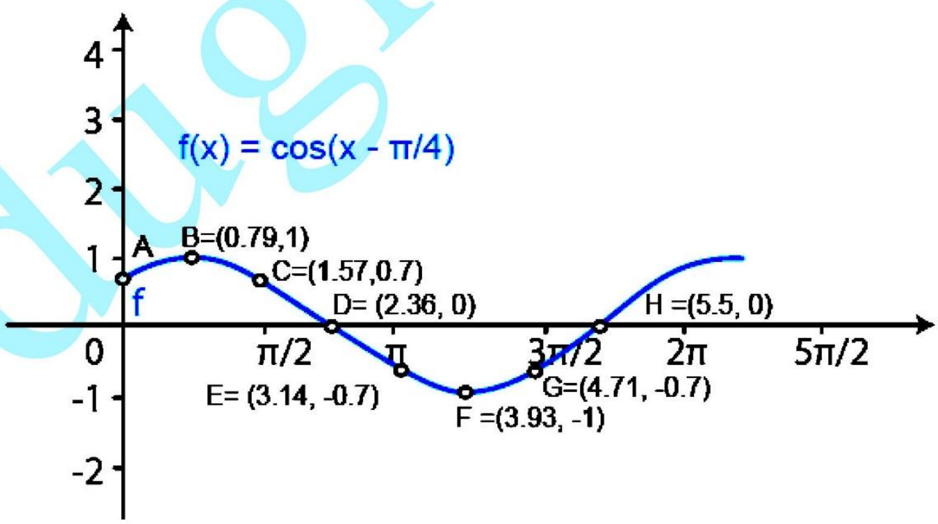

f(x) = sin(x/2)
g(x) = sin(x)
f
g
5
4
3
2
1
0
-1
-2
-3
-4
π/2
π
3π/2
2π
5π/2
3π
7π/2
4π

EXERCISE 6.2 PAGE NO: 6.8

1. Sketch the graphs of the following trigonometric functions:
(i) f (x) = cos (x – π/4)
(ii) g (x) = cos (x + π/4)
(iii) h (x) = $\cos^2$ 2x
(iv) ϕ (x) = 2 cos (x – π/6)
(v) ψ (x) = cos (3x)
(vi) u (x) = $\cos^2$ x/2
(vii) f (x) = cos πx
(viii) g (x) = cos 2π x
Solution:
(i) f (x) = cos (x – π/4)
We know that g (x) = cos x is a periodic function with period 2π.
So, f (x) = cos (x - π/4) is a periodic function with period π. So, we will draw the graph of f (x) = cos (x - π/4) in the interval [0, π]. The values of f (x) = cos (x - π/4) at various points in [0, π] are listed in the following table:

x	0 (A)	π/4 (B)	π/2 (C)	3π/4 (D)	π (E)	5π/4 (F)	3π/2 (G)	7π/4 (H)
f (x) = cos (x – π/4)	1/√2 = 0.7	1	1/√2 = 0.7	0	-1/√2 = -0.7	-1	-1/√2 = -0.7	0

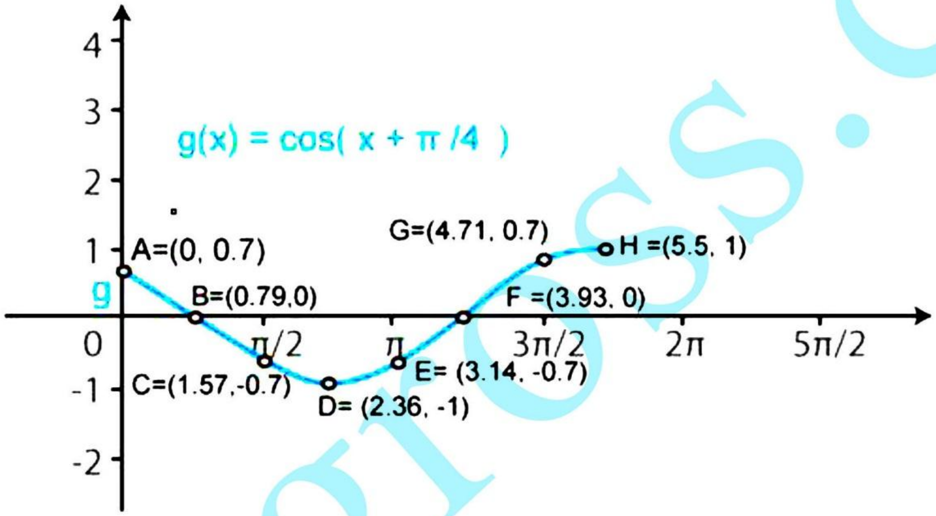
The required curve is:



(ii) g (x) = cos (x + π/4)
We know that f (x) = cos x is a periodic function with period 2π.
So, g (x) = cos (x + π/4) is a periodic function with period π. So, we will draw the graph of g (x) = cos (x + π/4) in the interval [0, π]. The values of g (x) = cos (x + π/4) at various points in [0, π] are listed in the following table:

x	0 (A)	π/4 (B)	π/2 (C)	3π/4 (D)	π (E)	5π/4 (F)	3π/2 (G)	7π/4 (H)
g (x) = cos (x + π/4)	1/√2 = 0.7	0	-1/√2 = -0.7	-1	-1/√2 = -0.7	0	1/√2 = 0.7	1

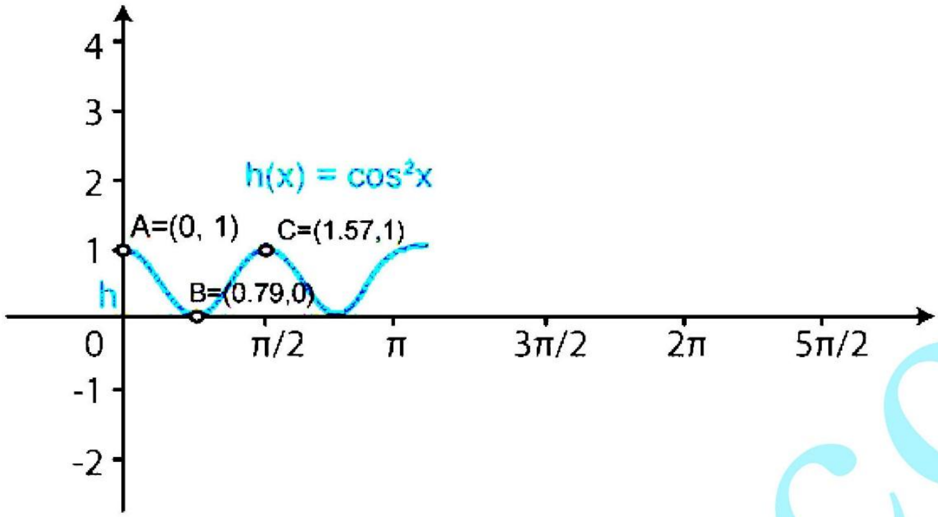
The required curve is:



(iii) h (x) = $\cos^2 2x$
We know that f (x) = cos x is a periodic function with period 2π.
So, h (x) = $\cos^2 2x$ is a periodic function with period π. So, we will draw the graph of h (x) = $\cos^2 2x$ in the interval [0, π]. The values of h (x) = $\cos^2 2x$ at various points in [0, π] are listed in the following table:

x	0 (A)	π/4 (B)	π/2 (C)	3π/4 (D)	π (E)	5π/4 (F)	3π/2 (G)
h (x) = $\cos^2 2x$	1	0	1	0	1	0	1

The required curve is:



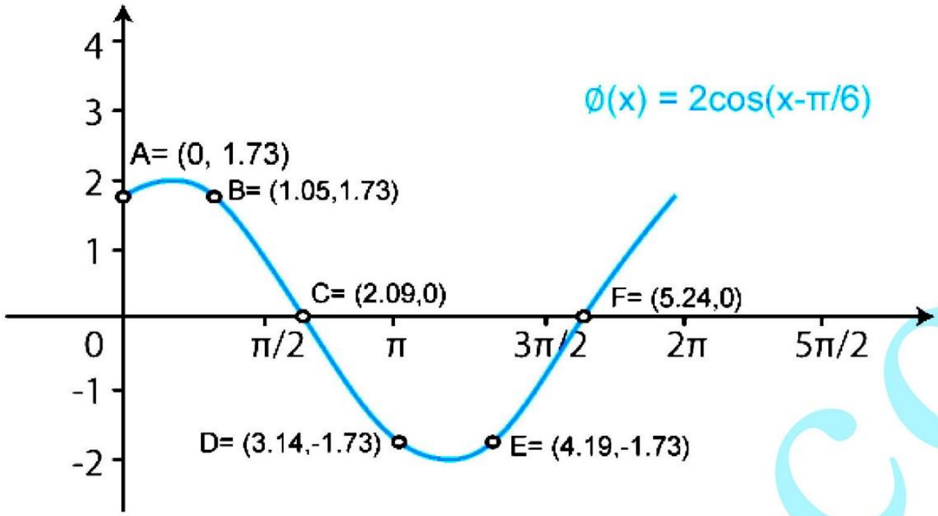
(iv) $\phi(x) = 2\cos(x - \pi/6)$

We know that $f(x) = \cos x$ is a periodic function with period 2π.

So, $\phi(x) = 2\cos(x - \pi/6)$ is a periodic function with period π. So, we will draw the graph of $\phi(x) = 2\cos(x - \pi/6)$ in the interval $[0, \pi]$. The values of $\phi(x) = 2\cos(x - \pi/6)$ at various points in $[0, \pi]$ are listed in the following table:

x	0 (A)	π/3 (B)	2π/3 (C)	π (D)	4π/3 (E)	5π/3 (F)
$\phi(x) = 2\cos(x - \pi/6)$	$\sqrt{3} = 1.73$	$\sqrt{3} = 1.73$	0	$-\sqrt{3} = -1.73$	$-\sqrt{3} = -1.73$	0

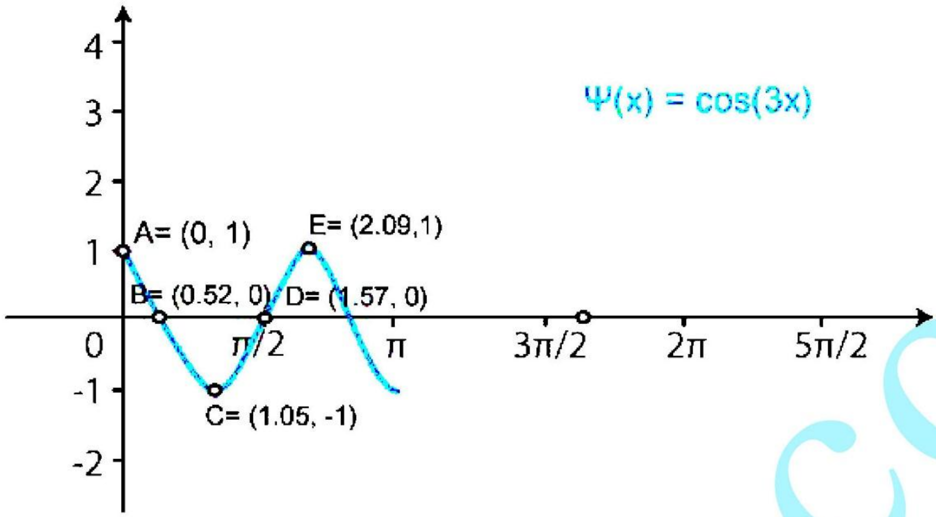
The required curve is:



(v) ψ (x) = cos (3x)
We know that f (x) = cos x is a periodic function with period 2π.
So, ψ (x) = cos (3x) is a periodic function with period 2π/3. So, we will draw the graph of ψ (x) = cos (3x) in the interval [0, 2π/3]. The values of ψ (x) = cos (3x) at various points in [0, 2π/3] are listed in the following table:

x	0 (A)	π/6 (B)	π/3 (C)	π/2 (D)	2π/3 (E)	5π/6 (F)
ψ (x) = cos (3x)	1	0	-1	0	1	0

The required curve is:



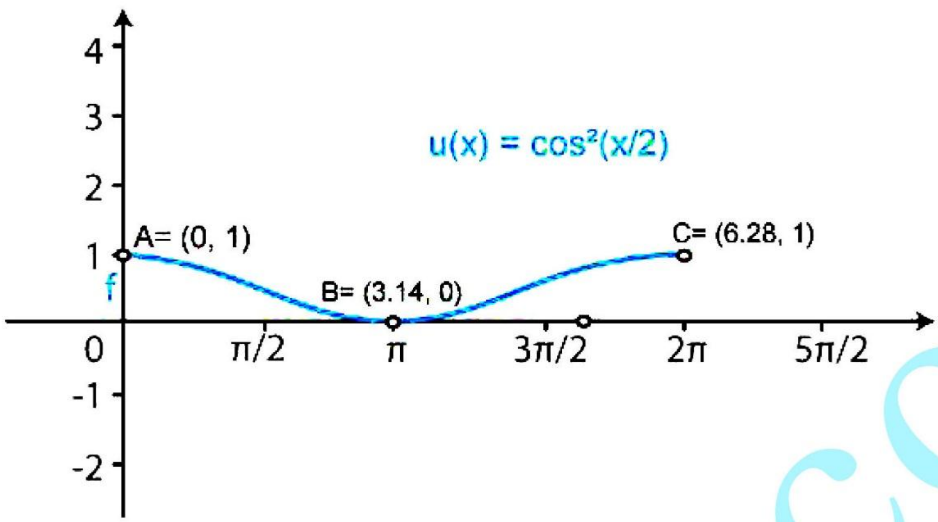
(vi) u (x) = $\cos^2 x/2$

We know that f (x) = cos x is a periodic function with period 2π.

So, u (x) = $\cos^2 (x/2)$ is a periodic function with period π. So, we will draw the graph of u (x) = $\cos^2 (x/2)$ in the interval [0, π]. The values of u (x) = $\cos^2 (x/2)$ at various points in [0, π] are listed in the following table:

x	0 (A)	π (B)	2π (C)	3π (D)
u (x) = $\cos^2 x/2$	1	0	1	0

The required curve is:



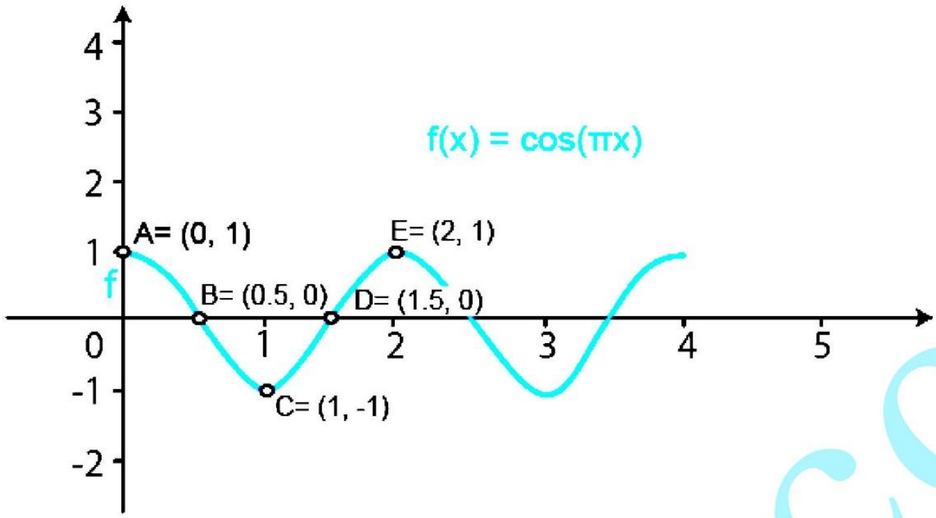
(vii) f (x) = cos πx

We know that g (x) = cos x is a periodic function with period 2π.

So, f (x) = cos (πx) is a periodic function with period 2. So, we will draw the graph of f (x) = cos (πx) in the interval [0, 2]. The values of f (x) = cos (πx) at various points in [0, 2] are listed in the following table:

x	0 (A)	1/2 (B)	1 (C)	3/2 (D)	2 (E)	5/2 (F)
f (x) = cos πx	1	0	-1	0	1	0

The required curve is:



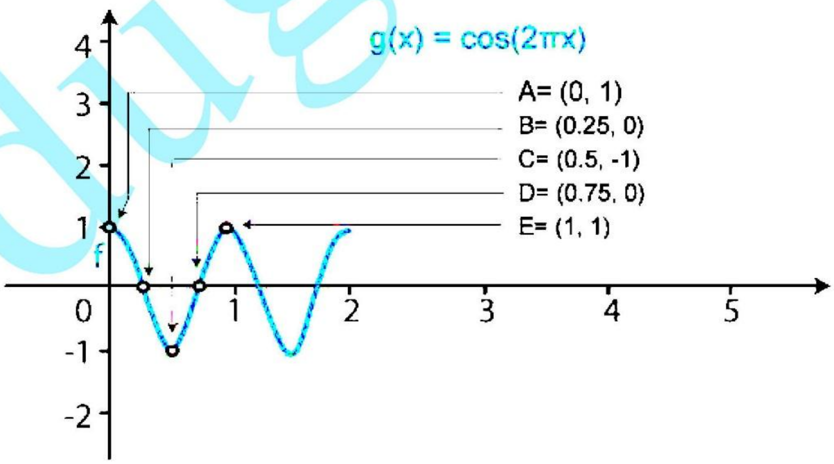
(viii) g (x) = cos 2π x

We know that f (x) = cos x is a periodic function with period 2π.

So, g (x) = cos (2πx) is a periodic function with period 1. So, we will draw the graph of g (x) = cos (2πx) in the interval [0, 1]. The values of g (x) = cos (2πx) at various points in [0, 1] are listed in the following table:

x	0 (A)	1/4 (B)	1/2 (C)	3/4 (D)	1 (E)	5/4 (F)	3/2 (G)	7/4 (H)	2
g (x) = cos 2π x	1	0	-1	0	1	0	-1	0	1

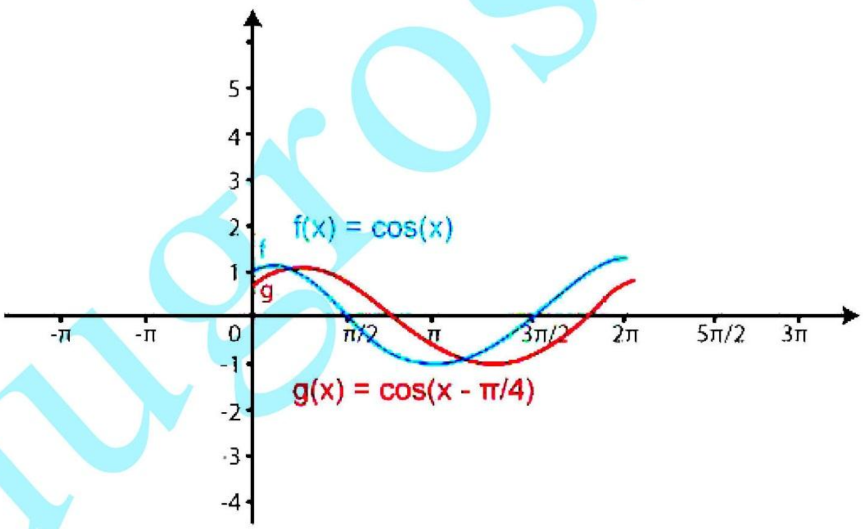
The required curve is:



2. Sketch the graphs of the following curves on the same scale and the same axes:
(i) y = cos x and y = cos (x – π/4)
(ii) y = cos 2x and y = cos (x - π/4)
(iii) y = cos x and y = cos x/2
(iv) y = $\cos^2$ x and y = cos x
Solution:
(i) y = cos x and y = cos (x – π/4)
We know that the functions y = cos x and y = cos (x - π/4) are periodic functions with periods π and π.
The values of these functions are tabulated below:

x	0	π/4	π/2	3π/4	π	5π/4	3π/2	7π/4
y = cos x	1	1/√2 = 0.7	0	-1/√2 = -0.7	-1	-1/√2 = -0.7	0	1
y = cos (x – π/4)	1/√2 = 0.7	1	1/√2 = 0.7	0	-1/√2 = -0.7	-1	-1/√2 = -0.7	0

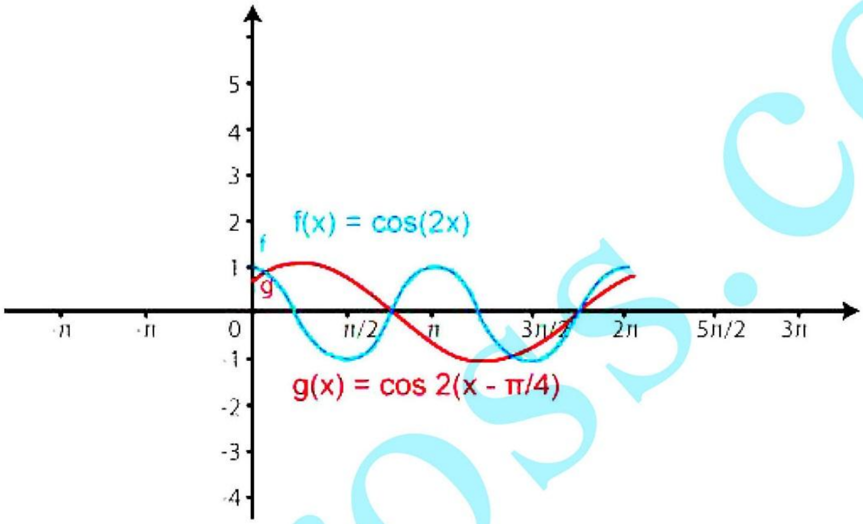
The required curve is:



(ii) y = cos 2x and y = cos 2(x - π/4)
We know that the functions y = cos 2x and y = cos 2(x - π/4) are periodic functions with periods π and π.
The values of these functions are tabulated below:

x	0	π/4	π/2	3π/4	π	5π/4	3π/2	7π/4
y = cos x	1	0	-1	0	1	0	-1	0
y = cos 2 (x – π/4)	0	1	0	-1	0	1	0	-1

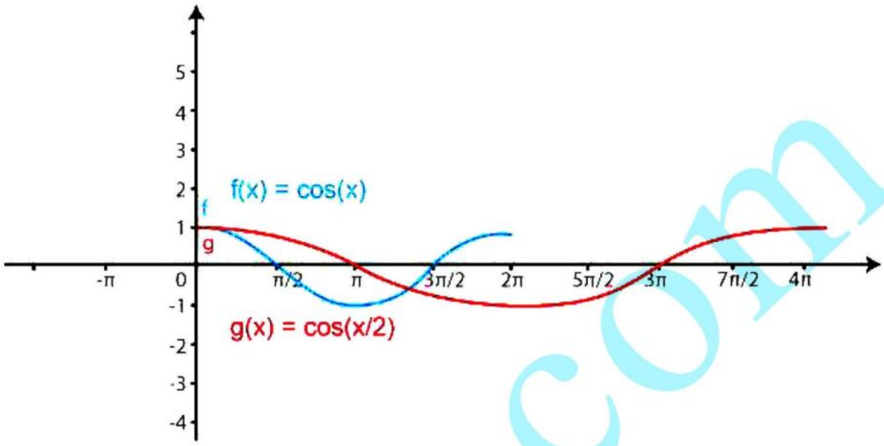
The required curve is:



(iii) y = cos x and y = cos x/2
We know that the functions y = cos x and y = cos (x/2) are periodic functions with periods π and π.
The values of these functions are tabulated below:

x	0	π/2	π	3π/2	2π
y = cos x	1	0	-1	0	1
y = cos x/2	1	1/√2 = 0.7	0	-1/√2 = -0.7	-1

The required curve is:



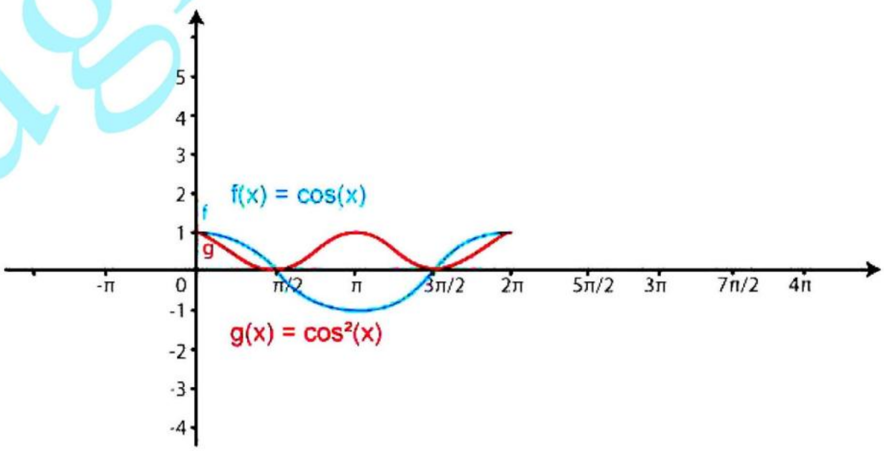
(iv) $y = \cos^2 x$ and $y = \cos x$

We know that the functions $y = \cos^2 x$ and $y = \cos x$ are periodic functions with period 2π.

The values of these functions are tabulated below:

x	0	π/2	π	3π/2	2π
$y = \cos^2 x$	1	0	1	0	1
y = cos x	1	0	-1	0	1

The required curve is:



EXERCISE 6.3 PAGE NO: 6.13

Sketch the graphs of the following functions:

1. f (x) = 2 cosec πx

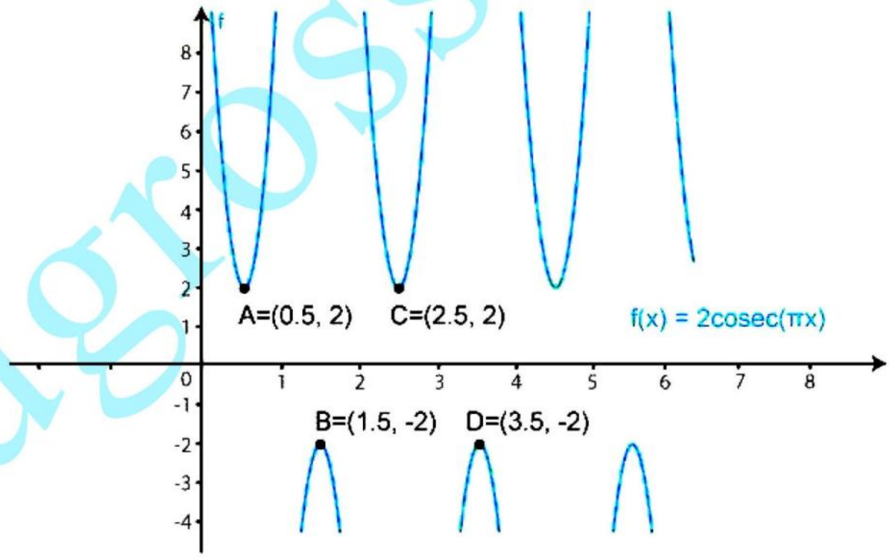
Solution:

We know that f (x) = cosec x is a periodic function with period 2π.

So, f (x) = 2 cosec (πx) is a periodic function with period 2. So, we will draw the graph of f (x) = 2 cosec (πx) in the interval [0, 2]. The values of f (x) = 2 cosec (πx) at various points in [0, 2] are listed in the following table:

x	0 (A)	1/2 (B)	1 (C)	-1 (D)	3/2 (E)	-2 (F)	2 (G)	5/2 (H)
f (x) = 2 cosec (πx)	∞	2	∞	-∞	-2	-∞	∞	2

The required curve is:



2. f (x) = 3 sec x

Solution:

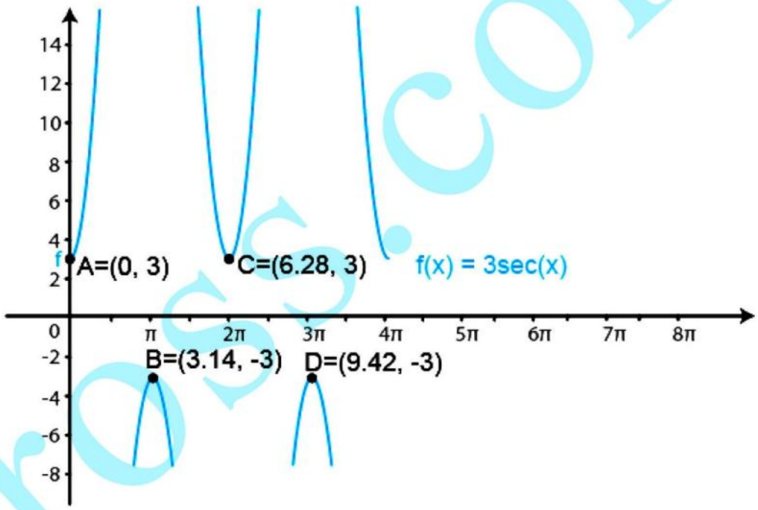
We know that f (x) = sec x is a periodic function with period π.

So, f (x) = 3 sec (x) is a periodic function with period π. So, we will draw the graph of f

(x) = 3 sec (x) in the interval [0, π]. The values of f (x) = 3 sec (x) at various points in [0, π] are listed in the following table:

x	0 (A)	π/2 (B)	-π/2 (C)	π (D)	-3π/2 (E)	3π/2 (F)	2π (G)	5π/2 (H)
f (x) = sec x	3	∞	-∞	-3	-∞	∞	3	∞

The required curve is:



3. f (x) = cot 2x

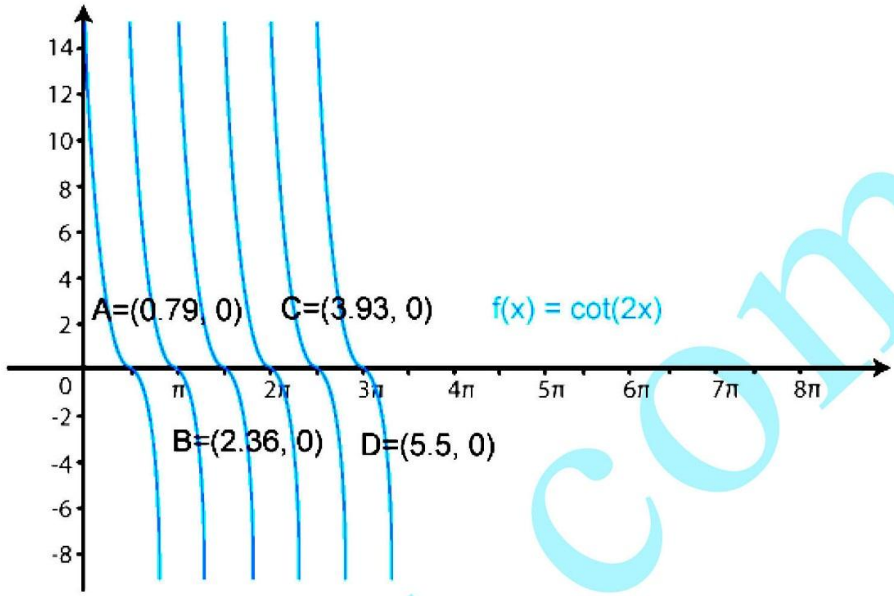
Solution:

We know that f (x) = cot x is a periodic function with period π.

So, f (x) = cot (2x) is a periodic function with period π. So, we will draw the graph of f (x) = cot (2x) in the interval [0, π]. The values of f (x) = cot (2x) at various points in [0, π] are listed in the following table:

x	0 (A)	π/4 (B)	-π/2 (C)	π/2 (D)	3π/4 (E)	-π (F)
f (x) = cot x	→∞	0	-∞	→∞	0	-∞

The required curve is:



4. f (x) = 2 sec πx

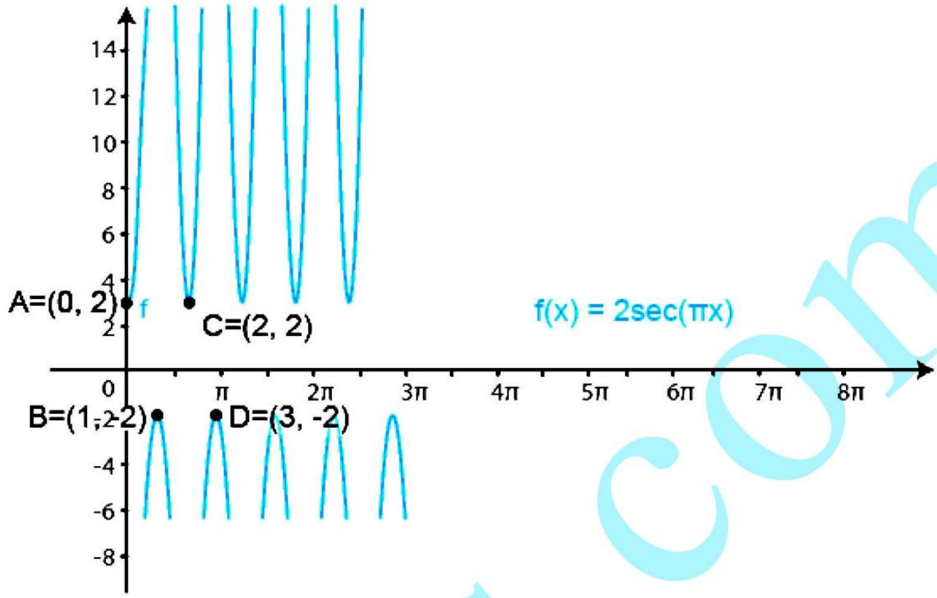
Solution:

We know that f (x) = sec x is a periodic function with period π.

So, f (x) = 2 sec (πx) is a periodic function with period 1. So, we will draw the graph of f (x) = 2 sec (πx) in the interval [0, 1]. The values of f (x) = 2 sec (πx) at various points in [0, 1] are listed in the following table:

x	0	1/2	-1/2	1	-3/2	3/2	2
f (x) = 2 sec (πx)	2	∞	→-∞	-2	-∞	∞	2

The required curve is:



5. f (x) = tan² x

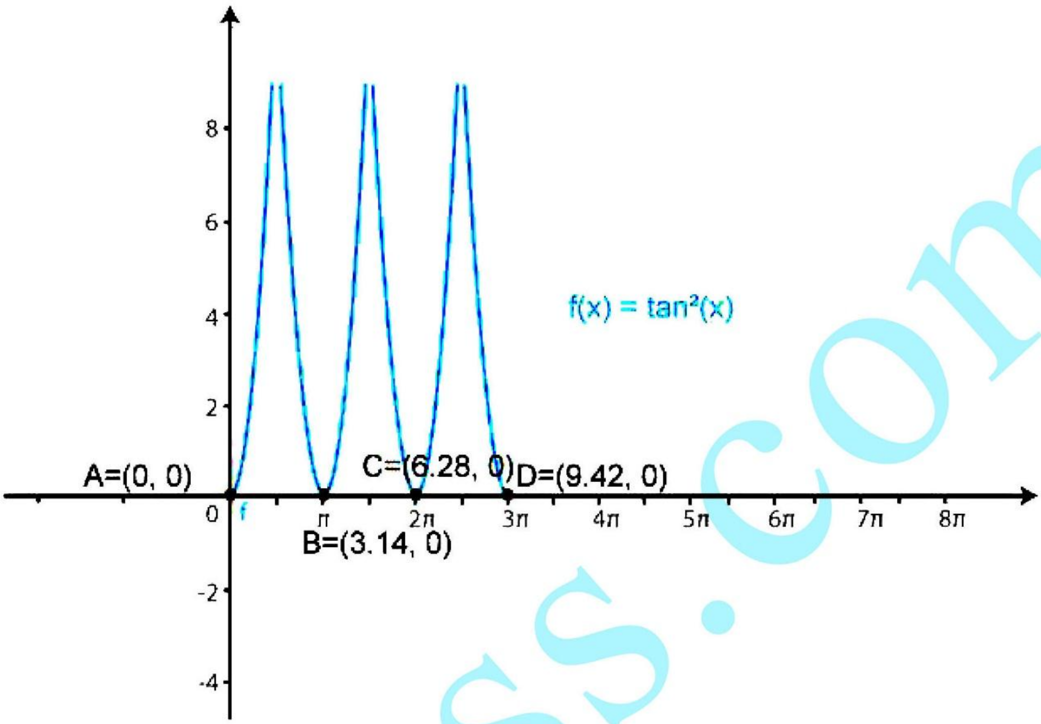
Solution:

We know that f (x) = tan x is a periodic function with period π.

So, f (x) = $\tan^2$ (x) is a periodic function with period π. So, we will draw the graph of f (x) = $\tan^2$ (x) in the interval [0, π]. The values of f (x) = $\tan^2$ (x) at various points in [0, π] are listed in the following table:

x	0 (A)	π/2 (B)	π/2 (C)	π (D)	3π/2 (E)	3π/2 (F)	2 π
f (x) = $\tan^2$ (x)	0	∞	→∞	0	∞	→∞	0

The required curve is:


f(x) = tan²(x)
A=(0, 0)
B=(3.14, 0)
C=(6.28, 0)
D=(9.42, 0)
8
6
4
2
0
-2
-4
π
2π
3π
4π
5π
6π
7π
8π