RD Sharma Solutions for Class 9 Maths Chapter 14 Quadrilaterals

Exercise 14.1 Page No: 14.4

Question 1: Three angles of a quadrilateral are respectively equal to 110^0, 50^0 and 40^0. Find its fourth angle.

Solution:

Three angles of a quadrilateral are 110^0, 50^0 and 40^0

Let the fourth angle be 'x'

We know, sum of all angles of a quadrilateral = 360^0

$110^0 + 50^0 + 40^0 + x^0 = 360^0$

=> $x = 360^0 – 200^0$

=> $x = 160^0$

Therefore, the required fourth angle is 160^0.

Question 2: In a quadrilateral ABCD, the angles A, B, C and D are in the ratio of 1:2:4:5. Find the measure of each angles of the quadrilateral.

Solution:

Let the angles of the quadrilaterals are A = x, B = 2x, C = 4x and D = 5x

We know, sum of all angles of a quadrilateral = 360^0

$A + B + C + D = 360^0$

$x + 2x + 4x + 5x = 360^0$

$12x = 360^0$

$x = 360^0/12 = 30^0$

Therefore,
$A = x = 30^0$

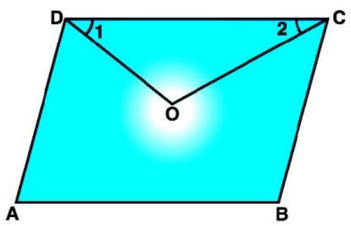
B = 2x = 60^0

C = 4x = 120^0

D = 5x = 150^0

Question 3: In a quadrilateral ABCD, CO and DO are the bisectors of ∠C and ∠D respectively. Prove that ∠COD = 1/2 (∠A + ∠B).

Solution:



In ΔDOC,

∠CDO + ∠COD + ∠DCO = 180^0 [Angle sum property of a triangle]

or 1/2∠CDA + ∠COD + 1/2∠DCB = 180^0

∠COD = 180^0 – 1/2(∠CDA + ∠DCB) .....(i)

Also
We know, sum of all angles of a quadrilateral = 360^0

∠CDA + ∠DCB = 360^0 - (∠DAB + ∠CBA) ......(ii)

Substituting (ii) in (i)

∠COD = 180^0 – 1/2{360^0 - (∠DAB + ∠CBA) }

We can also write, ∠DAB = ∠A and ∠CBA = ∠B

∠COD = 180^0 – 180^0 + 1/2(∠A + ∠B))

∠COD = 1/2(∠A + ∠B)
Hence Proved.

Question 4: The angles of a quadrilateral are in the ratio 3:5:9:13. Find all the angles of the quadrilateral.

Solution:

The angles of a quadrilateral are 3x, 5x, 9x and 13x respectively.

We know, sum of all interior angles of a quadrilateral = 360^0

Therefore, $3x + 5x + 9x + 13x = 360^0$

$30x = 360^0$

or $x = 12^0$

Hence, angles measures are

$3x = 3(12) = 36^0$

$5x = 5(12) = 60^0$

$9x = 9(12) = 108^0$

$13x = 13(12) = 156^0$

Exercise 14.2 Page No: 14.18

Question 1: Two opposite angles of a parallelogram are $(3x - 2)^0$ and $(50 - x)^0$. Find the measure of each angle of the parallelogram.

Solution:

Given**:** Two opposite angles of a parallelogram are $(3x - 2)^0$ and $(50 - x)^0$.

We know, opposite sides of a parallelogram are equal.

$(3x - 2)^0 = (50 - x)^0$

$3x + x = 50 + 2$

$4x = 52$

$x = 13$

Angle x is 13^0

Therefore,

$(3x-2)^0 = (3(13) - 2) = 37^0$

$(50-x)^0 = (50 - 13) = 37^0$

Adjacent angles of a parallelogram are supplementary.

$x + 37 = 180^0$

$x = 180^0 - 37^0 = 143^0$

Therefore, required angles are : 37^0, 143^0, 37^0 and 143^0.

Question 2: If an angle of a parallelogram is two-third of its adjacent angle, find the angles of the parallelogram.

Solution:

Let the measure of the angle be x. Therefore, measure of the adjacent angle is 2x/3.

We know, adjacent angle of a parallelogram is supplementary.

$x + 2x/3 = 180^0$

$3x + 2x = 540^0$

$5x = 540^0$

or $x = 108^0$

Measure of second angle is $2x/3 = 2(108^0)/3 = 72^0$
Similarly measure of 3rd and 4th angles are 108^0 and 72^0

Hence, four angles are 108^0, 72^0, 108^0, 72^0

Question 3: Find the measure of all the angles of a parallelogram, if one angle is 24^0 less than twice the smallest angle.
Solution:
Given: One angle of a parallelogram is 24^0 less than twice the smallest angle.
Let x be the smallest angle, then

$x + 2x – 24^0 = 180^0$

$3x – 24^0 = 180^0$

$3x = 108^0 + 24^0$

$3x = 204^0$

$x = 204^0/3 = 68^0$

So, $x = 68^0$

Another angle $= 2x – 24^0 = 2(68^0) – 24^0 = 112^0$

Hence, four angles are 68^0, 112^0, 68^0, 112^0.

Question 4: The perimeter of a parallelogram is 22cm. If the longer side measures 6.5cm what is the measure of the shorter side?

Solution:

Let x be the shorter side of a parallelogram.

Perimeter = 22 cm

Longer side = 6.5 cm

Perimeter = Sum of all sides = x + 6.5 + 6.5 + x

22 = 2 (x + 6.5)

11 = x + 6.5

or x = 11 – 6.5 = 4.5

Therefore, shorter side of a parallelogram is 4.5 cm

Exercise 14.3 Page No: 14.38

Question 1: In a parallelogram ABCD, determine the sum of angles ∠C and ∠D.
Solution:
In a parallelogram ABCD , ∠C and ∠D are consecutive interior angles on the same side of the transversal CD.

So, ∠C + ∠D = 180^0

Question 2: In a parallelogram ABCD, if ∠B = 135^0, determine the measures of its other angles.
Solution:
Given: In a parallelogram ABCD, if ∠B = 135^0

Here, ∠A = ∠C, ∠B = ∠D and ∠A + ∠B = 180^0

∠A + 135^0 = 180^0

∠A = 45^0

Answer:
∠A = ∠C = 45^0

∠B = ∠D = 135^0

Question 3: ABCD is a square. AC and BD intersect at O. State the measure of ∠AOB.
Solution:
We know, diagonals of a square bisect each other at right angle.

So, ∠AOB = 90^0

Question 4: ABCD is a rectangle with ∠ABD = 40^0. Determine ∠DBC.
Solution:
Each angle of a rectangle = 90^o

So, ∠ABC = 90^0

∠ABD = 40^0 (given)

Now, ∠ABD + ∠DBC = 90^0
40^0 + ∠DBC = 90^0
or ∠DBC = 50^0 .

Exercise 14.4 Page No: 14.55

Question 1: In a ΔABC, D, E and F are, respectively, the mid points of BC, CA and AB. If the lengths of sides AB, BC and CA are 7 cm, 8 cm and 9 cm, respectively, find the perimeter of ΔDEF.

Solution:
Given: AB = 7 cm, BC = 8 cm, AC = 9 cm

In ΔABC,

In a ΔABC, D, E and F are, respectively, the mid points of BC, CA and AB.

According to Midpoint Theorem:

EF = 1/2BC, DF = 1/2 AC and DE = 1/2 AB

Now, Perimeter of ΔDEF = DE + EF + DF

= 1/2 (AB + BC + AC)

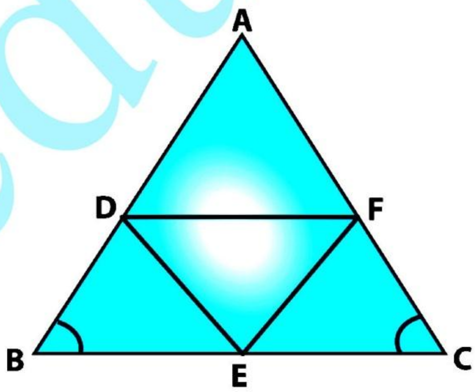
= 1/2 (7 + 8 + 9)

= 12

Perimeter of ΔDEF = 12cm

Question 2: In a ΔABC, ∠A = 50^0, ∠B = 60^0 and ∠C = 70^0. Find the measures of the angles of the triangle formed by joining the mid-points of the sides of this triangle.

Solution:



In ΔABC,

D, E and F are mid points of AB,BC and AC respectively.

In a Quadrilateral DECF:

By Mid-point theorem,

DE || AC => DE = AC/2

And CF = AC/2
=> DE = CF

Therefore, DECF is a parallelogram.

∠C = ∠D = 70^0
[Opposite sides of a parallelogram]

Similarly,

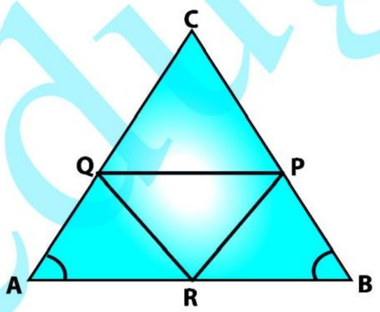
ADEF is a parallelogram, ∠A = ∠E = 50^0

BEFD is a parallelogram, ∠B = ∠F = 60^0

Hence, Angles of ΔDEF are: ∠D = 70^0, ∠E = 50^0, ∠F = 60^0.

Question 3: In a triangle, P, Q and R are the mid points of sides BC, CA and AB respectively. If AC = 21 cm, BC = 29 cm and AB = 30 cm, find the perimeter of the quadrilateral ARPQ.

Solution:



In ΔABC,

R and P are mid points of AB and BC

By Mid-point Theorem

RP || AC => RP = AC/2

In a quadrilateral, ARPQ

RP || AQ => RP = AQ
[A pair of side is parallel and equal]

Therefore, ARPQ is a parallelogram.

Now, AR = AB/2 = 30/2 = 15 cm
[AB = 30 cm (Given)]

AR = QP = 15 cm
[Opposite sides are equal]

And RP = AC/2 = 21/2 = 10.5 cm
[AC = 21 cm (Given)]

RP = AQ = 10.5cm
[Opposite sides are equal]

Now,

Perimeter of ARPQ = AR + QP + RP +AQ

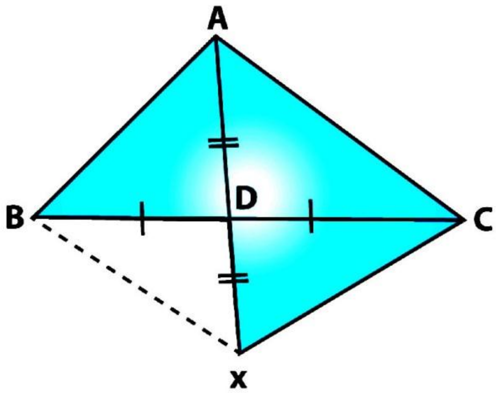
= 15 +15 +10.5 +10.5

= 51

Perimeter of quadrilateral ARPQ is 51 cm.

Question 4: In a ΔABC median AD is produced to X such that AD = DX. Prove that ABXC is a parallelogram.

Solution:



In a quadrilateral ABXC,

AD = DX [Given]

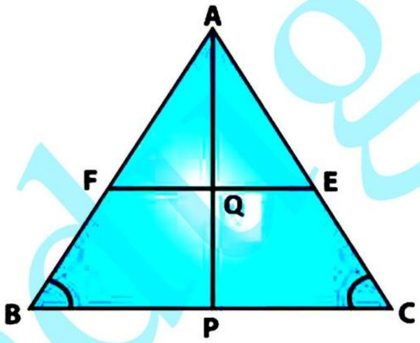
BD = DC [Given]

From figure, Diagonals AX and BC bisect each other.

ABXC is a parallelogram.
Hence Proved.

Question 5: In a ΔABC, E and F are the mid-points of AC and AB respectively. The altitude AP to BC intersects FE at Q. Prove that AQ = QP.

Solution:

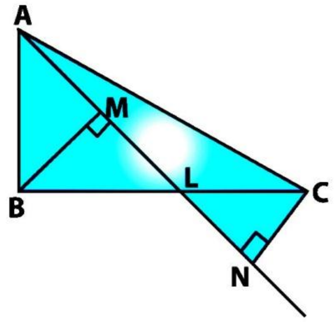


In a ΔABC
E and F are mid points of AC and AB (Given)
EF || FE => EF = BC/2 and
[By mid-point theorem]
In ΔABP
F is the mid-point of AB, again by mid-point theorem
FQ || BP

Q is the mid-point of AP
AQ = QP
Hence Proved.

Question 6: In a ΔABC, BM and CN are perpendiculars from B and C respectively on any line passing through A. If L is the mid-point of BC, prove that ML = NL.

Solution:



Given that,

In ΔBLM and ΔCLN

∠BML = ∠CNL = 90^0

BL = CL [L is the mid-point of BC]

∠MLB = ∠NLC [Vertically opposite angle]

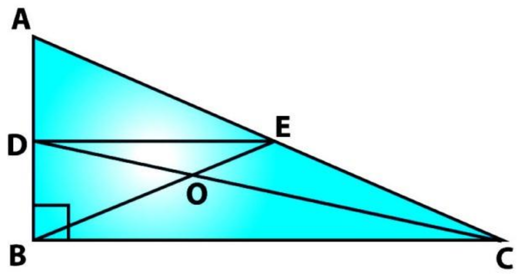
By ASA criterion:

ΔBLM ≅ ΔCLN

So, LM = LN [By CPCT]

Question 7: In figure, triangle ABC is a right-angled triangle at B. Given that AB = 9 cm, AC = 15 cm and D, E are the mid-points of the sides AB and AC respectively, calculate

(i) The length of BC **(ii) The area of ΔADE.**



Solution:

In ΔABC,
∠B=90^0 (Given)
AB = 9 cm, AC = 15 cm (Given)

By using Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$\Rightarrow 15^2 = 9^2 + BC^2$

$\Rightarrow BC^2 = 225 - 81 = 144$

or BC = 12

Again,

AD = DB = AB/2 = 9/2 = 4.5 cm [D is the mid−point of AB

D and E are mid-points of AB and AC

DE ∥ BC => DE = BC/2 [By mid−point theorem]

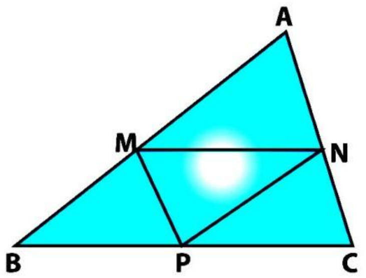
Now,
Area of ΔADE = 1/2 x AD x DE

= 1/2 x 4.5 x 6

=13.5

Area of ΔADE is 13.5 cm^2

Question 8: In figure, M, N and P are mid-points of AB, AC and BC respectively. If MN = 3 cm, NP = 3.5 cm and MP = 2.5 cm, calculate BC, AB and AC.



Solution:

Given: MN = 3 cm, NP = 3.5 cm and MP = 2.5 cm.

M and N are mid-points of AB and AC

By mid−point theorem, we have

MN||BC => MN = BC/2

or BC = 2MN

BC = 6 cm

[MN = 3 cm given)

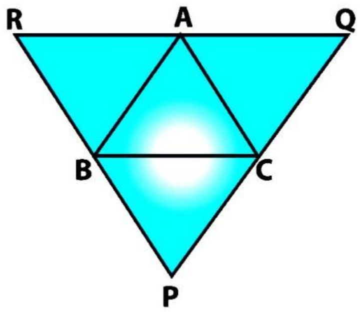
Similarly,

AC = 2MP = 2 (2.5) = 5 cm

AB = 2 NP = 2 (3.5) = 7 cm

Question 9: ABC is a triangle and through A, B, C lines are drawn parallel to BC, CA and AB respectively intersecting at P, Q and R. Prove that the perimeter of ΔPQR is double the perimeter of ΔABC.

Solution:



ABCQ and ARBC are parallelograms.

Therefore, BC = AQ and BC = AR

=>AQ = AR

=>A is the mid-point of QR

Similarly B and C are the mid points of PR and PQ respectively.

By mid–point theorem, we have

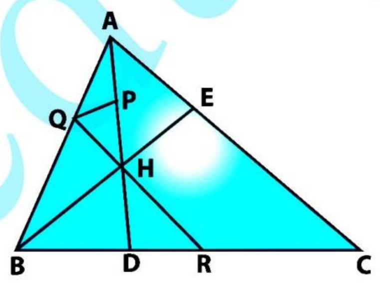
AB = PQ/2, BC = QR/2 and CA = PR/2

or PQ = 2AB, QR = 2BC and PR = 2CA

=>PQ + QR + RP = 2 (AB + BC + CA)

=> Perimeter of ΔPQR = 2 (Perimeter of ΔABC)
Hence proved.

Question 10: In figure, BE ⊥ AC, AD is any line from A to BC intersecting BE in H. P, Q and R are respectively the mid-points of AH, AB and BC. Prove that ∠PQR = 90^0.



Solution:

BE⊥AC and P, Q and R are respectively mid-point of AH, AB and BC. (Given)

In ΔABC, Q and R are mid-points of AB and BC respectively.

By Mid-point theorem:

QR || AC .....(i)

In ΔABH, Q and P are the mid-points of AB and AH respectively

QP || BH ....(ii)

But, BE⊥AC

From (i) and (ii) we have,

QP⊥QR

=>∠PQR = 90^0

Hence Proved.

Exercise VSAQs Page No: 14.62

Question 1: In a parallelogram ABCD, write the sum of angles A and B.

Solution:
In parallelogram ABCD, Adjacent angles of a parallelogram are supplementary.

Therefore, $\angle A + \angle B = 180^0$

Question 2: In a parallelogram ABCD, if $\angle D = 115^0$, then write the measure of $\angle A$.
Solution:

In a parallelogram ABCD,
$\angle D = 115^0$ (Given)

Since, $\angle A$ and $\angle D$ are adjacent angles of parallelogram.

We know, Adjacent angles of a parallelogram are supplementary.

$\angle A + \angle D = 180^0$

$\angle A = 180^0 - 115^0 = 65^0$

Measure of $\angle A$ is 65^0.

Question 3: PQRS is a square such that PR and SQ intersect at O. State the measure of $\angle POQ$.

Solution:

PQRS is a square such that PR and SQ intersect at O. (Given)

We know, diagonals of a square bisects each other at 90 degrees.

So, $\angle POQ = 90^0$

Question 4: In a quadrilateral ABCD, bisectors of angles A and B intersect at O such that $\angle AOB = 75°$, then write the value of $\angle C + \angle D$.

Solution:

$\angle AOB = 75^o$ (given)

In a quadrilateral ABCD, bisectors of angles A and B intersect at O, then

∠AOB = 1/2 (∠ADC + ∠ABC)

or ∠AOB = 1/2 (∠D + ∠C)

By substituting given values, we get

75° = 1/2 (∠D + ∠C)

or ∠C + ∠D = 150°

Question 5: The diagonals of a rectangle ABCD meet at O. If ∠BOC = 44°, find ∠OAD.

Solution:
ABCD is a rectangle and ∠BOC = 44° (given)

∠AOD = ∠BOC (vertically opposite angles)

∠AOD = ∠BOC = 44°

∠OAD = ∠ODA (Angles facing same side)

and OD = OA
Since sum of all the angles of a triangle is 180°, then

So, ∠OAD = 1/2 (180° - 44°) = 68°

Question 6: If PQRS is a square, then write the measure of ∠SRP.

Solution:
PQRS is a square.
=> All side are equal, and each angle is 90° degrees and diagonals bisect the angles.

So, ∠SRP = 1/2 (90°) = 45°

Question 7: If ABCD is a rectangle with ∠BAC = 32°, find the measure of ∠DBC.

Solution:

ABCD is a rectangle and ∠BAC=32° (given)

We know, diagonals of a rectangle bisects each other.
AO = BO

∠DBA = ∠BAC = 32° (Angles facing same side)

Each angle of a rectangle = 90 degrees

So, ∠DBC + ∠DBA = 90°

or ∠DBC + 32° = 90°

or ∠DBC = 58°

Question 8: If ABCD is a rhombus with ∠ABC = 56°, find the measure of ∠ACD.

Solution:
In a rhombus ABCD,
<ABC = 56°
So, <BCD = 2 (<ACD) (Diagonals of a rhombus bisect the interior angles)
or <ACD = 1/2 (<BCD) .....(1)

We know, consecutive angles of a rhombus are supplementary.

∠BCD + ∠ABC = 180°

∠BCD = 180° – 56° = 124°

Equation (1) => <ACD = 1/2 x 124° = 62°

Question 9: The perimeter of a parallelogram is 22 cm. If the longer side measure 6.5 cm, what is the measure of shorter side?

Solution:
Perimeter of a parallelogram = 22 cm. (Given)
Longer side = 6.5 cm
Let x be the shorter side.
Perimeter = 2x + 2×6.5
22 = 2x + 13
2x = 22 - 13 = 9
or x = 4.5
Measure of shorter side is 4.5 cm.

Question 10: If the angles of a quadrilateral are in the ratio 3:5:9:13, then find the measure of the smallest angle.

Solution:

Angles of a quadrilateral are in the ratio 3 : 5 : 9 : 13 (Given)

Let the sides are 3x, 5x, 9x, 13x

We know, sum of all the angles of a quadrilateral = 360°

$3x + 5x + 9x + 13x = 360^{\circ}$

$30x = 360^{\circ}$

$x = 12^{\circ}$

Length of smallest angle = $3x = 3(12) = 36^{\circ}$.