R D Sharma Solutions For Class 10 Maths Chapter 14 - Co-ordinate Geometry

Exercise 14.1

1. On which axis do the following points lie?
(i) P (5, 0)
(ii) Q (0, -2)
(iii) R (-4, 0)
(iv) S (0, 5)
Solution:

(i) P (5, 0) lies on x – axis
(ii) Q (0, -2) lies on y – axis (negation half)
(iii) R (-4, 0) lies on x – axis (negative half)
(iv) S (0, 5) lies on y – axis

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Exercise 14.2 Page No: 14.15

1. Find the distance between the following pair of points:
(i) (- 6, 7) and (-1, -5)
(ii) (a + b, b + c) and (a -b, c - b)
(iii) (a sin α, - b cos α) and (- a cos α, b sin α)
(iv) (a, 0) and (0, b)
Solution:

(i) Let the given points be P (- 6, 7) and Q (- 1, - 5)
Here,
$x_1 = -6$, $y_1 = 7$ and
$x_2 = -1$, $y_2 = -5$

$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$PQ = \sqrt{[-1-(-6)]^2 + (-5-7)^2}$

$PQ = \sqrt{(-1+6)^2 + (-5-7)^2}$

$PQ = \sqrt{(5)^2 + (-12)^2}$

$PQ = \sqrt{25+144}$

$PQ = \sqrt{(169)}$

PQ = 13

(ii) Let the given points be P (a + b, b + c) and Q (a - b, c - b)
Here,
$x_1 = a + b$, $y_1 = b + c$ and
$x_2 = a - b$, $y_2 = c - b$

$PQ = \sqrt{x_2 - x_1)^2 + (y_2 - y_1)^2}$

$PQ = \sqrt{[a-b-(a+b)]^2 + (c-b-(b+c))^2}$

$PQ = \sqrt{(a-b-a-b)^2 + (c-b-b-c)^2}$

$PQ = \sqrt{(-2b)^2 + (-2b)^2}$

$PQ = \sqrt{4b^2 + 4b^2}$

PQ = $\sqrt{8b^2}$

PQ = $\sqrt{4 \times 2b^2}$

PQ = $2\sqrt{2}b$

(iii) Let the given points be P(a sinα, – b cos α) and Q(-a cos α, b sin α) here
x_1 = a sin α, y_1 = - b cos α and
x_2 – a cos α, y_2 = b sin α

PQ = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

PQ = $\sqrt{(-a\cos a - a\sin a)^2 + [-b\sin a - (-b\cos a)]^2}$

PQ =
$\sqrt{(-\text{a}\cos\text{a})^2 + (-\text{a}\sin\text{a})^2 + 2(-\text{a}\cos\text{a})(-\text{a}\sin\text{a}) + (\text{b}\sin\text{a})^2 + (-\text{b}\cos\text{a})^2 - 2(\text{b}\sin\text{a})(-\text{b}\cos\text{a})}$

PQ = $\sqrt{\text{a}^2\cos^2\text{a} + \text{a}^2\sin^2\text{a} + 2\text{a}^2\cos\text{a}\sin\text{a} + \text{b}^2\sin^2\text{a} + \text{b}^2\sin\text{a}\cos\text{a}}$

PQ = $\sqrt{\text{a}^2(\cos^2\text{a} + \sin^2\text{a}) + 2\text{a}^2\cos\text{a}\sin\text{a} + \text{b}^2(\sin^2\text{a} + \cos^2\text{a}) + 2\text{b}^2\sin\text{a}\cos\text{a}}$

PQ = $\sqrt{\text{a}^2 \times 1 + 2\text{a}^2\cos\text{a}\sin\text{a} + \text{b}^2 \times 2\text{b}^2\sin\text{a}\cos\text{a}}$ [∴ $\sin^2 a + \cos^2 a = 1$]

PQ = $\sqrt{\text{a}^2 + \text{b}^2 + 2\text{a}^2\cos\text{a}\sin\text{a} + 2\text{b}^2\sin\text{a}\cos\text{a}}$

PQ = $\sqrt{(\text{a}^2 + \text{b}^2) + 2\cos\text{a}\sin\text{a}\,(\text{a}^2 + \text{b}^2)}$

PQ = $\sqrt{(\text{a}^2 + \text{b}^2)(1 + 2\cos\text{a}\sin\text{a})}$

(iv) Let the given points be P(a, 0) and Q (0, b)
Here,
x_1 = a, y_1 = 0, x_2 = 0, y_2 = b,

PQ = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

PQ = $\sqrt{(0 - a)^2 + (b - 0)^2}$

PQ = $\sqrt{(-a)^2 + (b)^2}$

PQ = $\sqrt{a^2 + b^2}$

2. Find the value of a when the distance between the points (3, a) and (4, 1) is √10.
Solution:

Let the given points be P (3, a) and Q(4, 1).
Here,
PQ = $\sqrt{10}$

PQ = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\Rightarrow \sqrt{10} = \sqrt{(4-3)^2 + (1-a)^2}$

$\Rightarrow \sqrt{10} = \sqrt{(1)^2 + (1-a)^2}$

$\Rightarrow \sqrt{10} = \sqrt{1 + 1 + a^2 - 2a}$ $[\because (a-b)^2 = a^2 + b^2 - 2ab]$

$\Rightarrow \sqrt{10} = \sqrt{2 + a^2 - 2a}$

On squaring on both sides, we have

$\Rightarrow (\sqrt{10})^2 = (\sqrt{2 + a^2 - 2a})^2$

$\Rightarrow 10 = 2 + a^2 - 2a$
$\Rightarrow a^2 - 2a + 2 - 10 = 0$
$\Rightarrow a^2 - 2a - 8 = 0$
By splitting the middle team,
$\Rightarrow a^2 - 4a + 2a - 8 = 0$
$\Rightarrow a(a - 4) + 2(a - 4) = 0$
$\Rightarrow (a - 4)(a + 2) = 0$
$\Rightarrow a = 4, a = -2$
Thus, there are 2 possible values for a which are 4 and -2.

3. If the points (2, 1) and (1, -2) are equidistant from the point (x, y), show that x + 3y = 0.
Solution:

Let the given points be P(2, 1) and Q(1,- 2) and R(x, y)

Also, PR = QR (given)

PR = $\sqrt{(x-2)^2 + (y-1)^2}$

$\Rightarrow$ PR = $\sqrt{(x^2+2)^2 - 2x \times 2 + y^2 + (1)^2 - 2 \times y \times 1}$

$\Rightarrow$ PR = $\sqrt{x^2 + 4 - 4x + y^2 + 1 - 2y}$

$\Rightarrow$ PR = $\sqrt{x^2 + 5 - 4x + y^2 - 2y}$

QR = $\sqrt{(x-1)^2 + (y+2)^2}$

$\Rightarrow$QR = $\sqrt{x^2 + 1 - 2x + y^2 + 4 + 4y}$

$\Rightarrow$QR = $\sqrt{x^2 + 5 - 2x + y^2 + 4y}$

But, PR = QR

$\Rightarrow \sqrt{x^2 + 5 - 4x + y^2 - 2y} = \sqrt{x^2 + 5 - 2x + y^2 + 4y}$

$\Rightarrow x^2 + 5 - 4x + y^2 - 2y = x^2 + 5 - 2x + y^2 + 4y$
$\Rightarrow x^2 + 5 - 4x + y^2 - 2y = x^2 + 5 - 2x + y^2 + 4y$
$\Rightarrow - 4x + 2x - 2y - 4y = 0$
$\Rightarrow - 2x - 6y = 0$
$\Rightarrow - 2(x + 3y) = 0$
$\Rightarrow -2(x + 3y) = 0$
$\Rightarrow x + 3y = 0/-2$
$\Rightarrow x + 3y = 0$

- Hence Proved.

4. Find the value of x, y if the distances of the point (x, y) from (- 3, 0) as well as from (3, 0) are 4.
Solution:

Let the given points be P(x, y), Q(-3, 0) and R(3, 0)

PQ = $\sqrt{(x+3)^2 + (y-0)^2}$

$\Rightarrow 4 = \sqrt{x^2 + 9 + 6x + y^2}$

On squaring on both sides, we get

$\Rightarrow (4)^2 = (\sqrt{x^2 + 9 + 6x + y^2})^2$

$\Rightarrow 16 = x^2 + 9 + 6x + y^2$
$\Rightarrow x^2 + y^2 = 7 - 6x$ …… (1)

PR = $(\sqrt{(x-3)^2+(y-0)^2})$

$\Rightarrow 4 = \sqrt{x^2+9-6x+y^2}$

On squaring on both sides,

$(4)^2 = (\sqrt{x^2+9-6x+y^2})^2$

$\Rightarrow 16 = x^2 + 9 - 6x + y^2$

$\Rightarrow x^2 + y^2 = 16 - 9 + 6x$

$\Rightarrow x^2 + y^2 = 7 + 6x$ …. (2)

Equating (1) and (2), we have

7 - 6x = 7 + 6x

$\Rightarrow 7 - 7 = 6x + 6x$

$\Rightarrow 0 = 12x$

$\Rightarrow x = 12$

Then, substituting the value of x = 0 in (2)

$x^2 + y^2 = 7 + 6x$

$0 + y^2 = 7 + 6 \times 0$

$y^2 = 7$

$y = \pm\sqrt{7}$

As y can have two values, the points are (12, √7) and (12, -√7).

5. The length of a line segment is of 10 units and the coordinates of one end-point are (2, -3). If the abscissa of the other end is 10, find the ordinate of the other end.
Solution:

Given,
Length of the line segment is 10 units.
Coordinates of one end-point are (2, -3) and the abscissa of the other end is 10.
So, let the ordinate of the other end be k.

Distance = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$10 = \sqrt{(10-2)^2+(k+3)^2}$

On squaring both sides, we get

$100 = (10-2)^2 + (k+3)^2$

$100 = 64 + k^2 + 6k + 9$

$k^2 + 6k - 27 = 0$

$k^2 + 9k - 3k - 27 = 0$

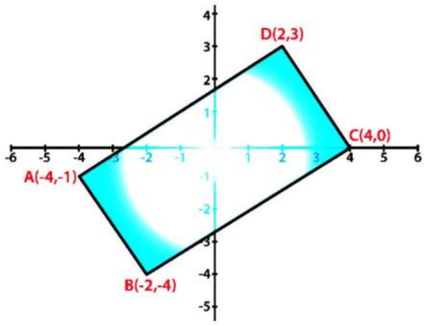
$k(k+9) - 3(k+9) = 0$

$(k-3)(k+9) = 0$

$k = 3, k = -9$

Therefore, the ordinates of the other end can be 3 or -9.

6. Show that the points A(- 4, -1), B(-2, - 4), C(4, 0) and D(2, 3) are the vertices points of a rectangle.
Solution:



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Given: Points A(- 4, -1), B(-2, - 4), C(4, 0) and D(2, 3)
Required to prove: the points are the vertices points of a rectangle.

Vertices of rectangle ABCD are: A(- 4, -1), B(-2, - 4), C(4, 0) and D(2, 3)

We know that,

Length of a side = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Length of side AB = $\sqrt{(-2+4)^2 + (-4+1)^2} = \sqrt{4+9} = \sqrt{13}$ units

Length of side BC = $\sqrt{(4+2)^2 + (0+4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$ units

Length of side CD = $\sqrt{(2-4)^2 + (3-0)^2} = \sqrt{4+9} = \sqrt{13}$ units

Length of side AD = $\sqrt{(2+4)^2 + (3+1)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$ units

Finding the diagonals,

Length of diagonal BD = $\sqrt{(2+2)^2 + (3+4)^2} = \sqrt{16+49} = \sqrt{65}$ units

Length of diagonal AC = $\sqrt{(4+4)^2 + (0+1)^2} = \sqrt{64+1} = \sqrt{65}$ units

As the opposite sides are equal and also the diagonals are equal.
Therefore, the given points are the vertices of a rectangle.

- Hence Proved

7. Show that the points A (1,- 2), B (3, 6), C (5, 10) and D (3, 2) are the vertices of a parallelogram.
Solution:

Given: Points A (1,- 2), B (3, 6), C (5, 10) and D (3, 2)
Required to prove: the points are the vertices points of a parallelogram.

Vertices of a parallelogram ABCD are: A (1, -2), B (3, 6), C (5, 10) and D (3, 2)
We know that,

Length of a side = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Length of side AB = $\sqrt{(3-1)^2 + (6+2)^2}$ = √(4+64)= √68 units

Length of side BC = $\sqrt{(5-3)^2 + (10-6)^2}$ = √(4+16) = √20 units

Length of side CD = $\sqrt{(3-5)^2 + (2-10)^2}$ = √(4+64)= √68 units

Length of side DA = $\sqrt{(3-1)^2 + (2+2)^2}$ = √(4+16) = √20 units

Finding the diagonals,

Length of diagonal BD = $\sqrt{(3-3)^2 + (2-6)^2}$ = √16= 4 units

Length of diagonal AC = $\sqrt{(5-1)^2 + (10+2)^2}$ = √(16+144) = √160 units

It's seen that the opposite sides of the quadrilateral formed by the given four points are equal i.e. (AB = CD) & (DA = BC)
Also, the diagonals BD & AC are found unequal.
Hence, the given points form a parallelogram.

- Hence Proved

8. Prove that the points A (1, 7), B (4, 2), C (-1, -1) and D (-4, 4) are the vertices of a square.
Solution:

Given: Points A (1, 7), B (4, 2), C (-1, -1) and D (-4, 4)
Required to prove: the points are the vertices points of a square.
Vertices of a square ABCD are: A (1, 7), B (4, 2), C (-1, -1) and D (-4, 4)
We know that,

Length of a side = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Length of side AB = $\sqrt{(4-1)^2 + (2-7)^2} = \sqrt{9+25} = \sqrt{34}$ units

Length of side BC = $\sqrt{(-1-4)^2 + (-2-1)^2} = \sqrt{25+9} = \sqrt{34}$ units

Length of side CD = $\sqrt{(-4+1)^2 + (4+1)^2} = \sqrt{9+25} = \sqrt{34}$ units

Length of side DA = $\sqrt{(-4-1)^2 + (4-7)^2} = \sqrt{25+9} = \sqrt{34}$ units

Finding the diagonals,

Length of diagonal BD = $\sqrt{(-4-4)^2 + (4-2)^2} = \sqrt{64+4} = \sqrt{68}$ units

Length of diagonal AC = $\sqrt{(-1-1)^2 + (-1-7)^2} = \sqrt{4+64} = \sqrt{68}$ units

As the opposite sides are equal and also the diagonals are equal the given vertices are therefore the vertices of a square.

- Hence Proved

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9. Prove that the points (3, 0), (6, 4) and (- 1, 3) are vertices of a right-angled isosceles triangle.
Solution:

Let the vertices of the triangle ABC be: A(3, 0), B(6, 4) and C (- 1, 3)
We know that,

Length of a side = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Length of side AB = $\sqrt{(6-3)^2 + (4-0)^2} = \sqrt{9+16} = \sqrt{25}$ units

Length of side BC = $\sqrt{(-1-6)^2 + (3-4)^2} = \sqrt{49+1} = \sqrt{50}$ units

Length of side AC = $\sqrt{(-1-3)^2 + (3-0)^2} = \sqrt{16+9} = \sqrt{25}$ units

It's seen that AB = AC, Thus, it's an isosceles triangle.
Verifying the Pythagoras theorem, we have
$BC^2 = AB^2 + AC^2$
$(\sqrt{50})^2 = (\sqrt{25})^2 + (\sqrt{25})^2$
50 = 25 + 25
50 = 50
As $BC^2 = AB^2 + AC^2$
Therefore, the given vertices are of a right-angled isosceles triangle.
- Hence Proved

10. Prove that (2, -2), (-2, 1) and (5, 2) are the vertices of a right angled triangle. Find the area of the triangle and the length of the hypotenuse.
Solution:

From given,
Let consider the vertices of a triangle ABC as: A(2, -2), B(-2, 1) and C(5, 2)
We know that,

Length of a side = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Length of side AB = $\sqrt{(-2-2)^2 + (1+2)^2} = \sqrt{16+9} = \sqrt{25}$ units

Length of side BC = $\sqrt{(5+2)^2 + (2-1)^2} = \sqrt{49+1} = \sqrt{50}$ units

Length of side AC = $\sqrt{(5-2)^2 + (2+2)^2} = \sqrt{9+16} = \sqrt{25}$ units

It's seen that AB = AC, thus the triangle is an isosceles triangle.
Verifying the Pythagoras theorem, we have
$BC^2 = AB^2 + AC^2$
$(\sqrt{50})^2 = (\sqrt{25})^2 + (\sqrt{25})^2$
50 = 25 + 25
50 = 50
As $BC^2 = AB^2 + AC^2$
Therefore, the given triangle is right angled triangle.
Now,

Area of right angled triangle = $\frac{1}{2} base \times altitude$

Area of right angled triangle = $\frac{1}{2} \times 5 \times 5 = \frac{25}{2}$ square units

And, the length of hypotenuse (BC) = $\sqrt{50} = 5\sqrt{2}$ units

- Hence Proved

11. Prove that the points (2a, 4a), (2a, 6a) and (2a + √3a, 5a) are the vertices of an equilateral triangle.
Solution:

From given,
Let's consider the vertices of a triangle ABC as: A(2 a, 4 a), B(2 a, 6 a) and C(2a + √3a, 5a)
We know that,

Length of a side = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Length of side AB = $\sqrt{(2a - 2a)^2 + (6a - 4a)^2} = \sqrt{(2a)^2}$ = 2a units

Length of side BC = $\sqrt{\left(2a + \sqrt{3}a - 2a\right)^2 + (5a - 6a)^2}$ = $\sqrt{(4a^2)}$ = 2a units

Length of side AC = $\sqrt{\left(2a + \sqrt{3}a - 2a\right)^2 + (5a - 4a)^2}$ = $\sqrt{(4a^2)}$ = 2a units

As all the sides are equal the triangle is an equilateral triangle.
Thus, the given vertices are of an equilateral triangle.

- Hence Proved

12. Prove that the points (2, 3), (-4, -6) and (1, 3/2) do not form a triangle.
Solution:

From given,
Let's consider the vertices of a triangle ABC as: A(2, 3), B(-4, -6) and C(1, 3/2)
We know that,

Length of a side = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Length of side AB = $\sqrt{(-4 - 2)^2 + (-6 - 3)^2} = \sqrt{36 + 81} = \sqrt{117}$ units

Length of side BC = $\sqrt{(1 + 4)^2 + \left(\frac{3}{2} + 6\right)^2} = \sqrt{25 + 56.25} = \sqrt{81.25}$ units

Length of side AC = $\sqrt{(1 - 2)^2 + \left(\frac{3}{2} - 3\right)^2} = \sqrt{1 + 2.25} = \sqrt{2.25}$ units

Thus, the given vertices do not form a triangle as the sum of two sides of a triangle is not greater than third side.

- Hence Proved

13. The points A (2, 9), B (a, 5) and C (5, 5) are the vertices of a triangle ABC right triangle ABC right angled at B. Find the values of a and hence the area of triangle ABC.
Solution:

Given,
A right triangle ABC, right angled at B.
Points A (2, 9), B (a, 5) and C (5, 5)
So, AC is the hypotenuse
Thus, from Pythagoras theorem we have
$AC^2 = AB^2 + BC^2$
$[(5 - 2)^2 + (5 - 9)^2] = [(a - 2)^2 + (5 - 9)^2] + [(5 - a)^2 + (5 - 5)^2]$
$[3^2 + (-4)^2] = [(a - 2)^2 + (-4)^2] + [(5 - a)^2 + 0]$
$9 + 16 = a^2 – 4a + 4 + 16 + 25 – 10a + a^2$
$2a^2 – 14a + 20 = 0$
$a^2 – 7a + 10 = 0$
$(a - 5)(a - 2) = 0$ [By factorization method]
So,
a = 5 or 2
Here, a = 5 is not possible as it coincides with point C. So, for a triangle to form the value of a = 2 is correct.
Thus, the coordinates of point B is (2, 5).
Now, the area of triangle ABC

$$\text{Area of } \Delta ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$\Delta = \frac{1}{2}[2(5 - 5) + 2(5 - 9) + 5(9 - 5)]$$

$$= \frac{1}{2}[2 \times 0 + 2(-4) + 5(4)]$$

$$= \frac{1}{2}(0 - 8 + 20) = \frac{1}{2} \times 12 = 6$$

Therefore, the area of triangle ABC is 6 sq. units

14. Show that the quadrilateral whose vertices are (2, -1), (3, 4), (-2, 3) and (-3, -2) is a rhombus.
Solution:

Let A(2, -1), B(3 ,4), C(-2, 3) and D(-3, -2)
Then we have,
Length of AB = $\sqrt{[(3 - 2)^2 + (4 – (-1))^2]} = \sqrt{[(1)^2 + (5)^2]} = \sqrt{[1 + 25]} = \sqrt{26}$ units
Length of BC = $\sqrt{[(3 – (-2))^2 + (4 – 3)^2]} = \sqrt{[(5)^2 + (1)^2]} = \sqrt{[25 + 1]} = \sqrt{26}$ units

Length of CD = $\sqrt{[(-2 - (-3))^2 + (3 - 2)^2]} = \sqrt{[(-5)^2 + (1)^2]} = \sqrt{[25 + 1]} = \sqrt{26}$ units
Length of AD = $\sqrt{[(-3 - 2)^2 + (-2 - (-1))^2]} = \sqrt{[(-5)^2 + (-1)^2]} = \sqrt{[25 + 1]} = \sqrt{26}$ units
As AB = BC = CD = AD
We can say that,
Quadrilateral ABCD is a rhombus.

15. Two vertices of an isosceles triangle are (2, 0) and (2, 5). Find the third vertex if the length of the equal sides is 3.
Solution:

Let the third vertex be C (x, y)
And, given A (2, 0) & B (2, 5)
We have,

Length of AB = $\sqrt{[(2 - 2)^2 + (5 - 0)^2]} = \sqrt{[(0)^2 + (5)^2]} = \sqrt{[0 + 25]} = 5$ units
Length of BC = $\sqrt{[(x - 2)^2 + (y - 5)^2]} = \sqrt{[x^2 - 4x + 4 + y^2 - 10y + 25]}$
$= \sqrt{[x^2 - 4x + y^2 - 10y + 29]}$ units
Length of AC = $\sqrt{[(x - 2)^2 + (y - 0)^2]} = \sqrt{[x^2 - 4x + 4 + y^2]}$ units
Given that,
AC = BC = 3
So, $AC^2 = BC^2 = 9$
$x^2 - 4x + 4 + y^2 = x^2 - 4x + y^2 - 10y + 29$
$10y = 25$
$y = 25/10 = 2.5$
And,
$AC^2 = 9$
$x^2 - 4x + 4 + y^2 = 9$
$x^2 - 4x + 4 + (2.5)^2 = 9$
$x^2 - 4x + 4 + 6.25 = 9$
$x^2 - 4x + 1.25 = 0$
$D = (-4)^2 - 4 \times 1 \times 1.25 = 16 - 5 = 11$
So, the roots are
$x = -(-4) + \sqrt{11}/ 2 = (4 + 3.31)/ 2 = 3.65$
And,
$x = -(-4) - \sqrt{11}/ 2 = (4 - 3.31)/ 2 = 0.35$
Therefore, the third vertex can be C (3.65, 2.5) or (0.35, 2.5)

16. Which point on x – axis is equidistant from (5, 9) and (-4, 6)?
Solution:

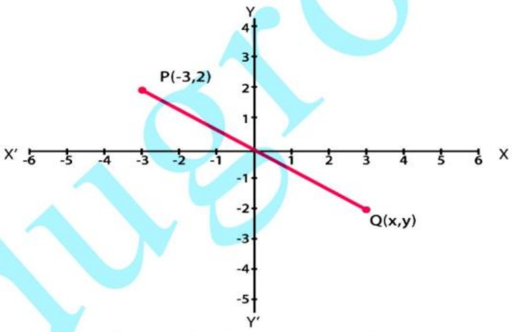
Let A (5, 9) and B (-4, 6) be the given points
Let the point on x - axis equidistant from the above points be C(x, 0)
Now, we have
AC = $\sqrt{[(x - 5)^2 + (0 - 9)^2]} = \sqrt{[x^2 - 10x + 25 + 81]} = \sqrt{[x^2 - 10x + 106]}$
And,
BC = $\sqrt{[(x - (-4))^2 + (0 - 6)^2]} = \sqrt{[x^2 + 8x + 16 + 36]} = \sqrt{[x^2 + 8x + 52]}$

As AC = BC (given condition)
So, $AC^2 = BC^2$
$x^2 - 10x + 106 = x^2 + 8x + 52$
$18x = 54$
$x = 3$
Therefore, the point on the x-axis is (3, 0)

17. Prove that the points (-2, 5), (0, 1) and (2, -3) are collinear.
Solution:

Let A (-2, 5), B(0, 1) and C (2, -3) be the given points
So, we have
$AB = \sqrt{[(0 - (-2))^2 + (1 - 5)^2]} = \sqrt{[(2)^2 + (-4)^2]} = \sqrt{[4 + 16]} = \sqrt{20} = 2\sqrt{5}$ units
$BC = \sqrt{[(2 - 0)^2 + (-3 - 1)^2]} = \sqrt{[(2)^2 + (-4)^2]} = \sqrt{[4 + 16]} = \sqrt{20} = 2\sqrt{5}$ units
$AC = \sqrt{[(2 - (-2))^2 + (-3 - 5)^2]} = \sqrt{[(4)^2 + (-8)^2]} = \sqrt{[16 + 64]} = \sqrt{80} = 4\sqrt{5}$ units
Now, it's seen that
AB + BC = AC
$2\sqrt{5} + 2\sqrt{5} = 4\sqrt{5}$
$4\sqrt{5} = 4\sqrt{5}$
Therefore, we can conclude that the given points (-2, 5), (0, 1) and (2, -3) are collinear

18. The coordinates of the point P are (-3, 2). Find the coordinates of the point Q which lies on the line joining P and origin such that OP = OQ.
Solution:



Let the coordinates of Q be taken as (x, y)
As Q lies on the line joining P and O(origin) with OP = OQ
Then, by mid-point theorem
$(x - 3)/2 = 0$
And,
$(y + 2)/ 2 = 0$
$\therefore x = 3, y = -2$
Therefore, the coordinates of point Q are (3, -2)

$d_1 = \sqrt{[(5-2)^2 + (8-4)^2]} = \sqrt{[(3)^2 + (4)^2]} = \sqrt{[9 + 16]} = \sqrt{25} = 5$ km

The position of the bank is (5, 8) and the position of the school is (13, 14).
So, the distance between the bank and the school,
$d_2 = \sqrt{[(13-5)^2 + (14-8)^2]} = \sqrt{[(8)^2 + (6)^2]} = \sqrt{[64 + 36]} = \sqrt{100} = 10$ km

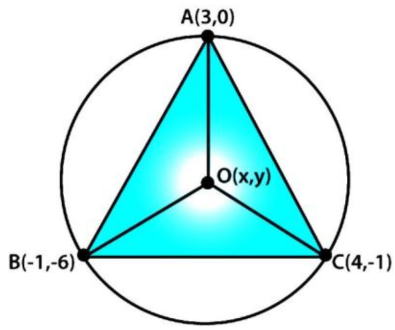
The position of the school is (13, 14) and the position of the office is (13, 26).
So, the distance between the school and the office,
$d_3 = \sqrt{[(13-13)^2 + (26-14)^2]} = \sqrt{[(0)^2 + (12)^2]} = \sqrt{144} = 12$ km
Let d be the total distance covered by Ayush
$d = d_1 + d_2 + d_3 = 5 + 10 + 12 = 27$ km
Let the D be the shortest distance from Ayush's house to the office,
$D = \sqrt{[(13-2)^2 + (26-4)^2]} = \sqrt{[(11)^2 + (22)^2]} = \sqrt{[121+ 484]} = \sqrt{605} = 24.6$ km
Thus, the extra distance covered by Ayush = d – D = 27 – 24.6 = 2.4 km

24. Find the value of k, if the point P(0, 2) is equidistant from (3, k) and (k, 5).
Solution:

Let the point P (0, 2) is equidistant from A (3, k) and B (k, 5)
So, PA = PB
$PA^2 = PB^2$
$(3 - 0)^2 + (k - 2)^2 = (k - 0)^2 + (5 - 2)^2$
$9 + k^2 + 4 - 4k - k^2 - 9 = 0$
$4 - 4k = 0$
$-4k = -4$
Therefore, the value of k = 1

25. If (-4, 3) and (4, 3) are two vertices of an equilateral triangle, find the coordinates of the third vertex, given that the origin lies in the (i) interior (ii) exterior of the triangle.
Solution:

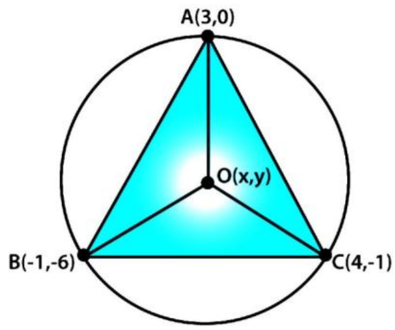
Let B (-4, 3) and C (4, 3) be the given two vertices of the equilateral triangle.
Let A (x, y) be the third vertex.
Then, we have
AB = BC = AC
Let us consider the part AB = BC
$AB^2 = BC^2$
$(-4 - x)^2 + (3 - y)^2 = (4 + 4)^2 + (3 - 3)^2$
$16 + x^2 + 8x + 9 + y^2 - 6y = 64$
$x^2 + y^2 + 8x - 6y = 39$
Now, let us consider AB = AC
$AB^2 = AC^2$
$(-4 - x)^2 + (3 - y)^2 = (4 - x)^2 + (3 - y)^2$
$16 + x^2 + 8x + 9 + y^2 - 18y = 16 + x^2 - 8x + 9 + y^2 - 6y$
$16x = 0$
$x = 0$



Let A(3, 0), B(-1, -6) and C(4 , -1) be the given points.
Let O(x , y) be the circumcenter of the triangle
Then, OA = OB = OC
$OA^2 = OB^2$
$(x - 3)^2 + (y - 0)^2 = (x + 1)^2 + (y + 6)^2$
$x^2 + 9 - 6x + y^2 = x^2 + 1 + 2x + y^2 + 36 + 12y$
-8x -12y = 28
2x + 3y = -7 …..(i) [After simplification]
Again,
$OB^2 = OC^2$
$(x + 1)^2 + (y + 6)^2 = (x - 4)^2 + (y + 1)^2$
$x^2 + 2x + 1 + y^2 + 36 + 12y = x^2 + 16 - 8x + y^2 + 1 + 2y$
10x + 10y = -20
x + y = -2 …..(ii) [After simplification]
Hence, the circumcenter of the triangle is (1, -3)
Circumradius = distance from any of the given points (say B)
$= \sqrt{[(1 + 1)^2 + (-3 + 6)^2]} = \sqrt{(4 + 9)}$
$= \sqrt{13}$ units

28. Find a point on the x-axis which is equidistant from the points (7, 6) and (-3, 4).
Solution:

Let A(7, 6) and B(-3, 4) be the given points.
Let P(x, 0) be the point on the x-axis such that PA = PB
So, $PA^2 = PB^2$
$(x - 7)^2 + (0 - 6)^2 = (x + 3)^2 + (0 - 4)^2$
$x^2 + 49 - 14x + 36 = x^2 + 9 + 6x + 16$
-20x = -60
x = 3
Therefore, the point on x-axis is (3, 0).



Let A(3, 0), B(-1, -6) and C(4 , -1) be the given points.
Let O(x , y) be the circumcenter of the triangle
Then, OA = OB = OC
$OA^2 = OB^2$
$(x - 3)^2 + (y - 0)^2 = (x + 1)^2 + (y + 6)^2$
$x^2 + 9 - 6x + y^2 = x^2 + 1 + 2x + y^2 + 36 + 12y$
-8x -12y = 28
2x + 3y = -7 …..(i) [After simplification]
Again,
$OB^2 = OC^2$
$(x + 1)^2 + (y + 6)^2 = (x - 4)^2 + (y + 1)^2$
$x^2 + 2x + 1 + y^2 + 36 + 12y = x^2 + 16 - 8x + y^2 + 1 + 2y$
10x + 10y = -20
x + y = -2 …..(ii) [After simplification]
Hence, the circumcenter of the triangle is (1, -3)
Circumradius = distance from any of the given points (say B)
$= \sqrt{[(1 + 1)^2 + (-3 + 6)^2]} = \sqrt{(4 + 9)}$
$= \sqrt{13}$ units

28. Find a point on the x-axis which is equidistant from the points (7, 6) and (-3, 4).
Solution:

Let A(7, 6) and B(-3, 4) be the given points.
Let P(x, 0) be the point on the x-axis such that PA = PB
So, $PA^2 = PB^2$
$(x - 7)^2 + (0 - 6)^2 = (x + 3)^2 + (0 - 4)^2$
$x^2 + 49 - 14x + 36 = x^2 + 9 + 6x + 16$
-20x = -60
x = 3
Therefore, the point on x-axis is (3, 0).

R D Sharma Solutions For Class 10 Maths Chapter 14 - Co-ordinate Geometry

Exercise 14.3 Page No: 14.28

1. Find the coordinates of the point which divides the line segment joining (-1, 3) and (4, - 7) internally in the ratio 3: 4.
Solution:

Let P(x, y) be the required point.
By section formula, we know that the coordinates are

$$x = \frac{mx_2 + nx_1}{m + n}$$

$$y = \frac{my_2 + ny_1}{m + n}$$

Here,
$x_1 = -1 y_1 = 3$
$x_2 = 4$ $y_2 = -7$
m: n = 3: 4
Then,

$$x = \frac{3 \times 4 + 4 \times (-1)}{3 + 4} \times 3$$

$$x = \frac{3 \times 4 + 4 \times (-1)}{3 + 4}$$

$$x = \frac{12 - 4}{7}$$

$$x = \frac{8}{7}$$

$$y = \frac{3 \times (-7) + 4 \times 3}{3 + 4}$$

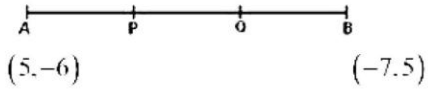
$$y = \frac{-21 + 12}{7}$$

$$y = \frac{-9}{7}$$

Therefore, the coordinates of P are (8/7, – 9/7)

2. Find the points of trisection of the line segment joining the points:
(i) (5, - 6) and (-7, 5)
(ii) (3, - 2) and (-3, - 4)
(iii) (2, - 2) and (-7, 4)
Solution:

(i) Let P and Q be the point of trisection of AB such that AP = PQ = QB



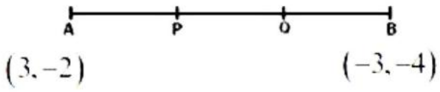
So, P divides AB internally in the ratio of 1: 2, thereby applying section formula, the coordinates of P will be

$$\left(\frac{2(-7)+(5)}{2+1}\right),\left(\frac{2(5)+1(-6)}{2+1}\right)\text{ i. e., }\left(1,-\frac{7}{3}\right)$$

Now, Q also divides AB internally in the ratio of 2:1 so their coordinates will be

$$\left(\frac{2(-7)+1(5)}{2+1}\right),\left(\frac{2(5)+1(-6)}{2+1}\right)\text{ i. e., }\left(-3,\frac{4}{3}\right)$$

(ii) Let P and Q be the points of trisection of AB such that AP = PQ = QB



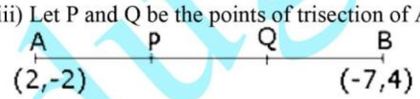
As, P divides AB internally in the ratio of 1: 2. Hence by applying section formula, the coordinates of P are

$$\left(\left(\frac{1(-3)+2(3)}{1+2}\right),\frac{1(-4)+(-2)}{1+2}\right)\text{ i. e., }\left(1,-\frac{8}{3}\right)$$

Now, Q also divides as internally in the ratio of 2: 1
So, the coordinates of Q are given by

$$\left(\left(\frac{2(-3)+1(3)}{2+1}\right),\frac{2(-4)+1(-2)}{2+1}\right)\text{ i. e., }\left(-1,\frac{-10}{3}\right)$$

(iii) Let P and Q be the points of trisection of AB such that AP = PQ = OQ



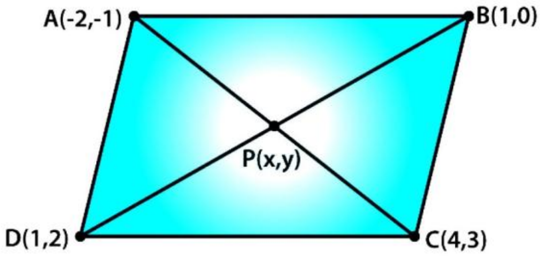
As, P divides AB internally in the ratio 1:2. So, the coordinates of P, by applying the section formula, are given by

$$\left(\left(\frac{1(-7)+2(2)}{1+2}\right),\left(\frac{1(4)+2(-2)}{1+2}\right)\right),\text{ i. e., }(-1,0)$$

Now. Q also divides AB internally in the ration 2: 1. And the coordinates of Q are given by

$$\left(\frac{2(-7)+1(2)}{2+1},\frac{2(4)+1(2)}{2+1}\right),\text{ i. e., }(-4,2)$$

3. Find the coordinates of the point where the diagonals of the parallelogram formed by joining the points (-2, -1), (1, 0), (4, 3) and (1, 2) meet.
Solution:



Let A(-2, -1), B(1, 0), C(4, 3) and D(1, 2) be the given points.
Let P(x, y) be the point of intersection of the diagonals of the parallelogram formed by the given points.
We know that, diagonals of a parallelogram bisect each other.

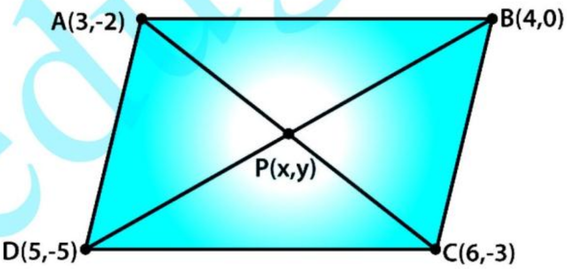
$$x = \frac{-2+4}{2}$$

$$\Rightarrow x = \frac{2}{2} = 1$$

$$y = \frac{-1+3}{2} = \frac{2}{2} = 1$$

Therefore, the coordinates of P are (1, 1)

4. Prove that the points (3, 2), (4, 0), (6, -3) and (5, -5) are the vertices of a parallelogram.
Solution:



Let A(3, -2), B(4, 0), C(6, -3) and D(5, -5)
Let P(x, y) be the point of intersection of diagonals AC and BD of ABCD.

The mid-point of AC is given by,

$$x = \frac{3 + 6}{2} = \frac{9}{2}$$

$$y = \frac{-2 - 3}{2} = \frac{-5}{2}$$

$$\text{Mid} - \text{point of AC } \left(\frac{9}{5}, \frac{-5}{2}\right)$$

Again, the mid-point of BD is given by,

$$x = \frac{5 + 4}{2} = \frac{9}{2}$$

$$y = \frac{-5 + 0}{2} = \frac{-5}{2}$$

Thus, we can conclude that diagonals AC and BD bisect each other.
And, we know that diagonals of a parallelogram bisect each other
Therefore, ABCD is a parallelogram.

5. If P(9a - 2, -b) divides the line segment joining A(3a + 1, -3) and B(8a, 5) in the ratio 3 : 1, find the values of a and b.

Solution:

Given that, P(9a – 2, -b) divides the line segment joining A(3a + 1, -3) and B(8a, 5) in the ratio 3:1
Then, by section formula
Coordinates of P are

$$9a - 2 = \frac{3(8a) + 1(3a + 1)}{3 + 1}$$

And,

$$-b = \frac{3(5) + 1(-3)}{3 + 1}$$

Solving for a, we have
(9a – 2) x 4 = 24a + 3a + 1
36a – 8 = 27a + 1
9a = 9
a = 1
Now, solving for b, we have
4 x –b = 15 – 3
-4b = 12
b = -3
Therefore, the values of a and b are 1 and -3 respectively.

6. If (a, b) is the mid-point of the line segment joining the points A (10, -6), B(k, 4) and a - 2b = 18, find the value of k and the distance AB.

Solution:

As (a, b) is the mid-point of the line segment A(10, -6) and B(k, 4)
So,
(a, b) = (10 + k / 2, -6 + 4/ 2)
a = (10 + k)/ 2 and b = -1
2a = 10 + k
k = 2a – 10
Given, a – 2b = 18
Using b = -1 in the above relation we get,
a – 2(-1) = 18
a = 18 – 2 = 16
So,
k = 2(16) - 10 = 32 – 10 = 22
Thus,
AB = $\sqrt{[(22 - 10)^2 + (4 + 6)^2]} = \sqrt{[(12)^2 + (10)^2]} = \sqrt{[144 + 100]} = 2\sqrt{61}$ units

7. Find the ratio in which the point (2, y) divides the line segment joining the points A(-2, 2) and B(3, 7). Also find the value of y.
Solution:

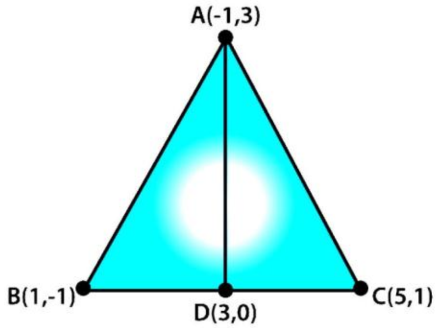
Let the point P(2, y) divide the line segment joining the points A(-2, 2) and B(3, 7) in the ratio k: 1
Then, the coordinates of P are given by

$$\left[\frac{3k + (-2) \times 1}{k + 1}, \frac{7k + 2 \times 1}{k + 1}\right]$$

$$= \left[\frac{3k - 2}{k + 1}, \frac{7k + 2}{k + 1}\right]$$

And, given the coordinates of P are (2, y)
So,
2 = (3k – 2)/ (k + 1) and y = (7k + 2)/ (k + 1)
Solving for k, we get
2(k + 1) = (3k – 2)
2k + 2 = 3k – 2
k = 4
Using k to find y, we have
y = (7(4) + 2)/ (4 + 1)
= (28 + 2)/5
= 30/5
y = 6
Therefore, the ratio id 4: 1 and y = 6

8. If A(-1, 3), B(1, -1) and C(5, 1) are the vertices of a triangle ABC, find the length of median through A.
Solution:



Let AD be the median through A.
As, AD is the median, D is the mid-point of BC
So, the coordinates of D are (1 + 5/ 2, -1 + 1/ 2) = (3 , 0)
Therefore,
Length of median AD = $\sqrt{[(3 + 1)^2 + (0 - 3)^2]} = \sqrt{[(4)^2 + (-3)^2]} = \sqrt{[16 + 9]} = \sqrt{25}$ = 5 units

9. If the points P,Q(x, 7), R, S(6, y) in this order divide the line segment joining A(2, p) and B (7, 10) in 5 equal parts, find x, y and p.
Solution:

From question, we have
AP = PQ = QR = RS = SB
So, Q is the mid-point of A and S
Then,
x = (2 + 6)/ 2 = 8/2 = 4
7 = (y + p)/ 2
y + p = 14 ….. (1)
Now, since S divides QB in the ratio 2: 1

$$y = \frac{2\times10+1\times7}{2+1} \Rightarrow \frac{20+7}{3} = \frac{27}{3} = 9$$

From (*i*), $y + p = 14 \Rightarrow 9 + p = 14$
So, p = 14 – 9 = 5
Therefore, x = 4, y = 9 and p = 5

10. If a vertex of a triangle be (1, 1) and the middle points of the sides through it be (-2, 3) and (5, 2) find the other vertices.
Solution:

Let A(1, 1) be the given vertex and D(-2, 3), E(5, 2) be the mid-points of AB and AC
Now, as D and E are the mid-points of AB and AC

$\frac{x_1+1}{2} = -2, \quad \frac{y_1+1}{2} = 3$

$x_1 + 1 = -4 \quad \Rightarrow y_1 + 1 = 6$
$x_1 = -5 \quad \Rightarrow y_1 = 5$

So, the coordinates of B are (-5, 5)
Again,

$\frac{x_2+1}{2} = 5, \quad \frac{y_2+1}{2} = 2$

$x_2 + 1 = 10 \quad \Rightarrow y_2 + 1 = 4$
$x_2 = 9 \quad \Rightarrow y_2 = 3$

So, the coordinates of C are (9, 3)
Therefore, the other vertices of the triangle are (-5, 5) and (9, 3).

11. (i) In what ratio is the segment joining the points (-2, -3) and (3, 7) divides by the y-axis? Also, find the coordinates of the point of division.
Solution:

Let P(-2, -3) and Q(9, 3) be the given points.
Suppose y-axis divides PQ in the ratio k: 1 at R(0, y)
So, the coordinates of R are given by

$\left[\frac{3k+(-2)\times 1}{k+1}, \frac{7k+(-3)\times 1}{k+1}\right]$

Now, equating

$\frac{3k+(-2)\times 1}{k+1} = 0$

3k – 2 = 0
k = 2/3
Therefore, the ratio is 2: 3
Putting k = 2/3 in the coordinates of R, we get
R (0, 1)

(ii) In what ratio is the line segment joining (-3, -1) and (-8, -9) divided at the point (-5, -21/5)?
Solution:
Let A(-3, -1) and B(-8, -9) be the given points.
And, let P be the point that divides AB in the ratio k: 1
So, the coordinates of P are given by

$\left[\frac{-8k-3}{k+1}, \frac{-9k-1}{k+1}\right]$

But, given the coordinates of P
On equating, we get
(-8k – 3)/ (k + 1) = -5
-8k – 3 = -5k – 5
3k = 2
k = 2/3
Thus, the point P divides AB in the ratio 2: 3

12. If the mid-point of the line joining (3, 4) and (k, 7) is (x, y) and 2x + 2y + 1 = 0 find the value of k.
Solution:

As (x , y) is the mid-point
x = (3 + k)/ 2 and y = (4 + 7)/ 2 = 11/2
Also,
Given that the mid-point lies on the line 2x + 2y + 1 = 0
2[(3 + k)/ 2] + 2(11/2) + 1 = 0
3 + k + 11 + 1 = 0
Thus, k = -15

13. Find the ratio in which the point P(3/4, 5/12) divides the line segments joining the point A(1/2, 3/2) and B(2, -5).
Solution:

Given,
Points A(1/2, 3/2) and B(2, -5)
Let the point P(3/4, 5/12) divide the line segment AB in the ratio k: 1
Then, we know that
P(3/4, 5/12) = (2k + ½)/ (k +1) , (2k + 3/2)/ (k + 1)
Now, equating the abscissa we get
¾ = (2k + ½)/ (k +1)
3(k + 1) = 4(2k + 1/2)
3k + 3 = 8k + 2
5k = 1
k = 1/5
Therefore, the ratio in which the point P(3/4, 5/12) divides is 1: 5

14. Find the ratio in which the line joining (-2, -3) and (5, 6) is divided by (i) x-axis (ii) y-axis. Also, find the coordinates of the point of division in each case.
Solution:
Let A(-2, -3) and B(5, 6) be the given points.
(i) Suppose x-axis divides AB in the ratio k: 1 at the point P
Then, the coordinates of the point of division are

$$\left[\frac{5k-2}{k+1}, \frac{6k-3}{k+1}\right]$$

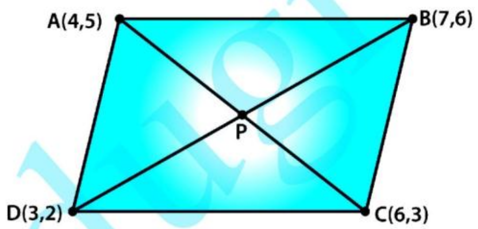
As, P lies in the x-axis, the y – coordinate is zero.
So,
6k – 3/ k + 1 = 0
6k – 3 = 0
k = ½
Thus, the required ratio is 1: 2
Using k in the coordinates of P
We get, P (1/3, 0)

(ii) Suppose y-axis divides AB in the ratio k: 1 at point Q
The, the coordinates of the point od division is given by

$$\left[\frac{5k-2}{k+1}, \frac{6k-3}{k+1}\right]$$

As, Q lies on the y-axis, the x – ordinate is zero.
So,
5k – 2/ k + 1 = 0
5k – 2 = 0
k = 2/5
Thus, the required ratio is 2: 5
Using k in the coordinates of Q
We get, Q (0, -3/7)

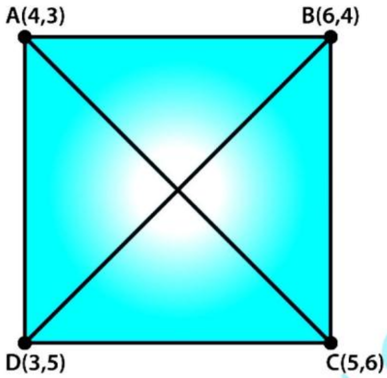
15. Prove that the points (4, 5), (7, 6), (6, 3), (3, 2) are the vertices of a parallelogram. Is it a rectangle?
Solution:



Let A (4, 5), B(7, 6), C(6, 3) and D(3, 2) be the given points.
And, P be the point of intersection of AC and BD.
Coordinates of the mid-point of AC are (4+6/2 , 5+3/2) = (5, 4)
Coordinates of the mid-point of BD are (7+3/2 , 6+2/2) = (5, 4)
Thus, it's clearly seen that the mid-point of AC and BD are same.
So, ABCD is a parallelogram.
Now,
AC = $\sqrt{[(6 - 4)^2 + (3 – 5)^2]} = \sqrt{[(2)^2 + (-2)^2]} = \sqrt{[4 + 4]} = \sqrt{8}$ units
And,
BD = $\sqrt{[(7 - 3)^2 + (6 – 2)^2]} = \sqrt{[(4)^2 + (4)^2]} = \sqrt{[16 + 16]} = \sqrt{32}$ units

Since, AC ≠ BD
Therefore, ABCD is not a rectangle.

16. Prove that (4, 3), (6, 4), (5, 6) and (3, 5) are the angular points of a square.
Solution:



Let A(4,3) , B(6,4) , C(5,6) and D(3,5) be the given points.
The distance formula is

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$AB = \sqrt{(4-6)^2 + (3-4)^2} = \sqrt{5}$$

$$BC = \sqrt{(6-5)^2 + (4-6)^2} = \sqrt{5}$$

$$CD = \sqrt{(5-3)^2 + (6-5)^2} = \sqrt{5}$$

$$AD = \sqrt{(4-3)^2 + (3-5)^2} = \sqrt{5}$$

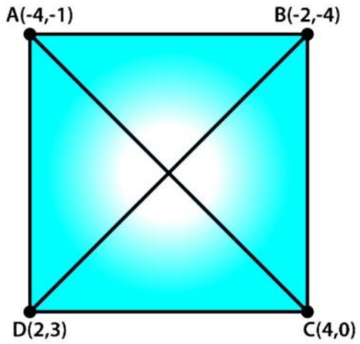
It's seen that the length of all the sides are same.
Now, the length of diagonals are

$$AC = \sqrt{(4-5)^2 + (3-6)^2} = \sqrt{10}$$

$$BD = \sqrt{(6-3)^2 + (4-5)^2} = \sqrt{10}$$

Also, the length of both the diagonals are same.
Therefore, we can conclude that the given points are the angular points of a square.

17. Prove that the points (-4, -1), (-2, -4), (4, 0) and (2, 3) are the vertices of a rectangle.
Solution:



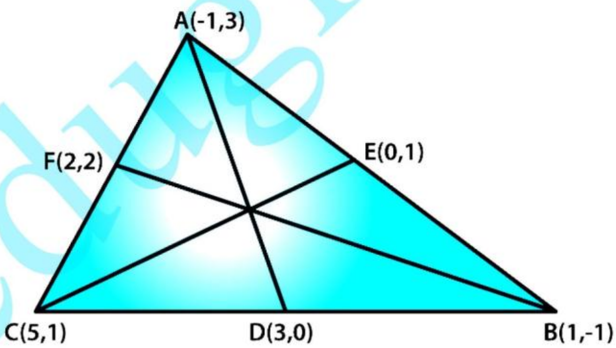
Let A(-4, -1), B(-2, -4), C(4, 0) and D(2, 3) be the given points.
Now,
Coordinates of the mid-point of AC are (-4 + 4/ 2, -1 + 0/2) = (0, -1/2)
Coordinates of the mid-point of BD are (-2 + 2/2, -4 + 3/2) = (0, -1/2)
Thus, it's seen that AC and BD have the same point.
And, we have diagonals

$$AC = \sqrt{(4+4)^2 + (0+1)^2} = \sqrt{65}$$

$$BD = \sqrt{(-2-2)^2 + (-4-3)^2} = \sqrt{65}$$

The length of diagonals are also same.
Therefore, the given points are the vertices of a rectangle.

18. Find the length of the medians of a triangle whose vertices are A(-1, 3), B(1, -1) and C(5, 1).
Solution:



Let AD, BF and CE be the medians of ΔABC
Coordinates of D are (5 + 1/ 2, 1 – 1/ 2) = (3, 0)
Coordinates of E are (-1 + 1/ 2, 3 – 1/ 2) = (0, 1)
Coordinates of F are (5 - 1/ 2, 1 + 3/ 2) = (2, 2)

Now,
Finding the length of the respectively medians:

$$\text{Length of } AD = \sqrt{(-1-3)^2 + (3-0)^2} = 5 \text{ units}$$

$$\text{Length of } BF = \sqrt{(2-1)^2 + (2+1)^2} = \sqrt{10} \text{ units}$$

$$\text{Length of } CE = \sqrt{(5-0)^2 + (1-1)^2} = 5 \text{ units}$$

19. Find the ratio in which the line segment joining the points A (3, -3) and B (-2, 7) is divided by x- axis. Also, find the coordinates of the point of division.
Solution:

Let the point on the x-axis be (x, 0). [y – coordinate is zero]
And, let this point divides the line segment AB in the ratio of k : 1.
Now using the section formula for the y-coordinate, we have
0 = (7k - 3)/(k + 1)
7k – 3 = 0
k = 3/7
Therefore, the line segment AB is divided by x-axis in the ratio 3: 7

20. Find the ratio in which the point P(x, 2) divides the line segment joining the points A (12, 5) and B (4, -3). Also, find the value of x.
Solution:

Let P divide the line joining A and B and let it divide the segment in the ratio k: 1
Now, using the section formula for the y – coordinate we have
2 = (-3k + 5)/ (k + 1)
2(k + 1) = -3k + 5
2k + 2 = -3k + 5
5k = 3
k = 3/5
Thus, P divides the line segment AB in the ratio of 3: 5
Using value of k, we get the x – coordinate as
x = 12 + 60/ 8 = 72/8 = 9
Therefore, the coordinates of point P is (9, 2)

21. Find the ratio in which the point P(-1, y) lying on the line segment joining A(-3, 10) and B(6, -8) divides it. Also find the value of y.
Solution:

Let P divide A(-3, 10) and B(6, -8) in the ratio of k: 1
Given coordinates of P as (-1, y)
Now, using the section formula for x – coordinate we have

-1 = 6k – 3/ k + 1
-(k + 1) = 6k – 3
7k = 2
k = 2/7
Thus, the point P divides AB in the ratio of 2: 7
Using value of k, to find the y-coordinate we have
y = (-8k + 10)/ (k + 1)
y = (-8(2/7) + 10)/ (2/7 + 1)
y = -16 + 70/ 2 + 7 = 54/9
y = 6
Therefore, the y-coordinate of P is 6

22. Find the coordinates of a point A, where AB is the diameter of circle whose center is (2, -3) and B is (1, 4).
Solution:

Let the coordinates of point A be (x, y)
If AB is the diameter, then the center in the mid-point of the diameter
So,
(2, -3) = (x + 1/ 2, y + 4/ 2)
2 = x + 1/2 and -3 = y + 4/ 2
4 = x + 1 and -6 = y + 4
x = 3 and y = -10
Therefore, the coordinates of A are (3, -10)

23. If the points (-2, 1), (1, 0), (x , 3) and (1, y) form a parallelogram, find the values of x and y.
Solution:

Let A(-2, 1), B(1, 0), C(x , 3) and D(1, y) be the given points of the parallelogram.
We know that the diagonals of a parallelogram bisect each other.
So, the coordinates of mid-point of AC = Coordinates of mid-point of BD

$$\left(\frac{x-2}{2}, \frac{3-1}{2}\right) = \left(\frac{1+1}{2}, \frac{y+0}{2}\right)$$

$$\left(\frac{x-2}{2}, 1\right) = \left(1, \frac{y}{2}\right)$$

$$\frac{x-2}{2} = 1 \quad \text{and} \quad \frac{y}{2} = 1$$

$$x - 2 = 2 \qquad \Rightarrow y = 2$$

$$x = 4 \qquad \Rightarrow y = 2$$

Therefore, the value of x is 4 and the value of y is 2.

24. The points A(2, 0), B(9, 1), C(11, 6) and D(4, 4) are the vertices of a quadrilateral ABCD. Determine whether ABCD is a rhombus or not.
Solution:

Given points are A(2, 0), B(9, 1), C(11, 6) and D(4, 4).
Coordinates of mid-point of AC are (11+2/ 2, 6+0/ 2) = (13/2, 3)
Coordinates of mid-point of BD are (9+4/ 2, 1+4/ 2) = (13/2, 5/2)
As the coordinates of the mid-point of AC ≠ coordinates of mid-point of BD, ABCD is not even a parallelogram.
Therefore, ABCD cannot be a rhombus too.

25. In what ratio does the point (-4,6) divide the line segment joining the points A(-6,10) and B(3,-8)?
Solution:

Let the point (-4, 6) divide the line segment AB in the ratio k: 1.
So, using the section formula, we have

$$(-4, 6) = \left(\frac{3k-6}{k+1}, \frac{-8k+10}{k+1}\right)$$

$$-4 = \frac{3k-6}{k+1}$$

$$-4k-4 = 3k-6$$

$$7k = 2$$

$$k : 1 = 2 : 7$$

The same can be checked for the y-coordinate also.
Therefore, the ratio in which the point (-4, 6) divides the line segment AB is 2: 7

26. Find the ratio in which the y-axis divides the line segment joining the points (5, -6) and (-1, -4). Also find the coordinates of the point of division.
Solution:

Let P(5, -6) and Q(-1, -4) be the given points.
Let the y-axis divide the line segment PQ in the ratio k: 1
Then, by using section formula for the x-coordinate (as it's zero) we have

$$\frac{-k+5}{k+1} = 0$$

$$-k+5 = 0$$

$$k = 5$$

Thus, the ratio in which the y-axis divides the given 2 points is 5: 1
Now, for finding the coordinates of the point of division
Putting k = 5, we get

$$\left(\frac{-5+5}{5+1}, \frac{-4\times5-6}{5+1}\right) = \left(0, \frac{-13}{3}\right)$$

Hence, the coordinates of the point of division are (0, -13/3)

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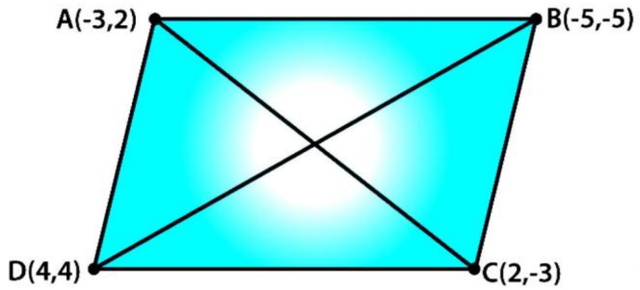
27. Show that A(-3, 2), B(-5, 5), C(2, -3) and D(4, 4) are the vertices of a rhombus.
Solution:



Given points are A(-3, 2), B(-5, 5), C(2, -3) and D(4, 4)
Now,
Coordinates of the mid-point of AC are (-3+2/ 2, 2-3/ 2) = (-1/2, -1/2)
And,
Coordinates of mid-point of BD are (-5+4/ 2, -5+4/ 2) = (-1/2, -1/2)
Thus, the mid-point for both the diagonals are the same. So, ABCD is a parallelogram.
Next, the sides

$AB = \sqrt{(-5+3)^2 + (-5-2)^2}$

$AB = \sqrt{4+49}$

$AB = \sqrt{53}$

$BC = \sqrt{(-5-2)^2 + (-5+3)^2}$

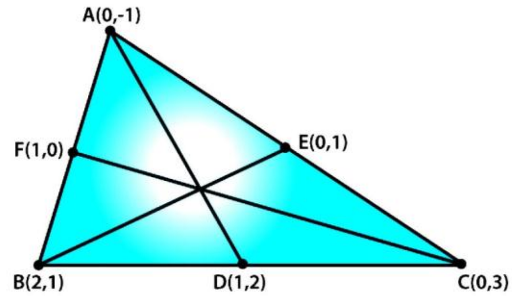
$BC = \sqrt{49+4}$

$BC = \sqrt{53}$

$AB = BC$

It's seen that ABCD is a parallelogram with adjacent sides equal.
Therefore, ABCD is a rhombus.

28. Find the lengths of the medians of a ΔABC having vertices at A(0, -1), B(2, 1) and C(0, 3).
Solution:



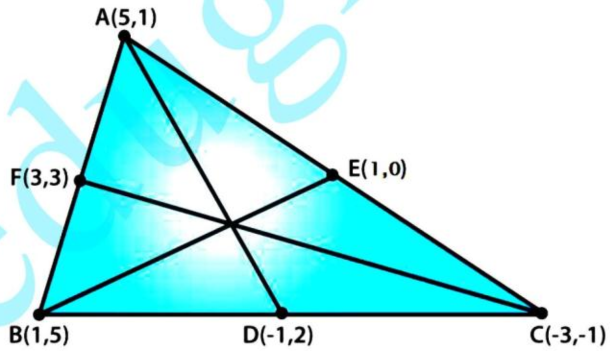
Let AD, BE and CF be the medians of ΔABC
Then,
Coordinates of D are (2+0/ 2, 1+3/ 2) = (1, 2)
Coordinates of E are (0/2, 3-1/ 2) = (0, 1)
Coordinates of F are (2+0/ 2, 1-1/ 2) = (1, 0)
Now, the length of the medians

Length of median $AD = \sqrt{(1-0)^2 + (2+1)^2} = \sqrt{10}$ units

Length of median $BE = \sqrt{(2-0)^2 + (1-1)^2} = 2$ units

Length of median $CF = \sqrt{(1-0)^2 + (0-3)^2} = \sqrt{10}$ units

29. Find the lengths of the median of a ΔABC having vertices at A(5, 1), B(1, 5) and C(-3, -1).
Solution:



Given vertices of ΔABC as A(5, 1), B(1, 5) and C(-3, -1).
Let AD, BE and CF be the medians
Coordinates of D are (1-3/ 2, 5-1/ 2) = (-1, 2)
Coordinates of E are (5-3/ 2, 1-1/2) = (1, 0)

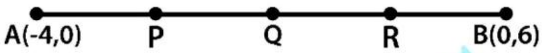
Coordinates of F are (5+1/ 2, 1+5/ 2) = (3, 3)
Now, the length of the medians

Length of median $AD = \sqrt{(5+1)^2 + (1-2)^2} = \sqrt{37}$ units

Length of median $BE = \sqrt{(1-1)^2 + (5-0)^2} = 5$ units

Length of median $CF = \sqrt{(3+3)^2 + (3+1)^2} = 2\sqrt{13} = \sqrt{52}$ units

30. Find the coordinates of the point which divide the line segment joining the points (-4, 0) and (0, 6) in four equal parts.
Solution:



Let A(-4, 0) and B(0, 6) be the given points
And, let P, Q and R be the points which divide AB is four equal points.
Now, we know that AP: PB = 1: 3
Using section formula the coordinates of P are

$$\left(\frac{1\times 0 + 3(-4)}{1+3}, \frac{1\times 6 + 3\times 0}{1+3}\right) = \left(-3, \frac{3}{2}\right)$$

And, it's seen that Q is the mid-point of AB
So, the coordinates of Q are

$$\left(\frac{-4+0}{2}, \frac{0+6}{2}\right) = (-2, 3)$$

Finally, the ratio of AR: BR is 3: 1
Then by using section formula the coordinates of R are

$$\left(\frac{3\times 0 + 1\times(-4)}{3+1}, \frac{3\times 6 + 1\times 0}{3+1}\right) = \left(-1, \frac{9}{2}\right)$$

Exercise 14.4 — Page No: 14.37

1. Find the centroid of the triangle whose vertices are:
(i) (1, 4), (-1, -1) and (3, -2) (ii) (-2, 3), (2, -1) and (4, 0)
Solution:

We know that the coordinates of the centroid of a triangle whose vertices are (x_1, y_1), (x_2, y_2), (x_3, y_3) are

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y + y_2 + y_3}{3}\right)$$

(i) So, the coordinates of the centroid of a triangle whose vertices are (1, 4), (-1, -1) and (3, -2) are

$$\left(\frac{1 - 1 + 3}{3}, \frac{4 - 1 - 2}{3}\right)$$

(1, 1/3)
Thus, centroid of the triangle is (1, 1/3)

(ii) So, the coordinates of the centroid of a triangle whose vertices are (-2, 3), (2, -1) and (4, 0) are

$$\left(\frac{2 - 2 + 4}{3}, \frac{3 - 1 + 0}{3}\right)$$

(4/3, 2/3)
Thus, centroid of the triangle is (4/3, 2/3)

2. Two vertices of a triangle are (1, 2), (3, 5) and its centroid is at the origin. Find the coordinates of the third vertex.
Solution:

Let the coordinates of the third vertex be (x, y)
Then, we know that the coordinates of centroid of the triangle are

$$\left(\frac{x + 1 + 3}{3}, \frac{y + 2 + 5}{3}\right)$$

Given that the centroid for the triangle is at the origin (0, 0)

$$\therefore \frac{x + 1 + 3}{3} = 0 \text{ and } \frac{y + 2 + 5}{3} = 0$$

$\Rightarrow x + 4 = 0 \quad \Rightarrow y + 7 = 0$
$\Rightarrow x = -4 \quad \Rightarrow y = -7$
Therefore, the coordinates of the third vertex is (-4, -7)

3. Find the third vertex of a triangle, if two of its vertices are at (-3, 1) and (0, -2) and the centroid is at the origin.

Solution:

Let the coordinates of the third vertex be (x, y)
Then, we know that the coordinates of centroid of the triangle are

$$\left(\frac{x - 3 - 0}{3}, \frac{y + 1 - 2}{3}\right)$$

Given that the centroid for the triangle is at the origin (0, 0)

$$\therefore \frac{x - 3 - 0}{3} = 0 \text{ and } \frac{y + 1 - 2}{3} = 0$$

$\Rightarrow x - 3 = 0 \quad \Rightarrow y - 1 = 0$
$\Rightarrow x = 3 \quad \Rightarrow y = 1$
Therefore, the coordinates of the third vertex is (3, 1)

4. A(3, 2) and B(-2, 1) are two vertices of a triangle ABC whose centroid G has the coordinates (5/3, -1/3). Find the coordinates of the third vertex C of the triangle.
Solution:

Let the coordinates of the third vertex C be (x, y)
Given, A(3, 2) and B(-2, 1) are two vertices of a triangle ABC
Then, we know that the coordinates of centroid of the triangle are

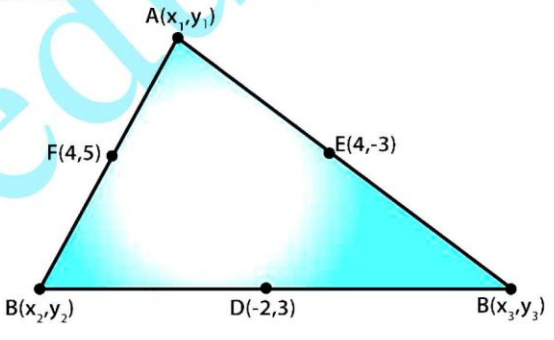
$$\left(\frac{x + 3 - 2}{3}, \frac{y + 2 + 1}{3}\right)$$

Given that the centroid for the triangle is (5/3, -1/3).

$$\therefore \frac{x + 3 - 2}{3} = 5/3 \text{ and } \frac{y + 2 + 1}{3} = -1/3$$

$\Rightarrow x + 1 = 5 \quad \Rightarrow y + 3 = -1$
$\Rightarrow x = 4 \quad \Rightarrow y = -4$
Therefore, the coordinates of the third vertex C is (4, -4)

5. If (-2, 3), (4, -3) and (4, 5) are the mid-points of the sides of a triangle, find the coordinates of its centroid.
Solution:



Let A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) be the vertices of triangle ABC.
Let D (-2, 3), E (4, -3) and F (4, 5) be the mid-points of sides BC, CA and AB respectively.
As D is the mid-point of BC

$\frac{x_2+x_3}{2}=-2$ and $\frac{y_2+y_3}{2}=3$

$x_2+x_3=-4$ and $y_2+y_3=6$ ……. (1)

Similarly E and F are the mid-points of AC and AB

$\frac{x_1+x_3}{2}=4$ and $\frac{y_1+y_3}{2}=-3$

$x_1+x_3=8$ and $y_1+y_3=-6$ …….. (2)

And,

$\frac{x_1+x_2}{2}=4$ and $\frac{y_1+y_2}{2}=5$

$x_1+x_2=8$ and $y_1+y_2=10$ …… (3)

From (1), (2) and (3), we have

$x_2+x_3+x_1+x_3+x_1+x_2=-4+8+8$ and

$y_2+y_3+y_1+y_3+y_1+y_2=6-6+10$

$2(x_1+x_2+x_3)=12$ and $2(y_1+y_2+y_3)=10$

$x_1+x_2+x_3=6$ and $y_1+y_2+y_3=5$ …….. (4)

Form (1) and (4), we get

$x_1-4=6$ and $y_1+6=5$

$x_1=10$ $\Rightarrow y_1=-1$

Thus, the coordinates of A are (10, -1)
From (2) and (4), we get

$x_2+8=6$ and $y_2-6=5$

$x_2=-2$ $\Rightarrow y_2=11$

Thus, the coordinates of B are (-2, 11)
From (3) and (4), we get

$x_3+8=6$ and $y_3+10=5$

$x_3=-2$ $\Rightarrow y_3=-5$

Thus, the coordinates of C are (-2, -5)
Hence, the vertices of triangle ABC are A (10, -1), B (-2, 11) and C (-2, -5).
Therefore, the coordinates of the centroid of triangle ABC are

$$\left(\frac{10-2-2}{3},\frac{-1+11-5}{3}\right)=\left(2,\frac{5}{3}\right)$$

R D Sharma Solutions For Class 10 Maths Chapter 14 - Co-ordinate Geometry

Exercise 14.5 Page No: 14.53

1. Find the area of a triangles whose vertices are
(i) (6, 3), (-3, 5) and (4, - 2)
(ii) [(at_1^2, at_1),(at_2^2, $2at_2$)(at_3^2, $2at_3$)]
(iii) (a, c + a), (a, c) and (-a, c - a)
Solution:

(i) Let A(6, 3), B(-3, 5) and C(4,-2) be the given points
We know that, area of a triangle is given by:
$1/2[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 + y_2)]$
Here,
$x_1 = 6, y_1 = 3, x_2 = -3, y_2 = 5, x_3 = 4, y_3 = -2$
So,
Area of ΔABC = 1/2 [6(5+2)+(-3)(- 2 -3)+ 4(3 - 5)]
=1/2 [6 × 7- 3 × (– 5) + 4(– 2)]
= 1/2[42 +15 - 8]
= 49/2 sq. units

(ii) Let A = $(x_1, y_1) = (at_1^2, 2at_1)$, B = $(x_2,y_2) = (at_2^2, 2at_2)$, C= $(x_3, y_3) = (at_3^2, 2at_2)$ be the given points.
Then,
The area of ΔABC is given by

$$= \frac{1}{2}[at_1^2(2at_2 - 2at_3) + at_2^2(2at_3 - 2at_1) + at_3^2(2at_1 - 2at_2)]$$

$$= \frac{1}{2}[at^2t_1^2t_2 - 2a^2t_1^2t_3 + 2a^2t_2^2t_3 - 2a^2t_2^2t_1 - 2a^2t_3^2t_2]$$

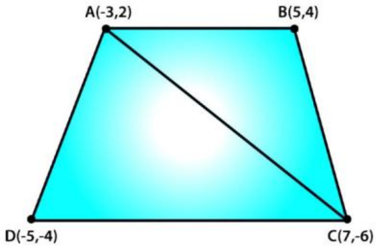
$$= \frac{1}{2} \times 2[a^2t_1^2(t_2 - t_3) + a^2t_2^2(t_3 - t_1) + a^2t_2^2(t_3 - t_1) + a^2 + t_3^2(t_1 - t_2)]$$

$$= a^2[t_1^2(t_2 - t_3) + t_2^2(t_3 - t_1) + t_3^2\ (t_1 - t_2)]$$

(iii) Let A = (x_1,y_1) = (a, c + a), B = (x_2, y_2) = (a, c) and C = (x_3, y_3) = (- a, c - a) be the given points
Then,
The area of ΔABC is given by
= 1/2[a (– {c - a}) + a(c – a - (c + a)) +(- a)(c + a - a)]
= 1/2 [a(c – c + a) + a(c – a – c - a) - a(c + a - c)]
= 1/2[a × a + ax(- 2a) – a × a]
= $1/2[a^2 - 2a^2 - a^2]$
= $1/2\times(-2a)^2$
= $- a^2$

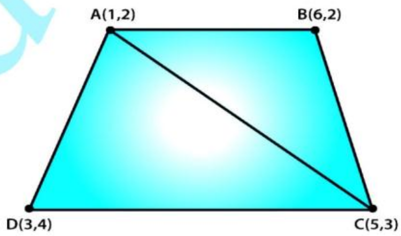
2. Find the area of the quadrilaterals, the coordinates of whose vertices are

(i) (-3, 2), (5, 4), (7, -6) and (-5, - 4)
(ii) (1, 2), (6, 2), (5, 3) and (3, 4)
(iii) (-4, -2), (-3, -5), (3, -2), (2, 3)
Solution:



(i)

Let A(-3, 2), B(5, 4), C(7,- 6) and D (-5, – 4) be the given points.
Area of ΔABC is given by
= 1/2[-3(4 + 6) + 5(- 6 - 2) + 7(2 - 4)]
= 1/2[-3×1 + 5×(-8) + 7(-2)]
= 1/2[- 30 – 40 -14]
= - 42
As the area cannot be negative,
The area of ΔADC = 42 square units
Now, area of ΔADC is given by
= 1/2[-3(– 6 + 4) + 7(- 4 - 2) + (- 5)(2 + 6)]
= 1/2[- 3(– 2) + 7(- 6) – 5 × 8]
= 1/2[6 – 42 - 40]
= 1/2 × – 76
= - 38
But, as the area cannot be negative,
The area of ΔADC = 38 square units
Thus, the area of quadrilateral ABCD = Ar. of ABC+ Ar. of ADC
= (42 + 38)
= 80 sq. units



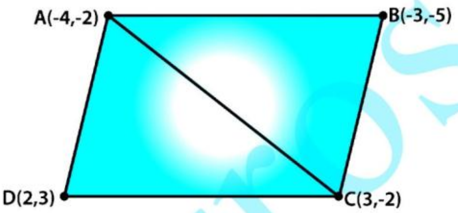
(ii)

Let A(1, 2) , B (6, 2) , C (5, 3) and (3, 4) be the given points

Firstly, area of ΔABC is given by
= 1/2[1(2 - 3) + 6(3 - 2) + 5(2 - 2)]
= 1/2[-1 + 6 × (1) + 0]
= 1/2[– 1 + 6]
= 5/2
Now, area of ΔADC is given by
= 1/2[1(3 - 4) + 5(4 - 2) + 3(2 - 3)]
= 1/2[-1 × 5 × 2 + 3(-1)]
= 1/2[-1 + 10 - 3]
= 1/2[6]
= 3
Thus, Area of quadrilateral ABCD = Area of ABC + Area of ADC

$$= \left(\frac{5}{2} + 3\right) \text{ sq. units}$$

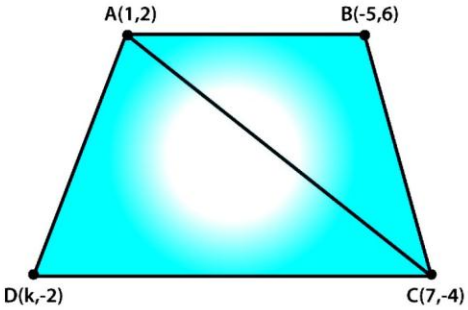
$$= \frac{11}{2} \text{ sq. units}$$



(iii)

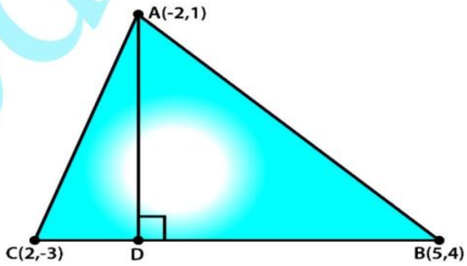
Let A (- 4, 2), B(– 3, – 5), C (3,- 2) and D(2, 3) be the given points
Firstly, area of ΔABC is given by
= 1/2|(- 4)(- 5 + 2) - 3(-2 + 2) + 3(- 2 + 5)|
= 1/2|(-4)(-3) – 3(0) + 3(3)|
= 21/2
Now, the area of ΔACD is given by
= 1/2|(– 4)(3 + 2) + 2(– 2 + 2) + 3(– 2 – 3)|
= 1/2|- 4(5) + 2(0) + 3(- 5)|= (- 35)/2
But, as the area can't negative,
The area of ΔADC = 35/2
Thus, the area of quadrilateral (ABCD) = ar(ΔABC) + ar(ΔADC)
= 21/2 + 35/2
= 56/2
= 28 sq. units

3. The four vertices of a quadrilateral are (1, 2), (-5, 6), (7, -4) and (k, -2) taken in order. If the area of the quadrilateral is zero, find the value of k.
Solution:



Let A(1, 2), B(-5, 6), C(7, -4) and D(k, -2) be the given points
Firstly, area of ΔABC is given by
= 1/2|(1)(6 + 4) - 5(-4 + 2) + 7(2 - 6)|
= 1/2|10 + 30 - 28|
= ½ x 12
= 6
Now, the area of ΔACD is given by
= 1/2|(1)(-4 + 2) + 7(– 2 - 2) + k(2 + 4)|
= 1/2|- 2 + 7x(-4) + k(6)|
= (- 30 + 6k)/2
= -15 + 3k
= 3k - 15
Thus, the area of quadrilateral (ABCD) = ar(ΔABC) + ar(ΔADC)
= 6 + 3k - 15
= 3k - 9
But, given area of quadrilateral is O.
So, 3k – 9 = 0
k = 9/3 = 3

4. The vertices of ΔABC are (-2, 1), (5, 4) and (2, -3) respectively. Find the area of the triangle and the length of the altitude through A.
Solution:



Let A(-2, 1), B(5, 4) and C(2, -3) be the vertices of ΔABC.
And let AD be the altitude through A.
Area of ΔABC is given by
= 1/2|(-2)(4 + 3) - 5(-3 - 1) + 2(1 - 4)|
= 1/2|-14 - 20 - 6|
= ½ x -40
= -20
But as the area cannot be negative,
The area of ΔABC = 20 sq. units
Now,

$$BC = \sqrt{(5-2)^2 + (4+3)^2}$$

$$BC = \sqrt{(3)^2 + (7)^2}$$

$$BC = \sqrt{58}$$

We know that, area of triangle
= ½ x Base x Altitude
20 = ½ x √58 x AD
AD = 40/ √58
Therefore, the altitude AD = 40/ √58

5. Show that the following sets of points are collinear.
(a) (2, 5), (4, 6) and (8, 8)   (ii) (1, -1), (2, 1) and (4, 5)
Solution:

Condition: For the 3 points to be collinear the area of the triangle formed with the 3 points has to be zero.

(a) Let A(2, 5), B(4, 6) and C(8, 8) be the given points
Then, the area of ΔABC is given by

$$= \frac{1}{2}\left[2(6-8) + 4(8-5) + 8(5-6)\right]$$

$$= \frac{1}{2}\left[2 \times (-2) + 4 \times 3 + 8 \times (-1)\right]$$

$$= \frac{1}{2}\left[-4 + 12 - 8\right]$$

$$= \frac{1}{2} \times 0$$

$$= 0$$

Since, the area (ΔABC) = 0 the given points (2, 5), (4, 6) and (8, 8) are collinear.

(b) Let A(1, -1), B(2, 1) and C(4, 5) be the given points
Then, the area of ΔABC is given by

$= \frac{1}{2}[1(1-5)+2(5+1)+4(-1-1)]$

$= \frac{1}{2}[-4+12-8]$

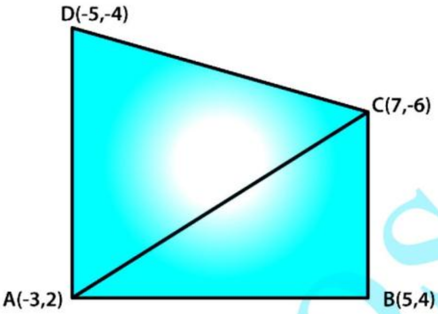
$= \frac{1}{2}\times 0$

$= 0$

Since, the area (ΔABC) = 0 the given points (1, -1), (2, 1) and (4, 5) are collinear.

6. Find the area of a quadrilateral ABCD, the coordinates of whose vertices are A (-3, 2), B (5, 4), C (7, 6) and D (-5, -4).

Solution:



Let's join AC. So, we have 2 triangles formed.
Now, the ar (ABCD) = Ar (ΔABC) + Ar (ΔACD)
Area of ΔABC is given by,

$\frac{1}{2}|-3(4+6)+5(-6-2)+7(2-4)|$

$= \frac{1}{2}|-30-40-14|$

$= \frac{1}{2}\times 84$

$= 42$ sq. units

Next, the area of ΔACD is given by,

$\frac{1}{2}|-3(-6+4)+7(-4-2)-5(2+6)|$

$= \frac{1}{2}|6-42-40|$

$= \frac{1}{2}\times 76$

$= 38$ sq. units

Thus, the area (ABCD) = 42 + 38 = 80 sq. units

R D Sharma Solutions For Class 10 Maths Chapter 14 - Co-ordinate Geometry

7. In △ABC, the coordinates of vertex A are (0, -1) and D(1, 0) and E(0, 1) respectively the mid-points of the sides AB and AC. If F is the mid-point of side BC, find the area of △DEF.
Solution:

Let B(a, b) and C(p, q) be the other two vertices of the △ABC
Now, we know that D is the mid-point of AB
So, coordinates of D = (0+a/ 2, -1+b/ 2)
(1, 0) = (a/2, b-1/2)
1 = a/2 and 0 = (b-1)/ 2
a = 2 and b = 1
Hence, the coordinates of B = (2, 1)
And, now
E is the mid-point of AC.
So, coordinates of E = (0+p/ 2, -1+q/ 2)
(0, 1) = (p/2 , (q -1)/ 2)
p/2 = 0 and 1 = (q - 1)/2
p = 0 and 2 = q -1
p = 0 and q = 3
Hence, the coordinates of C = (0, 3)
Again, F is the mid-point of BC
Coordinates of F = (2+0/ 2, 1+3/ 2) = (1, 2)
Thus, the area of △DEF is given by

$$= \frac{1}{2}|1(1-2)+0(2-0)+1(0-1)|$$
$$= \frac{1}{2}|-1+0-1|$$
$$= \frac{1}{2} \times 2$$
$$= 1 \text{ sq. unit}$$

8. Find the area of the triangle PQR with Q (3, 2) and the mid-points of the sides through Q being (2, -1) and (1, 2).
Solution:

Let the coordinates of P and R be (x_1, y_1) and (x_2, y_2) respectively.
And, let the points E and F be the mid-points of PQ and QR respectively.

$$\frac{x_1+3}{2} = 1, \frac{y_1+2}{2} = 2 \text{ and } \frac{x_2+3}{2} = 2, \frac{y_2+2}{2} = -1$$

$x_1 + 3 = 2$, $y_1 + 2 = 4$ and $x_2 + 3 = 4$, $y_2 + 2 = -2$
$x_1 = -1$, $y_1 = 2$ and $x_2 = 1$, $y_2 = -4$
Hence, the coordinates of P and R are (-1, 2) and (1, 0) respectively.
Therefore, the area of △PQR is given by

$= \frac{1}{2}|(3\times2+(-1)\times(-4)+1\times2)-(-1\times2+1\times2+3\times(-4))|$

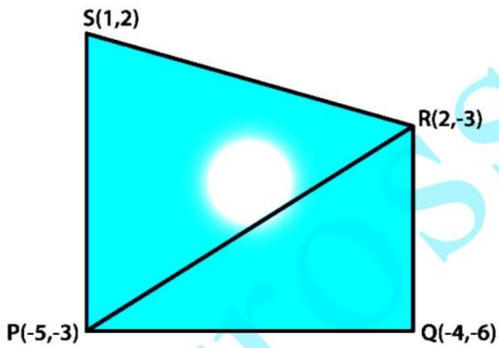
$= \frac{1}{2}|(6+4+2)-(-2+2-12)|$

$= \frac{1}{2}\times|24|$

$= \frac{1}{2}\times24$

= 12 sq. units

9. If P(-5, -3), Q(-4, -6), R(2, -3) and S(1, 2) are the vertices of a quadrilateral PQRS, find its area.
Solution:



First, let's join P and R.
Then,
Area of △PSR is given by

$= \frac{1}{2}|-5(2+3)+1(-3+3)+2(-3-2)|$

$= \frac{1}{2}|-5\times5+1\times0+2\times(-5)|$

$= \frac{1}{2}|-25+0-10|$

$= \frac{1}{2}|-35|$

$= \frac{35}{2}$

And, now
Area of △PQR is given by

$= \frac{1}{2}|-5(-6+3)-4(-3+3)+2(-3+6)|$

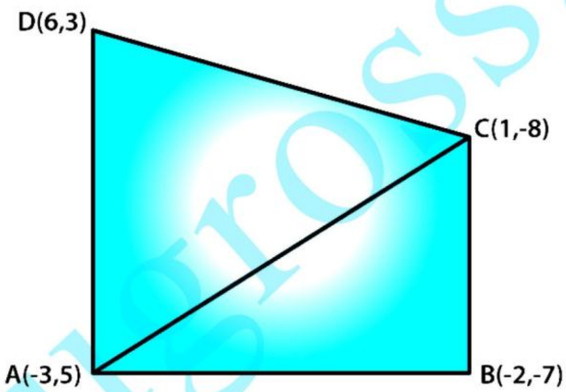
$= \frac{1}{2}|-5\times(-3)-4\times 0+2\times 3|$

$= \frac{1}{2}|15+0+6|$

$= \frac{21}{2}$

Thus,
Area of quad. PQRS = Area of △PSR + Area of △PQR
= 35/2 + 21/2
= 56/2
= 28 sq. units

10. If A (-3, 5), B(-2, -7), C(1, -8) and D(6, 3) are the vertices of a quadrilateral ABCD, find its area.
Solution:



Let's join A and C.
So, we get △ABC and △ADC
Hence,
The Area of quad. ABCD = Area of △ABC + Area of △ADC

$= \frac{1}{2}|-3(-7-(-8))+(-2)(-8-5)+1(5-(-7))|+\frac{1}{2}|-3(3+8)+6(-8-5)+1(5-3)|$

$= \frac{1}{2}|-3(1)+(-2)(-13)+1(12)|+\frac{1}{2}|-3(11)+6(-13)+1(2)|$

$= \frac{1}{2}|-3+26+12|+\frac{1}{2}|-33-78+2|$

$= \frac{1}{2}|35|+\frac{1}{2}|-109|$

$= \frac{1}{2}\times 35+\frac{1}{2}\times 109$

$= \frac{35+109}{2}$

$= \frac{144}{2}$

= 72 sq. units

Therefore, the area of the quadrilateral ABCD is 72 sq. units

11. For what value of a the points (a, 1), (1, -1) and (11, 4) are collinear?
Solution:

Let A (a, 1), B (1, -1) and C (11, 4) be the given points
Then the area of △ABC is given by,

$= \frac{1}{2}\{a(-1-4)+1(4-1)+11(1+1)\}$

$= \frac{1}{2}\{-5a+3+22\}$

$= \frac{1}{2}\{-5a+25\}$

We know that for the points to be collinear the area of △ABC has to be zero.
½(-5a + 25) = 0
5a = 25
∴ a = 5

12. Prove that the points (a, b), (a_1, b_1) and (a-a_1, b-b_1) are collinear if $ab_1 = a_1b$
Solution:

Let A (a, b), B (a_1, b_1) and C (a-a_1, b-b_1) be the given points.
So, the area of △ABC is given by,

$= \frac{1}{2}\{a[b_1 - (b - b_1)] + a_1(b - b_1 - b) + (a - a_1)(b - b_1)\}$

$= \frac{1}{2}\{a(b_1 - b + b_1) + a_1(-b_1) + ab - ab_1 - a_1b + a_1b_1\}$

$= \frac{1}{2}\{ab_1 - ab + ab_1 - a_1b_1 + ab - ab_1 - a_1b + a_1b_1\}$

$= \frac{1}{2}\{ab_1 - a_1b\}$

So, only if $ab_1 = a_1b$ the area becomes zero
$\triangle ABC = \frac{1}{2}(0) = 0$
Therefore, the given points are collinear if $ab_1 = a_1b$

13. If the vertices of a triangle are (1,-3), (4,p) and (-9, 7) and its area is 15 sq. units, find the value (s) of p.
Solution:

Let A(1,-3), B(4,p) and C(-9, 7) be the vertices of $\triangle ABC$
Area of $\triangle ABC$ = 15 sq. units

$15 = \frac{1}{2}|1(p-7) + 4(7+3) - 9(-3-p)|$

$15 = \frac{1}{2}|p - 7 + 40 + 27 + 9p|$

$15 = \frac{1}{2}|10p + 60|$

When modulus is removed, two cases arise:

$30 = 10p + 60 \text{ or } 30 = -10p - 60$

$10p = -30 \text{ or } 10p = -90$

$p = -3 \text{ or } p = -9$

14. If (x, y) be on the line joining the two points (1, -3) and (-4, 2). Prove that x + y + 2 = 0
Solution:

Let A (x, y), B (1, -3) and C (-4, 2) be the given points.
Area of $\triangle ABC$ is given by,

$= \frac{1}{2}\{x(-3-2) + 1(2-y) + (-4)(y+3)\}$

$= \frac{1}{2}\{-5x + 2 - y - 4y - 12\}$

$= \frac{1}{2}\{-5x - 5y - 10\}$

As, the three points lie on the same line (that means they are collinear).
Then, the area of $\triangle ABC = 0$
$\frac{1}{2}(-5x - 5y - 10) = 0$

-5x – 5y – 10 = 0
-5(x + y + 2) = 0
x + y + 2 = 0

- Hence proved

15. Find the value of k if points (k, 3), (6, -2) and (-3, 4) are collinear.
Solution:

Let A (k, 3), B (6, -2) and C (-3, 4) be the given points.
Then, the area of △ABC is given by,

$$= \frac{1}{2}\{k(-2-4)+6(4-3)+(-3)(3+2)\}$$

$$= \frac{1}{2}\{-6k+6-15\}$$

$$= \frac{1}{2}\{-6k-9\}$$

As, the points are collinear.
Area of △ABC has to be zero.
½ x (-6k – 9) = 0
-6k – 9 = 0
k = -9/6
∴ k = -3/2

16. Find the value of k, if points A(7, -2), B(5, 1) and C(3, 2k) are collinear.
Solution:

Given,
Points A(7, -2), B(5, 1) and C(3, 2k)
Then, the area of △ABC is given by,

$$= \frac{1}{2}\{7(1-2k)+5(2k+2)+3(-2-1)\}$$

$$= \frac{1}{2}\{7-14k+10k+10-9\}$$

$$= \frac{1}{2}\{-4k+8\}$$

As, the points are collinear.
Area of △ABC has to be zero.
½ (-4k + 8) = 0
-4k + 8 = 0
-4k = -8
∴ k = 2