EXERCISE 4.1 PAGE NO: 4.15

1. Find the degree measure corresponding to the following radian measures (Use π = 22/7)
(i) 9π/5 (ii) -5π/6 (iii) (18π/5)^c (iv) (-3)^c (v) 11^c (vi) 1^c

Solution:

We know that π rad = 180° ⇒ 1 rad = 180°/ π

(i) 9π/5

$[(180/\pi) \times (9\pi/5)]^o$

Substituting the value of π = 22/7

$[180/22 \times 7 \times 9 \times 22/(7\times5)]$

$(36 \times 9)^\circ$

324°

∴ Degree measure of 9π/5 is 324°

(ii) -5π/6

$[(180/\pi) \times (-5\pi/6)]^o$

Substituting the value of π = 22/7

$[180/22 \times 7 \times -5 \times 22/(7\times6)]$

$(30 \times -5)^\circ$

$-(150)^\circ$

∴ Degree measure of -5π/6 is -150°

(iii) (18π/5)

$[(180/\pi) \times (18\pi/5)]^o$

Substituting the value of π = 22/7

$[180/22 \times 7 \times 18 \times 22/(7\times5)]$

$(36 \times 18)^\circ$

648°

∴ Degree measure of 18π/5 is 648°

(iv) $(-3)^c$

$[(180/\pi) \times (-3)]^o$

Substituting the value of π = 22/7

$[180/22 \times 7 \times -3]^o$

$(-3780/22)^o$

$(-171\ 18/22)^o$

$(-171^o(18/22 \times 60)')$

$(-171^o\ (49\ 1/11)')$

$(-171^o\ 49'\ (1/11 \times 60)')$
$-\ (171°\ 49'\ 5.45'')$
$\approx -\ (171°\ 49'\ 5'')$
∴ Degree measure of $(-3)^c$ is $-171°\ 49'\ 5''$

(v) 11^c
$(180/\pi \times 11)^o$
Substituting the value of $\pi = 22/7$
$(180/22 \times 7 \times 11)^o$
$(90 \times 7)°$
$630°$
∴ Degree measure of 11^c is $630°$

(vi) 1^c
$(180/\pi \times 1)^o$
Substituting the value of $\pi = 22/7$
$(180/22 \times 7 \times 1)^o$
$(1260/22)^o$
$(57\ 3/11)^o$
$(57^o\ (3/11 \times 60)')$
$(57^o\ (16\ 4/11)')$
$(57^o\ 16'\ (4/11 \times 60)')$
$(57^o\ 16'\ 21.81'')$
$\approx (57^o\ 16'\ 21'')$
∴ Degree measure of 1^c is $57^o\ 16'\ 21''$

2. Find the radian measure corresponding to the following degree measures:
(i) 300^o (ii) 35^o (iii) -56^o (iv)135^o (v) -300^o
(vi) $7^o\ 30'$ (vii) $125^o\ 30'$ (viii) $-47^o\ 30'$
Solution:
We know that $180° = \pi$ rad $\Rightarrow 1° = \pi/180$ rad
(i) $300°$
$(300 \times \pi/180)$ rad
$5\pi/3$
∴ Radian measure of 300^o is $5\pi/3$

(ii) $35°$
$(35 \times \pi/180)$ rad
$7\pi/36$

∴ Radian measure of 35° is 7π/36

(iii) -56°
(-56 × π/180) rad
-14π/45
∴ Radian measure of -56° is -14π/45

(iv) 135°
(135 × π/180) rad
3π/4
∴ Radian measure of 135° is 3π/4

(v) -300°
(-300 × π/180) rad
-5π/3
∴ Radian measure of -300° is -5π/3

(vi) 7° 30'
We know that, 30' = (1/2) °
7° 30' = (7 1/2) °
= (15/2)°
= (15/2 × π/180) rad
= π/24
∴ Radian measure of 7° 30' is π/24

(vii) 125° 30'
We know that, 30' = (1/2) °
125° 30' = (125 1/2) °
= (251/2)°
= (251/2 × π/180) rad
= 251π/360
∴ Radian measure of 125° 30' is 251π/360

(viii) -47° 30'
We know that, 30' = (1/2) °
-47° 30' = - (47 1/2) °
= - (95/2)°
= - (95/2 × π/180) rad
= - 19π/72

∴ Radian measure of -47° 30' is - 19π/72

3. The difference between the two acute angles of a right-angled triangle is 2π/5 radians. Express the angles in degrees.

Solution:

Given the difference between the two acute angles of a right-angled triangle is 2π/5 radians.

We know that π rad = 180° ⇒ 1 rad = 180°/ π

Given:

2π/5

(2π/5 × 180/ π) °

Substituting the value of π = 22/7

(2×22/(7×5) × 180/22 × 7)

(2/5 × 180) °

72°

Let one acute angle be x° and the other acute angle be 90° - x°.

Then,

x° - (90° - x°) = 72°

2x° - 90° = 72°

2x° = 72° + 90°

2x° = 162°

x° = 162°/ 2

x° = 81° and

90° - x° = 90° - 81°

= 9°

∴ The angles are 81° and 9°

4. One angle of a triangle is 2/3x grades, and another is 3/2x degrees while the third is πx/75 radians. Express all the angles in degrees.

Solution:

Given:

One angle of a triangle is 2x/3 grades and another is 3x/2 degree while the third is πx/75 radians.

We know that, 1 grad = (9/10) °

2/3x grad = (9/10) (2/3x) °

= 3/5x°

We know that, π rad = 180° ⇒ 1 rad = 180°/ π

Given: πx/75

$(\pi x/75 \times 180/\pi)^{o}$
$(12/5x)^{o}$

We know that, the sum of the angles of a triangle is 180°.
$3/5x^o + 3/2x^o + 12/5x^o = 180^o$
$(6+15+24)/10x^o = 180^o$
Upon cross-multiplication we get,
$45x^o = 180^o \times 10^o$
$= 1800^o$
$x^o = 1800^o/45^o$
$= 40^o$
∴ The angles of the triangle are:
$3/5x^o = 3/5 \times 40^o = 24^o$
$3/2x^o = 3/2 \times 40^o = 60^o$
$12/5\ x^o = 12/5 \times 40^o = 96^o$

5. Find the magnitude, in radians and degrees, of the interior angle of a regular:
(i) Pentagon (ii) Octagon (iii) Heptagon (iv) Duodecagon.
Solution:
We know that the sum of the interior angles of a polygon = $(n – 2)\ \pi$
And each angle of polygon = sum of interior angles of polygon / number of sides
Now, let us calculate the magnitude of
(i) Pentagon
Number of sides in pentagon = 5
Sum of interior angles of pentagon = $(5 – 2)\ \pi = 3\pi$
∴ Each angle of pentagon = $3\pi/5 \times 180^o/\ \pi = 108^o$

(ii) Octagon
Number of sides in octagon = 8
Sum of interior angles of octagon = $(8 – 2)\ \pi = 6\pi$
∴ Each angle of octagon = $6\pi/8 \times 180^o/\ \pi = 135^o$

(iii) Heptagon
Number of sides in heptagon = 7
Sum of interior angles of heptagon = $(7 – 2)\ \pi = 5\pi$
∴ Each angle of heptagon = $5\pi/7 \times 180^o/\ \pi = 900^o/7 = 128^o\ 34'\ 17''$

(iv) Duodecagon
Number of sides in duodecagon = 12

Sum of interior angles of duodecagon = (12 – 2) π = 10π
∴ Each angle of duodecagon = 10π/12 × 180°/ π = 150°

6. The angles of a quadrilateral are in A.P., and the greatest angle is 120°. Express the angles in radians.
Solution:
Let the angles of quadrilateral be (a – 3d) °, (a – d) °, (a + d) ° and (a + 3d) °.
We know that, the sum of angles of a quadrilateral is 360°.
a – 3d + a – d + a + d + a + 3d = 360°
4a = 360°
a = 360/4
= 90°
Given:
The greatest angle = 120°
a + 3d = 120°
90° + 3d = 120°
3d = 120° - 90°
3d = 30°
d = 30°/3
= 10°

∴ The angles are:
(a – 3d) ° = 90° - 30° = 60°
(a – d) ° = 90° - 10° = 80°
(a + d) ° = 90° + 10° = 100°
(a + 3d) ° = 120°

Angles of quadrilateral in radians:
(60 × π/180) rad = π/3
(80 × π/180) rad = 4π/9
(100 × π/180) rad = 5π/9
(120 × π/180) rad = 2π/3

7. The angles of a triangle are in A.P., and the number of degrees in the least angle is to the number of degrees in the mean angle as 1:120. Find the angle in radians.
Solution:
Let the angles of the triangle be (a – d) °, a° and (a + d) °.
We know that, the sum of the angles of a triangle is 180°.
a – d + a + a + d = 180°

3a = 180°
a = 60°
Given:
Number of degrees in the least angle / Number of degrees in the mean angle = 1/120
(a-d)/a = 1/120
(60-d)/60 = 1/120
(60-d)/1 = 1/2
120-2d = 1
2d = 119
d = 119/2
= 59.5

∴ The angles are:
(a – d) ° = 60° – 59.5° = 0.5°
a° = 60°
(a + d) ° = 60° + 59.5° = 119.5°

Angles of triangle in radians:
(0.5 × π/180) rad = π/360
(60 × π/180) rad = π/3
(119.5 × π/180) rad = 239π/360

8. The angle in one regular polygon is to that in another as 3:2 and the number of sides in first is twice that in the second. Determine the number of sides of two polygons.
Solution:
Let the number of sides in the first polygon be 2x and
The number of sides in the second polygon be x.
We know that, angle of an n-sided regular polygon = [(n-2)/n] π radian
The angle of the first polygon = [(2x-2)/2x] π = [(x-1)/x] π radian
The angle of the second polygon = [(x-2)/x] π radian
Thus,
[(x-1)/x] π / [(x-2)/x] π = 3/2
(x-1)/(x-2) = 3/2
Upon cross-multiplication we get,
2x – 2 = 3x – 6
3x-2x = 6-2
x = 4
∴ Number of sides in the first polygon = 2x = 2(4) = 8

Number of sides in the second polygon = x = 4

9. The angles of a triangle are in A.P. such that the greatest is 5 times the least. Find the angles in radians.

Solution:

Let the angles of the triangle be (a – d)°, a° and (a + d)°.
We know that, the sum of angles of triangle is 180°.
a – d + a + a + d = 180°
3a = 180°
a = 180°/3
= 60°

Given:
Greatest angle = 5 × least angle
Upon cross-multiplication,
Greatest angle / least angle = 5
(a+d)/(a-d) = 5
(60+d)/(60-d) = 5
By cross-multiplying we get,
60 + d = 300 – 5d
6d = 240
d = 240/6
= 40
Hence, angles are:
(a – d) ° = 60° – 40° = 20°
a° = 60°
(a + d) ° = 60° + 40° = 100°

∴ Angles of triangle in radians:
(20 × π/180) rad = π/9
(60 × π/180) rad = π/3
(100 × π/180) rad = 5π/9

10. The number of sides of two regular polygons is 5:4 and the difference between their angles is 9°. Find the number of sides of the polygons.

Solution:

Let the number of sides in the first polygon be 5x and
The number of sides in the second polygon be 4x.
We know that, angle of an n-sided regular polygon = [(n-2)/n] π radian

The angle of the first polygon = $[(5x-2)/5x]\ 180^o$
The angle of the second polygon = $[(4x-1)/4x]\ 180^o$
Thus,
$[(5x-2)/5x]\ 180^o - [(4x-1)/4x]\ 180^o = 9$
$180^o\ [(4(5x-2) – 5(4x-2))/20x] = 9$
Upon cross-multiplication we get,
$(20x – 8 – 20x + 10)/20x = 9/180$
$2/20x = 1/20$
$2/x = 1$
$x = 2$
∴Number of sides in the first polygon = $5x = 5(2) = 10$
Number of sides in the second polygon = $4x = 4(2) = 8$