

## EXERCISE 22.1

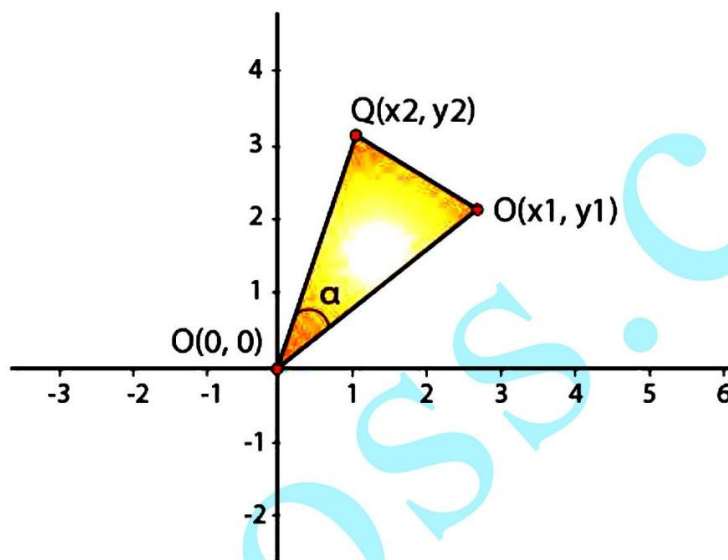
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1. If the line segment joining the points  $P(x_1, y_1)$  and  $Q(x_2, y_2)$  subtends an angle  $\alpha$  at the origin  $O$ , prove that :  $OP \cdot OQ \cos \alpha = x_1 x_2 + y_1 y_2$ .

**Solution:**

Given,

Two points  $P$  and  $Q$  subtends an angle  $\alpha$  at the origin as shown in figure:



From figure we can see that points  $O$ ,  $P$  and  $Q$  forms a triangle.  
Clearly in  $\triangle OPQ$  we have:

$$\cos \alpha = \frac{OP^2 + OQ^2 - PQ^2}{2OP.OQ} \quad \{\text{from cosine formula}\}$$

$$2 OP.OQ \cos \alpha = OP^2 + OQ^2 - PQ^2 \dots \text{equation (1)}$$

We know that the, coordinates of O are  $(0, 0) \Rightarrow x_2 = 0$  and  $y_2 = 0$

Coordinates of P are  $(x_1, y_1) \Rightarrow x_1 = x_1$  and  $y_1 = y_1$

By using distance formula we have:

$$\begin{aligned} OP &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(x_1 - 0)^2 + (y_1 - 0)^2} \\ &= \sqrt{x_1^2 + y_1^2} \end{aligned}$$

$$\begin{aligned} \text{Similarly, } OQ &= \sqrt{(x_2 - 0)^2 + (y_2 - 0)^2} \\ &= \sqrt{x_2^2 + y_2^2} \end{aligned}$$

$$\text{And, } PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\therefore OP^2 + OQ^2 - PQ^2 = x_1^2 + y_1^2 + x_2^2 + y_2^2 - \{(x_2 - x_1)^2 + (y_2 - y_1)^2\}$$

By using  $(a-b)^2 = a^2 + b^2 - 2ab$

$$\therefore OP^2 + OQ^2 - PQ^2 = 2x_1 x_2 + 2y_1 y_2 \dots \text{Equation (2)}$$

So now from equation (1) and (2) we have:

$$2OP.OQ \cos \alpha = 2x_1 x_2 + 2y_1 y_2$$

$$OP.OQ \cos \alpha = x_1 x_2 + y_1 y_2$$

Hence Proved.

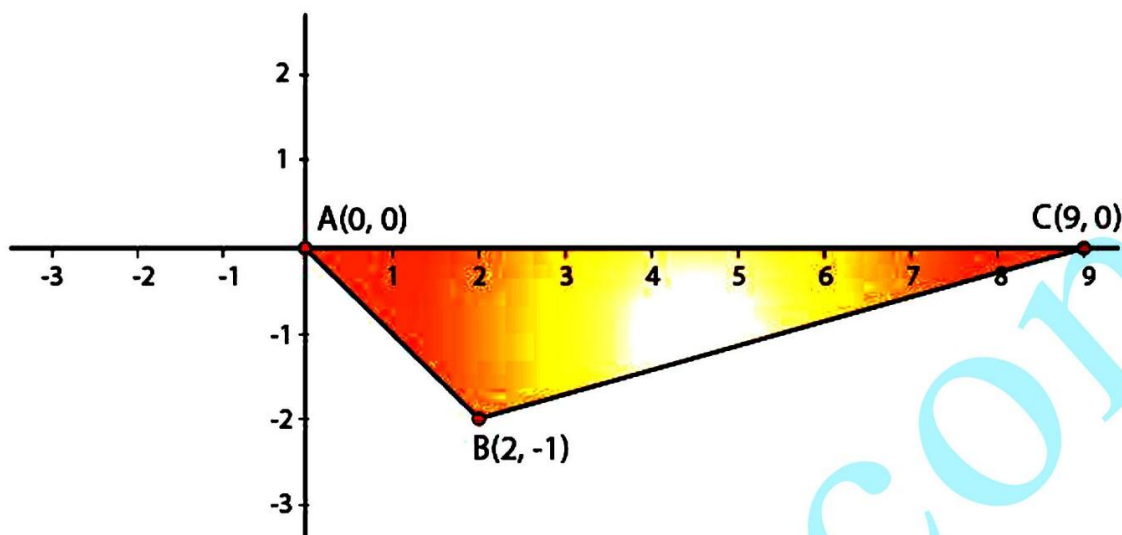
**2. The vertices of a triangle ABC are A(0, 0), B (2, -1) and C (9, 0). Find cos B.**

**Solution:**

Given:

The coordinates of triangle.

From the figure,



By using cosine formula,

In  $\triangle ABC$ , we have:

$$\cos B = \frac{AB^2 + BC^2 - AC^2}{2AB \cdot BC}$$

Now by using distance formula we have:

$$AB = \sqrt{(2-0)^2 + (-1-0)^2} = \sqrt{5}$$

$$BC = \sqrt{(9-2)^2 + (0-(-1))^2} = \sqrt{7^2 + 1^2} = \sqrt{50}$$

$$\text{And, } AC = \sqrt{(9-0)^2 + (0-0)^2} = 9$$

Now substitute the obtained values in the cosine formula, we get

$$\therefore \cos B = \frac{(\sqrt{5})^2 + (\sqrt{50})^2 - 9^2}{2\sqrt{5}\sqrt{50}} = \frac{55-81}{2\sqrt{5}\sqrt{2 \times 25}} = \frac{-26}{10\sqrt{10}} = \frac{-13}{5\sqrt{10}}$$

3. Four points A (6, 3), B (-3, 5), C (4, -2) and D (x, 3x) are given in such a way

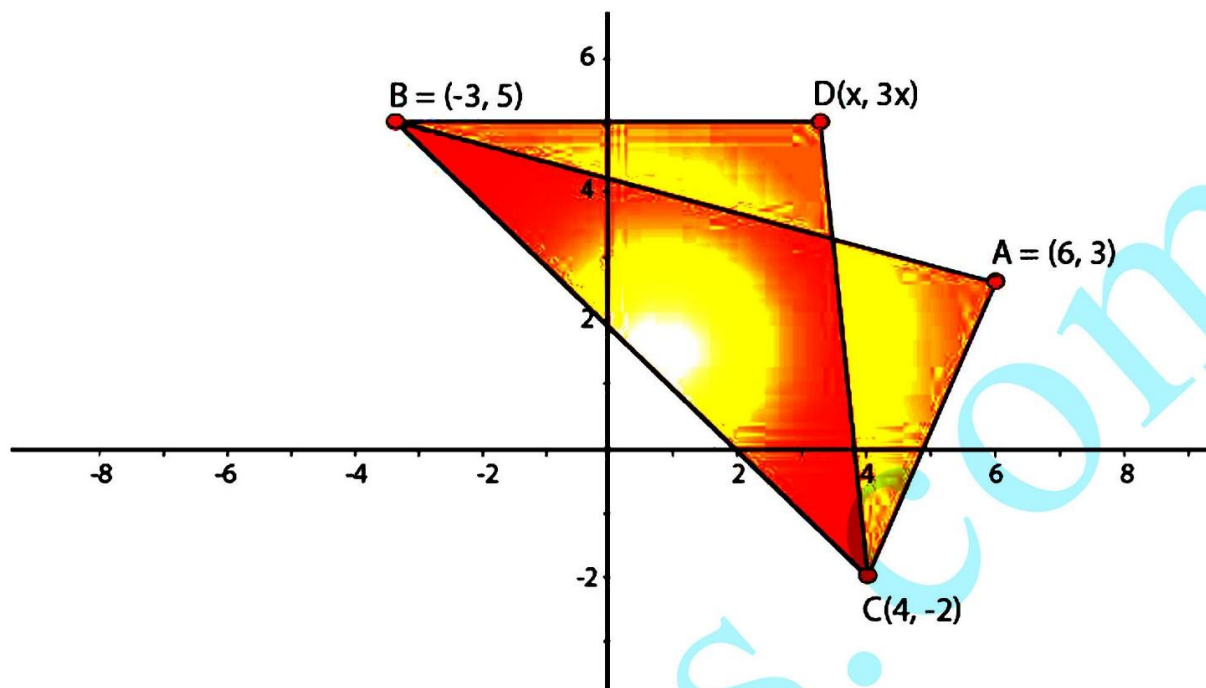
that  $\frac{\triangle DBC}{\triangle ABC} = \frac{1}{2}$ , find x.

**Solution:**

**Given:**

The coordinates of triangle are shown in the below figure.

$$\text{Also, } \frac{\triangle DBC}{\triangle ABC} = \frac{1}{2}$$



Now let us consider Area of a  $\Delta PQR$

Where,  $P(x_1, y_1)$ ,  $Q(x_2, y_2)$  and  $R(x_3, y_3)$  be the 3 vertices of  $\Delta PQR$ .

So, Area of  $(\Delta PQR) = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$$\begin{aligned} \text{Area of } (\Delta DBC) &= \frac{1}{2} [x(5 - (-2)) + (-3)(-2 - 3x) + 4(3x - 5)] \\ &= \frac{1}{2} [7x + 6 + 9x + 12x - 20] = 14x - 7 \end{aligned}$$

$$\begin{aligned} \text{Similarly, area of } (\Delta ABC) &= \frac{1}{2} [6(5 - (-2)) + (-3)(-2 - 3) + 4(3 - 5)] \\ &= \frac{1}{2} [42 + 15 - 8] = \frac{49}{2} = 24.5 \end{aligned}$$

$$\therefore \frac{\Delta DBC}{\Delta ABC} = \frac{1}{2} = \frac{14x-7}{24.5}$$

$$24.5 = 28x - 14$$

$$28x = 38.5$$

$$x = 38.5/28$$

$$= 1.375$$

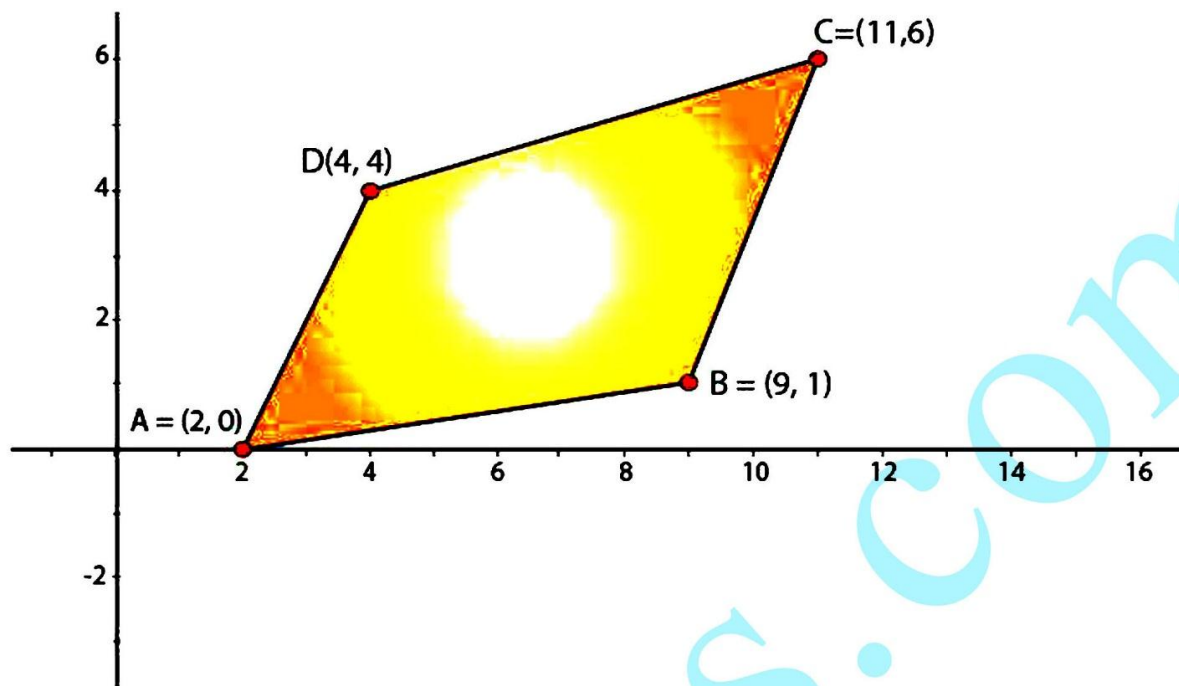
4. The points A (2, 0), B (9, 1), C (11, 6) and D (4, 4) are the vertices of a quadrilateral ABCD. Determine whether ABCD is a rhombus or not.

**Solution:**

Given:

The coordinates of 4 points that form a quadrilateral is shown in the below figure





Now by using distance formula, we have:

$$AB = \sqrt{(9 - 2)^2 + (1 - 0)^2} = \sqrt{7^2 + 1} = \sqrt{50}$$

$$BC = \sqrt{(11 - 9)^2 + (6 - 1)^2} = \sqrt{2^2 + 5^2} = \sqrt{29}$$

It is clear that,  $AB \neq BC$  [quad ABCD does not have all 4 sides equal.]

$\therefore$  ABCD is not a Rhombus

## EXERCISE 22.2

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1. Find the locus of a point equidistant from the point (2, 4) and the y-axis.

**Solution:**

Let P (h, k) be any point on the locus and let A (2, 4) and B (0, k).

Then, PA = PB

$$PA^2 = PB^2$$

By using distance formula:

$$\text{Distance of (h, k) from (2, 4)} = \sqrt{(h-2)^2 + (k-4)^2}$$

$$\text{Distance of (h, k) from (0, k)} = \sqrt{(h-0)^2 + (k-k)^2}$$

So both the distances are same.

$$\therefore \sqrt{(h-2)^2 + (k-4)^2} = \sqrt{(h-0)^2 + (k-k)^2}$$

By squaring on both the sides we get,

$$(h-2)^2 + (k-4)^2 = (h-0)^2 + (k-k)^2$$

$$h^2 + 4 - 4h + k^2 - 8k + 16 = h^2 + 0$$

$$k^2 - 4h - 8k + 20 = 0$$

Replace (h, k) with (x, y)

$\therefore$  The locus of point equidistant from (2, 4) and y-axis is

$$y^2 - 4x - 8y + 20 = 0$$

2. Find the equation of the locus of a point which moves such that the ratio of its distance from (2, 0) and (1, 3) is 5: 4.

**Solution:**

Let P (h, k) be any point on the locus and let A (2, 0) and B (1, 3).

So then, PA/ BP = 5/4

$$PA^2 = BP^2 = 25/16$$

$$\text{Distance of (h, k) from (2, 0)} = \sqrt{(h-2)^2 + (k-0)^2}$$

$$\text{Distance of (h, k) from (1, 3)} = \sqrt{(h-1)^2 + (k-3)^2}$$

So,

$$\frac{\sqrt{(h-2)^2 + (k-0)^2}}{\sqrt{(h-1)^2 + (k-3)^2}} = \frac{5}{4}$$

By squaring on both the sides we get,

$$16\{(h-2)^2 + k^2\} = 25\{(h-1)^2 + (k-3)^2\}$$

$$16\{h^2 + 4 - 4h + k^2\} = 25\{h^2 - 2h + 1 + k^2 - 6k + 9\}$$

$$9h^2 + 9k^2 + 14h - 150k + 186 = 0$$

Replace (h, k) with (x, y)

∴ The locus of a point which moves such that the ratio of its distance from (2, 0) and (1, 3) is 5: 4 which is

$$9x^2 + 9y^2 + 14x - 150y + 186 = 0$$

3. A point moves as so that the difference of its distances from (ae, 0) and (-ae, 0) is 2a, prove that the equation to its locus is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1, \text{ where } b^2 = a^2(e^2 - 1).$$

**Solution:**

Let P (h, k) be any point on the locus and let A (ae, 0) and B (-ae, 0).

Where, PA - PB = 2a

$$\text{Distance of (h, k) from (ae, 0)} = \sqrt{(h - ae)^2 + (k - 0)^2}$$

$$\text{Distance of (h, k) from (-ae, 0)} = \sqrt{(h - (-ae))^2 + (k - 0)^2}$$

So,

$$\sqrt{(h - ae)^2 + (k - 0)^2} - \sqrt{(h - (-ae))^2 + (k - 0)^2} = 2a$$

$$\sqrt{(h - ae)^2 + (k - 0)^2} = 2a + \sqrt{(h + ae)^2 + (k - 0)^2}$$

By squaring on both the sides we get:

$$(h - ae)^2 + (k - 0)^2 = \left\{ 2a + \sqrt{(h + ae)^2 + (k - 0)^2} \right\}^2$$

$$\Rightarrow h^2 + a^2e^2 - 2aeh + k^2 = 4a^2 + \{(h + ae)^2 + k^2\} + 4a\sqrt{(h + ae)^2 + (k - 0)^2}$$

$$\Rightarrow h^2 + a^2e^2 - 2aeh + k^2$$

$$= 4a^2 + h^2 + 2aeh + a^2e^2 + k^2 + 4a\sqrt{(h + ae)^2 + (k - 0)^2}$$

$$-4aeh - 4a^2 = 4a\sqrt{(h + ae)^2 + (k - 0)^2}$$

$$-4a(eh + a) = 4a\sqrt{(h + ae)^2 + (k - 0)^2}$$

Now again let us square on both the sides we get,

$$(eh + a)^2 = (h + ae)^2 + (k - 0)^2$$

$$e^2h^2 + a^2 + 2aeh = h^2 + a^2e^2 + 2aeh + k^2$$

$$h^2(e^2 - 1) - k^2 = a^2(e^2 - 1)$$

$$\frac{h^2}{a^2} - \frac{k^2}{a^2(e^2 - 1)} = 1$$

$$\frac{h^2}{a^2} - \frac{k^2}{b^2} = 1 \text{ [where, } b^2 = a^2(e^2 - 1)]$$



Now let us replace (h, k) with (x, y)

The locus of a point such that the difference of its distances from (ae, 0) and (-ae, 0) is 2a.

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{Where } b^2 = a^2(e^2 - 1)$$

Hence proved.

**4. Find the locus of a point such that the sum of its distances from (0, 2) and (0, -2) is 6.**

**Solution:**

Let P (h, k) be any point on the locus and let A (0, 2) and B (0, -2).

Where, PA + PB = 6

$$\text{Distance of (h, k) from (0, 2)} = \sqrt{(h-0)^2 + (k-2)^2}$$

$$\text{Distance of (h, k) from (0, -2)} = \sqrt{(h-0)^2 + (k-(-2))^2}$$

So,

$$\sqrt{(h)^2 + (k-2)^2} + \sqrt{(h)^2 + (k+2)^2} = 6$$

$$\sqrt{(h)^2 + (k-2)^2} = 6 - \sqrt{(h)^2 + (k+2)^2}$$

By squaring on both the sides we get,

$$h^2 + (k-2)^2 = \left\{ 6 - \sqrt{h^2 + (k+2)^2} \right\}^2$$

$$\Rightarrow h^2 + 4 - 4k + k^2 = 36 + \{h^2 + k^2 + 4k + 4\} - 12\sqrt{h^2 + (k+2)^2}$$

$$\Rightarrow -8k - 36 = -12\sqrt{h^2 + (k+2)^2}$$

$$\Rightarrow -4(2k + 9) = -12\sqrt{h^2 + (k+2)^2}$$

Now, again let us square on both the sides we get,

$$(2k + 9)^2 = \left\{ 3\sqrt{h^2 + (k+2)^2} \right\}^2$$

$$4k^2 + 81 + 36k = 9(h^2 + k^2 + 4k + 4)$$

$$9h^2 + 5k^2 = 45$$

By replacing (h, k) with (x, y)

$\therefore$  The locus of a point is

$$9x^2 + 5y^2 = 45$$

**5. Find the locus of a point which is equidistant from (1, 3) and x-axis.**

**Solution:**

Let P (h, k) be any point on the locus and let A (1, 3) and B (h, 0).

Where, PA = PB



$$\text{Distance of } (h, k) \text{ from } (1, 3) = \sqrt{(h-1)^2 + (k-3)^2}$$

$$\text{Distance of } (h, k) \text{ from } (h, 0) = \sqrt{(h-h)^2 + (k-0)^2}$$

It is given that both distance are same.

So,

$$\sqrt{(h-1)^2 + (k-3)^2} = \sqrt{(h-h)^2 + (k-0)^2}$$

Now, let us square on both the sides we get,

$$(h-1)^2 + (k-3)^2 = (h-h)^2 + (k-0)^2$$

$$h^2 + 1 - 2h + k^2 - 6k + 9 = k^2 + 0$$

$$h^2 - 2h - 6k + 10 = 0$$

By replacing  $(h, k)$  with  $(x, y)$ ,

$\therefore$  The locus of point equidistant from  $(1, 3)$  and x-axis is

$$x^2 - 2x - 6y + 10 = 0$$

**6. Find the locus of a point which moves such that its distance from the origin is three times is distance from x-axis.**

**Solution:**

Let P  $(h, k)$  be any point on the locus and let A  $(0, 0)$  and B  $(h, 0)$ .

Where,  $PA = 3PB$

$$\text{Distance of } (h, k) \text{ from } (0, 0) = \sqrt{(h-0)^2 + (k-0)^2}$$

$$\text{Distance of } (h, k) \text{ from } (h, 0) = \sqrt{(h-h)^2 + (k-0)^2}$$

So, where  $PA = 3PB$

$$\therefore \sqrt{(h-0)^2 + (k-0)^2} = 3\sqrt{(h-h)^2 + (k-0)^2}$$

Now by squaring on both the sides we get,

$$h^2 + k^2 = 9k^2$$

$$h^2 = 8k^2$$

By replacing  $(h, k)$  with  $(x, y)$

$\therefore$  The locus of point is  $x^2 = 8y^2$

## EXERCISE 22.3

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1. What does the equation  $(x - a)^2 + (y - b)^2 = r^2$  become when the axes are transferred to parallel axes through the point  $(a - c, b)$ ?

**Solution:**

Given:

The equation,  $(x - a)^2 + (y - b)^2 = r^2$

The given equation  $(x - a)^2 + (y - b)^2 = r^2$  can be transformed into the new equation by changing  $x$  by  $x - a + c$  and  $y$  by  $y - b$ , i.e. substitution of  $x$  by  $x + a$  and  $y$  by  $y + b$ .

$$((x + a - c) - a)^2 + ((y - b) - b)^2 = r^2$$

$$(x - c)^2 + y^2 = r^2$$

$$x^2 + c^2 - 2cx + y^2 = r^2$$

$$x^2 + y^2 - 2cx = r^2 - c^2$$

Hence, the transformed equation is  $x^2 + y^2 - 2cx = r^2 - c^2$

2. What does the equation  $(a - b)(x^2 + y^2) - 2abx = 0$  become if the origin is shifted to the point  $(ab / (a - b), 0)$  without rotation?

**Solution:**

Given:

The equation  $(a - b)(x^2 + y^2) - 2abx = 0$

The given equation  $(a - b)(x^2 + y^2) - 2abx = 0$  can be transformed into new equation by changing  $x$  by  $[X + ab / (a - b)]$  and  $y$  by  $Y$

$$(a - b) \left[ \left( X + \frac{ab}{a - b} \right)^2 + Y^2 \right] - 2ab \times \left( X + \frac{ab}{a - b} \right) = 0$$

Upon expansion we get,

$$(a - b) \left( X^2 + \frac{a^2 b^2}{(a - b)^2} + \frac{2abX}{a - b} + Y^2 \right) - 2abX - \frac{2a^2 b^2}{a - b} = 0$$

Now let us simplify,

$$(a - b)(X^2 + Y^2) + \frac{a^2 b^2}{a - b} + 2abX - 2abX - \frac{2a^2 b^2}{a - b} = 0$$

$$(a - b)(X^2 + Y^2) - \frac{a^2 b^2}{a - b} = 0$$

By taking LCM we get,

$$(a - b)^2 (X^2 + Y^2) = a^2 b^2$$

Hence, the transformed equation is  $(a - b)^2 (X^2 + Y^2) = a^2 b^2$

3. Find what the following equations become when the origin is shifted to the point

(1, 1)?

(i)  $x^2 + xy - 3x - y + 2 = 0$

(ii)  $x^2 - y^2 - 2x + 2y = 0$

(iii)  $xy - x - y + 1 = 0$

(iv)  $xy - y^2 - x + y = 0$

**Solution:**

(i)  $x^2 + xy - 3x - y + 2 = 0$

Firstly let us substitute the value of x by x + 1 and y by y + 1

Then,

$$(x + 1)^2 + (x + 1)(y + 1) - 3(x + 1) - (y + 1) + 2 = 0$$

$$x^2 + 1 + 2x + xy + x + y + 1 - 3x - 3 - y - 1 + 2 = 0$$

Upon simplification we get,

$$x^2 + xy = 0$$

∴ The transformed equation is  $x^2 + xy = 0$ .

(ii)  $x^2 - y^2 - 2x + 2y = 0$

Let us substitute the value of x by x + 1 and y by y + 1

Then,

$$(x + 1)^2 - (y + 1)^2 - 2(x + 1) + 2(y + 1) = 0$$

$$x^2 + 1 + 2x - y^2 - 1 - 2y - 2x - 2 + 2y + 2 = 0$$

Upon simplification we get,

$$x^2 - y^2 = 0$$

∴ The transformed equation is  $x^2 - y^2 = 0$ .

(iii)  $xy - x - y + 1 = 0$

Let us substitute the value of x by x + 1 and y by y + 1

Then,

$$(x + 1)(y + 1) - (x + 1) - (y + 1) + 1 = 0$$

$$xy + x + y + 1 - x - 1 - y - 1 + 1 = 0$$

Upon simplification we get,

$$xy = 0$$

∴ The transformed equation is  $xy = 0$ .

(iv)  $xy - y^2 - x + y = 0$

Let us substitute the value of x by x + 1 and y by y + 1

Then,

$$(x + 1)(y + 1) - (y + 1)^2 - (x + 1) + (y + 1) = 0$$

$$xy + x + y + 1 - y^2 - 1 - 2y - x - 1 + y + 1 = 0$$

Upon simplification we get,



$$xy - y^2 = 0$$

∴ The transformed equation is  $xy - y^2 = 0$ .

**4. At what point the origin be shifted so that the equation  $x^2 + xy - 3x + 2 = 0$  does not contain any first-degree term and constant term?**

**Solution:**

Given:

The equation  $x^2 + xy - 3x + 2 = 0$

We know that the origin has been shifted from (0, 0) to (p, q)

So any arbitrary point (x, y) will also be converted as (x + p, y + q).

The new equation is:

$$(x + p)^2 + (x + p)(y + q) - 3(x + p) + 2 = 0$$

Upon simplification,

$$x^2 + p^2 + 2px + xy + py + qx + pq - 3x - 3p + 2 = 0$$

$$x^2 + xy + x(2p + q - 3) + y(q - 1) + p^2 + pq - 3p - q + 2 = 0$$

For no first degree term, we have  $2p + q - 3 = 0$  and  $p - 1 = 0$ , and

For no constant term we have  $p^2 + pq - 3p - q + 2 = 0$ .

By solving these simultaneous equations we have  $p = 1$  and  $q = 1$  from first equation.

The values  $p = 1$  and  $q = 1$  satisfies  $p^2 + pq - 3p - q + 2 = 0$ .

Hence, the point to which origin must be shifted is  $(p, q) = (1, 1)$ .

**5. Verify that the area of the triangle with vertices (2, 3), (5, 7) and (-3 -1) remains invariant under the translation of axes when the origin is shifted to the point (-1, 3).**

**Solution:**

Given:

The points (2, 3), (5, 7), and (-3, -1).

The area of triangle with vertices  $(x_1, y_1)$ ,  $(x_2, y_2)$ , and  $(x_3, y_3)$  is

$$= \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$\text{The area of given triangle} = \frac{1}{2} [2(7+1) + 5(-1-3) - 3(3-7)]$$

$$= \frac{1}{2} [16 - 20 + 12]$$

$$= \frac{1}{2} [8]$$

$$= 4$$

Origin shifted to point (-1, 3), the new coordinates of the triangle are (3, 0), (6, 4), and (-2, -4) obtained from subtracting a point (-1, 3).

$$\text{The new area of triangle} = \frac{1}{2} [3(4-(-4)) + 6(-4-0) - 2(0-4)]$$

$$= \frac{1}{2} [24-24+8]$$

$$= \frac{1}{2} [8]$$

$$= 4$$

Since the area of the triangle before and after the translation after shifting of origin



remains same, i.e. 4.

∴ We can say that the area of a triangle is invariant to shifting of origin.

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