

**EXERCISE 12.1****PAGE NO: 12.3****1. If  $P(n)$  is the statement “ $n(n+1)$  is even”, then what is  $P(3)$ ?****Solution:**

Given:

 $P(n) = n(n+1)$  is even.

So,

$$\begin{aligned}P(3) &= 3(3+1) \\&= 3(4) \\&= 12\end{aligned}$$

Hence,  $P(3) = 12$ ,  $P(3)$  is also even.**2. If  $P(n)$  is the statement “ $n^3 + n$  is divisible by 3”, prove that  $P(3)$  is true but  $P(4)$  is not true.****Solution:**

Given:

 $P(n) = n^3 + n$  is divisible by 3We have  $P(n) = n^3 + n$ 

So,

$$\begin{aligned}P(3) &= 3^3 + 3 \\&= 27 + 3 \\&= 30\end{aligned}$$

 $P(3) = 30$ , So it is divisible by 3Now, let's check with  $P(4)$ 

$$\begin{aligned}P(4) &= 4^3 + 4 \\&= 64 + 4 \\&= 68\end{aligned}$$

 $P(4) = 68$ , so it is not divisible by 3Hence,  $P(3)$  is true and  $P(4)$  is not true.**3. If  $P(n)$  is the statement “ $2^n \geq 3n$ ”, and if  $P(r)$  is true, prove that  $P(r+1)$  is true.****Solution:**

Given:

 $P(n) = “2^n \geq 3n”$  and  $p(r)$  is true.We have,  $P(n) = 2^n \geq 3n$ Since,  $P(r)$  is true

So,

$$2^r \geq 3r$$

RD Sharma Solutions for Class 11 Maths Chapter 12 –  
Mathematical Induction

Now, let's multiply both sides by 2

$$2 \times 2^r \geq 3r \times 2$$

$$2^{r+1} \geq 6r$$

$$2^{r+1} \geq 3r + 3r \text{ [since } 3r > 3 = 3r + 3r \geq 3 + 3r]$$

$$\therefore 2^{r+1} \geq 3(r+1)$$

Hence,  $P(r+1)$  is true.

**4. If  $P(n)$  is the statement “ $n^2 + n$  is even”, and if  $P(r)$  is true, then  $P(r+1)$  is true**

**Solution:**

Given:

$P(n) = n^2 + n$  is even and  $P(r)$  is true, then  $r^2 + r$  is even

Let us consider  $r^2 + r = 2k \dots (i)$

Now,  $(r+1)^2 + (r+1)$

$$r^2 + 1 + 2r + r + 1$$

$$(r^2 + r) + 2r + 2$$

$$2k + 2r + 2 \text{ [from equation (i)]}$$

$$2(k+r+1)$$

$$2\mu$$

$$\therefore (r+1)^2 + (r+1) \text{ is Even.}$$

Hence,  $P(r+1)$  is true.

**5. Given an example of a statement  $P(n)$  such that it is true for all  $n \in \mathbb{N}$ .**

**Solution:**

Let us consider

$$P(n) = 1 + 2 + 3 + \dots + n = n(n+1)/2$$

So,

$P(n)$  is true for all natural numbers.

Hence,  $P(n)$  is true for all  $n \in \mathbb{N}$ .

**6. If  $P(n)$  is the statement “ $n^2 - n + 41$  is prime”, prove that  $P(1)$ ,  $P(2)$  and  $P(3)$  are true. Prove also that  $P(41)$  is not true.**

**Solution:**

Given:

$$P(n) = n^2 - n + 41 \text{ is prime.}$$

$$P(n) = n^2 - n + 41$$

$$P(1) = 1 - 1 + 41$$

$$= 41$$

$P(1)$  is Prime.

Similarly,

RD Sharma Solutions for Class 11 Maths Chapter 12 –  
Mathematical Induction

$$\begin{aligned}P(2) &= 2^2 - 2 + 41 \\&= 4 - 2 + 41 \\&= 43\end{aligned}$$

P (2) is prime.

Similarly,

$$\begin{aligned}P(3) &= 3^2 - 3 + 41 \\&= 9 - 3 + 41 \\&= 47\end{aligned}$$

P (3) is prime

Now,

$$\begin{aligned}P(41) &= (41)^2 - 41 + 41 \\&= 1681\end{aligned}$$

P (41) is not prime

Hence, P (1), P(2), P (3) are true but P (41) is not true.

**EXERCISE 12.2****PAGE NO: 12.27****Prove the following by the principle of mathematical induction:****1.  $1 + 2 + 3 + \dots + n = n(n+1)/2$  i.e., the sum of the first  $n$  natural numbers is  $n(n+1)/2$ .****Solution:**Let us consider  $P(n) = 1 + 2 + 3 + \dots + n = n(n+1)/2$ For,  $n = 1$ LHS of  $P(n) = 1$ RHS of  $P(n) = 1(1+1)/2 = 1$ 

So, LHS = RHS

Since,  $P(n)$  is true for  $n = 1$ Let us consider  $P(n)$  be the true for  $n = k$ , so $1 + 2 + 3 + \dots + k = k(k+1)/2 \dots (i)$ 

Now,

$$\begin{aligned}
 (1 + 2 + 3 + \dots + k) + (k + 1) &= k(k+1)/2 + (k+1) \\
 &= (k+1)(k/2 + 1) \\
 &= [(k+1)(k+2)]/2 \\
 &= [(k+1)((k+1)+1)]/2
 \end{aligned}$$

 $P(n)$  is true for  $n = k + 1$  $P(n)$  is true for all  $n \in \mathbb{N}$ 

So, by the principle of Mathematical Induction

Hence,  $P(n) = 1 + 2 + 3 + \dots + n = n(n+1)/2$  is true for all  $n \in \mathbb{N}$ .**2.  $1^2 + 2^2 + 3^2 + \dots + n^2 = [n(n+1)(2n+1)]/6$** **Solution:**Let us consider  $P(n) = 1^2 + 2^2 + 3^2 + \dots + n^2 = [n(n+1)(2n+1)]/6$ For,  $n = 1$  $P(1) = [1(1+1)(2+1)]/6$  $1 = 1$  $P(n)$  is true for  $n = 1$ Let  $P(n)$  is true for  $n = k$ , so $P(k): 1^2 + 2^2 + 3^2 + \dots + k^2 = [k(k+1)(2k+1)]/6$ Let's check for  $P(n) = k + 1$ , so

$$\begin{aligned}
 P(k) &= 1^2 + 2^2 + 3^2 + \dots + k^2 + (k+1)^2 = [k+1(k+2)(2k+3)]/6 \\
 &= 1^2 + 2^2 + 3^2 + \dots + k^2 + (k+1)^2 \\
 &= [k+1(k+2)(2k+3)]/6 + (k+1)^2
 \end{aligned}$$



RD Sharma Solutions for Class 11 Maths Chapter 12 –  
Mathematical Induction

$$\begin{aligned}
 &= (k+1) [(2k^2 + k)/6 + (k+1)/1] \\
 &= (k+1) [2k^2 + k + 6k + 6]/6 \\
 &= (k+1) [2k^2 + 7k + 6]/6 \\
 &= (k+1) [2k^2 + 4k + 3k + 6]/6 \\
 &= (k+1) [2k(k+2) + 3(k+2)]/6 \\
 &= [(k+1) (2k+3) (k+2)] / 6
 \end{aligned}$$

P (n) is true for  $n = k + 1$

Hence, P (n) is true for all  $n \in \mathbb{N}$ .

### 3. $1 + 3 + 3^2 + \dots + 3^{n-1} = (3^n - 1)/2$

**Solution:**

Let  $P(n) = 1 + 3 + 3^2 + \dots + 3^{n-1} = (3^n - 1)/2$

Now, For  $n = 1$

$$P(1) = 1 = (3^1 - 1)/2 = 2/2 = 1$$

P (n) is true for  $n = 1$

Now, let's check for P (n) is true for  $n = k$

$$P(k) = 1 + 3 + 3^2 + \dots + 3^{k-1} = (3^k - 1)/2 \dots (i)$$

Now, we have to show P (n) is true for  $n = k + 1$

$$P(k+1) = 1 + 3 + 3^2 + \dots + 3^k = (3^{k+1} - 1)/2$$

Then,  $\{1 + 3 + 3^2 + \dots + 3^{k-1}\} + 3^k$

$$= (3k - 1)/2 + 3^k \text{ using equation (i)}$$

$$= (3k - 1 + 2 \times 3^k)/2$$

$$= (3 \times 3^k - 1)/2$$

$$= (3^{k+1} - 1)/2$$

P (n) is true for  $n = k + 1$

Hence, P (n) is true for all  $n \in \mathbb{N}$ .

### 4. $1/1.2 + 1/2.3 + 1/3.4 + \dots + 1/n(n+1) = n/(n+1)$

**Solution:**

Let  $P(n) = 1/1.2 + 1/2.3 + 1/3.4 + \dots + 1/n(n+1) = n/(n+1)$

For,  $n = 1$

$$P(n) = 1/1.2 = 1/1+1$$

$$1/2 = 1/2$$

P (n) is true for  $n = 1$

Let's check for P (n) is true for  $n = k$ ,

$$1/1.2 + 1/2.3 + 1/3.4 + \dots + 1/k(k+1) + k/(k+1) (k+2) = (k+1)/(k+2)$$

Then,

$$1/1.2 + 1/2.3 + 1/3.4 + \dots + 1/k(k+1) + k/(k+1) (k+2)$$

$$= 1/(k+1)/(k+2) + k/(k+1)$$

$$\begin{aligned}
 &= 1/(k+1) [k(k+2)+1]/(k+2) \\
 &= 1/(k+1) [k^2 + 2k + 1]/(k+2) \\
 &= 1/(k+1) [(k+1) (k+1)]/(k+2) \\
 &= (k+1) / (k+2)
 \end{aligned}$$

P (n) is true for  $n = k + 1$

Hence, P (n) is true for all  $n \in \mathbb{N}$ .

**5.  $1 + 3 + 5 + \dots + (2n - 1) = n^2$  i.e., the sum of first n odd natural numbers is  $n^2$ .**

**Solution:**

Let P (n):  $1 + 3 + 5 + \dots + (2n - 1) = n^2$

Let us check P (n) is true for  $n = 1$

$$P(1) = 1 = 1^2$$

$$1 = 1$$

P (n) is true for  $n = 1$

Now, Let's check P (n) is true for  $n = k$

$$P(k) = 1 + 3 + 5 + \dots + (2k - 1) = k^2 \dots (i)$$

We have to show that

$$1 + 3 + 5 + \dots + (2k - 1) + 2(k + 1) - 1 = (k + 1)^2$$

Now,

$$\begin{aligned}
 1 + 3 + 5 + \dots + (2k - 1) + 2(k + 1) - 1 &= k^2 + (2k + 1) \\
 &= k^2 + 2k + 1 \\
 &= (k + 1)^2
 \end{aligned}$$

P (n) is true for  $n = k + 1$

Hence, P (n) is true for all  $n \in \mathbb{N}$ .

**6.  $1/2.5 + 1/5.8 + 1/8.11 + \dots + 1/(3n-1)(3n+2) = n/(6n+4)$**

**Solution:**

Let P (n) =  $1/2.5 + 1/5.8 + 1/8.11 + \dots + 1/(3n-1)(3n+2) = n/(6n+4)$

Let us check P (n) is true for  $n = 1$

$$P(1): 1/2.5 = 1/6.1+4 \Rightarrow 1/10 = 1/10$$

P (1) is true.

Now,

Let us check for P (k) is true, and have to prove that P (k + 1) is true.

$$P(k): 1/2.5 + 1/5.8 + 1/8.11 + \dots + 1/(3k-1)(3k+2) = k/(6k+4)$$

$$\begin{aligned}
 P(k+1): 1/2.5 + 1/5.8 + 1/8.11 + \dots + 1/(3k-1)(3k+2) + 1/(3k+3-1)(3k+3+2) \\
 : k/(6k+4) + 1/(3k+2)(3k+5) \\
 : [k(3k+5)+2] / [2(3k+2)(3k+5)]
 \end{aligned}$$

RD Sharma Solutions for Class 11 Maths Chapter 12 –  
Mathematical Induction

$$: (k+1) / (6(k+1)+4)$$

$P(k+1)$  is true.

Hence proved by mathematical induction.

$$7. \frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \dots + \frac{1}{(3n-2)(3n+1)} = \frac{n}{3n+1}$$

**Solution:**

$$\text{Let } P(n) = \frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \dots + \frac{1}{(3n-2)(3n+1)} = \frac{n}{3n+1}$$

Let us check for  $n = 1$ ,

$$P(1): \frac{1}{1.4} = \frac{1}{4}$$

$$\frac{1}{4} = \frac{1}{4}$$

$P(n)$  is true for  $n = 1$ .

Now, let us check for  $P(n)$  is true for  $n = k$ , and have to prove that  $P(k+1)$  is true.

$$P(k) = \frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \dots + \frac{1}{(3k-2)(3k+1)} = \frac{k}{3k+1} \dots (i)$$

So,

$$\begin{aligned} & \left[ \frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \dots + \frac{1}{(3k-2)(3k+1)} \right] + \frac{1}{(3k+1)(3k+4)} \\ &= \frac{k}{(3k+1)} + \frac{1}{(3k+1)(3k+4)} \\ &= \frac{1}{(3k+1)} \left[ \frac{k}{1} + \frac{1}{(3k+4)} \right] \\ &= \frac{1}{(3k+1)} \left[ \frac{k(3k+4)+1}{(3k+4)} \right] \\ &= \frac{1}{(3k+1)} \left[ \frac{3k^2 + 4k + 1}{(3k+4)} \right] \\ &= \frac{1}{(3k+1)} \left[ \frac{3k^2 + 3k + k + 1}{(3k+4)} \right] \\ &= \frac{[3k(k+1) + (k+1)]}{[(3k+4)(3k+1)]} \\ &= \frac{[(3k+1)(k+1)]}{[(3k+4)(3k+1)]} \\ &= \frac{(k+1)}{(3k+4)} \end{aligned}$$

$P(n)$  is true for  $n = k+1$

Hence,  $P(n)$  is true for all  $n \in \mathbb{N}$ .

$$8. \frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \dots + \frac{1}{(2n+1)(2n+3)} = \frac{n}{3(2n+3)}$$

**Solution:**

$$\text{Let } P(n) = \frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \dots + \frac{1}{(2n+1)(2n+3)} = \frac{n}{3(2n+3)}$$

Let us check for  $n = 1$ ,

$$P(1): \frac{1}{3.5} = \frac{1}{3(2.1+3)}$$

$$: \frac{1}{15} = \frac{1}{15}$$

$P(n)$  is true for  $n = 1$ .

Now, let us check for  $P(n)$  is true for  $n = k$ , and have to prove that  $P(k+1)$  is true.

$$P(k) = \frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \dots + \frac{1}{(2k+1)(2k+3)} = \frac{k}{3(2k+3)} \dots (i)$$

So,

$$\begin{aligned} & \frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \dots + \frac{1}{(2k+1)(2k+3)} + \frac{1}{[2(k+1)+1][2(k+1)+3]} \\ & \frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \dots + \frac{1}{(2k+1)(2k+3)} + \frac{1}{(2k+3)(2k+5)} \end{aligned}$$

Now substituting the value of  $P(k)$  we get,



RD Sharma Solutions for Class 11 Maths Chapter 12 –  
Mathematical Induction

$$\begin{aligned}
 &= k/3(2k+3) + 1/(2k+3)(2k+5) \\
 &= [k(2k+5)+3] / [3(2k+3)(2k+5)] \\
 &= (k+1) / [3(2(k+1)+3)]
 \end{aligned}$$

P (n) is true for  $n = k + 1$

Hence, P (n) is true for all  $n \in \mathbb{N}$ .

$$9. \frac{1}{3 \cdot 7} + \frac{1}{7 \cdot 11} + \frac{1}{11 \cdot 15} + \dots + \frac{1}{(4n-1)(4n+3)} = \frac{n}{3(4n+3)}$$

**Solution:**

$$\text{Let } P(n) = \frac{1}{3 \cdot 7} + \frac{1}{7 \cdot 11} + \frac{1}{11 \cdot 15} + \dots + \frac{1}{(4n-1)(4n+3)} = \frac{n}{3(4n+3)}$$

Let us check for  $n = 1$ ,

$$\begin{aligned}
 P(1): \frac{1}{3 \cdot 7} &= \frac{1}{(4 \cdot 1 - 1)(4 + 3)} \\
 &: \frac{1}{21} = \frac{1}{21}
 \end{aligned}$$

P (n) is true for  $n = 1$ .

Now, let us check for P (n) is true for  $n = k$ , and have to prove that P (k + 1) is true.

$$P(k): \frac{1}{3 \cdot 7} + \frac{1}{7 \cdot 11} + \frac{1}{11 \cdot 15} + \dots + \frac{1}{(4k-1)(4k+3)} = \frac{k}{3(4k+3)} \dots (i)$$

So,

$$\frac{1}{3 \cdot 7} + \frac{1}{7 \cdot 11} + \frac{1}{11 \cdot 15} + \dots + \frac{1}{(4k-1)(4k+3)} + \frac{1}{(4k+3)(4k+7)}$$

Substituting the value of P (k) we get,

$$\begin{aligned}
 &= \frac{k}{(4k+3)} + \frac{1}{(4k+3)(4k+7)} \\
 &= \frac{1}{(4k+3)} \left[ \frac{k(4k+7)+3}{3(4k+7)} \right] \\
 &= \frac{1}{(4k+3)} \left[ \frac{4k^2 + 7k + 3}{3(4k+7)} \right] \\
 &= \frac{1}{(4k+3)} \left[ \frac{4k^2 + 3k + 4k + 3}{3(4k+7)} \right] \\
 &= \frac{1}{(4k+3)} \left[ \frac{4k(k+1) + 3(k+1)}{3(4k+7)} \right] \\
 &= \frac{1}{(4k+3)} \left[ \frac{(4k+3)(k+1)}{3(4k+7)} \right] \\
 &= \frac{(k+1)}{3(4k+7)}
 \end{aligned}$$

P (n) is true for  $n = k + 1$

Hence, P (n) is true for all  $n \in \mathbb{N}$ .

$$10. 1 \cdot 2 + 2 \cdot 2^2 + 3 \cdot 2^3 + \dots + n \cdot 2^n = (n-1) 2^{n+1} + 2$$

**Solution:**

$$\text{Let } P(n) = 1 \cdot 2 + 2 \cdot 2^2 + 3 \cdot 2^3 + \dots + n \cdot 2^n = (n-1) 2^{n+1} + 2$$

Let us check for  $n = 1$ ,

$$\begin{aligned}
 P(1): 1 \cdot 2 &= 0 \cdot 2^0 + 2 \\
 &: 2 = 2
 \end{aligned}$$

P (n) is true for  $n = 1$ .

Now, let us check for P (n) is true for  $n = k$ , and have to prove that P (k + 1) is true.

$$P(k): 1 \cdot 2 + 2 \cdot 2^2 + 3 \cdot 2^3 + \dots + k \cdot 2^k = (k-1) 2^{k+1} + 2 \dots (i)$$

So,

$$\{1 \cdot 2 + 2 \cdot 2^2 + 3 \cdot 2^3 + \dots + k \cdot 2^k\} + (k+1) 2^{k+1}$$



Now, substituting the value of  $P(k)$  we get,

$$\begin{aligned} &= [(k-1)2^{k+1} + 2] + (k+1)2^{k+1} \text{ using equation (i)} \\ &= (k-1)2^{k+1} + 2 + (k+1)2^{k+1} \\ &= 2^{k+1}(k-1+k+1) + 2 \\ &= 2^{k+1} \times 2k + 2 \\ &= k \times 2^{k+2} + 2 \end{aligned}$$

$P(n)$  is true for  $n = k + 1$

Hence,  $P(n)$  is true for all  $n \in \mathbb{N}$ .

### 11. $2 + 5 + 8 + 11 + \dots + (3n - 1) = 1/2 n (3n + 1)$

**Solution:**

Let  $P(n) = 2 + 5 + 8 + 11 + \dots + (3n - 1) = 1/2 n (3n + 1)$

Let us check for  $n = 1$ ,

$$\begin{aligned} P(1): 2 &= 1/2 \times 1 \times 4 \\ &: 2 = 2 \end{aligned}$$

$P(n)$  is true for  $n = 1$ .

Now, let us check for  $P(n)$  is true for  $n = k$ , and have to prove that  $P(k + 1)$  is true.

$$P(k) = 2 + 5 + 8 + 11 + \dots + (3k - 1) = 1/2 k (3k + 1) \dots (i)$$

So,

$$2 + 5 + 8 + 11 + \dots + (3k - 1) + (3k + 2)$$

Now, substituting the value of  $P(k)$  we get,

$$\begin{aligned} &= 1/2 \times k (3k + 1) + (3k + 2) \text{ by using equation (i)} \\ &= [3k^2 + k + 2 (3k + 2)] / 2 \\ &= [3k^2 + k + 6k + 2] / 2 \\ &= [3k^2 + 7k + 2] / 2 \\ &= [3k^2 + 4k + 3k + 2] / 2 \\ &= [3k(k+1) + 4(k+1)] / 2 \\ &= [(k+1) (3k+4)] / 2 \end{aligned}$$

$P(n)$  is true for  $n = k + 1$

Hence,  $P(n)$  is true for all  $n \in \mathbb{N}$ .

### 12. $1.3 + 2.4 + 3.5 + \dots + n. (n+2) = 1/6 n (n+1) (2n+7)$

**Solution:**

Let  $P(n): 1.3 + 2.4 + 3.5 + \dots + n. (n+2) = 1/6 n (n+1) (2n+7)$

Let us check for  $n = 1$ ,

$$\begin{aligned} P(1): 1.3 &= 1/6 \times 1 \times 2 \times 9 \\ &: 3 = 3 \end{aligned}$$

$P(n)$  is true for  $n = 1$ .

Now, let us check for  $P(n)$  is true for  $n = k$ , and have to prove that  $P(k + 1)$  is true.

RD Sharma Solutions for Class 11 Maths Chapter 12 –  
Mathematical Induction

$$P(k): 1.3 + 2.4 + 3.5 + \dots + k.(k+2) = \frac{1}{6} k(k+1)(2k+7) \dots (i)$$

So,

$$1.3 + 2.4 + 3.5 + \dots + k.(k+2) + (k+1)(k+3)$$

Now, substituting the value of  $P(k)$  we get,

$$= \frac{1}{6} k(k+1)(2k+7) + (k+1)(k+3) \text{ by using equation (i)}$$

$$= (k+1) \left[ \frac{k(2k+7)}{6} + \frac{(k+3)}{1} \right]$$

$$= (k+1) \left[ \frac{(2k^2 + 7k + 6k + 18)}{6} \right] / 6$$

$$= (k+1) \left[ \frac{2k^2 + 13k + 18}{6} \right] / 6$$

$$= (k+1) \left[ \frac{2k^2 + 9k + 4k + 18}{6} \right] / 6$$

$$= (k+1) \left[ \frac{2k(k+2) + 9(k+2)}{6} \right] / 6$$

$$= (k+1) \left[ \frac{(2k+9)(k+2)}{6} \right] / 6$$

$$= \frac{1}{6} (k+1)(k+2)(2k+9)$$

$P(n)$  is true for  $n = k + 1$

Hence,  $P(n)$  is true for all  $n \in \mathbb{N}$ .

$$13. 1.3 + 3.5 + 5.7 + \dots + (2n - 1)(2n + 1) = \frac{n(4n^2 + 6n - 1)}{3}$$

**Solution:**

$$\text{Let } P(n): 1.3 + 3.5 + 5.7 + \dots + (2n - 1)(2n + 1) = \frac{n(4n^2 + 6n - 1)}{3}$$

Let us check for  $n = 1$ ,

$$P(1): (2.1 - 1)(2.1 + 1) = \frac{1(4.1^2 + 6.1 - 1)}{3}$$

$$: 1 \times 3 = \frac{1(4+6-1)}{3}$$

$$: 3 = 3$$

$P(n)$  is true for  $n = 1$ .

Now, let us check for  $P(n)$  is true for  $n = k$ , and have to prove that  $P(k + 1)$  is true.

$$P(k): 1.3 + 3.5 + 5.7 + \dots + (2k - 1)(2k + 1) = \frac{k(4k^2 + 6k - 1)}{3} \dots (i)$$

So,

$$1.3 + 3.5 + 5.7 + \dots + (2k - 1)(2k + 1) + (2k + 1)(2k + 3)$$

Now, substituting the value of  $P(k)$  we get,

$$= \frac{k(4k^2 + 6k - 1)}{3} + (2k + 1)(2k + 3) \text{ by using equation (i)}$$

$$= \frac{[k(4k^2 + 6k - 1) + 3(4k^2 + 6k + 2k + 3)]}{3}$$

$$= \frac{[4k^3 + 6k^2 - k + 12k^2 + 18k + 6k + 9]}{3}$$

$$= \frac{[4k^3 + 18k^2 + 23k + 9]}{3}$$

$$= \frac{[4k^3 + 4k^2 + 14k^2 + 14k + 9k + 9]}{3}$$

$$= \frac{[(k+1)(4k^2 + 8k + 4 + 6k + 6 - 1)]}{3}$$

$$= \frac{[(k+1)4[(k+1)^2 + 6(k+1) - 1]]}{3}$$

$P(n)$  is true for  $n = k + 1$

Hence,  $P(n)$  is true for all  $n \in \mathbb{N}$ .

$$14. 1.2 + 2.3 + 3.4 + \dots + n(n+1) = \frac{[n(n+1)(n+2)]}{3}$$

RD Sharma Solutions for Class 11 Maths Chapter 12 –  
Mathematical Induction**Solution:**

Let  $P(n): 1.2 + 2.3 + 3.4 + \dots + n(n+1) = [n(n+1)(n+2)] / 3$

Let us check for  $n = 1$ ,

$P(1): 1(1+1) = [1(1+1)(1+2)] / 3$

$$: 2 = 2$$

$P(n)$  is true for  $n = 1$ .

Now, let us check for  $P(n)$  is true for  $n = k$ , and have to prove that  $P(k+1)$  is true.

$P(k): 1.2 + 2.3 + 3.4 + \dots + k(k+1) = [k(k+1)(k+2)] / 3 \dots (i)$

So,

$$1.2 + 2.3 + 3.4 + \dots + k(k+1) + (k+1)(k+2)$$

Now, substituting the value of  $P(k)$  we get,

$$= [k(k+1)(k+2)] / 3 + (k+1)(k+2) \text{ by using equation (i)}$$

$$= (k+2)(k+1)[k/2 + 1]$$

$$= [(k+1)(k+2)(k+3)] / 3$$

$P(n)$  is true for  $n = k + 1$

Hence,  $P(n)$  is true for all  $n \in \mathbb{N}$ .

**15.  $1/2 + 1/4 + 1/8 + \dots + 1/2^n = 1 - 1/2^n$** **Solution:**

Let  $P(n): 1/2 + 1/4 + 1/8 + \dots + 1/2^n = 1 - 1/2^n$

Let us check for  $n = 1$ ,

$P(1): 1/2^1 = 1 - 1/2^1$

$$: 1/2 = 1/2$$

$P(n)$  is true for  $n = 1$ .

Now, let us check for  $P(n)$  is true for  $n = k$ , and have to prove that  $P(k+1)$  is true.

Let  $P(k): 1/2 + 1/4 + 1/8 + \dots + 1/2^k = 1 - 1/2^k \dots (i)$

So,

$$1/2 + 1/4 + 1/8 + \dots + 1/2^k + 1/2^{k+1}$$

Now, substituting the value of  $P(k)$  we get,

$$= 1 - 1/2^k + 1/2^{k+1} \text{ by using equation (i)}$$

$$= 1 - ((2-1)/2^{k+1})$$

$P(n)$  is true for  $n = k + 1$

Hence,  $P(n)$  is true for all  $n \in \mathbb{N}$ .

**16.  $1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = 1/3 n(4n^2 - 1)$** **Solution:**

Let  $P(n): 1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = 1/3 n(4n^2 - 1)$

Let us check for  $n = 1$ ,

$P(1): (2.1 - 1)^2 = 1/3 \times 1 \times (4 - 1)$



RD Sharma Solutions for Class 11 Maths Chapter 12 –  
Mathematical Induction

$$: 1 = 1$$

$P(n)$  is true for  $n = 1$ .

Now, let us check for  $P(n)$  is true for  $n = k$ , and have to prove that  $P(k + 1)$  is true.

$$P(k): 1^2 + 3^2 + 5^2 + \dots + (2k - 1)^2 = \frac{1}{3} k (4k^2 - 1) \dots (i)$$

So,

$$1^2 + 3^2 + 5^2 + \dots + (2k - 1)^2 + (2k + 1)^2$$

Now, substituting the value of  $P(k)$  we get,

$$= \frac{1}{3} k (4k^2 - 1) + (2k + 1)^2 \text{ by using equation (i)}$$

$$= \frac{1}{3} k (2k + 1) (2k - 1) + (2k + 1)^2$$

$$= (2k + 1) \left[ \frac{k(2k-1)}{3} + (2k+1) \right]$$

$$= (2k + 1) \left[ \frac{2k^2 - k + 3(2k+1)}{3} \right]$$

$$= (2k + 1) \left[ \frac{2k^2 - k + 6k + 3}{3} \right]$$

$$= \frac{(2k+1)(2k^2 + 5k + 3)}{3}$$

$$= \frac{(2k+1)(2k(k+1)) + 3(k+1)}{3}$$

$$= \frac{(2k+1)(2k+3)(k+1)}{3}$$

$$= \frac{(k+2)}{2} \left[ \frac{4k^2 + 6k + 2k + 3}{2} \right]$$

$$= \frac{(k+2)}{2} \left[ \frac{4k^2 + 8k - 1}{2} \right]$$

$$= \frac{(k+2)}{2} \left[ \frac{4(k+1)^2 - 1}{2} \right]$$

$P(n)$  is true for  $n = k + 1$

Hence,  $P(n)$  is true for all  $n \in \mathbb{N}$ .

$$17. a + ar + ar^2 + \dots + ar^{n-1} = a \left[ \frac{r^n - 1}{r - 1} \right], r \neq 1$$

**Solution:**

$$\text{Let } P(n): a + ar + ar^2 + \dots + ar^{n-1} = a \left[ \frac{r^n - 1}{r - 1} \right]$$

Let us check for  $n = 1$ ,

$$P(1): a = a \left[ \frac{r^1 - 1}{r - 1} \right]$$

$$: a = a$$

$P(n)$  is true for  $n = 1$ .

Now, let us check for  $P(n)$  is true for  $n = k$ , and have to prove that  $P(k + 1)$  is true.

$$P(k): a + ar + ar^2 + \dots + ar^{k-1} = a \left[ \frac{r^k - 1}{r - 1} \right] \dots (i)$$

So,

$$a + ar + ar^2 + \dots + ar^{k-1} + ar^k$$

Now, substituting the value of  $P(k)$  we get,

$$= a \left[ \frac{r^k - 1}{r - 1} \right] + ar^k \text{ by using equation (i)}$$

$$= a \left[ \frac{r^k - 1 + r^k(r-1)}{r-1} \right]$$

$$= a \left[ \frac{r^k - 1 + r^{k+1} - r^k}{r-1} \right]$$

$$= a \left[ \frac{r^{k+1} - 1}{r-1} \right]$$

$P(n)$  is true for  $n = k + 1$

Hence,  $P(n)$  is true for all  $n \in \mathbb{N}$ .



RD Sharma Solutions for Class 11 Maths Chapter 12 –  
Mathematical Induction

$$18. a + (a + d) + (a + 2d) + \dots + (a + (n-1)d) = n/2 [2a + (n-1)d]$$

**Solution:**

$$\text{Let } P(n): a + (a + d) + (a + 2d) + \dots + (a + (n-1)d) = n/2 [2a + (n-1)d]$$

Let us check for  $n = 1$ ,

$$P(1): a = \frac{1}{2} [2a + (1-1)d]$$

$$: a = a$$

$P(n)$  is true for  $n = 1$ .

Now, let us check for  $P(n)$  is true for  $n = k$ , and have to prove that  $P(k + 1)$  is true.

$$P(k): a + (a + d) + (a + 2d) + \dots + (a + (k-1)d) = k/2 [2a + (k-1)d] \dots (i)$$

So,

$$a + (a + d) + (a + 2d) + \dots + (a + (k-1)d) + (a + (k)d)$$

Now, substituting the value of  $P(k)$  we get,

$$= k/2 [2a + (k-1)d] + (a + kd) \text{ by using equation (i)}$$

$$= [2ka + k(k-1)d + 2(a+kd)] / 2$$

$$= [2ka + k^2d - kd + 2a + 2kd] / 2$$

$$= [2ka + 2a + k^2d + kd] / 2$$

$$= [2a(k+1) + d(k^2 + k)] / 2$$

$$= (k+1)/2 [2a + kd]$$

$P(n)$  is true for  $n = k + 1$

Hence,  $P(n)$  is true for all  $n \in \mathbb{N}$ .

$$19. 5^{2n} - 1 \text{ is divisible by } 24 \text{ for all } n \in \mathbb{N}$$

**Solution:**

$$\text{Let } P(n): 5^{2n} - 1 \text{ is divisible by } 24$$

Let us check for  $n = 1$ ,

$$P(1): 5^2 - 1 = 25 - 1 = 24$$

$P(n)$  is true for  $n = 1$ . Where,  $P(n)$  is divisible by 24

Now, let us check for  $P(n)$  is true for  $n = k$ , and have to prove that  $P(k + 1)$  is true.

$$P(k): 5^{2k} - 1 \text{ is divisible by } 24$$

$$: 5^{2k} - 1 = 24\lambda \dots (i)$$

We have to prove,

$$5^{2k+1} - 1 \text{ is divisible by } 24$$

$$5^{2(k+1)} - 1 = 24\mu$$

So,

$$= 5^{2(k+1)} - 1$$

$$= 5^{2k} \cdot 5^2 - 1$$

$$= 25 \cdot 5^{2k} - 1$$

$$= 25 \cdot (24\lambda + 1) - 1 \text{ by using equation (1)}$$

$$= 25.24\lambda + 24$$

$$= 24\lambda$$

$P(n)$  is true for  $n = k + 1$

Hence,  $P(n)$  is true for all  $n \in \mathbb{N}$ .

### 20. $3^{2n} + 7$ is divisible by 8 for all $n \in \mathbb{N}$

**Solution:**

Let  $P(n)$ :  $3^{2n} + 7$  is divisible by 8

Let us check for  $n = 1$ ,

$$P(1): 3^2 + 7 = 9 + 7 = 16$$

$P(n)$  is true for  $n = 1$ . Where,  $P(n)$  is divisible by 8

Now, let us check for  $P(n)$  is true for  $n = k$ , and have to prove that  $P(k + 1)$  is true.

$P(k)$ :  $3^{2k} + 7$  is divisible by 8

$$: 3^{2k} + 7 = 8\lambda$$

$$: 3^{2k} = 8\lambda - 7 \dots (i)$$

We have to prove,

$3^{2(k+1)} + 7$  is divisible by 8

$$3^{2k+2} + 7 = 8\mu$$

So,

$$= 3^{2(k+1)} + 7$$

$$= 3^{2k} \cdot 3^2 + 7$$

$$= 9 \cdot 3^{2k} + 7$$

$$= 9 \cdot (8\lambda - 7) + 7 \text{ by using equation (i)}$$

$$= 72\lambda - 63 + 7$$

$$= 72\lambda - 56$$

$$= 8(9\lambda - 7)$$

$$= 8\mu$$

$P(n)$  is true for  $n = k + 1$

Hence,  $P(n)$  is true for all  $n \in \mathbb{N}$ .

### 21. $5^{2n+2} - 24n - 25$ is divisible by 576 for all $n \in \mathbb{N}$

**Solution:**

Let  $P(n)$ :  $5^{2n+2} - 24n - 25$  is divisible by 576

Let us check for  $n = 1$ ,

$$P(1): 5^{2 \cdot 1 + 2} - 24 \cdot 1 - 25$$

$$: 625 - 49$$

$$: 576$$

$P(n)$  is true for  $n = 1$ . Where,  $P(n)$  is divisible by 576

Now, let us check for  $P(n)$  is true for  $n = k$ , and have to prove that  $P(k + 1)$  is true.

RD Sharma Solutions for Class 11 Maths Chapter 12 –  
Mathematical Induction

P (k):  $5^{2k+2} - 24k - 25$  is divisible by 576

$$: 5^{2k+2} - 24k - 25 = 576\lambda \dots (i)$$

We have to prove,

$5^{2k+4} - 24(k+1) - 25$  is divisible by 576

$$5^{(2k+2)+2} - 24(k+1) - 25 = 576\mu$$

So,

$$= 5^{(2k+2)+2} - 24(k+1) - 25$$

$$= 5^{(2k+2)} \cdot 5^2 - 24k - 24 - 25$$

$$= (576\lambda + 24k + 25)25 - 24k - 49 \text{ by using equation (i)}$$

$$= 25 \cdot 576\lambda + 576k + 576$$

$$= 576(25\lambda + k + 1)$$

$$= 576\mu$$

P (n) is true for  $n = k + 1$

Hence, P (n) is true for all  $n \in \mathbb{N}$ .

**22.  $3^{2n+2} - 8n - 9$  is divisible by 8 for all  $n \in \mathbb{N}$**

**Solution:**

Let P (n):  $3^{2n+2} - 8n - 9$  is divisible by 8

Let us check for  $n = 1$ ,

$$P(1): 3^{2 \cdot 1 + 2} - 8 \cdot 1 - 9$$

$$: 81 - 17$$

$$: 64$$

P (n) is true for  $n = 1$ . Where, P (n) is divisible by 8

Now, let us check for P (n) is true for  $n = k$ , and have to prove that P (k + 1) is true.

P (k):  $3^{2k+2} - 8k - 9$  is divisible by 8

$$: 3^{2k+2} - 8k - 9 = 8\lambda \dots (i)$$

We have to prove,

$3^{2k+4} - 8(k+1) - 9$  is divisible by 8

$$3^{(2k+2)+2} - 8(k+1) - 9 = 8\mu$$

So,

$$= 3^{2(k+1)} \cdot 3^2 - 8(k+1) - 9$$

$$= (8\lambda + 8k + 9)9 - 8k - 8 - 9$$

$$= 72\lambda + 72k + 81 - 8k - 17 \text{ using equation (1)}$$

$$= 72\lambda + 64k + 64$$

$$= 8(9\lambda + 8k + 8)$$

$$= 8\mu$$

P (n) is true for  $n = k + 1$

Hence, P (n) is true for all  $n \in \mathbb{N}$ .

**23.  $(ab)^n = a^n b^n$  for all  $n \in \mathbb{N}$** **Solution:**Let  $P(n): (ab)^n = a^n b^n$ Let us check for  $n = 1$ , $P(1): (ab)^1 = a^1 b^1$  $: ab = ab$  $P(n)$  is true for  $n = 1$ .Now, let us check for  $P(n)$  is true for  $n = k$ , and have to prove that  $P(k + 1)$  is true. $P(k): (ab)^k = a^k b^k \dots (i)$ 

We have to prove,

 $(ab)^{k+1} = a^{k+1} \cdot b^{k+1}$ 

So,

$$= (ab)^{k+1}$$

$$= (ab)^k (ab)$$

$$= (a^k b^k) (ab) \text{ using equation (1)}$$

$$= (a^{k+1}) (b^{k+1})$$

 $P(n)$  is true for  $n = k + 1$ Hence,  $P(n)$  is true for all  $n \in \mathbb{N}$ .**24.  $n(n + 1)(n + 5)$  is a multiple of 3 for all  $n \in \mathbb{N}$ .****Solution:**Let  $P(n): n(n + 1)(n + 5)$  is a multiple of 3Let us check for  $n = 1$ , $P(1): 1(1 + 1)(1 + 5)$ 

$$: 2 \times 6$$

$$: 12$$

 $P(n)$  is true for  $n = 1$ . Where,  $P(n)$  is a multiple of 3Now, let us check for  $P(n)$  is true for  $n = k$ , and have to prove that  $P(k + 1)$  is true. $P(k): k(k + 1)(k + 5)$  is a multiple of 3

$$: k(k + 1)(k + 5) = 3\lambda \dots (i)$$

We have to prove,

 $(k + 1)[(k + 1) + 1][(k + 1) + 5]$  is a multiple of 3

$$(k + 1)[(k + 1) + 1][(k + 1) + 5] = 3\mu$$

So,

$$= (k + 1) [(k + 1) + 1] [(k + 1) + 5]$$

$$= (k + 1) (k + 2) [(k + 1) + 5]$$

$$= [k(k + 1) + 2(k + 1)] [(k + 5) + 1]$$

$$= k(k + 1)(k + 5) + k(k + 1) + 2(k + 1)(k + 5) + 2(k + 1)$$

$$= 3\lambda + k^2 + k + 2(k^2 + 6k + 5) + 2k + 2$$



## RD Sharma Solutions for Class 11 Maths Chapter 12 – Mathematical Induction

$$\begin{aligned}
 &= 3\lambda + k^2 + k + 2k^2 + 12k + 10 + 2k + 2 \\
 &= 3\lambda + 3k^2 + 15k + 12 \\
 &= 3(\lambda + k^2 + 5k + 4) \\
 &= 3\mu
 \end{aligned}$$

$P(n)$  is true for  $n = k + 1$

Hence,  $P(n)$  is true for all  $n \in \mathbb{N}$ .

**25.  $7^{2n} + 2^{3n-3} \cdot 3n - 1$  is divisible by 25 for all  $n \in \mathbb{N}$**

**Solution:**

Let  $P(n)$ :  $7^{2n} + 2^{3n-3} \cdot 3n - 1$  is divisible by 25

Let us check for  $n = 1$ ,

$$P(1): 7^2 + 2^0 \cdot 3^0$$

$$: 49 + 1$$

$$: 50$$

$P(n)$  is true for  $n = 1$ . Where,  $P(n)$  is divisible by 25

Now, let us check for  $P(n)$  is true for  $n = k$ , and have to prove that  $P(k + 1)$  is true.

$P(k)$ :  $7^{2k} + 2^{3k-3} \cdot 3k - 1$  is divisible by 25

$$: 7^{2k} + 2^{3k-3} \cdot 3k - 1 = 25\lambda \dots (i)$$

We have to prove that:

$$7^{2k+1} + 2^{3k} \cdot 3k \text{ is divisible by 25}$$

$$7^{2k+2} + 2^{3k} \cdot 3k = 25\mu$$

So,

$$= 7^{2(k+1)} + 2^{3k} \cdot 3k$$

$$= 7^{2k} \cdot 7^1 + 2^{3k} \cdot 3k$$

$$= (25\lambda - 2^{3k-3} \cdot 3k - 1) 49 + 2^{3k} \cdot 3k \text{ by using equation (i)}$$

$$= 25\lambda \cdot 49 - 2^{3k}/8 \cdot 3k/3 \cdot 49 + 2^{3k} \cdot 3k$$

$$= 24 \times 25 \times 49\lambda - 2^{3k} \cdot 3k \cdot 49 + 24 \cdot 2^{3k} \cdot 3k$$

$$= 24 \times 25 \times 49\lambda - 25 \cdot 2^{3k} \cdot 3k$$

$$= 25(24 \cdot 49\lambda - 2^{3k} \cdot 3k)$$

$$= 25\mu$$

$P(n)$  is true for  $n = k + 1$

Hence,  $P(n)$  is true for all  $n \in \mathbb{N}$ .