EXERCISE 2.1 PAGE NO: 2.8

(1) (i) If (a/3 + 1, b – 2/3) = (5/3, 1/3), find the values of a and b.
(ii) If (x + 1, 1) = (3y, y - 1), find the values of x and y.
Solution:
Given:
(a/3 + 1, b – 2/3) = (5/3, 1/3)
By the definition of equality of ordered pairs,
Let us solve for a and b
a/3 + 1 = 5/3 and b – 2/3 = 1/3
a/3 = 5/3 – 1 and b = 1/3 + 2/3
a/3 = (5-3)/3 and b = (1+2)/3
a/3 = 2/3 and b = 3/3
a = 2(3)/3 and b = 1
a = 2 and b = 1
∴ Values of a and b are, a = 2 and b = 1

(ii) If (x + 1, 1) = (3y, y - 1), find the values of x and y.
Given:
(x + 1, 1) = (3y, y - 1)
By the definition of equality of ordered pairs,
Let us solve for x and y
x + 1 = 3y and 1 = y - 1
x = 3y - 1 and y = 1 + 1
x = 3y – 1 and y = 2
Since, y = 2 we can substitute in
x = 3y – 1
= 3(2) – 1
= 6 – 1
= 5
∴ Values of x and y are, x = 5 and y = 2

2. If the ordered pairs (x, - 1) and (5, y) belong to the set {(a, b): b = 2a - 3}, find the values of x and y.
Solution:
Given:
The ordered pairs (x, - 1) and (5, y) belong to the set {(a, b): b = 2a - 3}
Solving for first order pair
(x, - 1) = {(a, b): b = 2a - 3}

x = a and -1 = b
By taking b = 2a – 3
If b = - 1 then 2a = - 1 + 3
= 2
a = 2/2
= 1
So, a = 1
Since x = a, x = 1
Similarly, solving for second order pair
(5, y) = {(a, b): b = 2a - 3}
5 = a and y = b
By taking b = 2a - 3
If a = 5 then b = 2×5 – 3
= 10 – 3
= 7
So, b = 7
Since y = b, y = 7
∴ Values of x and y are, x = 1 and y = 7

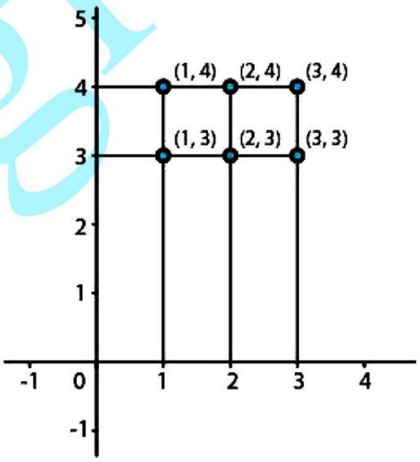
3. If a ∈ {- 1, 2, 3, 4, 5} and b ∈ {0, 3, 6}, write the set of all ordered pairs (a, b) such that a + b = 5.
Solution:
Given: a ∈ {- 1, 2, 3, 4, 5} and b ∈ {0, 3, 6},
To find: the ordered pair (a, b) such that a + b = 5
Then the ordered pair (a, b) such that a + b = 5 are as follows
(a, b) ∈ {(- 1, 6), (2, 3), (5, 0)}

4. If a ∈ {2, 4, 6, 9} and b ∈ {4, 6, 18, 27}, then form the set of all ordered pairs (a, b) such that a divides b and a<b.
Solution:
Given:
a ∈ {2, 4, 6, 9} and b ∈{4, 6, 18, 27}
Here,
2 divides 4, 6, 18 and is also less than all of them
4 divides 4 and is also less than none of them
6 divides 6, 18 and is less than 18 only
9 divides 18, 27 and is less than all of them
∴ Ordered pairs (a, b) are (2, 4), (2, 6), (2, 18), (6, 18), (9, 18) and (9, 27)

5. If A = {1, 2} and B = {1, 3}, find A x B and B x A.
Solution:
Given:
A = {1, 2} and B = {1, 3}
A × B = {1, 2} × {1, 3}
= {(1, 1), (1, 3), (2, 1), (2, 3)}
B × A = {1, 3} × {1, 2}
= {(1, 1), (1, 2), (3, 1), (3, 2)}

6. Let A = {1, 2, 3} and B = {3, 4}. Find A x B and show it graphically
Solution:
Given:
A = {1, 2, 3} and B = {3, 4}
A x B = {1, 2, 3} × {3, 4}
= {(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)}

Steps to follow to represent A × B graphically,
Step 1: One horizontal and one vertical axis should be drawn
Step 2: Element of set A should be represented in horizontal axis and on vertical axis elements of set B should be represented
Step 3: Draw dotted lines perpendicular to horizontal and vertical axes through the elements of set A and B
Step 4: Point of intersection of these perpendicular represents A × B



7. If A = {1, 2, 3} and B = {2, 4}, what are A x B, B x A, A x A, B x B, and (A x B) ∩ (B x A)?

Solution:

Given:

A = {1, 2, 3} and B = {2, 4}

Now let us find: A × B, B × A, A × A, (A × B) ∩ (B × A)

A × B = {1, 2, 3} × {2, 4}

= {(1, 2), (1, 4), (2, 2), (2, 4), (3, 2), (3, 4)}

B × A = {2, 4} × {1, 2, 3}

= {(2, 1), (2, 2), (2, 3), (4, 1), (4, 2), (4, 3)}

A × A = {1, 2, 3} × {1, 2, 3}

= {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)}

B × B = {2, 4} × {2, 4}

= {(2, 2), (2, 4), (4, 2), (4, 4)}

Intersection of two sets represents common elements of both the sets

So,

(A × B) ∩ (B × A) = {(2, 2)}

EXERCISE 2.2 PAGE NO: 2.12

1. Given A = {1, 2, 3}, B = {3, 4}, C = {4, 5, 6}, find (A x B) ∩ (B x C).
Solution:
Given:
A = {1, 2, 3}, B = {3, 4} and C = {4, 5, 6}
Let us find: (A × B) ∩ (B × C)
(A × B) = {1, 2, 3} × {3, 4}
= {(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)}

(B × C) = {3, 4} × {4, 5, 6}
= {(3, 4), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6)}

∴ (A × B) ∩ (B × C) = {(3, 4)}

2. If A = {2, 3}, B = {4, 5}, C = {5, 6} find A x (B ∪ C), (A x B) ∪ (A x C).
Solution:
Given: A = {2, 3}, B = {4, 5} and C = {5, 6}
Let us find: A x (B ∪ C) and (A x B) ∪ (A x C)
(B ∪ C) = {4, 5, 6}
A × (B ∪ C) = {2, 3} × {4, 5, 6}
= {(2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)}

(A × B) = {2, 3} × {4, 5}
= {(2, 4), (2, 5), (3, 4), (3, 5)}

(A × C) = {2, 3} × {5, 6}
= {(2, 5), (2, 6), (3, 5), (3, 6)}

∴ (A × B) ∪ (A × C) = {(2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)}
A × (B ∪ C) = {(2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)}

3. If A = {1, 2, 3}, B = {4}, C = {5}, then verify that:
(i) A x (B ∪ C) = (A x B) ∪ (A x C)
(ii) A x (B ∩ C) = (A x B) ∩ (A x C)
(iii) A x (B – C) = (A x B) – (A x C)
Solution:
Given:
A = {1, 2, 3}, B = {4} and C = {5}

(i) A × (B ∪ C) = (A × B) ∪ (A × C)
Let us consider LHS: (B ∪ C)
(B ∪ C) = {4, 5}
A × (B ∪ C) = {1, 2, 3} × {4, 5}
= {(1, 4), (1, 5), (2, 4), (2, 5), (3, 4), (3, 5)}

Now, RHS
(A × B) = {1, 2, 3} × {4}
= {(1, 4), (2, 4), (3, 4)}

(A × C) = {1, 2, 3} × {5}
= {(1, 5), (2, 5), (3, 5)}

(A × B) ∪ (A × C) = {(1, 4), (2, 4), (3, 4), (1, 5), (2, 5), (3, 5)}
∴ LHS = RHS

(ii) A × (B ∩ C) = (A × B) ∩ (A × C)
Let us consider LHS: (B ∩ C)
(B ∩ C) = ∅ (No common element)
A × (B ∩ C) = {1, 2, 3} × ∅
= ∅

Now, RHS
(A × B) = {1, 2, 3} × {4}
= {(1, 4), (2, 4), (3, 4)}

(A × C) = {1, 2, 3} × {5}
= {(1, 5), (2, 5), (3, 5)}

(A × B) ∩ (A × C) = ∅
∴ LHS = RHS

(iii) A × (B − C) = (A × B) − (A × C)
Let us consider LHS: (B − C)
(B − C) = ∅
A × (B − C) = {1, 2, 3} × ∅
= ∅

Now, RHS
$(A \times B) = \{1, 2, 3\} \times \{4\}$
$= \{(1, 4), (2, 4), (3, 4)\}$

$(A \times C) = \{1, 2, 3\} \times \{5\}$
$= \{(1, 5), (2, 5), (3, 5)\}$

$(A \times B) - (A \times C) = \emptyset$
$\therefore$ LHS = RHS

4. Let A = {1, 2}, B = {1, 2, 3, 4}, C = {5, 6} and D = {5, 6, 7, 8}. Verify that:
(i) A x C ⊂ B x D
(ii) A x (B ∩ C) = (A x B) ∩ (A x C)
Solution:
Given:
$A = \{1, 2\}$, $B = \{1, 2, 3, 4\}$, $C = \{5, 6\}$ and $D = \{5, 6, 7, 8\}$

(i) A x C ⊂ B x D
Let us consider LHS A x C
$A \times C = \{1, 2\} \times \{5, 6\}$
$= \{(1, 5), (1, 6), (2, 5), (2, 6)\}$

Now, RHS
$B \times D = \{1, 2, 3, 4\} \times \{5, 6, 7, 8\}$
$= \{(1, 5), (1, 6), (1, 7), (1, 8), (2, 5), (2, 6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8)\}$

Since, all elements of $A \times C$ is in $B \times D$.
$\therefore$We can say $A \times C \subset B \times D$

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$
Let us consider LHS $A \times (B \cap C)$
$(B \cap C) = \emptyset$
$A \times (B \cap C) = \{1, 2\} \times \emptyset$
$= \emptyset$

Now, RHS
$(A \times B) = \{1, 2\} \times \{1, 2, 3, 4\}$
$= \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4)\}$

(A × C) = {1, 2} × {5, 6}
= {(1, 5), (1, 6), (2, 5), (2, 6)}
Since, there is no common element between A × B and A × C
(A × B) ∩ (A × C) = Ø
∴ A × (B ∩ C) = (A × B) ∩ (A × C)

5. If A = {1, 2, 3}, B = {3, 4} and C = {4, 5, 6}, find
(i) A x (B ∩ C)
(ii) (A x B) ∩ (A x C)
(iii) A x (B ∪ C)
(iv) (A x B) ∪ (A x C)
Solution:
Given:
A = {1, 2, 3}, B = {3, 4} and C = {4, 5, 6}
(i) A × (B ∩ C)
(B ∩ C) = {4}
A × (B ∩ C) = {1, 2, 3} × {4}
= {(1, 4), (2, 4), (3, 4)}

(ii) (A × B) ∩ (A × C)
(A × B) = {1, 2, 3} × {3, 4}
= {(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)}

(A × C) = {1, 2, 3} × {4, 5, 6}
= {(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)}
(A × B) ∩ (A × C) = {(1, 4), (2, 4), (3, 4)}

(iii) A × (B ∪ C)
(B ∪ C) = {3, 4, 5, 6}
A × (B ∪ C) = {1, 2, 3} × {3, 4, 5, 6}
= {(1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 5), (3, 6)}

(iv) (A × B) ∪ (A × C)
(A × B) = {1, 2, 3} × {3, 4}
= {(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)}

(A × C) = {1, 2, 3} × {4, 5, 6}
= {(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)}

(A × B) ∪ (A × C) = {(1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 5), (3, 6)}

6. Prove that:
(i) (A ∪ B) x C = (A x C) = (A x C) ∪ (B x C)
(ii) (A ∩ B) x C = (A x C) ∩ (B x C)
Solution:
(i) (A ∪ B) x C = (A x C) = (A x C) ∪ (B x C)
Let (x, y) be an arbitrary element of (A ∪ B) × C
(x, y) ∈ (A ∪ B) C
Since, (x, y) are elements of Cartesian product of (A ∪ B) × C
x ∈ (A ∪ B) and y ∈ C
(x ∈ A or x ∈ B) and y ∈ C
(x ∈ A and y ∈ C) or (x ∈ Band y ∈ C)
(x, y) ∈ A × C or (x, y) ∈ B × C
(x, y) ∈ (A × C) ∪ (B × C) … (1)

Let (x, y) be an arbitrary element of (A × C) ∪ (B × C).
(x, y) ∈ (A × C) ∪ (B × C)
(x, y) ∈ (A × C) or (x, y) ∈ (B × C)
(x ∈ A and y ∈ C) or (x ∈ B and y ∈ C)
(x ∈ A or x ∈ B) and y ∈ C
x ∈ (A ∪ B) and y ∈ C
(x, y) ∈ (A ∪ B) × C … (2)
From 1 and 2, we get: (A ∪ B) × C = (A × C) ∪ (B × C)

(ii) (A ∩ B) x C = (A x C) ∩ (B x C)
Let (x, y) be an arbitrary element of (A ∩ B) × C.
(x, y) ∈ (A ∩ B) × C
Since, (x, y) are elements of Cartesian product of (A ∩ B) × C
x ∈ (A ∩ B) and y ∈ C
(x ∈ A and x ∈ B) and y ∈ C
(x ∈ A and y ∈ C) and (x ∈ Band y ∈ C)
(x, y) ∈ A × C and (x, y) ∈ B × C
(x, y) ∈ (A × C) ∩ (B × C) … (1)

Let (x, y) be an arbitrary element of (A × C) ∩ (B × C).
(x, y) ∈ (A × C) ∩ (B × C)
(x, y) ∈ (A × C) and (x, y) ∈ (B × C)

(x ∈A and y ∈ C) and (x ∈ Band y ∈ C)
(x ∈A and x ∈ B) and y ∈ C
x ∈ (A ∩ B) and y ∈ C
(x, y) ∈ (A ∩ B) × C … (2)
From 1 and 2, we get: (A ∩ B) × C = (A × C) ∩ (B × C)

7. If A x B ⊆ C x D and A ∩ B ∈ ∅, Prove that A ⊆ C and B ⊆ D.
Solution:
Given:
A × B ⊆ C x D and A ∩ B ∈ ∅
A × B ⊆ C x D denotes A × B is subset of C × D that is every element A × B is in C × D.
And A ∩ B ∈ ∅ denotes A and B does not have any common element between them.
A × B = {(a, b): a ∈ A and b ∈ B}
∴We can say (a, b) ⊆ C × D [Since, A × B ⊆ C x D is given]
a ∈ C and b ∈ D
a ∈ A = a ∈ C
A ⊆ C
And
b ∈ B = b ∈ D
B ⊆ D
Hence proved.

EXERCISE 2.3 PAGE NO: 2.20

1. If A = {1, 2, 3}, B = {4, 5, 6}, which of the following are relations from A to B? Give reasons in support of your answer.
(i) {(1, 6), (3, 4), (5, 2)}
(ii) {(1, 5), (2, 6), (3, 4), (3, 6)}
(iii) {(4, 2), (4, 3), (5, 1)}
(iv) A × B
Solution:
Given,
A = {1, 2, 3}, B = {4, 5, 6}
A relation from A to B can be defined as:
A × B = {1, 2, 3} × {4, 5, 6}
= {(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)}

(i) {(1, 6), (3, 4), (5, 2)}
No, it is not a relation from A to B. The given set is not a subset of A × B as (5, 2) is not a part of the relation from A to B.

(ii) {(1, 5), (2, 6), (3, 4), (3, 6)}
Yes, it is a relation from A to B. The given set is a subset of A × B.

(iii) {(4, 2), (4, 3), (5, 1)}
No, it is not a relation from A to B. The given set is not a subset of A × B.

(iv) A × B
A × B is a relation from A to B and can be defined as:
{(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6),(3, 4),(3, 5),(3, 6)}

2. A relation R is defined from a set A = {2, 3, 4, 5} to a set B = {3, 6, 7, 10} as follows: (x, y) R x is relatively prime to y. Express R as a set of ordered pairs and determine its domain and range.
Solution:
Relatively prime numbers are also known as co-prime numbers. If there is no integer greater than one that divides both (that is, their greatest common divisor is one).

Given: (x, y) ∈ R = x is relatively prime to y
Here,
2 is co-prime to 3 and 7.

3 is co-prime to 7 and 10.
4 is co-prime to 3 and 7.
5 is co-prime to 3, 6 and 7.
∴ R = {(2, 3), (2, 7), (3, 7), (3, 10), (4, 3), (4, 7), (5, 3), (5, 6), (5, 7)}
Domain of relation R = {2, 3, 4, 5}
Range of relation R = {3, 6, 7, 10}

3. Let A be the set of first five natural and let R be a relation on A defined as follows: (x, y) R x ≤ y
Express R and R^{-1} as sets of ordered pairs. Determine also
(i) the domain of R^{-1}
(ii) The Range of R.
Solution:
A is set of first five natural numbers.
So, A= {1, 2, 3, 4, 5}
Given: (x, y) R x ≤ y
1 is less than 2, 3, 4 and 5.
2 is less than 3, 4 and 5.
3 is less than 4 and 5.
4 is less than 5.
5 is not less than any number A
∴ R = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (2, 2), (2, 3), (2, 4), (2, 5), (3, 3), (3, 4), (3, 5), (4, 4), (4, 5), (5, 5)}

"An inverse relation is the set of ordered pairs obtained by interchanging the first and second elements of each pair in the original relation. If the graph of a function contains a point (a, b), then the graph of the inverse relation of this function contains the point (b, a)".

∴ R^{-1} = {(1, 1), (2, 1), (3, 1), (4, 1), (5, 1), (2, 2), (3, 2), (4, 2), (5, 2), (3, 3), (4, 3), (5, 3), (4, 4), (5, 4) (5, 5)}

(i) Domain of R^{-1} = {1, 2, 3, 4, 5}
(ii) Range of R = {1, 2, 3, 4, 5}

4. Find the inverse relation R^{-1} in each of the following cases:
(i) R= {(1, 2), (1, 3), (2, 3), (3, 2), (5, 6)}
(ii) R= {(x, y) : x, y ∈ N; x + 2y = 8}
(iii) R is a relation from {11, 12, 13} to (8, 10, 12} defined by y = x – 3

Solution:

(i) Given:

R= {(1, 2), (1, 3), (2, 3), (3, 2), (5, 6)}

So, R^{-1} = {(2, 1), (3, 1), (3, 2), (2, 3), (6, 5)}

(ii) Given,

R= {(x, y): x, y ∈ N; x + 2y = 8}

Here, x + 2y = 8

x = 8 – 2y

As y ∈ N, Put the values of y = 1, 2, 3,…… till x ∈ N

When, y = 1, x = 8 – 2(1) = 8 – 2 = 6

When, y = 2, x = 8 – 2(2) = 8 – 4 = 4

When, y = 3, x = 8 – 2(3) = 8 – 6 = 2

When, y = 4, x = 8 – 2(4) = 8 – 8 = 0

Now, y cannot hold value 4 because x = 0 for y = 4 which is not a natural number.

∴ R = {(2, 3), (4, 2), (6, 1)}

R^{-1} = {(3, 2), (2, 4), (1, 6)}

(iii) Given,

R is a relation from {11, 12, 13} to (8, 10, 12} defined by y = x – 3

Here,

x = {11, 12, 13} and y = (8, 10, 12}

y = x – 3

When, x = 11, y = 11 – 3 = 8 ∈ (8, 10, 12}

When, x = 12, y = 12 – 3 = 9 ∉ (8, 10, 12}

When, x = 13, y = 13 – 3 = 10 ∈ (8, 10, 12}

∴ R = {(11, 8), (13, 10)}

R^{-1} = {(8, 11), (10, 13)}

5. Write the following relations as the sets of ordered pairs:

(i) A relation R from the set {2, 3, 4, 5, 6} to the set {1, 2, 3} defined by x = 2y.

(ii) A relation R on the set {1, 2, 3, 4, 5, 6, 7} defined by (x, y) ∈ R ⇔ x is relatively prime to y.

(iii) A relation R on the set {0, 1, 2,…,10} defined by 2x + 3y = 12.

(iv) A relation R form a set A = {5, 6, 7, 8} to the set B = {10, 12, 15, 16, 18} defined by (x, y) R x divides y.

Solution:

(i) A relation R from the set {2, 3, 4, 5, 6} to the set {1, 2, 3} defined by x = 2y.

Let A = {2, 3, 4, 5, 6} and B = {1, 2, 3}

Given, x = 2y where y = {1, 2, 3}
When, y = 1, x = 2(1) = 2
When, y = 2, x = 2(2) = 4
When, y = 3, x = 2(3) = 6
∴ R = {(2, 1), (4, 2), (6, 3)}

(ii) A relation R on the set {1, 2, 3, 4, 5, 6, 7} defined by (x, y) ∈ R ⇔ x is relatively prime to y.
Given:
(x, y) R x is relatively prime to y
Here,
2 is co-prime to 3, 5 and 7.
3 is co-prime to 2, 4, 5 and 7.
4 is co-prime to 3, 5 and 7.
5 is co-prime to 2, 3, 4, 6 and 7.
6 is co-prime to 5 and 7.
7 is co-prime to 2, 3, 4, 5 and 6.
∴ R ={(2, 3), (2, 5), (2, 7), (3, 2), (3, 4), (3, 5), (3, 7), (4, 3), (4, 5), (4, 7), (5, 2), (5, 3), (5, 4), (5, 6), (5, 7), (6, 5), (6, 7), (7, 2), (7, 3), (7, 4), (7, 5), (7, 6), (7, 7)}

(iii) A relation R on the set {0, 1, 2,…, 10} defined by 2x + 3y = 12.
Given,
(x, y) R 2x + 3y = 12
Where x and y = {0, 1, 2,…, 10}
2x + 3y = 12
2x = 12 – 3y
x = (12-3y)/2
When, y = 0, x = (12-3(0))/2 = 12/2 = 6
When, y = 2, x = (12-3(2))/2 = (12-6)/2 = 6/2 = 3
When, y = 4, x = (12-3(4))/2 = (12-12)/2 = 0/2 = 0
∴ R = {(0, 4), (3, 2), (6, 0)}

(iv) A relation R form a set A = {5, 6, 7, 8} to the set B = {10, 12, 15, 16, 18} defined by (x, y) ∈ R ⇔ x divides y.
Given,
(x, y) R x divides y
Where, x = {5, 6, 7, 8} and y = {10, 12, 15, 16, 18}
Here,
5 divides 10 and 15.

6 divides 12 and 18.
7 divides none of the value of set B.
8 divides 16.
∴ R = {(5, 10), (5, 15), (6, 12), (6, 18), (8, 16)}

6. Let R be a relation in N defined by (x, y) ∈ R ⇔ x + 2y = 8. Express R and R^{-1} as sets of ordered pairs.
Solution:
Given,
(x, y) R x + 2y = 8 where x ∈ N and y ∈ N
x + 2y= 8
x = 8 – 2y
Putting the values y = 1, 2, 3,…… till x ∈ N
When, y = 1, x = 8 – 2(1) = 8 – 2 = 6
When, y = 2, x = 8 – 2(2) = 8 – 4 = 4
When, y = 3, x = 8 – 2(3) = 8 – 6 = 2
When, y = 4, x = 8 – 2(4) = 8 – 8 = 0
Now, y cannot hold value 4 because x = 0 for y = 4 which is not a natural number.
∴ R = {(2, 3), (4, 2), (6, 1)}
R^{-1} = {(3, 2), (2, 4), (1, 6)}

7. Let A = {3, 5} and B = {7, 11}. Let R = {(a, b): a ∈ A, b ∈ B, a-b is odd}. Show that R is an empty relation from A into B.
Solution:
Given,
A = {3, 5} and B = {7, 11}
R = {(a, b): a ∈ A, b ∈ B, a-b is odd}
On putting a = 3 and b = 7,
a – b = 3 – 7 = -4 which is not odd
On putting a = 3 and b = 11,
a – b = 3 – 11 = -8 which is not odd
On putting a = 5 and b = 7:
a – b = 5 – 7 = -2 which is not odd
On putting a = 5 and b = 11:
a – b = 5 – 11 = -6 which is not odd
∴ R = { } = Φ
R is an empty relation from A into B.
Hence proved.

8. Let A = {1, 2} and B = {3, 4}. Find the total number of relations from A into B.
Solution:
Given,
A= {1, 2}, B= {3, 4}
n (A) = 2 (Number of elements in set A).
n (B) = 2 (Number of elements in set B).
We know,
n (A × B) = n (A) × n (B)
= 2 × 2
= 4 [since, n(x) = a, n(y) = b. total number of relations = 2^{ab}]
∴ Number of relations from A to B are $2^4 = 16$.

9. Determine the domain and range of the relation R defined by
(i) R = {(x, x+5): x ∈ {0, 1, 2, 3, 4, 5}
(ii) R= {(x, x^3): x is a prime number less than 10}
Solution:
(i) R = {(x, x+5): x ∈ {0, 1, 2, 3, 4, 5}
Given,
R = {(x, x+5): x ∈ {0, 1, 2, 3, 4, 5}
∴ R = {(0, 0+5), (1, 1+5), (2, 2+5), (3, 3+5), (4, 4+5), (5, 5+5)}
R = {(0, 5), (1, 6), (2, 7), (3, 8), (4, 9), (5, 10)}
So,
Domain of relation R = {0, 1, 2, 3, 4, 5}
Range of relation R = {5, 6, 7, 8, 9, 10}

(ii) R= {(x, x^3): x is a prime number less than 10}
Given,
R = {(x, x^3): x is a prime number less than 10}
Prime numbers less than 10 are 2, 3, 5 and 7
∴ R = {(2, 2^3), (3, 3^3), (5, 5^3), (7, 7^3)}
R = {(2, 8), (3, 27), (5, 125), (7, 343)}
So,
Domain of relation R = {2, 3, 5, 7}
Range of relation R = {8, 27, 125, 343}

10. Determine the domain and range of the following relations:
(i) R= {a, b): a ∈ N, a < 5, b = 4}
(ii) S= {a, b): b = |a-1|, a ∈ Z and |a| ≤ 3}
Solution:

(i) R= {a, b): a ∈ N, a < 5, b = 4}
Given,
R= {a, b): a ∈ N, a < 5, b = 4}
Natural numbers less than 5 are 1, 2, 3 and 4
a = {1, 2, 3, 4} and b = {4}
R = {(1, 4), (2, 4), (3, 4), (4, 4)}
So,
Domain of relation R = {1, 2, 3, 4}
Range of relation R = {4}

(ii) S= {a, b): b = |a-1|, a ∈ Z and |a| ≤ 3}
Given,
S= {a, b): b = |a-1|, a ∈ Z and |a| ≤ 3}
Z denotes integer which can be positive as well as negative
Now, |a| ≤ 3 and b = |a-1|
∴ a = {-3, -2, -1, 0, 1, 2, 3}
For, a = -3, -2, -1, 0, 1, 2, 3 we get,
S = {(-3, |-3 – 1|), (-2, |-2 – 1|), (-1, |-1 – 1|), (0, |0 – 1|), (1, |1 – 1|), (2, |2 – 1|), (3, |3 – 1|)}
S = {(-3, |-4|), (-2, |-3|), (-1, |-2|), (0, |-1|), (1, |0|), (2, |1|), (3, |2|)}
S = {(-3, 4), (-2, 3), (-1, 2), (0, 1), (1, 0), (2, 1), (3, 2)}
b = 4, 3, 2, 1, 0, 1, 2
So,
Domain of relation S = {0, -1, -2, -3, 1, 2, 3}
Range of relation S = {0, 1, 2, 3, 4}

11. Let A = {a, b}. List all relations on A and find their number.
Solution:
The total number of relations that can be defined from a set A to a set B is the number of possible subsets of A × B. If n (A) = p and n (B) = q, then n (A × B) = pq.
So, the total number of relations is 2^{pq}.
Now,
A × A = {(a, a), (a, b), (b, a), (b, b)}
Total number of relations are all possible subsets of A × A:
[{(a, a), (a, b), (b, a), (b, b)}, {(a, a), (a, b)}, {(a, a), (b, a)},{(a, a), (b, b)}, {(a, b), (b, a)}, {(a, b), (b, b)}, {(b, a), (b, b)}, {(a, a), (a, b), (b, a)}, {(a, b), (b, a), (b, b)}, {(a, a), (b, a), (b, b)}, {(a, a), (a, b), (b, b)}, {(a, a), (a, b), (b, a), (b, b)}]
n (A) = 2 ⇒ n (A × A) = 2 × 2 = 4
∴ Total number of relations = 2^4 = 16